SECOND-ORDER CYCLIC STATISTICS BASED BLIND CHANNEL IDENTIFICATION AND EQUALIZATION *

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ABSTRACT

Blind channel identification and equalization based on second-order statistics by subspace fitting and linear prediction have received a lot of attention lately. On the other hand, the use of cyclic statistics in fractionally sampled channels has also raised considerable interest. We propose to use these statistics in subspace fitting and linear prediction for (possibly multiuser) channel identification. The main benefit expected is to get rid of the dependence on the color of the additive noise, due to the properties of the cyclocorrelations. We also present some simulations to illustrate the effectiveness of the method.

1. PROBLEM POSITION

We consider a communication system with p emitters and a receiver constituted of an array of M antennas. The signals received are oversampled by a factor m w.r.t. the symbol rate. The channel is FIR of duration NT/m where T is the symbol duration. The received signal can be written as :

$$\mathbf{x}(n) = \sum_{k=-\infty}^{\infty} \mathbf{h}(k)\mathbf{u}(n-k) + \mathbf{v}(n) = \sum_{k=-\infty}^{\infty} \mathbf{h}(n-km)\mathbf{a}_k + \mathbf{v}(n)$$

where

$$\mathbf{u}(n) = \sum_{k=-\infty}^{\infty} \mathbf{a}_k \delta(n-km)$$

The received signal $\mathbf{x}(n)$ and noise $\mathbf{v}(n)$ are a $M \times 1$ vectors. $\mathbf{x}(n)$ is cycloctationary with period m whereas $\mathbf{v}(n)$ is assumed not to be cyclostationary with period m. $\mathbf{h}(k)$ has dimension $M \times p$, $\mathbf{a}(k)$ and $\mathbf{u}(k)$ have dimensions $p \times 1$.

2. CYCLIC STATISTICS

$\mathbf{x}(n)$ is cycloctationary with period m whereas $\mathbf{v}(n)$ is assumed not to be cyclostationary with period m. Hence, the correlations :

$$\mathbf{R}_{xx}(n,\tau) = \mathrm{E}\left\{\mathbf{x}(n)\mathbf{x}^H(n-\tau)\right\}$$

are cyclic in n with period m(^{H} denotes complex conjugate tranpose). One can easily express them as:

$$\mathbf{R}_{xx}(n,\tau) = \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} \mathbf{h}(n-\alpha m)\mathbf{R}_{aa}(\beta)\mathbf{h}^H(n-\alpha m+\beta m-\tau) + \mathbf{R}_{vv}(\tau)$$

We then express the k^{th} cyclocorrelation as :

$$\mathbf{R}_{xx}^{\{k\}}(\tau) \triangleq \frac{1}{m}\sum_{l=0}^{m-1}\mathbf{R}_{xx}(l,\tau)e^{-\jmath\frac{2\pi l k}{m}} = \mathrm{E}^k\left\{\mathbf{x}(l)\mathbf{x}^H(l-\tau)\right\}$$

whose value is :

$$\begin{aligned}\mathbf{R}_{xx}^{\{k\}}(\tau) &= \tfrac{1}{m}\sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty}\mathbf{h}(\alpha)\mathbf{R}_{aa}(\beta) \\ &\qquad \mathbf{h}^H(\alpha+\beta m-\tau)e^{-\jmath\frac{2\pi\alpha k}{m}} + \mathbf{R}_{vv}(\tau)\delta(k)\end{aligned}$$

