# Userwise Distortionless Pathwise Interference Cancellation for the DS-CDMA Uplink *

Christian Fischer and Dirk T.M. Slock

Mobile Communications Dept.
Institut Eurécom, 2229 Route des Crêtes, BP 193
06904 Sophia Antipolis Cedex, France
{cfischer,slock}@eurecom.fr

**ABSTRACT:** One of the main problems with linear multiuser detectors for DS-CDMA systems with large spreading factors and time-varying multipath propagation is that typically not enough data is available to estimate the detector's parameters well. Pathwise processing is an approach that allows a separation between fastly varying and slowly varying parameters. In this approach, which was introduced by Matti Latva-aho, the scarce training data are used to estimate the few fastly varying parameters while the whole received signal can be used to estimate the slowly varying parameters. We present some refinements to the original pathwise processing approach to avoid signal cancellation due to correlation between paths in slowly varying environments. We also consider the extension to spatio-temporal processing and propose the introduction of structural constraints in the detector filters to reduce complexity and facilitate the practical implementation.

## I. DATA MODEL

For the received signal model, we assume the $K$ users to be transmitting linearly modulated signals over a linear, specular multipath channel with additive gaussian noise in an asynchronous fashion. Furthermore, we assume that the basestation receiver utilizes an antenna array with $Q$ elements.

### A. Channel Model

A specular model for the spatio-temporal channel is assumed. The channel impulse response is characterised for users $k \in [1 \dots K]$ by

$$\mathbf{h}_k(t) = \sum_{m=1}^{M} A_{k,m}\mathbf{h}(\theta_{k,m})\delta(t-\tau_{k,m})$$

where $\mathbf{h}_k$ and $\mathbf{h}_{k,m} = \mathbf{h}(\theta_{k,m})$ are vectors of dimension $Q$, the number of sensors employed at the receiver. $\mathbf{h}_{k,m}$ defines the response of the antenna array and is a function of the Direction of Arrival (DoA), $\theta_{k,m}$, of the signal. For identifiability reasons, we chose the anntenna response vector to have unity power, $\mathbf{h}_{k,m}^H\mathbf{h}_{k,m} = 1$. Further, the specular channel is characterised by $A_{k,m}$ and $\tau_{k,i}$, the complex amplitude and the path delays, respectively. $M$ is the number of specular paths. The channel parameters can be divided into two classes: fast and slowly varying parameters. The slowly varying parameters are the delays, $\tau_{k,m}$, the DoA, $\theta_{k,m}$, and the short-term path power, $\mathrm{E}|A_{k,m}|^2$. Hence, the fast varying parameters are the complex phases and amplitudes, $A_{k,m}$.

### B. Signal Model

The received continous-time signal before sampling can be written as

$$\mathbf{y}(t) = \sum_{k=1}^{K}\Big\{\sum_{n=-\infty}^{\infty}\sum_{m=1}^{M}(A_{k,m}a_k[n]) \times \sum_{l=0}^{L-1} s_k[l]\mathbf{h}_{k,m}p(t-\tau_{k,m}-lT_c-nT)+\mathbf{n}(t)\Big\} \quad (1)$$

$\mathbf{y}(t)$ and the Additive White Gaussian Noise (AWGN), $\mathbf{n}(t)$, are vector signals due to the use of multiple sensors and are of dimensions $Q \times 1$. $a_k[n], p(t)$ are the transmitted symbols for user $k$ and the pulseshaping filter, respectively. At the receiver front-end, the received signal (1)is lowpass-filtered and sampled at $1/T_s$, where $T_s = T_c/J = T/LJ$ and $T_c$ is the chip period, $T$ the symbol period and $J$ the oversampling factor. The spreading codes, $s_k(.)$ are assumed to be periodic of length $LT_c = T$. We obtain the discrete-time signal model

$$\mathbf{y}[n] = \sum_{q=-\infty}^{\infty}\tilde{\mathbf{P}}_q\tilde{\mathbf{S}}\tilde{\mathbf{H}}\tilde{\mathbf{A}}\mathbf{a}[n-q]+\mathbf{v}[n] \quad (2)$$

