ON SPATIO-FREQUENTIAL SMOOTHING FOR JOINT ANGLES AND TIMES OF ARRIVAL ESTIMATION OF MULTIPATHS

Ahmad Bazzi⋆†   *Dirk T.M. Slock*⋆   *Lisa Meilhac*†

⋆ EURECOM Mobile Comm's Dept., 450 route des Chappes, 06410 Biot - Sophia Antipolis, France
Email: {bazzi,slock}@eurecom.fr
†CEVA-RivieraWaves, 400, avenue Roumanille Les Bureaux, Bt 6, 06410 Biot - Sophia Antipolis, France
Email:{ahmad.bazzi, lisa.meilhac}@ceva-dsp.com

ABSTRACT

A natural extension of the "Spatial" smoothing preprocessing technique is presented and analysed. It is well known that subspace methods do not work properly in the presence of coherent sources. In this paper, a "Spatio-Frequential" smoothing technique is described when the transmit OFDM symbol is received through multiple coherent signals using a uniform linear antenna array. After this pre-processing technique, one could efficiently apply any 2-dimensional subspace method to jointly estimate the angles and times of arrival of the incoming coherent signals. Simulation results demonstrate the potential of the proposed 2D smoothing method over existing separate spatial or frequential smoothing techniques.

Index Terms— Joint Estimation, Angle of Arrival, Time of Arrival, Spatio-Frequential, Smoothing

1. INTRODUCTION

The problem of Joint Angle and Delay of arrival Estimation, also known as JADE, is a well-known and challenging problem in the context of array signal processing. In a parametric approach [1], the system is very model-sensitive to perturbations, such as array calibration [2] and timing synchronisation [3]. Many algorithms have been developed to jointly estimate the angles and times of arrival of multiple paths, such as those based on Maximum Likelihood (ML), [4] and [5]. In [4], the authors manage to derive an iterative technique that transforms the ML problem into two sets of simple one-dimensional optimisation problems. In [5], the algorithm presented is a generalisation of the Iterative Quadratic ML (IQML) algorithm to the case of JADE. Under some conditions, algorithms based on ML are able to resolve coherent sources, which is the case of an indoor environment, where the received signal is a sum of scaled and delayed versions of the transmitted one.

Subspace algorithms are based on extraction of signal and noise subspaces. These algorithms are computationally much more efficient than ML techniques, but their performance is suboptimal compared to ML. The classical ones are MUSIC (derived independently in [6] and [7]), ROOT-MUSIC [8] which is a root finding version of MUSIC, and ESPRIT [9] that is based on shift-invariance properties of the data covariance matrix. Indeed, these algorithms were first designed for estimating the angles of arrival of multiple non-coherent sources. Extensions of the classical subspace methods

were derived to jointly estimate angles and times of arrival of multiple signals, such as [10–17]. These methods will be referred to as 2D subspace techniques.

In the case of *coherent* sources, i.e. the received signal is a sum of scaled and delayed version of the transmitted signal, all subspace methods fail to estimate angles or times of arrival. Therefore, pre-processing techniques, such as *Spatial* smoothing, have been developed to cope with this issue [18] so as to estimate the angles of arrival using subspace techniques. The same preprocessing technique was applied in an attempt of estimating times of arrival using OFDM subcarriers in a single receive antenna case [19], thus the name frequency smoothing. *Therefore, it only seems natural to propose a Spatio-Frequential smoothing technique to "decorrelate" the coherent signals so that one could efficiently estimate angles and times of arrival using 2D subspace techniques. The advantage of Spatio-Frequential smoothing is also discussed.* The idea of 2D-Smoothing has been used in the context of 2D-arrays [20, 21] (i.e. rectangular arrays). However, this paper describes a new 2D preprocessing technique that is applicable to Single-Input-Multiple-Output (SIMO) systems that use multi-carrier transmit signals.

The paper is divided as follows: Section 2 presents the system model, assumptions, and problem formulation. A Recap of the JADE-MUSIC [10], or 2D-MUSIC, algorithm is given in Section 3. The *Spatio-Frequential* smoothing technique is described and analysed in Section 4, followed by simulations in Section 5. We conclude the paper in Section 6.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^\dagger$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation. $\otimes$ represents the *Kronecker* product. $\|\mathbf{X}\|_2$ is the *Frobenius* norm of matrix $\mathbf{X}$. The matrix $\mathbf{I}_N$ is the identity matrix of dimensions $N \times N$. The symbol ∎ indicates the end of a proof.