We can introduce a cyclic correlation matrix as :

$$\mathbf{R}_{xx}^{\{k\}} \triangleq \begin{bmatrix} \mathbf{R}_{xx}^{\{k\}}(0) & \mathbf{R}_{xx}^{\{k\}}(1) & \cdots & \mathbf{R}_{xx}^{\{k\}}(K-1) \\ \mathbf{R}_{xx}^{\{k\}}(-1) & \mathbf{R}_{xx}^{\{k\}}(0) & \cdots & \mathbf{R}_{xx}^{\{k\}}(K-2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{xx}^{\{k\}}(1-K) & \mathbf{R}_{xx}^{\{k\}}(2-K) & \cdots & \mathbf{R}_{xx}^{\{k\}}(0) \end{bmatrix}$$

$$= \mathcal{T}_K(\mathbf{H}_N\mathbf{D}_{DFT}^{\{k\}})\mathbf{R}_{uu}^{\{k\}}\mathcal{T}_K^H(\mathbf{H}_N) + \delta(k)\mathbf{R}_{vv}$$

where $\mathbf{R}_{uu}^{\{k\}} = \mathbf{R}_{aa} \otimes I_m$ *, where $\otimes$ is a block kronecker product, where the first matrix is a block matrix and the second matrix is a elementwise matrix.

*

$$\begin{aligned} r_{uu}(n,\tau) &= \mathrm{E}\left\{\mathbf{u}(n)\mathbf{u}(n-\tau)\right\} \\ &= \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(n-im)\delta(n-jm-\tau)\mathbf{a}(i)\mathbf{a}^H(j)\end{aligned}$$

$r_{uu}^{\{k\}}(\tau) =$

$$\textstyle\sum_{l=0}^{m-1}\sum_i\sum_j \underbrace{\delta(l-im)}_{\Rightarrow l=i.m=0}\delta(n-jm-\tau)\mathbf{a}(i)\mathbf{a}^H(j)r_{aa}(j-i)w^{kl}$$

where $w = e^{-\imath\frac{2\pi}{m}}$

$$r_{uu}^{\{k\}}(\tau) = \sum_{j=-\infty}^{\infty}\delta(jm+\tau)r_{aa}(j)$$

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$\mathcal{T}_K(\mathbf{H}_N)$ is the convolution matrix of $\mathbf{H}_N = [\mathbf{h}(0)\mathbf{h}(1)\cdots\mathbf{h}(N-1)]$ and

$$\mathbf{D}_{DFT}^{\{k\}} = \text{blockdiag}[\mathbf{I}_{p\times p}|e^{-\jmath\frac{2\pi k}{m}}\mathbf{I}_{p\times p}|\cdots|e^{-\jmath\frac{2\pi(N-1)k}{m}}\mathbf{I}_{p\times p}]$$

This shows that the cyclic correlations at cycle frequency $k \neq 0$ are not affected by the additive non k/T cyclostationary noise, whatever its color.

3. CHANNEL ESTIMATION BY SUBSPACE FITTING.

3.1. The classical way

Let's come back to the $p = 1$ case and write $\mathbf{H}'_N = \mathbf{H}_N\mathbf{D}_{DFT}^{\{k\}}$, the index k being deduced from the context.

One can write the (compact form of the) SVD of the cyclocorrelation matrix $\mathbf{R}_{xx}^{\{k\}} = \mathbf{U}\mathbf{D}\mathbf{V}^H$ with the relations:

$$\text{range}\{\mathbf{U}\} = \text{range}\{\mathcal{T}_K(\mathbf{H}'_N)\}$$

and

$$\text{range}\{\mathbf{V}\} = \text{range}\{\mathcal{T}_K(\mathbf{H}_N)\}$$

We have assumed that $\mathcal{T}_K(\mathbf{H}'_N)\}$ is full rank, which is the usual condition (indeed, one can easily verify than $\text{rank}\{\mathcal{T}_K(\mathbf{H}_N)\} = \text{rank}\{\mathcal{T}_K(\mathbf{H}'_N)\}$). We can then solve the classical subspace fitting problem :

$$\min_{\mathbf{H}'_N, T} ||\mathcal{T}_K(\mathbf{H}'_N) - \mathbf{U}T||_F^2$$

If we introduce $\mathbf{U}^\perp$ such that $[\mathbf{U}\mathbf{U}^\perp]$ is a unitary matrix, this leads to