where $\mathbf{y}[n] = [\mathbf{y}[n+0\cdot T_c/J]\dots\mathbf{y}[n+(LJ-1)\cdot T_c/J]]^T$, i.e. we stacked all samples of the received signal for the duration of a symbol period $T$ into $\mathbf{y}[n]$. $\mathbf{a}[n] = [a_1(n)a_2(n)\dots a_K(n)]^T$ contains the data symbols of all $K$ users for a given $n$, $^T$ indicating the matrix transpose, $\tilde{\mathbf{A}} = diag(\mathbf{A}_1\dots\mathbf{A}_K)$ is the block diagonal matrix containing the complex amplitude coefficients for each user such that $\mathbf{A}_k = [A_{k,1}^H\dots A_{k,M}^H]^H$, $\tilde{\mathbf{H}} = diag(\mathbf{H}_1\dots\mathbf{H}_K)$ where $\mathbf{H}_k = diag(\mathbf{h}_{k,1}\dots\mathbf{h}_{k,M})$

*Eurécom's research is partially supported by its industrial partners: Ascom, Swisscom, Thomson-CSF, IBM France, CEGETEL, Motorola, France Télécom, Hitachi Europe and Texas Instruments. The research of Christian Fischer is supported by the Swiss National Science Foundation

where both $\mathbf{H}_k$ and $\tilde{\mathbf{H}}$ are block diagonal matrices and $\mathbf{h}_{k,m}$ is a column vector. $\tilde{\mathbf{S}} = diag(\mathbf{S}_1 \ldots \mathbf{S}_K)$ where $\mathbf{S}_k = [\mathbf{I}_M \otimes (\mathbf{s}_k \otimes \mathbf{I}_Q)]; \mathbf{s}_k = [s_k[0] \ldots s_k[L-1]]^T$ represent the spreading code vector, $\mathbf{I}_M, \mathbf{I}_Q$ denote identity matrices of dimensions $M \times M$ and $Q \times Q$, respectively. $\otimes$ signifies the Kronecker product. $\tilde{\mathbf{P}}_n = [\mathbf{p}_{n,1} \ldots \mathbf{p}_{n,K}]; \mathbf{p}_{n,k} = [\mathbf{p}_{n,k,1} \ldots \mathbf{p}_{n,k,M}]$ and

$$\mathbf{p}_{n,k,m} = \begin{bmatrix} \mathbf{p}_{n,k,m,1,1} & \cdots & \mathbf{p}_{n,k,m,1,L-1} \\ \vdots & \ddots & \vdots \\ \mathbf{p}_{n,k,m,LJ-1,1} & \cdots & \mathbf{p}_{n,k,m,LJ-1,L-1} \end{bmatrix}$$

where $\mathbf{p}_{n,k,m,r,l} = [p\,(nT + (r/J - l)T_c - \tau_{k,m}) \otimes \mathbf{I}_Q]$. Depending on the pulseshaping filter used, the received signal $\mathbf{y}[n]$ can be approximated using a finite impulse response (FIR) concatenation of the pulseshape, $p(t)$, to $\pm uT$ hence rendering the overall channel response finite. Due to the delay spread of the multipath channel, $\mathbf{h}_k(t)$, the transmitted symbols are spread out in time over the duration of possibly several symbol periods. Assuming that the maximum delay spread, $\tau_{max}$ experienced in the channel, $\mathbf{h}_k(t)$ is known and given the asynchronism between transmitter and receiver, a processing window of length $b = \lceil(\tau_{max} + 2uT)/T\rceil + 1$ symbol periods for the receiving filter will guarantee to capture the entire contribution of a certain data symbol, $a_k[n]$. It is therefore often advantageous to use samples from the received signal over the duration of several symbol periods rather than just one, thereby also increasing the available data for interference cancellation. To this end, let us stack $N$ vectors $\mathbf{y}[n]$ into a vector $\mathbf{Y}[n]$ which represents the received signal samples over a duration of $NT$, such that

$$\mathbf{Y}[n] = \begin{bmatrix} \mathbf{y}[n] \\ \vdots \\ \mathbf{y}[n-N+1] \end{bmatrix}$$

$$\mathbf{Y}[n] = \mathbf{PSHA}\mathbf{a}_n + \mathbf{V}[n] \qquad (3)$$

where $\mathbf{a}_n = \left[\mathbf{a}[n]^T \ldots \mathbf{a}[n-N-b+2]^T\right]^T$, $\mathbf{A} = \mathbf{I}_{N+b-1} \otimes \tilde{\mathbf{A}}, \mathbf{H} = \mathbf{I}_{N+b-1} \otimes \tilde{\mathbf{H}}, \mathbf{S} = \mathbf{I}_{N+b-1} \otimes \tilde{\mathbf{S}}$ and $\tilde{\mathbf{P}}$ is a banded block Toeplitz matrix of dimensions $NLPQ \times KMQL(N+b-1)$, as shown in (4).

