ROBUST PILOT DECONTAMINATION: A JOINT ANGLE AND POWER DOMAIN APPROACH

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ABSTRACT

In this paper we propose a novel robust channel estimation algorithm exploiting path diversity in both angle and power domains, relying on a suitable combination of the spatial filtering and amplitude based projection. The proposed approach is able to cope with a wide range of system and topology scenarios, including those where interference channel may overlap with desired channels in terms of multipath angles of arrival (AoA) or exceed them in terms of received power. We establish the analytical conditions under which the proposed channel estimator is fully decontaminated.

***Index Terms*—massive MIMO, pilot decontamination, channel estimation, covariance, eigenvalue decomposition**

I. INTRODUCTION

Massive MIMO is believed to be one of the key enablers of the future 5th generation (5G) wireless systems thanks to its potential to substantially enhance spectral and energy efficiencies [1], [2] compared to traditional MIMO with fewer antennas. However, one of the fundamental issues is the acquisition of the Channel State Information (CSI). In practice, CSI is typically acquired based on non-orthogonal pilot sequences sent by users, resulting in *pilot contamination* [3], [4], which has a detrimental impact on the actual achievable spectral and energy efficiencies. Research efforts towards alleviating pilot interference span from smart design of pilot reuse schemes (e.g. [5], [6]) to channel estimation techniques based on coordinated pilot allocation (e.g. [7], [8]), to methods relying on multi-cell joint processing (e.g. [9]), to nonlinear channel estimation techniques [10], [11].

Two key features of massive MIMO channels are of particular interest here: 1) channels of different users tend to be pairwise orthogonal as the number of antennas increases, leading to a specific subspace structure for the received data vectors that depend on these channels [11] and 2) the channel covariance matrix exhibits a low-rankness property whenever the multipath impinging on the MIMO array spans a finite angular spread [7], [12], [13]. The blind signal subspace estimation in [11] capitalizes on the first property. The second property has been utilized in [7], [12]–[15] assuming the knowledge of the long-term channel covariance matrices. In this work we propose a novel approach exploiting these two key features in a *combined* manner so as to mitigate pilot contamination with much higher robustness.

Some initial algorithms on pilot decontamination exploiting the short-term and long-term statistics of user channels has been reported in [16]. The channel estimation scheme proposed in this paper differs in the following aspects: 1) The algorithms in [16] discriminates users with different AoA by applying a minimum mean square error (MMSE) estimator to training signals. Here a spatial filter pre-processes all data signals to help the subsequent amplitude projection. 2) The amplitude projection subspace in [16] was suffering from interference also from users with orthogonal pilots. In this paper we pre-process the received data to mitigate interference from users that have orthogonal training sequences. 3) Unlike in [16], the proposed method in this paper is robust even when different user channels overlap in both power and angular domains. These substantial improvements that remove the drawbacks of both angular and amplitude algorithms were not present in [16].

Our contributions are as follows: 1) We propose a novel channel estimation scheme called "covariance-aided amplitude based projection". It combines the merits of linear MMSE estimator and amplitude based projection method; yet it can be shown to have significant gains over these known schemes. 2) We give asymptotic analysis on this proposed method and provide a condition to achieve full decontamination which is weaker compared to the previous methods. 3) In the case of Uniform Linear Array (ULA), we identify a sufficient propagation condition to satisfy the uniformly boundedness of the largest eigenvalue of channel covariance, which is useful in random matrix methods [17].

II. SIGNAL AND CHANNEL MODELS

We consider an L-cell network, operating in time-division duplexing (TDD) mode. Each base station (BS) is equipped with M antennas. There are K single-antenna users in each

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cell simultaneously served by their BSs. Each BS estimates the channels of its K users during a coherence time interval. The pilot sequences inside each cell are assumed orthogonal to each other. However the same pilot pool is reused in other cells. The pilot sequence assigned to the k-th user in a certain cell is denoted by

$$\mathbf{s}_k = [\ s_{k1} \quad s_{k2} \quad \cdots \quad s_{k\tau}\ ]^T, \tag{1}$$

where τ is the length of a pilot. Without loss of generality we assume unitary average power of pilot symbols.