$$\min_{\mathbf{H}'_N} \mathbf{H}_N^{\prime t}\left[\sum_{i=1}^{D^\perp}\mathcal{T}_N(\mathbf{U}_i^{\perp Ht})\mathcal{T}_N^H(\mathbf{U}_i^{\perp Ht})\right]\mathbf{H}_N^{\prime Ht}$$

where $\mathbf{U}_i^\perp$ is a $K \times 1$ block vector with $M \times 1$ blocks, $D^\perp = N+K-1$ and superscript ^{t} denotes the transposition of the blocks of a block matrix. $\mathbf{H}_N^{\prime Ht}$ is then the eigenvector corresponding to the minimum eigenvalue of the matrix between brackets.

The case $p > 1$ can be (partially) solved in a manner similar to [Slo94] and [Lou96].

3.2. A low complexity algorithm

The above derivation relies on the orthogonality between the signal and noise subspaces. When $k \neq 0$, we have :

$$\mathbf{R}_{xx}^{\{k\}} = \mathcal{T}_K(\mathbf{H}'_N)\mathbf{R}_{uu}^{\{k\}}\mathcal{T}_K^H(\mathbf{H}_N)$$

where the noise contribution to the covariance expression disappears. From this, we observe that the left column space of $\mathcal{T}_K(\mathbf{H}'_N)$ and of $\mathbf{R}_{xx}^{\{k\}}$ are the same. This leads to the following subspace fitting criterion:

$$\min_{\mathbf{H}_N,\mathbf{Q}} ||\mathcal{T}_K(\mathbf{H}'_N) - \hat{\mathbf{R}}_{xx}^{\{k\}}\mathbf{B}\mathbf{Q}||_F^2 \quad (1)$$

The matrix $\mathbf{B}$ has the same dimensions as $\mathcal{T}_K(\mathbf{H}_N)$ and is fixed. Its choice influences the quality of the channel estimate. The criterion is separable in $\mathbf{H}_N$ and $\mathbf{Q}$. Minimizing w.r.t. $\mathbf{Q}$ gives :

$$\mathbf{Q} = (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathcal{T}_K(\mathbf{H}'_N), \qquad \mathbf{F} = \hat{\mathbf{R}}_{xx}^{\{k\}}\mathbf{B}$$

Substitution in (1) leads to :

$$\min_{\mathbf{H}_N} ||\mathbf{P}_{\mathbf{F}}^\perp\mathcal{T}_K(\mathbf{H}'_N)||_F^2$$

where $\mathbf{P}_{\mathbf{F}}^\perp = \mathbf{I} - \mathbf{P}_{\mathbf{F}}$ and $\mathbf{P}_{\mathbf{F}} = \mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H$. With the constraint $||\mathbf{H}_N|| = ||\mathbf{H}'_N|| = 1$, we get:

$$\begin{aligned}\hat{\mathbf{H}'_N} &= \arg\max_{||\mathbf{H}_N||=1} \mathcal{T}_K(\mathbf{H}'_N)^H\mathbf{P}_{\mathbf{F}}\mathcal{T}_K(\mathbf{H}'_N)\\ &= \arg\max_{||\mathbf{H}_N||=1} \mathbf{H}_N^{\prime t}\mathcal{F}\mathbf{H}_N^{\prime H^t}\end{aligned}$$

where $\mathcal{F}$ can easily be constructed from $\mathbf{P}_{\mathbf{F}}$ and ^{t} is the usual block transpose operator. The solution is thus the maximum eigenvector of $\mathcal{F}$. As in [KOS96], the choice of $\mathbf{B}$ is set to $\mathbf{B} = \mathcal{T}_K(\mathbf{H}_N)$, leading to a two step algorithm, the first step where $\mathbf{B}$ is an arbitrary selection matrix, yielding a consistent estimate of the channel, the second step where $\mathbf{B} = \mathcal{T}_K(\hat{\mathbf{H}}_N)$.