$$\mathbf{P} = \begin{bmatrix} \tilde{\mathbf{P}}_{-u} & \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{P}}_{-u+1} & \ddots & \mathbf{0} \\ \vdots & \ddots & \tilde{\mathbf{P}}_{-u} \\ \tilde{\mathbf{P}}_{u-1} & \ddots & \tilde{\mathbf{P}}_{-u+1} \\ \tilde{\mathbf{P}}_{u} & \ddots & \ddots \\ \mathbf{0} & \ddots & \tilde{\mathbf{P}}_{u-1} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{P}}_{u} \end{bmatrix} \qquad (4)$$

## II. PATHWISE INTERFERENCE CANCELLATION

In linear multiuser detection approaches, there are two different ways of handling multipath channels. The Interference Cancellation (IC) can either take place prior or after the various mulitpath components are recombined. These two methods are known as *precombining interference cancellation* and the more common *postcombining interference cancellation*, respectively, as defined in [1][2][3]. From (3), the received signal can be factored into two components, one of them relying soly on slow parameters as defined in section A, the other component relying on the fast parameters, namely, the product of the data symbols with the complex path amplitudes, $A_{k,m}a_k[n]$. This observation motivates *pathwise interference cancellation* (PWIC) which only requires the knowledge of the slowly varying parameters as opposed to the more common *postcombining* approach which requires complete knowledge of the channel. Hence, in the pathwise scenario, the interference cancellation takes place between individual multipath components, typically, before they are spatio-temporally recombined. The obvious advantage of an interference cancelling filter that relies only on slow parameters, $\tau_{k,m}$ and $\mathbf{h}_{k,m}$ as a function of the DoA, $\theta_{k,m}$ is that the adaptation requirements of the filter also will be based on the rate of change of the slow parameters which are easier to estimate as well as to relax the update rate of the adaptive interference cancelling filter, hence reducing the complexity of the filter. Furthermore, a pathwise filtering approach allows improved channel parameter estimation since the estimated path components contain the signal of interest with an improves SINR compared to the received signal $\mathbf{Y}[n]$.

### *A. Precombining LMMSE PWIC*

The original *precombing interference cancellation* was proposed by Latva-aho [1], the motivation being an adaptive filter implementation of an interference cancelling scheme that relies only on slowly varying parameters of the channel as well as the estimation of the channel coefficients. Namely, the fast varying parameters, $A_{k,m}$, can be estimated using scarce training data and are not required for the IC-filter design. The slow parameters can also be estimated over a longer duration. This allows to find filters for each path $m$ of a user $k$ by employing filters $\mathbf{F}_{k,m}$ to each path and to cancel both Interuser Interference (IUI) as well as Intersymbol Interference (ISI), caused by the multipath propagation channels. The filter coefficient can typically be derived using a Linearly Constrained Minimum Variance (LCMV) or Minimum Output Energy (MOE) approach. Using carefully chosen constraints which guarantee the contribution of the target path to be present in the filter output, such an approach is in principle equivalent to maximizing the Signal to Interference plus Noise ratio (SINR) at the filter output. Since in a RAKE receiver, the treatment of the received signal is naturally pathwise, in the sense that there exists a 'finger' or pulseshaped matched filter in cascade with a correlator, matched to the spreading code of the user of interest, the precombining approach lends itself as an extension to the classical receiver in DS-CDMA, the RAKE receiver. It is hence possible to envisage two ways of proceeding with the pathwise interference cancellation, namely by using the correlator outputs of the RAKE(i), as suggested above, or to use the received signal directly(ii). It may be noted here, that it is in fact not important that the interference cancellation be necessarily before spatio-temporal recombining of the multipath components but that there is a pathwise treatment. Approach (i) is inherently attractive since the entry vector size to the filter in this case is proportional to $KM$ whereas in approach (ii), the entry vector is proportional to $LKM$. This is particularly true in the case where the number of users, $K$, is small compared to the processing gain, $L$ and hence promises reduced complexity. However, approach (i) is more difficult to formulate in a discrete-time processing context, as well as to present the inconvenience of signal structural change with a varying no. of users and/or number of paths. Approach (i) is a true multi-user approach and is used by Latva-aho [3] to present the filter theory, but approach (ii) is used in the context of adaptive filtering, since it allows to follow a single-user approach, in the sense that only the information relative to the user of interest, $k$, is required. Appraoch (ii) can hence be formulated such that the IC-filter, $\mathbf{F}_{k,m}$ for path $m$ of user $k$ works directly on the received signal given in (3), such that filter output can be written as $\mathbf{F}_{k,m}\mathbf{Y}[n]$. The LCMV optimisation criterion is hence given by