The $M \times 1$ channel vector between the k-th user located in the l-th cell and the j-th base station is denoted by $\mathbf{h}_{lk}^{(j)}$. The following general channel model [18] is adopted:

$$\mathbf{h}_{lk}^{(j)} = \mathbf{R}_{lk}^{(j)\frac{1}{2}} \mathbf{h_W}_{lk}^{(j)}, \tag{2}$$

where $\mathbf{h_W}_{lk}^{(j)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is an i.i.d. $M \times 1$ Gaussian vector with unit variance, and $\mathbf{R}_{lk}^{(j)} \triangleq \mathbb{E}\{\mathbf{h}_{lk}^{(j)} \mathbf{h}_{lk}^{(j)H}\}$ is the channel covariance matrix.

The received training signal at base station j is

$$\mathbf{Y}^{(j)} = \sum_{l=1}^{L} \mathbf{H}_l^{(j)} \mathbf{S} + \mathbf{N}^{(j)}, \tag{3}$$

where $\mathbf{N}^{(j)} \in \mathbb{C}^{M \times \tau}$ is the spatially and temporally white additive Gaussian noise (AWGN) with zero-mean and element-wise variance σ_n^2, and

$$\mathbf{H}_l^{(j)} \triangleq [\mathbf{h}_{l1}^{(j)} | \mathbf{h}_{l2}^{(j)} | \cdots | \mathbf{h}_{lK}^{(j)}] \tag{4}$$

$$\mathbf{S} \triangleq \left[\mathbf{s}_1 | \mathbf{s}_2 | \quad \cdots | \quad \mathbf{s}_K\right]^T. \tag{5}$$

The received uplink data signal at base station j is:

$$\mathbf{W}^{(j)} = \sum_{l=1}^{L} \mathbf{H}_l^{(j)} \mathbf{X}_l + \mathbf{Z}^{(j)}, \tag{6}$$

where $\mathbf{X}_l \in \mathbb{C}^{K \times C}$ is the transmitted symbols of all users in cell l-th. The symbols are i.i.d. with zero-mean and unit average element-wise variance. $\mathbf{Z}^{(j)} \in \mathbb{C}^{M \times C}$ is the AWGN noise with zero-mean and element-wise variance σ_n^2.

III. MMSE CHANNEL ESTIMATION

We briefly recall the MMSE channel estimator in a multi-cell single-user per cell setting. For simplicity, we assume cell j is the target cell, and $\mathbf{h}_j^{(j)} \in \mathbb{C}^{M \times 1}$ is the desired channel. An MMSE estimator for $\mathbf{h}_j^{(j)}$ is given by

$$\widehat{\mathbf{h}}_j^{(j)\text{MMSE}} = \mathbf{R}_j^{(j)} \left(\tau (\sum_{l=1}^{L} \mathbf{R}_l^{(j)}) + \sigma_n^2 \mathbf{I}_M \right)^{-1} \overline{\mathbf{S}}^H \mathbf{y}^{(j)} \tag{7}$$

where the pilot matrix $\overline{\mathbf{S}}$ is defined by $\overline{\mathbf{S}} \triangleq \mathbf{s} \otimes \mathbf{I}_M$, and $\mathbf{y}^{(j)} = \text{vec}(\mathbf{Y}^{(j)})$, with $\otimes$ and vec denoting the Kronecker product and the vectorization respectively.

As shown in [7], [13], for a base station equipped with a ULA, the above MMSE estimator can fully eliminate the effects of interfering channels when $M \to \infty$, under the condition that the multipath AoAs of interference and desired channels have disjoint angular supports, or, equivalently, their covariance matrices are orthogonal.

IV. AMPLITUDE BASED PROJECTION

Interestingly, angle is not the only domain where interference can be discriminated upon. In [11], [19] a pilot decontamination approach exploited the empirical instantaneous covariance matrix built from the received data (6). This approach is briefly reviewed below. The eigenvalue decomposition (EVD) of $\mathbf{W}^{(j)} \mathbf{W}^{(j)H}/C$ is written as

$$\frac{1}{C} \mathbf{W}^{(j)} \mathbf{W}^{(j)H} = \mathbf{U}^{(j)} \mathbf{\Lambda}^{(j)} \mathbf{U}^{(j)H}, \tag{8}$$