2. SYSTEM MODEL

2.1. Analytic Formulation

Consider an OFDM symbol $s(t)$ composed of M subcarriers, impinging an array of N antennas via q multipath components, each arriving at AoAs $\{\theta_1, \ldots, \theta_q\}$ and ToAs $\{\tau_1, \ldots, \tau_q\}$. In baseband, we could write the received signal of the lth OFDM symbol at the nth antenna as:

$$r_n(t,l) = \sum_{i=1}^{q} \gamma_i(l) a_n(\theta_i) s(t - \tau_i) + n_n(t,l) \qquad (1)$$

with

$$s(t) = \begin{cases} \sum_{m=0}^{M-1} b_m e^{j2\pi m \triangle_f t} & \text{if } t \in [0, T] \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

where $T = \frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, b_m is the modulated information onto the mth subcarrier. $a_n(\theta)$ is the nth antenna response to an incoming signal at angle θ, the form of $a_n(\theta)$ depends on the array geometry. $\gamma_i(l)$ is the complex attenuation of the ith multipath component. The term $n_n(t, l)$ is background noise. Now, plugging (2) in (1) and sampling $r_n(t, l)$ at regular intervals of $k \triangleq k\frac{T}{M}$, we get $r_{n,k}(l) \triangleq r_n(k\frac{T}{M}, l)$ as:

$$r_{n,k}(l) = \sum_{i=1}^{q} \sum_{m=0}^{M-1} b_m \gamma_i(l) a_n(\theta_i) e^{j2\pi m(\frac{k}{M} - \triangle_f \tau_i)} + n_{n,k}(l) \quad (3)$$

Applying an M-point DFT to M samples collected from (3) ($k = 0, \dots, M-1$), we get:

$$R_{n,m}(l) = \sum_{k=0}^{M-1} r_{n,k}(l) e^{-j2\pi m \frac{k}{M}}$$
$$= b_m \sum_{i=1}^{q} \gamma_i(l) a_n(\theta_i) e^{-j2\pi m \triangle_f \tau_i} + N_{n,m}(l) \quad (4)$$

We assume prior knowledge of the symbols b_m. This is a valid assumption in Wi-Fi 802.11 systems since a fixed preamble is transmitted for channel estimation and other purposes. Thus, multiplying each $R_{n,m}(l)$ by $\frac{b_m^*}{|b_m|^2}$ and re-writing (4) in a compact form, we have

$$\mathbf{x}(l) = \mathbf{A}\boldsymbol{\gamma}(l) + \mathbf{n}(l), \qquad l = 1 \dots L \quad (5)$$

where $\mathbf{x}(l)$ and $\mathbf{n}(l)$ are $NM \times 1$ vectors given by

$$\mathbf{x}(l) = [R_{1,1}(l) \dots R_{1,M}(l) \dots R_{N,1}(l) \dots R_{N,M}(l)]^T \quad (6)$$

$$\mathbf{n}(l) = [N_{1,1}(l) \dots N_{1,M}(l) \dots N_{N,1}(l) \dots N_{N,M}(l)]^T \quad (7)$$

and $\boldsymbol{\gamma}(l)$ is the multipath vector of size $q \times 1$

$$\boldsymbol{\gamma}(l) = [\gamma_1(l) \dots \gamma_q(l)]^T \quad (8)$$

The matrix $\mathbf{A}$ is an $MN \times q$ matrix, i.e.

$$\mathbf{A} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1) \dots \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)] \quad (9)$$

with

$$\mathbf{a}(\theta) = [a_1(\theta) \dots a_N(\theta)]^T \quad (10)$$

and

$$\mathbf{c}(\tau) = [c_1(\tau) \dots c_M(\tau)]^T \quad (11)$$

In the rest of this paper, we consider a uniform linear antenna array spaced at half a wavelength, i.e. $a_n(\theta) = e^{-j\pi(n-1)sin(\theta)}$; also, the subcarriers are uniformly spaced at each $\triangle_f$, i.e. $c_m(\tau) = e^{-j2\pi\triangle_f(m-1)\tau}$.

2.2. Assumptions and Problem Statement

We assume the following:

- **A1**: $\mathbf{A}$ is full column rank.
- **A2**: The number of multipath components q is known.
- **A3**: The vector $\mathbf{n}(l)$ is additive Gaussian noise of zero mean and covariance $\sigma^2 \mathbf{I}_{MN}$, assumed to be white over space, frequencies, and symbols; we also assume that the noise is independent from the multipath coefficients.

Condition **A1** is valid as long as:

- **A1.1**: $q < MN$.
- **A1.2**: We consider that $\forall i \neq j$, $(\theta_i, \tau_i) \neq (\theta_j, \tau_j)$, that is all paths have distinct ToAs and AoAs, simultaneously, but it may happen that two, or more, paths arrive with the same ToAs, but different AoAs.
- **A1.3**: Let q^θ be the number of distinct AoAs, i.e. $\theta^1, \dots, \theta^{q^\theta}$; and let the following integers $Q_1, \dots, Q_{q^\theta}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\theta} Q_i = q$. This condition states that $\max_i Q_i < N$. That is the maximum number of paths arriving at the same time, i.e. $\max_i Q_i$, should be less than N.
- **A1.4**: Similar to **A1.3**, let q^τ be the number of distinct ToAs, i.e. $\tau^1, \dots, \tau^{q^\tau}$; and let the following integers $P_1, \dots, P_{q^\tau}$ denote their corresponding multiplicity. This condition states that $\max_i P_i < M$.

