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Technical areas:

Wireless networks, Gilbert-Elliot channel, Rayleigh fading,  
Multicast communication, FEC

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*Abstract—*

Wireless channels are highly affected by unpredictable factors such as cochannel interference, adjacent channel interference, propagation path loss, shadowing and multipath fading. The unreliability of media degrades the transmission quality seriously. Forward Error Correction (FEC) schemes are frequently used in wireless environments to reduce the high bit error rate of the channel. We take a Gilbert-Elliot (GE) model to capture the error characteristics of a fading channel and we provide an analytical study of the performance of FEC for multicast communication. The obtained results are then compared to a Binary Symmetric Channel (BSC) model where errors are independent. Reed-Solomon erasure codes are used throughout this study because of their appropriate characteristics in terms of powerful coding and implementation simplicity.

**keywords**—Wireless networks, Gilbert-Elliot channel, Rayleigh fading, Multicast communication, FEC.

## I. INTRODUCTION

Multicasting is the process of delivering a packet to several destinations using a single transmission [1]. The advantage of multicast communication is its efficient savings in bandwidth and network resources since the sender can transmit the data with a single transmission to all receivers. Multicast applications are becoming more and more popular. Examples of such applications include audio and video conferencing, distributed games, and computer supported collaborative work (CSCW). The key idea of these systems is in multicast data transmission. Due to these advantages, it is important that future wireless networks can support multicast communications.

Most of the work done for multicast communication has been based on a fixed Internet environment. In fixed Internet, packets are most likely dropped due to congestion while in wireless, the unreliability of media is the major factor causing packet loss. In fact, wireless channels are highly affected by unpredictable factors such as cochannel interference, adjacent channel interference, propagation path loss, shadowing and multipath fading. End-to-end error recovery mechanisms do not necessarily work well in the presence of wireless links and different kinds of mechanisms are required to guarantee reliability at the traversed wireless links. These are our basic motivations for the study of error recovery mechanisms for multicast communication in wireless environments.

Basically, there are two main error recovery mechanisms: Automatic Repeat Request (ARQ) and Forward Error Correction (FEC). ARQ tries to retransmit the lost packets while FEC transmits some redundant data with the original ones. FEC is frequently used in wireless environments but it can not assure full reliability unless coupled with ARQ.

Most of the reliable multicast protocols propose the use of ARQ [2], [3]. However, the use of simple ARQ for reliable multicast transmission toward a large group may cause a high retransmission rate at the sender even if each receiver has a low error rate. The use of FEC in this case can reduce the retransmission rate tremendously [4], [5].

In this context, we focus on the performance evaluation of FEC for multicast communication in wireless environments. [6] calculated the average number of transmissions for a packet in a multicast group with an ARQ-based error recovery mechanism. The sender retransmits a packet as long as there is at least one receiver that has not received the packet correctly. [4] calculated the average number of transmissions in a multicast group with a FEC-based error recovery mechanism. Two loss models have been considered: independent loss and burst loss. However, no mathematical expression has been derived for the average number of transmissions in the burst loss model. Both works have considered end-to-end error recovery in fixed Internet.

In this paper, we present an analytical study of the performance of FEC for multicast communication in a Gilbert-Elliot (GE) channel. We use Reed-Solomon erasure codes because of their appropriate characteristics in terms of powerful coding and implementation simplicity. We make numerical analysis for a set of Reed-Solomon erasure codes in a GE model and we compare the results with a Binary Symmetric Channel (BSC) model.

The rest of the paper is organized as follows. Section 2 provides a brief description of packet level FEC and Reed-Solomon erasure codes. Section 3 presents the channel models used throughout this study. Section 4 provides the performance evaluation of FEC for multicast communication. Section 5 shows the numerical results and finally, section 6 provides concluding remarks.

## II. CODING ASPECTS

### A. Bit-level versus Packet Level FEC

In a system that uses FEC for error control, the sender and the receiver use a mutually agreed code to protect the data. This code can be represented by  $C(n, k)$ . The code adds  $h = (n - k)$  redundant symbols to the  $k$  information symbols in order to correct the errors found in the received codeword of  $n$  symbols. Redundancy level of a coding scheme is defined as the ratio of  $h/k$  and it represents the amount of redundancy added to the original information.