The main benefit of this procedure is the low complexity of the algorithm, compared to the first one, as we don't need to make an eigendecomposition of the complete cyclic covariance matrix.

As noted in [KOS96], the use of the conjugate correlations (or conjugate cyclic correlations) for BPSK and MSK modulations can lead to the same conclusions for the $k = 0$ case.

Let's further note that, for $k = 0$, the use of this algorithm is also possible if we consider $\tilde{\mathbf{R}}_{xx}^{\{0\}} = \mathbf{R}_{xx}^{\{0\}} - \mathbf{R}_{vv}$, in the case of a known or estimated noise covariance matrix.

4. LINEAR PREDICTION

In this section, we assume uncorrelated symbol sequences.

Consider vectors of signals

$$\mathbf{X}_{Lm}(n) = [\mathbf{x}(n)^H\mathbf{x}(n-1)^H\cdots\mathbf{x}(n-Lm+1)^H]^H,$$

$\mathbf{U}_{Lm+N-1}(n) = [\mathbf{u}(n)^H\cdots\mathbf{u}(n-Lm-N+1)^H]^H$, then we have the equation $\mathbf{X}_{Lm}(n) = \mathcal{T}_{Lm}(\mathbf{H}_N)\mathbf{U}_{Lm+N-1}(n)$. The linear prediction equations are then :

$$\hat{\mathbf{x}}(n)|_{\mathbf{X}_{Lm(n-1)}} = \mathbf{p}_1\mathbf{x}(n-1) + \cdots + \mathbf{p}_{Lm}\mathbf{x}(n-Lm)$$

$$\tilde{\mathbf{x}}(n)|_{\mathbf{X}_{Lm(n-1)}} = \mathbf{x}(n) - \hat{\mathbf{x}}(n)|_{\mathbf{X}_{Lm(n-1)}}$$

$$\tilde{\mathbf{x}}(n)|_{\mathbf{X}_{Lm(n-1)}} = \underbrace{[I_M - \mathbf{P}_{Lm}]}_{=\bar{\mathbf{P}}_{Lm}}\mathbf{X}_{Lm+1}(n)$$

To find the prediction filter, we express orthogonality of the prediction error to the past using the cyclocorrelation. This gives the normal equations :

Hence

$$\mathbf{R}_{uu,Lm}^{\{k\}} = \mathbf{R}_{aa,L} \otimes \mathbf{I}_m$$

$$\mathrm{E}^k\left\{\tilde{\mathbf{x}}(n)\mathbf{X}^H_{Lm+1}(n)\right\} = [I_M - \mathbf{P}_{Lm}]R^{\{k\}}_{xx,Lm+1} = [\sigma^{2\{k\}}_{\tilde{x},Lm} 0 \cdots 0]$$

Where

$$\begin{aligned}\sigma^{2\{k\}}_{\tilde{x},Lm} &\triangleq \mathrm{E}^k\left\{\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n)\right\} \\ &= [I_M - \mathbf{P}_{Lm}]\mathrm{E}^k\left\{\mathbf{X}_{Lm+1}(n)\mathbf{X}^H_{Lm+1}(n)\right\}[I_M - \mathbf{P}_{Lm}]^H \\ &= \bar{\mathbf{P}}_{Lm}\mathbf{R}^{\{k\}}_{xx,Lm+1}\bar{\mathbf{P}}^H_{Lm}\end{aligned}$$

The prediction error can be expressed as : $\tilde{\mathbf{x}}(n) = \mathbf{h}(0)\mathbf{u}(n) + (\overline{\mathbf{H}}_N - \mathbf{P}_{Lm}\mathcal{T}(\mathbf{H}_N))\mathbf{U}_{Lm+N-1}(n-1)$, where $\overline{\mathbf{H}}_N = [\mathbf{h}(1)\cdots\mathbf{h}(N)0\cdots 0]$.