$$\mathbf{F}_{k,m} = \arg \min_{\mathbf{F}_{k,m}\mathbf{E}_{k,m}=1} \mathbf{F}_{k,m}\mathbf{R}_{YY}\mathbf{F}_{k,m}^H \tag{5}$$

where $\mathbf{E}_{k,m}$ is the constraint vector, chosen such that it represents the contribution of the path of interest, $A_{k,m}a_k[n-d]$, in $\mathbf{Y}[n]$, i.e the column in $\mathbf{PSH}$ corresponding to $A_{k,m}a_k[n-d]$ in $\mathbf{Aa}$. $d$ denotes some delay with respect to the input signal time index $n$, typically chosen such that the symbols contribution corresponds roughly to the middle portion of the received vector $\mathbf{Y}_n$. This leads to the solution of

$$\mathbf{F}_{k,m} = (\mathbf{E}_{k,m}^H\mathbf{R}_{YY}^{-1}\mathbf{E}_{k,m})^{-1}\mathbf{E}_{k,m}^H\mathbf{R}_{YY}^{-1} \tag{6}$$

Hence, it can be seen that the $\mathbf{E}_{k,m}$ and therefore $\mathbf{F}_{k,m}$ only depends on the slowly varying parameters as defined earlier in section A.

### *B. User-wise Distortionless PWIC (1)*

In the approach described in section A it is supposed that the estimation time for $R_{YY}$ is such that the complex amplitudes $A_{k,m}$ of the paths vary strongly over the estimation time of $R_{YY}$ so that the coefficients $A_{k,m}$ can be considered mutually independent and hence decorrelated between different paths for a given user $k$. If this decorrelation is perfect, the approach of section A is optimal in the sense that it corresponds to a maximization of the SINR of each path. However, if it cannot be assumed that the mobile terminal moves sufficiently, the performance of the approach given in [3] will be limited severely as the signal $A_{k,m}a_k[n-d]$ for the path $m$ of the user $k$ can be strongly correlated with the other paths $i \neq m$ since they belong to the same data symbol. It is now possible to resolve this problem by requiring that the filter $\mathbf{F}_{k,m}$ for path $m$ blocks the contribution of the other paths $i \neq m$ according to the following LCMV criteria:

$$\mathbf{F}_{k,m} = \arg \min_{\mathbf{F}_{k,m}\mathbf{E}_{k,j}=\delta_{m,j}} \mathbf{F}_{k,m}\mathbf{R}_{YY}\mathbf{F}_{k,m}^H \tag{7}$$

where the number of vector constraints has become equal to the number of paths, $M$, of user $k$. Stacking the filters $\mathbf{F}_{k,m} : m \in \{1 \ldots M\}$ into a matrix $\underline{\mathbf{F}}_k = [\mathbf{F}_{k,1}^H \ldots \mathbf{F}_{k,M}^H]^H$ and $\mathbf{E}_{k,m} : m \in \{1 \ldots M\}$ into $\underline{\mathbf{E}}_k = [\mathbf{E}_{k,1} \ldots \mathbf{E}_{k,M}]$, the LCMV criteria can be rewritten as

$$\underline{\mathbf{F}}_k = \arg \min_{\underline{\mathbf{F}}_k\underline{\mathbf{E}}_k=\mathbf{I}_M} \underline{\mathbf{F}}_k\mathbf{R}_{YY}\underline{\mathbf{F}}_k^H \tag{8}$$

with solution

$$\underline{\mathbf{F}}_k = (\underline{\mathbf{E}}_k^H\mathbf{R}_{YY}^{-1}\underline{\mathbf{E}}_k)^{-1}\underline{\mathbf{E}}_k^H\mathbf{R}_{YY}^{-1} \tag{9}$$