where $\mathbf{U}^{(j)} \in \mathbb{C}^{M \times M} = [\mathbf{u}_1^{(j)} | \mathbf{u}_2^{(j)} | \cdots | \mathbf{u}_M^{(j)}]$ is a unitary matrix and $\mathbf{\Lambda}^{(j)} = \text{diag}\{\lambda_1^{(j)}, \cdots, \lambda_M^{(j)}\}$ with its diagonal entries sorted in a non-increasing order. By extracting the first K columns of $\mathbf{U}^{(j)}$, we obtain an orthogonal basis

$$\mathbf{E}^{(j)} \triangleq [\mathbf{u}_1^{(j)} | \mathbf{u}_2^{(j)} | \cdots | \mathbf{u}_K^{(j)}] \in \mathbb{C}^{M \times K}. \tag{9}$$

The basic idea in [11], [19] is to use the orthogonal basis $\mathbf{E}^{(j)}$ as an estimate of the subspace spanned by $\mathbf{H}_j^{(j)}$, which includes all desired user channels in cell j. Then, by projecting the received signal onto the subspace spanned by $\mathbf{E}^{(j)}$, most of the signal of interest is preserved. In contrast, the interference signal is canceled out thanks to the asymptotic property that the user channels are pairwise orthogonal as the number of antennas tends to infinity. Thus the estimate of the multi-user channel $\mathbf{H}_j^{(j)}$ is given by:

$$\widehat{\mathbf{H}}_j^{(j)\text{AM}} = \frac{1}{\tau} \mathbf{E}^{(j)} \mathbf{E}^{(j)H} \mathbf{Y}^{(j)} \mathbf{S}^H. \tag{10}$$

This estimate is labeled “AM” for “Amplitude”. For a finite number of antennas, the short-term fading realization can cause the interference subspace to spill over the desired one. An enhanced version can somewhat mitigate this problem by selecting a possibly larger number ($\kappa^{(j)}$) of dominant eigenvectors to form $\mathbf{E}^{(j)}$, where $\kappa^{(j)}$ is the number of eigenvalues in $\mathbf{\Lambda}^{(j)}$ that are greater than $\mu \lambda_K^{(j)}$. μ is a design parameter. See section VI for details on the choice of μ.

V. COVARIANCE-AIDED AMPLITUDE BASED PROJECTION

Note that both previous methods perform well only in some restricted user/channel topologies. We now propose a channel estimation method designed to combine the useful properties of the previous methods, aiming at enhanced robustness in a realistic cellular scenario.

V-A. Single user per cell

For ease of exposition we first consider a simplified scenario where intra-cell interference is ignored by assuming that each cell has only one user, i.e., $K = 1$. The users in different cells share the same pilot sequence $\mathbf{s}$. Then with proper modifications we will generalize this method to the setting of multiple users per cell in section V-B.

The objective is to combine long-term statistics which include spatial distribution information together with short-term empirical covariance which contains instantaneous amplitude and direction channel information. The intuition is that such a long-term statistical spatial filter may bring the residual interference to a level such that the instantaneous projection-based channel estimator can be effectively applied. In order to carry out this intuition, we introduce a statistical filter $\boldsymbol{\Xi}_j$, which is based on channel covariance matrices in a way similar to the MMSE estimator in (7).

$$\boldsymbol{\Xi}_j = \left(\sum_{l=1}^{L} \mathbf{R}_l^{(j)} + \sigma_n^2 \mathbf{I}_M \right)^{-1} \mathbf{R}_j^{(j)}, \tag{11}$$

Note that the linear filter $\boldsymbol{\Xi}_j$ allows to discriminate against the interference in angular domain by projecting away from multipath AoAs that are occupied by interference. Note also that the choice of spatial filter $\boldsymbol{\Xi}_j$ is justified from the fact that the full information of desired channel $\mathbf{h}_j^{(j)}$ is preserved, as $\mathbf{h}_j^{(j)}$ lies in the the signal space of $\mathbf{R}_j^{(j)}$. In fact, the desired channel is recoverable using the linear transformation $\boldsymbol{\Xi}_j{}'$:

$$\boldsymbol{\Xi}_j{}' \triangleq \mathbf{R}_j^{(j)\dagger} \left(\sum_{l=1}^{L} \mathbf{R}_l^{(j)} + \sigma_n^2 \mathbf{I}_M \right), \tag{12}$$