Techniques for estimating the number of coherent sources could be done by the Minimum Description Length (MDL) criterion as described in [22], or by a Modified MDL (MMDL) criterion as recently presented in [23]. However, we assume knowledge of the number of sources, i.e. q is known. Any further assumptions will be mentioned. Now, we address our problem: *Given $\{\mathbf{x}(l)\}_{l=1}^{L}$ and q coherent sources, preprocess the data $\{\mathbf{x}(l)\}_{l=1}^{L}$ so as to estimate the signal parameters $\{(\theta_i, \tau_i)\}_{i=1}^{q}$ using a 2D subspace technique.*

3. THE JADE-MUSIC ALGORITHM: A RECAP

The spatio-frequential covariance matrix is given by

$$\mathbf{R}_{xx} = E\{\mathbf{x}(l)\mathbf{x}^\dagger(l)\} = \mathbf{A}\mathbf{R}_{\gamma\gamma}\mathbf{A}^\dagger + \sigma^2 \mathbf{I}_{MN} \quad (12)$$

where $\mathbf{R}_{\gamma\gamma}$ is the covariance matrix of $\boldsymbol{\gamma}(l)$. The matrix given in (12) is, usually, estimated through a sample average over snapshots, and is known as the sample covariance matrix, i.e.

$$\mathbf{R}_{xx} \simeq \frac{1}{L} \sum_{l=1}^{L} \mathbf{x}(l)\mathbf{x}^\dagger(l) \quad (13)$$

In what follows, $\mathbf{R}_{xx}$ will be referred to as the sample covariance matrix, and not the true one. We denote $\lambda_1 > \lambda_2 > \dots > \lambda_{MN}$ the eigenvalues of $\mathbf{R}_{xx}$. Their corresponding eigenvectors are named $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{MN}$. The sample covariance matrix in (13) is an input to most subspace algorithms for estimating the signal parameters, i.e. AoAs $\{\theta_1 \dots \theta_q\}$ and ToAs $\{\tau_1 \dots \tau_q\}$. One of these algorithms is the JADE-MUSIC algorithm, which is a 2-D version of MUSIC. This algorithm is summarised as follows:

(1) Apply an eigenvalue decomposition of $\mathbf{R}_{xx}$.
(2) Form the noise subspace matrix, i.e. $\mathbf{U}_n = [\mathbf{u}_{q+1} \dots \mathbf{u}_{MN}]$.
(3) Search for $\{\hat{\theta}_i, \hat{\tau}_i\}_{i=1}^{q}$ by

$$\{(\widehat{\theta}_i, \widehat{\tau}_i)\}_{i=1}^{q} = \arg\max_{\theta, \tau} \left\{ \frac{1}{\|\mathbf{U}_n^\dagger[\mathbf{a}(\theta) \otimes \mathbf{c}(\tau)]\|_F^2} \right\} \quad (14)$$

The MUSIC algorithm is one of many subspace techniques, i.e. the extraction of a signal or noise subspace is required for further processing. Subspace techniques assume that the matrix $\mathbf{R}_{\gamma\gamma}$ is full rank, otherwise the estimated subspaces do not reflect the true ones (See [18] for a mathematical argument). Furthermore, rank deficiency of $\mathbf{R}_{\gamma\gamma}$ is due to coherence of multiple signals, or to insufficient number of snapshots, i.e. $L < q$. The spatial smoothing
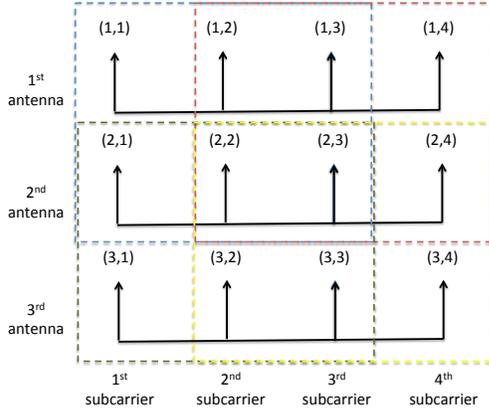
Fig. 1: A spatio-frequential array of $N = 3$ antennas and $M = 4$ subcarriers partitioned into $N_p = 2$ and $M_p = 3$, hence a total of $K_M K_N = 4$ subarrays.

pre-processing technique is known to "decorrelate" the sources, and therefore attain full rank of the matrix $\mathbf{R}_{\gamma\gamma}$.