Forward error correction can be done at many levels from bit level up to packet level. In a bit level FEC, a bit is considered as a symbol while in packet level FEC, a symbol is a packet. Bit level FEC is basically implemented at the physical layer of almost all wireless networks. It is typically done by means of a Digital Signal Processor (DSP) chip or a specific Integrated Circuit (IC). It is designed to correct bit errors as its name indicates.

Packet level FEC consists of producing  $h$  redundant packets from  $k$  original ones. Packet level FEC is based on erasure coding. In coding theory, an error is defined as a corrupted symbol in an unknown position while an erasure is a corrupted symbol in a known position. The error correcting capability of a code can be increased if the decoder can exploit the erasure information [7].

Packet level FEC is mostly interesting in the context of multicast communication. Its interest lies on the fact that a single redundant packet can recover the loss of different information packets at different receivers.

### B. Reed-Solomon Erasure Code

A Reed-Solomon erasure (RSE) code is a Reed-Solomon code with symbols defined over the Galois Field  $GF(2^m)$ , designed to correct only erasures. It has the capacity to correct  $h$  erasures with only  $h$  redundant symbols. This characteristic makes this kind of code particularly powerful to cope with transmission packet losses.

We take  $k$  data packets of length  $L$  each. In the sender side, the RSE encoder takes these  $k$  packets and generates  $h$  redundant packets to form a coded block of  $n = k + h$  packets. If the receiver gets at least  $k$  packets out of  $k + h$  transmitted packets correctly, it can reconstruct the original data. Here the loss unit is a packet and a packet payload is considered as a symbol. Thanks to the packet sequence numbers, the location of lost packets can be easily detected.

### C. Implementation Issues

RSE coders with large symbol size are difficult to implement. McAuley proposed a hardware architecture for RSE codes in [8] using a symbol size  $m = 8$  and  $m = 32$ . Rizzo proposed a software implementation of RSE codes in [9]. The maximum efficiency of his coding scheme is achieved with a symbol size not larger than half the word size of the processor due to fast table lookups.

Normally, the packet size is on the orders of hundreds or thousands of bits. In this case, we need to consider a packet size of  $L = l.m$  where  $l$  is an integer. The coding can then be implemented using parallel RSE coders.

Since the number of elements of the  $GF(2^m)$  with a symbol size of  $m$  is limited to  $2^m$ , it is important to choose a RSE code with  $n < 2^m$ . If we take  $m = 8$ , we will have a maximum block length  $n = 255$ .

## III. CHANNEL MODEL

### A. Gilbert-Elliot model

Two state Markov models have been extensively used in the literature to capture the bursty nature of the error sequences generated by a wireless channel. Previous studies [10], [11] show that a first order Markov chain such as a two state Markov model provides a good approximation in modeling the error process in fading channels. Two state Markov model was first used by Gilbert [12]. Elliot generalized the Gilbert model slightly in [13]. We take a GE model, as shown in figure 1, to characterize the error sequences in a fading channel.

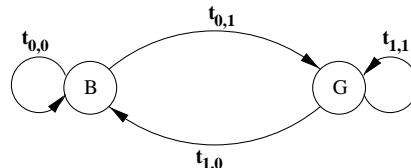


Fig. 1. Gilbert-Elliot model

The model consists of two states. State  $G$  corresponds to a Good state where errors occur with a low probability  $e_G$ . State  $B$  corresponds to the Bad state where errors occur with a high probability  $e_B$ . One of the advantages of this model is the facility to map its parameters to real physical quantities in case of a Rayleigh fading channel.