In this approach, the filter will let pass all the paths, $m$ of user $k$ without distortion and allows for zero-forcing. The estimate of the signal will be obtained by maximum ratio combining, $\hat{a}_k[n-d] = \sum_{m=1}^{M} A_{k,m}^*\mathbf{F}_{k,m}\mathbf{Y}[n]$. This PWIC approach is also suitable to the estimation of the complex amplitude coefficients, $A_{k,m}$, since they are contained in the filter outputs at improved SINR as compared to the unprocessed signal $\mathbf{Y}[n]$. The complex coefficient estimation hence can be achieved through the use of a training sequence according to the following Least-Square (LS) criterion

$$\hat{A}_{k,m} = \arg \min_{A_{k,m}} \sum_n ||A_{k,m}a_k[n-d] - \mathbf{F}_{k,m}\mathbf{Y}[n]||^2 \tag{10}$$

The disadvantage of this method lies therein that it does require the knowledge of the antenna response vector $\mathbf{h}_{k,m}$ but does not permit the estimation thereof since the spatial recombination is implicit in the interference cancelling filter. Hence, the estimation of $\mathbf{h}_{k,m}$ would have to be obtained independently from a different source. In the next section, we show an alternative which allows spatial recombination after interference cancellation.

### *C. User-wise Distortionless PWIC (2)*

In order to allow also the estimation of the channel response vectors, $\mathbf{h}_{k,m}$, the approach in section B can be extended directly, so as to achieve explicit spatial recombination after IC-filtering. This requires the filter to become a matrix filter, $\underline{\mathbf{F}}_{k,m}$, instead of a vector filter unlike (7), further increasing the degrees of freedom available. Let us define

$$\mathbf{E}_{k,m} = \underline{\mathbf{E}}_{k,m}\mathbf{h}_{k,m} \tag{11}$$

where $\underline{\mathbf{E}}_{k,m}$ is a matrix, containing the contribution of $\mathbf{h}_{k,m}A_{k,m}a_k[n-d]$ in $\mathbf{PS}$ of equation (3), the spreading and the pulseshaping matrix, as detailed in section B. We can then write the LCMV criteria as

$$\underline{\mathbf{F}}_{k,m} = \arg \min_{\underline{\mathbf{F}}_{k,m}\underline{\mathbf{E}}_{k,j}=\mathbf{I}_Q\delta_{m,j}} \underline{\mathbf{F}}_{k,m}\mathbf{R}_{YY}\underline{\mathbf{F}}_{k,m}^H \tag{12}$$

if we now stack the filters $\underline{\mathbf{F}}_{k,m}$ and the constraint matrices $\underline{\mathbf{E}}_{k,m}$ as in section B, we obtain $\underline{\underline{\mathbf{F}}}_k$ and $\underline{\underline{\mathbf{E}}}_k$ and the LCMV criterion can be written as

$$\underline{\underline{\mathbf{F}}}_k = \arg \min_{\underline{\underline{\mathbf{F}}}_k\underline{\underline{\mathbf{E}}}_k=\mathbf{I}_{QM}} \underline{\underline{\mathbf{F}}}_k\mathbf{R}_{YY}\underline{\underline{\mathbf{F}}}_k^H \tag{13}$$

leading to

$$\underline{\underline{\mathbf{F}}}_k = (\underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \underline{\underline{\mathbf{E}}}_k)^{-1} \underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \tag{14}$$

The symbol estimate is therefore given by $\hat{a}_k[n-d] = \sum_m^M A_{k,m}^* \mathbf{h}_{k,m}^H \underline{\mathbf{F}}_{k,m} \mathbf{Y}[n]$. This method clearly allows for the estimation of a path's channels response, requiring only the knowledge of the delays, $\tau_{k,m}$, and the spreading code, $\mathbf{s}_k$, for the user of interest, $k$, to adapt the interference cancelling filter $\underline{\underline{\mathbf{F}}}_k$. The anntena array response, $\mathbf{h}_{k,m}$, can be estimated over the duration of several bursts where as the complex channel coefficients, $A_{k,m}$, can be obtained by estimation over a much shorter time interval. The estimates can be found by

$$\min_{\mathbf{A}_k, \mathbf{h}_{k,m} : \mathbf{h}_{k,m}^H \mathbf{h}_{k,m} = 1} \sum_n \|\mathbf{G}_k \mathbf{A}_k a_k[n-d] - \underline{\underline{\mathbf{F}}}_k \mathbf{Y}[n]\|^2 \tag{15}$$

where $\mathbf{A}_k = [A_{k,1}^H \ldots A_{k,M}^H]^H$ and $\mathbf{G}_k = diag(\mathbf{h}_{k,1} \ldots \mathbf{h}_{k,M})$. Due to the extra degrees of freedom compared to the approach of section B this approach allows even more powerful interference cancellation. On the other hand, with the extension of the degrees of freedom, this also means that the complexity is higher. We therefore suggest an alternative possiblity in section D.