The cyclovariance of the error becomes :

$$\mathrm{E}^k\left\{\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n)\right\} = \frac{\sigma_a^2}{m}\mathbf{h}(0)\mathbf{h}^H(0) + \frac{\sigma_a^2}{m}\overline{\overline{\mathbf{H}}}'_N\overline{\overline{\mathbf{H}}}^H_N$$

where $\overline{\overline{\mathbf{H}}}_N = \overline{\mathbf{H}}_N + \mathbf{P}_{Lm}\mathcal{T}(\mathbf{H}_N)$. Minimizing this cyclovariance with respect to $\mathbf{P}_{Lm}$ leads to $\overline{\overline{\mathbf{H}}}_N = 0$.

Hence, from the prediction quantities, we can determine a zero forcing (ZF) equalizer and the channel similarly to [SP94].

5. SIMULATIONS

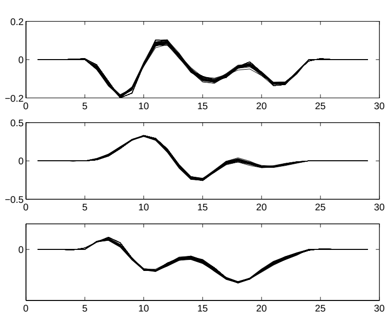
5.1. Subspace fitting

In this section, we restrict ourselves to the $p = 1$ case, using a randomly generated real channel of length 5T, following a raised cosine with 90 % excess bandwith.

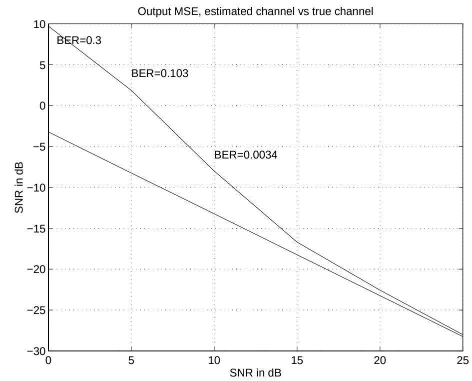
The receiver has $M = 3$ antennas and we oversample by $m = 3$. We draw the NRMSE of the channel, defined as

$$\mathrm{NRMSE} = \sqrt{\frac{1}{50}\sum_{l=1}^{50}||\hat{\mathbf{h}}^{(l)} - \mathbf{h}||^2_F/||\mathbf{h}||^2_F}$$

where $\hat{\mathbf{h}}^{(l)}$ is the estimated channel in the l^{th} trial. In the figures below, the NRMSE in dB has been calculated as $10 * \log 10(NRMSE)$. The estimation is based on the classic subspace fitting of the $k = 1$ cyclocorrelation matrix. This matrix is calculated from a burst of 100 QAM-4 symbols (note that if we used real sources, we would have used the conjugate cyclocorrelation, wich is another means of getting rid of the noise, provided it is circular). For these simulations, we used 50 Monte-Carlo runs and a channel length of 18 (neglecting the 5 first and 6 last near zero values of the channel). The estimations for an SNR of 20 dB are reproduced hereunder.



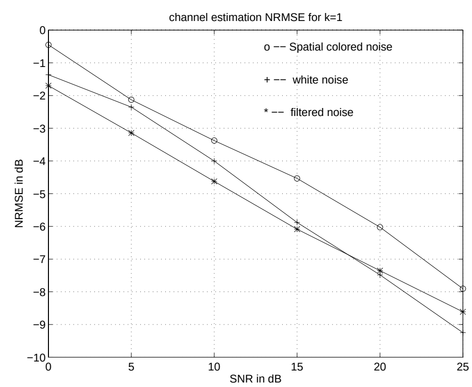
This channel estimate is then followed by a linear MMSE-ZF multichannel equalizer to give us the MSE at the output in the figure below and an estimation of the BER (for SNR= 0,5 and 10 dB), this latter is obtained by simulation on 100000 QAM-4 symbols for each channel estimate : i.e. 10^7 bits for each SNR.