If the received SNR is above a certain threshold  $\lambda_T$ , the channel is in the good state ( $G$ ). It is in the bad state ( $B$ ) if the received SNR is below  $\lambda_T$ . Using the level crossing rate and the SNR density function, the parameters of the model can be found in terms of physical quantities [15] [16]. Assuming that the channel fades slowly with respect to the symbol interval  $T$ , the transition probabilities of the Markov chain can be calculated as:

$$t_{0,1} = \frac{f_d T \sqrt{2\pi \frac{\lambda_T}{\bar{\lambda}}}}{\exp(\frac{\lambda_T}{\bar{\lambda}}) - 1} \quad (1)$$

$$t_{1,0} = f_d T \sqrt{2\pi \frac{\lambda_T}{\bar{\lambda}}} \quad (2)$$

$$t_{0,0} = 1 - t_{0,1} \quad (3)$$

$$t_{1,1} = 1 - t_{1,0} \quad (4)$$

where  $\bar{\lambda}$  is the average SNR and  $f_d$  is the maximum doppler frequency given by  $f_d = \frac{v f_c}{c}$  with  $v$  the vehicle speed,  $f_c$  the carrier frequency and  $c$  the speed of light ( $3 \times 10^8$  m/s). The steady state probabilities  $\pi_G$  and  $\pi_B$  can be found as:

$$\pi_G = \exp\left(\frac{-\lambda_T}{\bar{\lambda}}\right) \quad (5)$$

$$\pi_B = 1 - \exp\left(\frac{-\lambda_T}{\bar{\lambda}}\right) \quad (6)$$

The error probabilities  $e_G$  and  $e_B$  of each state can be related to the received SNR according to the modulation scheme used

in the system. Simplified expressions for  $e_G$  and  $e_B$  are provided in [16] for a BPSK scheme. The average error rate of the model can be found as  $p = e_G\pi_G + e_B\pi_B$ .

It is important to note that the correlation property of the fading process depends only on  $f_dT$ . If the value  $f_dT < 0.1$ , the fading process is very correlated and is considered as slow fading. In this case, the assumption that the losses are independent is not correct. For the values of  $f_dT > 0.2$ , two samples of the channel are almost independent and the fading process is considered as fast fading [10].

### B. Binary Symmetric channel

The Binary Symmetric Channel is an independent error model where every transmitted bit has exactly the same error probability as the other bits. The error process is a geometric process with the parameter  $e_b$ . The probability that a bit is transmitted erroneously is  $e_b$  and the probability that a bit is transmitted correctly is  $1 - e_b$ .

## IV. PERFORMANCE EVALUATION OF FEC FOR MULTICAST COMMUNICATION

In order to better understand the effect of FEC on multicast communication, we make an analytical study of FEC in this section. We take *efficiency* as a measure of performance of FEC and we define it as the inverse of average number of transmissions required by all receivers to receive a packet correctly. The efficiency gives us an indication of the used bandwidth. For our analysis, we take the models defined in the previous section. Throughout this study, we suppose that the the loss events at different receivers are independent.

### A. Gilbert-Elliot Channel

We consider a GE model with packets taken as the symbols of the model and we assume that the channel is constant during a packet interval. For typical data rates (e.g. more than 64 Kb/s) and for environments commonly considered (e.g. carrier frequency of about 1-2 GHz and typical pedestrian and vehicular speeds), this assumption is reasonable. Therefore, without the loss of generality, we can apply the same GE model to packets with  $T$  taken as packet interval and  $p_G$  and  $p_B$  as packet loss rates in Good and Bad states respectively.

$$p_G = 1 - (1 - e_G)^L \quad (7)$$

$$p_B = 1 - (1 - e_B)^L \quad (8)$$

Let us consider first the scenario where a sender multicasts data to  $R$  receivers using an ARQ scheme. The sender retransmits the original packet if there is at least one receiver that has not received the packet correctly. We define  $L_r$  as the number of losses perceived by a receiver. We assume that the state transitions occur at the beginning of a time slot of unit length and then a packet is transmitted. The probability to have exactly  $l$  losses  $P(L_r = l)$  is the sum of  $P_G(L_r = l)$ , the probability of a receiver to have exactly  $l$  losses with the channel ending in state  $G$ , and  $P_B(L_r = l)$ , the probability of a receiver to have exactly  $l$  losses with the channel ending in state  $B$ .