*D. User-wise Distortionless PWIC (3)*

Due to the matrix constraints and the corresponding complexity in section C it may desirable to achieve a similar approach, based on vector constraints only, reducing the degrees of freedom thereby. Multiplying the constraint of (12) with $\mathbf{h}_{k,m}$, rembembering the definition of $\underline{\mathbf{E}}_{k,m}$ as given in (11), it's clear that we can derive from (12) in the form of a vector constrained problem

$$\underline{\mathbf{F}}_{k,m} = \arg \min_{\underline{\mathbf{F}}_{k,m} \mathbf{E}_{k,j} = \mathbf{h}_{k,m} \delta_{m,j}} \underline{\mathbf{F}}_{k,m} \mathbf{R}_{YY} \underline{\mathbf{F}}_{k,m}^H \tag{16}$$

Again, by stacking the filters as well as the constraint vectors for all the paths $m$ as in section B, we can reformulate the problem as

$$\underline{\underline{\mathbf{F}}}_k = \arg \min_{\underline{\underline{\mathbf{F}}}_k \underline{\underline{\mathbf{E}}}_k = \mathbf{G}_k} \underline{\underline{\mathbf{F}}}_k \mathbf{R}_{YY} \underline{\underline{\mathbf{F}}}_k^H \tag{17}$$

with solution

$$\underline{\underline{\mathbf{F}}}_k = \mathbf{G}_k (\underline{\mathbf{E}}_k^H \mathbf{R}_{YY}^{-1} \underline{\mathbf{E}}_k)^{-1} \underline{\mathbf{E}}_k^H \mathbf{R}_{YY}^{-1} \tag{18}$$

Although this method allows for the estimation of the channel parameters $\mathbf{h}_{k,m}, A_{k,m}$, it also requires the antenna response vectors, $\mathbf{h}_{k,m}$, for the filter computation. Note that this filter is equivalent to the case in UPWIC(1). The channel coefficients can be estimated using the following LS criterion

$$\min_{\mathbf{A}_k, \mathbf{h}_{k,m} : \mathbf{h}_{k,m}^H \mathbf{h}_{k,m} = 1} \sum_n \|\mathbf{G}_k \mathbf{A}_k a_k[n-d] - \underline{\underline{\mathbf{F}}}_k \mathbf{Y}[n]\|^2 \tag{19}$$

*E. User-wise Distortionless PWIC (4)*

In order to allow channel estimation while limiting the filter $\mathbf{F}_{k,m}$ to be a vector as in section B, a further variation can be found, similar to the approach in section D by using the following LCMV approach

$$\mathbf{F}_{k,m} = \arg \min_{\mathbf{F}_{k,m} \underline{\mathbf{E}}_{k,j} = \mathbf{h}_{k,m}^H \delta_{mj}} \mathbf{F}_{k,m} \mathbf{R}_{YY} \mathbf{F}_{k,m}^H \tag{20}$$

This leads to a solution

$$\underline{\mathbf{F}}_k = \mathbf{G}_k^H (\underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \underline{\underline{\mathbf{E}}}_k)^{-1} \underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \tag{21}$$

As in section D, this approach requires an iterative implementation due to the fact that the antenna response vector, $\mathbf{h}_{k,m}$ is required to find $\mathbf{F}_{k,m}$, while it is possible to estimate $\mathbf{h}_{k,m}$ from the filter outputs. This can be achieved in the following way:

$$\begin{aligned}\hat{\mathbf{A}}_k, \hat{\mathbf{h}}_{k,m} = \arg \min_{\mathbf{A}_k, \mathbf{h}_{k,m} : \mathbf{h}_{k,m}^H \mathbf{h}_{k,m} = 1} \sum_n \|\mathbf{A}_k a_k[n-d] \quad (22)\\ -\mathbf{G}_k^H (\underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \underline{\underline{\mathbf{E}}}_k)^{-1} \underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \mathbf{Y}[n]\|^2\end{aligned}$$