where $\mathbf{A}^\dagger$ denotes the Moore-Penrose pseudoinverse of $\mathbf{A}$, as can be seen from the following equality

$$\boldsymbol{\Xi}_j' \boldsymbol{\Xi}_j \mathbf{h}_j^{(j)} = \mathbf{R}_j^{(j)\dagger} \mathbf{R}_j^{(j)} \mathbf{h}_j^{(j)} = \mathbf{V}_j \mathbf{V}_j^H \mathbf{h}_j^{(j)} = \mathbf{h}_j^{(j)}, \tag{13}$$

where the columns of $\mathbf{V}_j$ are the eigenvectors of $\mathbf{R}_j^{(j)}$ corresponding to non-zero eigenvalues.

The spatial filter is applied to the received data signal as

$$\widetilde{\mathbf{W}}_j \triangleq \boldsymbol{\Xi}_j \mathbf{W}^{(j)}. \tag{14}$$

The amplitude-based method as shown in section IV can now be applied on the filtered received data to get rid of the residual interference. Take the eigenvector corresponding to the largest eigenvalue of the matrix $\widetilde{\mathbf{W}}_j \widetilde{\mathbf{W}}_j^H / C$:

$$\widetilde{\mathbf{u}}_{j1} = \mathbf{e}_1 \{ \frac{1}{C} \widetilde{\mathbf{W}}_j \widetilde{\mathbf{W}}_j^H \}. \tag{15}$$

Hence $\widetilde{\mathbf{u}}_{j1}$ can be considered as an estimate of the direction of the vector $\boldsymbol{\Xi}_j \mathbf{h}_j^{(j)}$.

We then cancel the effect of $\boldsymbol{\Xi}_j$ using $\boldsymbol{\Xi}_j'$ and obtain an estimate of the direction of $\mathbf{h}_j^{(j)}$ as follows:

$$\overline{\mathbf{u}}_{j1} = \frac{\boldsymbol{\Xi}_j' \widetilde{\mathbf{u}}_{j1}}{\left\| \boldsymbol{\Xi}_j' \widetilde{\mathbf{u}}_{j1} \right\|_2}, \tag{16}$$

where $\|\cdot\|_2$ denotes the ℓ^2 norm when the argument is a vector, and the spectral norm when the argument is a matrix.

Finally, the phase and amplitude ambiguities of the desire channel can be resolved by projecting the least squares (LS) estimate onto the subspace spanned by $\overline{\mathbf{u}}_{j1}$:

$$\widehat{\mathbf{h}}_j^{(j)\mathrm{CA}} = \frac{1}{\tau} \overline{\mathbf{u}}_{j1} \overline{\mathbf{u}}_{j1}^H \mathbf{Y}^{(j)} \mathbf{s}^*, \tag{17}$$

where "CA" stands for the covariance-aided amplitude domain projection. The asymptotic performance of this new estimator is now analyzed theoretically. In order to facilitate the analysis, we introduce the following condition:

Condition C1: The spectral norm of $\mathbf{R}_l^{(j)}$ is uniformly bounded:

$$\forall M \in \mathbb{Z}^+ \text{ and } \forall l \in \{1, \ldots, L\}, \exists \zeta, \text{s.t.} \left\| \mathbf{R}_l^{(j)} \right\|_2 < \zeta, \tag{18}$$

where $\mathbb{Z}^+$ is the set of positive integers, and ζ is a constant.

The corresponding physical condition is now investigated in the example of a ULA with a typical antenna spacing D (less than or equal to half wavelength).

Proposition 1 *Let Φ be the AoA support of a certain user. Let $p(\theta)$ be the probability density function of AoA of that user. If $p(\theta) < +\infty, \forall \theta \in \Phi$, and Φ lies in a closed interval that does not include the parallel directions with respect to the array, i.e., $0, \pi \notin \Phi$, then, the spectral norm of the user's covariance $\mathbf{R}$ is uniformly bounded:*

$$\forall M \in \mathbb{Z}^+, \exists \zeta, s.t.\ \|\mathbf{R}\|_2 < \zeta, \tag{19}$$

where ζ is a constant.