$$P(L_r = l) = P_G(L_r = l) + P_B(L_r = l), \quad (9)$$

$$P_G(L_r = l) = \begin{cases} (1 - p_G)\pi_G & l = 0, \\ p_G\pi_G & l = 1, \\ P_G(L_r = l - 1)t_{1,1}p_G \\ + P_B(L_r = l - 1)t_{0,1}p_G & l = 2, 3, \dots \end{cases} \quad (10)$$

$$P_B(L_r = l) = \begin{cases} (1 - p_B)\pi_B & l = 0, \\ p_B\pi_B & l = 1, \\ P_G(L_r = l - 1)t_{1,0}p_B \\ + P_B(L_r = l - 1)t_{0,0}p_B & l = 2, 3, \dots \end{cases} \quad (11)$$

Next, we define  $M_r$  as the number of transmissions required for a correct reception of a packet by a receiver  $r$  and  $M$  as the number of transmissions required for a correct reception of a packet by all receivers. The average number of transmissions,  $E[M]$ , as well as the efficiency of the scheme,  $Eff$ , can be calculated as follows:

$$P(M_r \leq m) = 1 - P(L_r = m), \quad (12)$$

$$P(M \leq m) = (1 - P(L_r = m))^R \quad (13)$$

$$E[M] = \sum_{m=1}^{\infty} mP(M = m) = \sum_{m=1}^{\infty} P(M \geq m) \quad (14)$$

$$Eff = \frac{1}{E[M]} = \frac{1}{\sum_{m=1}^{\infty} [1 - (1 - P(L_r = m))^R]} \quad (15)$$

Now, we consider the case where the sender uses a Reed-Solomon erasure code which generates a FEC block of  $n$  packets containing  $k$  original packets and  $h$  redundant packets. We represent such a code by  $RSE(n, k)$ . In this case, the sender sends  $k$  original packets followed by  $h$  redundant ones. Each receiver can recover from loss if it receives correctly  $k$  packets out of  $k + h$  transmitted packets.

[17] calculated the probability to have  $i$  packet losses in  $j$  transmissions,  $P(i, j)$  in a Gilbert-Elliot model using recursion. Let  $P_B(i, j)$  be the probability to have  $i$  losses in  $j$  transmissions with the channel ending in state  $B$  and  $P_G(i, j)$  be the probability to have  $i$  losses in  $j$  transmissions with the channel ending in state  $G$ . As before, we assume that state transitions occur at the beginning of a time slot of unit length and then a packet is transmitted. From [17], we have:

$$P(i, j) = P_G(i, j) + P_B(i, j) \quad (16)$$

$$P_G(i, j) = P_G(i, j - 1)t_{1,1}(1 - p_G) \\ + P_B(i, j - 1)t_{0,1}(1 - p_G) \\ + P_G(i - 1, j - 1)t_{1,1}p_G \\ + P_B(i - 1, j - 1)t_{0,1}p_G \quad (17)$$

$$P_B(i, j) = P_B(i, j - 1)t_{0,0}(1 - p_B) \\ + P_G(i, j - 1)t_{1,0}(1 - p_B) \\ + P_B(i - 1, j - 1)t_{0,0}p_B \\ + P_G(i - 1, j - 1)t_{1,0}p_B \quad (18)$$

for  $i = 0, 1, 2, \dots, j$  and  $j = 1, 2, 3, \dots$

Let's define  $Q(L_r = l)$  as the probability to have exactly  $l$  losses when using FEC. This probability is again the sum of  $Q_G(L_r = l)$ , the probability of a receiver to perceive exactly  $l$  losses with the channel ending in state  $G$ , and  $Q_B(L_r = l)$ , the probability of a receiver to perceive exactly  $l$  losses with the channel ending in state  $B$ .

$$Q(L_r = l) = Q_G(L_r = l) + Q_B(L_r = l), \quad (19)$$

In the presence of FEC, a packet is retransmitted if it is lost by the FEC receiver and if more than  $h-1$  out of the other  $n-1$  packets of the FEC block are lost. In the same way, a packet is considered to be correctly received if it has not been lost or if it has been lost but there are at least  $h-1$  packets out of the other  $n-1$  packets of the FEC block that have been correctly received.