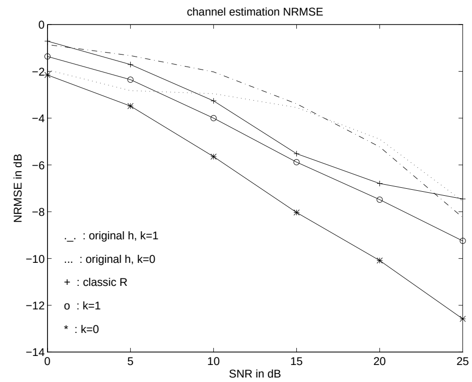
Furthermore, we simulated two colored noise scenarios, the first one in a spatially correlated noise with correlation matrix $\begin{pmatrix} 1 & .7 & .49 \\ .7 & 1 & .7 \\ .49 & .7 & 1 \end{pmatrix}$

the other with a spatio-temporally filtered noise by

$$hn = \begin{pmatrix} .346 & -.180 & .057 & -.057 & -.365 & -.358 \\ .298 & .068 & -.231 & .053 & .090 & -.229 \\ .387 & -.100 & .413 & .172 & .032 & .079 \end{pmatrix}$$

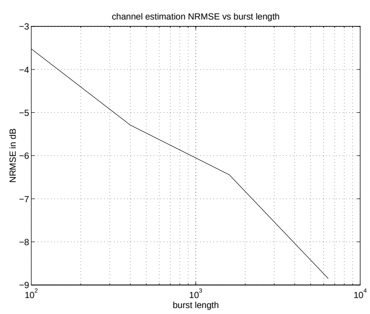


For comparison purposes, we also made estimations with $k = 0$ and for the classical subspace fitting based on the covariance matrix with $M * m$ channels. We illustrate the effect of the (raised cosine transmission) filter on the overall channel estimation performance. The tree bottom curves correspond to the estimation of the convolution of channel and filter. The two top curves corresond to the estimation of the (original) channel alone (no tranmission filter present).



5.2. Linear prediction

Simulations on this method proved to be rather poor (for $k=1$), in the sense that we have to use rather large bursts of symbols to get a proper estimation. For illustration, we include the evolution of the NRMSE as the burst length grows for a well-behaved channel (of length $2T$ and randomly generated). This simulation is done with SNR=25, M=3, m=3.



6. CONCLUSION

We have proposed two new channel identification schemes based on cyclic statistics which are independent of the color of the additive noise. The subspace fitting gives good performance, but the benefit of the independence on the noise color is to be moderated by the loss due to the rather low cyclic spectral power at $k \neq 0$.

On the other hand, our approach, with $k=0$ gives better results than the classical scheme. This is mostly due to the fact that we can better refine the channel length estimate (and should be preceded by a good channel length estimation algorithm). Indeed, if we use a channel of the form (where $M=2$ and $m=2$)

$$\begin{pmatrix} \epsilon & * & * & * & * & * & \epsilon \\ \epsilon & * & * & * & * & * & \epsilon \end{pmatrix}$$

where ϵ is a near zero value, the cyclocorrelation approach can afford to restrict to the central part of the channel, but the classical approach will try to find the $M*m$ multichannel :

$$\begin{pmatrix} \epsilon & * & * \\ * & * & * \\ \epsilon & * & * \\ * & * & * \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} * & * & * \\ * & * & \epsilon \\ * & * & * \\ * & * & \epsilon \end{pmatrix}$$

with 2 more (near zero) parameters to estimate, which will globally give a worse estimation. The cyclic correlation approach with $k=0$ simply corresponds to the use of a mean correlation matrix estimated at $\frac{T}{m}$ separated lags.

Unfortunately, at least in mobile environments, where the burst lengths are short, the linear prediction method proposed gives poor results. When long data pieces are available, it could work. A possible direction to improve this method is the use of the best linear predictor proposed by [Mia93], where an equivalent multivariate stationary process is built, which catches more prediction properties as the classical multivariate process obtained by vectorization of the oversampled signals.

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