*F. User-wise Distortionless PWIC(5)*

From the filter expression for UDPWIC(4) given in equation (21) and the filter equation for UDPWIC(2), equation (14), it can be seen that the two solutions are identical in the case where we use $\mathbf{G}^H$ for the spatial recombination in (14). That is to say that

$$\underline{\mathbf{F}}_k = \mathbf{G}_k^H \underline{\underline{\mathbf{F}}}_k$$

Using $\mathbf{q}$ as a generic spatio-temporal recombination vector, we can express the SINR at the symbol estimator output from

$$\hat{a}_k[n-d] = \mathbf{q}^H \underline{\underline{\mathbf{F}}}_k \mathbf{Y}[n] \tag{23}$$

and

$$\mathbf{Y}[n] = \underline{\underline{\mathbf{E}}}_k \mathbf{G}_k \mathbf{A}_k a_k[n-d] + \nu[n]$$

where $\underline{\underline{\mathbf{E}}}_k \mathbf{G}_k \mathbf{A}_k$ is the signal term and $\nu[n]$ represents the noise and interference term as

$$SINR = \frac{\sigma_a^2 \mathbf{q}^H \mathbf{G}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{G}_k^H \mathbf{q}}{\mathbf{q}^H (\underline{\underline{\mathbf{F}}}_k \mathbf{R}_{YY} \underline{\underline{\mathbf{F}}}_k^H - \sigma_a^2 \mathbf{G}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{G}_k^H) \mathbf{q}}$$

In order to maximize the above SINR w.r.t. $\mathbf{q}$, the problem can be reformulated into a generalized eigenvalue problem of the following form:

$$\mathbf{q}_{max} = \arg \max_{\mathbf{q}} \frac{\mathbf{q}^H \mathbf{G}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{G}_k^H \mathbf{q}}{\mathbf{q}^H \underline{\underline{\mathbf{F}}}_k \mathbf{R}_{YY} \underline{\underline{\mathbf{F}}}_k^H \mathbf{q}}$$

with solution

$$\mathbf{q}_{max} = \mathbf{A}_k^H \mathbf{G}_k^H (\underline{\underline{\mathbf{F}}}_k \mathbf{R}_{YY} \underline{\underline{\mathbf{F}}}_k^H)^{-1}$$

Upon backsubstitution into equation (23) we find

$$\hat{a}_k[n-d] = \mathbf{A}_k^H \mathbf{G}_k^H \mathcal{F}_k \mathbf{Y}[n]$$

where

$$\mathcal{F}_k = \underline{\underline{\mathbf{E}}}_k^H \mathbf{R}_{YY}^{-1} \qquad (24)$$

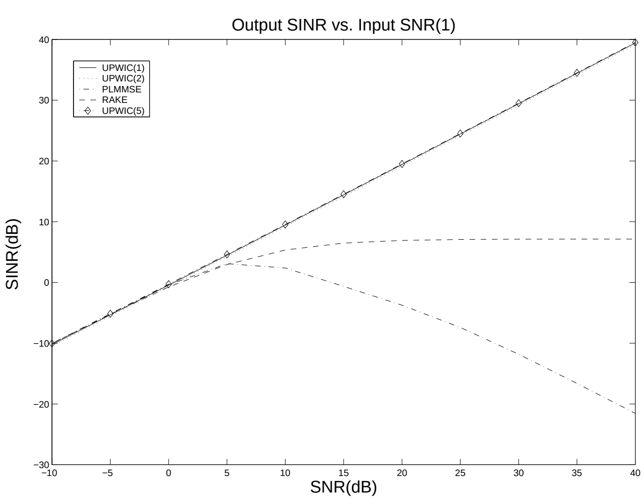
is a matrix filter. Note that the filter only simplifies in the case where the estimation interval of $\mathbf{R}_{YY}$ used in the construction of $\underline{\underline{\mathbf{F}}}_k$ is equal to the estimation interval of $\mathbf{A}_k$ and hence $q$. In the case where the filter is constructed with an $\mathbf{R}_{YY}$ that is averaged over several realisations of $\mathbf{A}_k$, the $\mathbf{R}_{YY}$ used in $q$, will have to be computed seperately and we will use equation (23). This filter is substantially less complex to compute than the filters given in equations (21) and (14) while also maximising the output SINR. It is worth noting that this is neither the case for UDPWIC(2/4) nor UDPWIC(1/3) unless the interference plus noise covariance matrix is identity. Furthermore, this approach allows the filter to be constructed with a minimum of a priori knowledge, in particular the path delays and the spreading code of user $k$, while still allowing the estimation of the channel coefficients.