Proof: Due to lack of space, we refer to [20] for the full proof. □

Theorem 1 *Given condition C1, if the following inequality holds true:*

$$\alpha_j^{(j)} > \alpha_l^{(j)}, \forall l \neq j, \tag{20}$$

where

$$\alpha_l^{(j)} \triangleq \lim_{M \to \infty} \frac{1}{M} \operatorname{tr}\{ \boldsymbol{\Xi}_j \mathbf{R}_l^{(j)} \boldsymbol{\Xi}_j^H \}, \forall l = 1, \ldots, L, \tag{21}$$

then, the estimation error of (17) vanishes:

$$\lim_{M, C \to \infty} \frac{\left\| \widehat{\mathbf{h}}_j^{(j)CA} - \mathbf{h}_j^{(j)} \right\|_2^2}{\left\| \mathbf{h}_j^{(j)} \right\|_2^2} = 0. \tag{22}$$

Proof: Due to page limit, we give a sketch of the proof only. More details can be found in [20]. Define

$$\Gamma \triangleq \lim_{C \to \infty} \left(\frac{1}{C} \widetilde{\mathbf{W}}_j \widetilde{\mathbf{W}}_j^H \right) \tag{23}$$

$$= \underline{\mathbf{h}}_j^{(j)} \underline{\mathbf{h}}_j^{(j)H} + \sum_{l \neq j} \underline{\mathbf{h}}_l^{(j)} \underline{\mathbf{h}}_l^{(j)H} + \sigma_n^2 \boldsymbol{\Xi}_j \boldsymbol{\Xi}_j^H, \tag{24}$$

where $\underline{\mathbf{h}}_l^{(j)} \triangleq \mathbf{\Xi}_j \mathbf{h}_l^{(j)}, l = 1, \ldots, L$. Due to the asymptotic pairwise orthogonality between $\underline{\mathbf{h}}_j^{(j)}$, and $\underline{\mathbf{h}}_l^{(j)}, l \neq j$, which is true under Condition C1, we can prove that

$$\lim_{M\to\infty} \frac{1}{M} \underline{\mathbf{h}}_j^{(j)H} \underline{\mathbf{h}}_l^{(j)} = 0, \forall l \neq j. \tag{25}$$

Then we show that $\alpha_j^{(j)}$ is the asymptotic dominant eigenvalue of Γ/M, with the corresponding asymptotic eigenvector being $\underline{\mathbf{h}}_j^{(j)}/\|\underline{\mathbf{h}}_j^{(j)}\|_2$. Afterwards, we can show that $\mathbf{\Xi}_j'$ cancels the effect of $\mathbf{\Xi}_j$ so that $\overline{\mathbf{u}}_{j1}$ converges to $\mathbf{h}_j^{(j)}/\|\mathbf{h}_j^{(j)}\|_2$ up to a random phase. Finally, after projecting the LS estimate onto $\overline{\mathbf{u}}_{j1}$, we can prove the estimation error converges to zero. □

Note that condition (20) can be replaced equivalently by

$$\left\|\mathbf{\Xi}_j \mathbf{h}_j^{(j)}\right\|_2 > \left\|\mathbf{\Xi}_j \mathbf{h}_l^{(j)}\right\|_2, \forall l \neq j. \tag{26}$$

V-B. Generalization to multiple users per cell

Now we generalize the covariance-aided amplitude based projection into multi-user setting where K users are served simultaneously in each cell. We consider the estimation of user channel $\mathbf{h}_{jk}^{(j)}$ in the reminder of this section. Define

$$\widehat{\mathbf{H}}_{j\backslash k}^{(j)} \triangleq [\widehat{\mathbf{h}}_{j1}^{(j)} | \cdots | \widehat{\mathbf{h}}_{j(k-1)}^{(j)} | \widehat{\mathbf{h}}_{j(k+1)}^{(j)} | \cdots | \widehat{\mathbf{h}}_{jK}^{(j)}], \tag{27}$$

as a sub-matrix of $\widehat{\mathbf{H}}_j^{(j)}$, which is an LS estimate of $\mathbf{H}_j^{(j)}$.