$$Q_G(L_r = l) = \begin{cases} \sum_{i=0}^{h-1} [P_G(i, n-1)t_{1,1}p_G + P_B(i, n-1)t_{0,1}p_G] + \sum_{i=0}^{n-1} [P_G(i, n-1)t_{1,1}(1-p_G) + P_B(i, n-1)t_{0,1}(1-p_G)] & l = 0 \\ \sum_{i=h}^{n-1} [P_G(i, n-1)t_{1,1}p_G + P_B(i, n-1)t_{0,1}p_G] & l = 1, 2, 3, \dots \end{cases} \quad (20)$$

$$Q_B(L_r = l) = \begin{cases} \sum_{i=0}^{h-1} [P_G(i, n-1)t_{1,0}p_B + P_B(i, n-1)t_{0,0}p_B] + \sum_{i=0}^{n-1} [P_G(i, n-1)t_{1,0}(1-p_B) + P_B(i, n-1)t_{0,0}(1-p_B)] & l = 0 \\ \sum_{i=h}^{n-1} [P_G(i, n-1)t_{1,0}p_B + P_B(i, n-1)t_{0,0}p_B] & l = 1, 2, 3, \dots \end{cases} \quad (21)$$

where the initial values for  $P(i, j)$  are

$$P_G(0, 0) = \begin{cases} \pi_G & l = 0, 1 \\ Q_G(L_r = l-1) & l = 2, 3, \dots \end{cases}$$

$$P_B(0, 0) = \begin{cases} \pi_B & l = 0, 1 \\ Q_B(L_r = l-1) & l = 2, 3, \dots \end{cases}$$

and  $P_B(i, 0) = P_G(i, 0) = 0$  for  $i \neq 0$ . It is clear that with these initial values, all numerical values are steady states results.

The efficiency is then calculated from the following equation by using equations (20) and (21) to find  $Q(L_r = l)$ .

$$Eff = \frac{1}{E[M]} = \frac{k}{n} \frac{1}{\sum_{m=1}^{\infty} [1 - (1 - Q(L_r = m-1))^R]} \quad (22)$$

## B. BSC channel

We consider a packet of  $L$  bits transmitted on a BSC channel with the error probability  $e_b$ . The packet loss rate and the efficiency in the case of an ARQ error recovery mechanism can be calculated as [6]:

$$p = 1 - (1 - e_b)^L \quad (23)$$

$$Eff = \frac{1}{E[M]} = \frac{1}{\sum_{m=1}^{\infty} (1 - (1 - p^{(m-1)})^R)} \quad (24)$$

For a FEC based error recovery scheme, using an  $RSE(n, k)$ , the perceived packet loss rate by each receiver,  $q$ , and the efficiency of the scheme are calculated as follows [4]:

$$q = p \left( 1 - \sum_{j=0}^{h-1} \binom{n-1}{j} p^j (1-p)^{n-j-1} \right) \quad (25)$$

$$Eff = \frac{1}{E[M]} = \frac{k}{n} \frac{1}{\sum_{m=1}^{\infty} (1 - (1 - q^{(m-1)})^R)} \quad (26)$$

## V. NUMERICAL ANALYSIS

Figure 2 shows the efficiency as a function of bit error rate in a group of 1000 wireless receivers in a GE model. A pedestrian speed of 3 km/h has been chosen which corresponds to a doppler frequency of 2.5 Hz for a carrier frequency of 900 MHz. We took a data rate of 1 Mb/s and a packet size of 1024 bits corresponding to a packet interval of 1.024 msec. For these values, we have a slow fading channel ( $f_d T < 0.1$ ). Threshold SNR,  $\lambda_T$ , is set to be  $0.1\lambda$  so that an SNR 10 dB below the average SNR causes a transition to the bad state. We used the BPSK modulation scheme throughout our analysis.