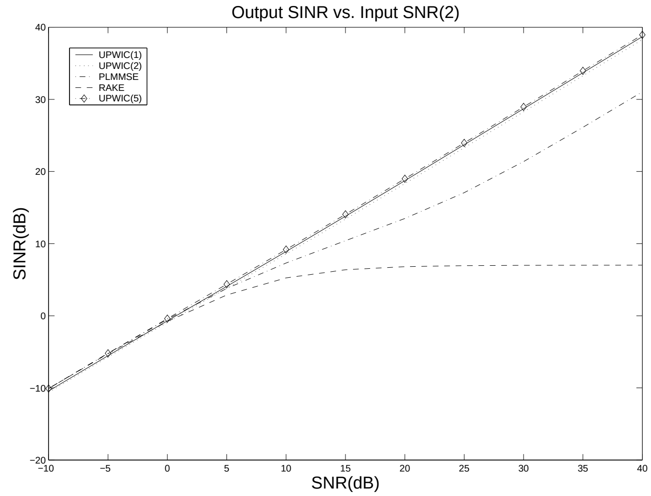
### *G. Structural Filter Constraints*

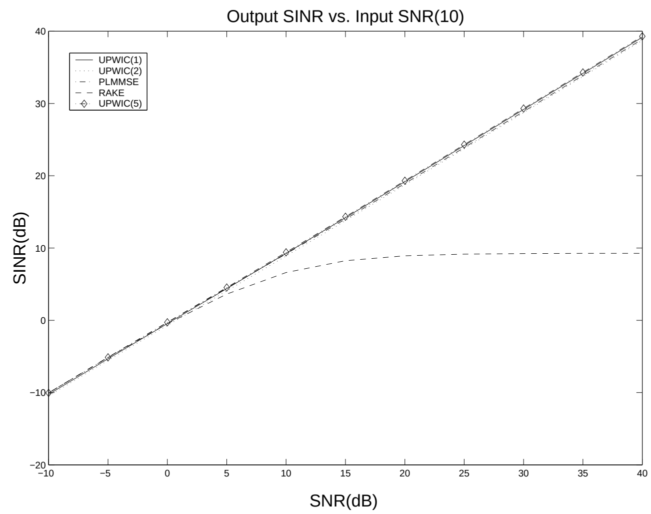
So far, the filters shown in the preceding sections had no structural constraints imposed on them, other than being FIR. It is however possible, to define an a priori structural constraint on the filter $\mathbf{F}_{k,m}$ with the aim of further reducing the complexity and/or improve the performances. Possible constraints are to define the filter $\mathbf{F}_{k,m}$ to be the cascade of a free, shorter filter and a pulseshaped matched filter, $p^*(-t)$ or even a cascade of the pulseshaped matched filter as well as the spreading code correlator and a free filter part.

## III. **SIMULATION RESULTS**

We consider a scenario with $L = 8$, $K = 2$, $M = 2$ and SIR=-10dB. Three cases are shown, in which $R_{YY}$ is averaged over 1, 2 and 10 slots, respectively. The fast parameters are drawn randomly in each slot, while the slow parameters are constant. The simulations show that the original approach by Matti Latva-Aho (PLMMSE curves) suffers from signal cancellation when the fast parameters do not vary, whereas the new approaches are fairly insensitive to the speed of variation of the fast parameters.




## References


[1] Matti Latva-aho. *Advanced Receivers for Wideband CDMA Systems*. PhD thesis, Oulu University, Finland, September 1998.

[2] Matti Latva-aho and Markus Juntti. LMMSE Detection for DS-CDMA Systems in Fading Channels. In *IEEE Trans. Commun.*, February 2000.

[3] Markus Juntti Matti Latva-aho and Kimmo Kansanen. LMMSE Receivers for DS-CDMA Systems in Frequency-Selective Channels. In *FRAMES Workshop*, January 1999.