We propose to first neutralize the intra-cell interference with a Zero-Forcing (ZF) filter $\mathbf{T}_{jk}$ based on the training based LS estimate $\widehat{\mathbf{H}}_{j\backslash k}^{(j)}$, and then apply the spatial filter $\mathbf{\Xi}_{jk}$. After these two filters, the data signal is now:

$$\widetilde{\mathbf{W}}_{jk} \triangleq \mathbf{\Xi}_{jk} \mathbf{T}_{jk} \mathbf{W}^{(j)}, \tag{28}$$

where

$$\mathbf{T}_{jk} \triangleq \mathbf{I}_M - \widehat{\mathbf{H}}_{j\backslash k}^{(j)} (\widehat{\mathbf{H}}_{j\backslash k}^{(j)H} \widehat{\mathbf{H}}_{j\backslash k}^{(j)})^{-1} \widehat{\mathbf{H}}_{j\backslash k}^{(j)H}, \tag{29}$$

$$\mathbf{\Xi}_{jk} \triangleq \left(\sum_{l=1}^{L} \mathbf{R}_{lk}^{(j)} + \sigma_n^2 \mathbf{I}_M \right)^{-1} \mathbf{R}_{jk}^{(j)}. \tag{30}$$

The rest of this method proceeds as in the single user setting and thus omitted. The interested reader is referred [20] for more details. Note that in this method, we build the ZF type filter $\mathbf{T}_{jk}$ based on a rough LS estimate. Further improvements can be attained with higher quality estimate at the cost of higher complexity.

VI. NUMERICAL RESULTS

In simulations, we have $L = 7$ hexagonally shaped adjacent cells in the network. The radius of each cell is 1000 meters. The BS antennas form a ULA, with half wavelength antenna spacing. The length of pilot sequence is $\tau = 10$. The multipath angle of arrival of any channel follows a uniform distribution centered at the direction corresponding to line of sight. The angular spread is 30 degrees. The path loss exponent is $\gamma = 2$. The SNR of the desired user at BS is 0dB. The length of data sequence is $C = 500$. Two performance metrics are considered: the normalized channel estimation error, and the uplink per-cell rate when MRC receiver is used. In the figures, "Pure amplitude" denotes the case when we apply the generalized amplitude based projection method only, with the design parameter $\mu = 0.2$. "CA estimator" denotes the proposed method. Fig. 1 and Fig. 2 show the channel estimation performance and the corresponding uplink per-cell rate for a 7-cell network with each cell having only one user. Our proposed method shows the best performance, thus proving its robustness. For more simulations please refer to the full-length version [20].

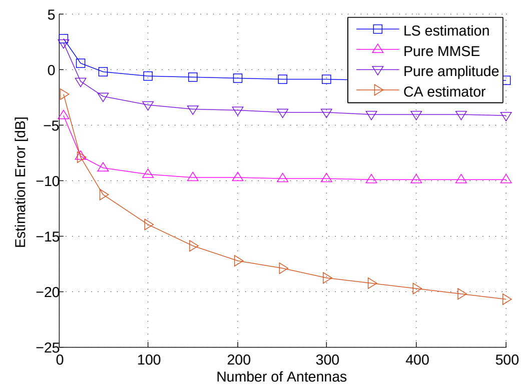

Fig. 1. Estimation performance vs. M.

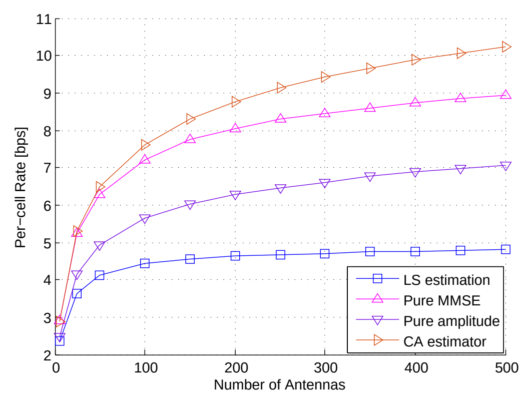

Fig. 2. Per-cell rate vs. M.

VII. CONCLUSIONS

In this paper we proposed a robust channel estimation algorithm exploiting path diversity in both angle and power domains. The method is robust even when the desired channel and the interference channels overlap in multipath AoA and are not separable just in terms of power. Asymptotic analysis shows the condition under which the channel estimation error converges to zero.

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