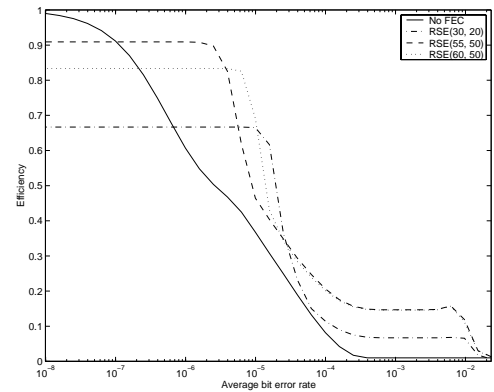


Fig. 2. Efficiency as a function of average bit error rate, R=1000

Generally, we can make the conclusion that FEC outperforms ARQ except for very low bit error rates. If the bit error rate is not high, the best efficiency can be obtained by choosing the number of information packets  $k$  as high as possible with a few redundant packets. If the bit error rate goes high, the number of redundant packets must be increased. Nevertheless, if even the maximum number of available redundant packets can not increase the efficiency anymore, we must decrease the number

of information packets  $k$  while keeping the number of redundant packets at its maximum. We observe that there is no one best code. Depending on the bit error rate of the channel, the efficiency of a code varies. Therefore, we can only select one best code for a range of bit error rates. If the bit error rate changes, the choice of best code changes also. However, for a very high bit error rate, even a coding scheme can not help. This motivates the use of adaptive FEC schemes where the parameters of FEC vary dynamically according to the wireless channel state.

Figure 3 illustrates the efficiency as a function of number of wireless receivers with a bit error rate of about  $10^{-5}$ . We can observe that the number of receivers has an important impact on the efficiency if an ARQ scheme is used. The efficiency of ARQ reduces sharply if the number of receivers increases. The use of FEC, however, reduces the impact of number of receivers on efficiency but its redundancy level must be chosen carefully. From figure 3 we can observe that the RSE(30, 20) maintains a constant efficiency for different number of receivers while the efficiency of the RSE(60, 50) starts degrading for high number of receivers.

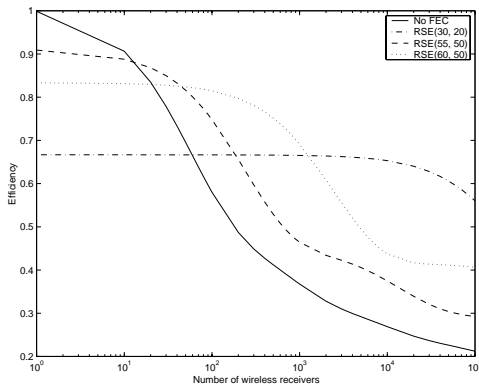


Fig. 3. Efficiency as a function of number of wireless receivers,  $p = 10^{-5}$

Figure 4 depicts the efficiency of the two codes RSE(30, 20) and RSE(60, 50) in the GE and BSC channel models.

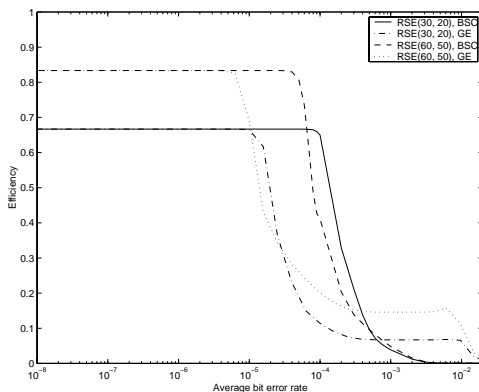


Fig. 4. Efficiency as a function of average bit error rate for different channel models,  $R=1000$

From this figure, we can conclude that when the fading process is slow, the choice of an independent error model leads

to unrealistic results. Another important conclusion that is that RSE codes perform better in case of BSC channel. Their efficiencies decrease when the channel is more correlated.

## VI. CONCLUSION

In this paper, we studied the performance of FEC for multicast communication in wireless networks. We took two different models to capture the error characteristics of a fading channel, a GE model and a BSC model. According to the numerical results obtained, we observed that the BSC model is not a good estimation of the channel in case of correlated errors. The GE model provides the necessary correlation property of the error process in the presence of slow fading. We also concluded that FEC outperforms ARQ for multicast applications even for low bit error rates. We saw that there is no unique best code. Depending on the bit error rate of the channel and the number of receivers, the efficiency of a code varies. Therefore, we can only designate one best code for a certain range of bit error rates and in the presence of a certain number of receivers. Nevertheless, for very high bit error rates, even a coding scheme can not help.

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