In the following section, we present a 2D version of smoothing, i.e. *spatio-frequential* smoothing, and we show its advantage over conventional *spatial* or *frequential* smoothing techniques.

4. THE SPATIO-FREQUENTIAL PREPROCESSING TECHNIQUE

Recall that equation (5) gives the information on all subcarriers at all antennas. We shall use the notation (n, m) to index the mth subcarrier and n^{th} antenna. Let the spatio-frequential array $\{(i, j)\}_{i=1...N}^{j=1...M}$ of size MN be divided into overlapping subarrays of size $M_p N_p$ (M_p and N_p being the number of subcarriers and antennas in the subarrays, respectively). Indeed, one could check that the total number of overlapping subarrays is equal to $K_M K_N$, where $K_M = M - M_p + 1$ and $K_N = N - N_p + 1$.

To visualise how the subarrays are formed, we refer the reader to figure 1, where a setting of $N = 3$ antennas and $M = 4$ subcarriers is partitioned into overlapping subarrays of sizes $N_p = 2$ and $M_p = 3$, and therefore a total of $K_M K_N = 4$ subarrays.

Since the effective number of subcarriers and antennas used now are M_p and N_p, respectively, then (5) becomes

$$\mathbf{x}_{m,n}(l) = \bar{\mathbf{A}} \mathbf{D}_\tau^{m-1} \mathbf{D}_\theta^{n-1} \boldsymbol{\gamma}(l) + \mathbf{n}_{m,n}(l) \quad (15)$$

where $\bar{\mathbf{A}} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1) \ldots \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)]$ is an $M_p N_p \times q$ matrix. The vectors $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ are the same as in (10) and (11) but with sizes N_p and M_p instead of N and M, respectively. $\mathbf{D}_\tau^{m-1}$ and $\mathbf{D}_\theta^{n-1}$ are the $(m-1)^{th}$ and $(n-1)^{th}$ power of the diagonal $q \times q$ matrices $\mathbf{D}_\tau$ and $\mathbf{D}_\theta$, given by

$$\mathbf{D}_\tau = \text{diag} \{c_2(\tau_1) \ldots c_2(\tau_q)\} \quad (16a)$$

$$\mathbf{D}_\theta = \text{diag} \{a_2(\theta_1) \ldots a_2(\theta_q)\} \quad (16b)$$

This means that $\mathbf{x}_{m,n}(l)$ is an $M_p N_p \times 1$ received vector on the subarray $\{(i, j)\}_{j=n...N_p+n-1}^{i=m...M_p+m-1}$. The covariance matrix of $\mathbf{x}_{m,n}(l)$

in (15) after averaging over time snapshots is given as

$$\mathbf{R}_{m,n} = \bar{\mathbf{A}} \mathbf{D}_\tau^{m-1} \mathbf{D}_\theta^{n-1} \mathbf{R}_{\gamma\gamma} \mathbf{D}_\theta^{\dagger\,n-1} \mathbf{D}_\tau^{\dagger\,m-1} \bar{\mathbf{A}}^\dagger + \sigma^2 \mathbf{I}_{M_p N_p} \quad (17)$$

The *spatio-frequential smoothed covariance* matrix is given by

$$\bar{\mathbf{R}} = \frac{1}{K_M K_N} \sum_{m=1}^{K_M} \sum_{n=1}^{K_N} \mathbf{R}_{m,n} \quad (18)$$

$\bar{\mathbf{R}}$ could also be written as

$$\bar{\mathbf{R}} = \bar{\mathbf{A}} \bar{\mathbf{R}}_{\gamma\gamma} \bar{\mathbf{A}}^\dagger + \sigma^2 \mathbf{I}_{M_p N_p} \quad (19)$$

where

$$\bar{\mathbf{R}}_{\gamma\gamma} = \frac{1}{K_M K_N} \sum_{m=1}^{K_M} \sum_{n=1}^{K_N} \mathbf{D}_\tau^{m-1} \mathbf{D}_\theta^{n-1} \mathbf{R}_{\gamma\gamma} \mathbf{D}_\theta^{\dagger\,n-1} \mathbf{D}_\tau^{\dagger\,m-1} \quad (20)$$

In a single carrier case, i.e. $M = M_p = 1$, it has been proven that the spatial smoothing technique ensures full rank of $\bar{\mathbf{R}}_{\gamma\gamma}$ [18], given that $q \leq K_N$.

Analogously, in the single antenna but multi-carrier case, i.e. $N = N_p = 1$, the same technique has been applied in [19] and was referred to as frequency smoothing, in order to acheive full rank of $\bar{\mathbf{R}}_{\gamma\gamma}$, when $q \leq K_M$. However, in the general multi-antenna and multi-carrier case, we have the following:

Theorem: If the number of subarrays formed jointly over space and frequency is greater than the number of multipath components, i.e. $q \leq K_M K_N$, and the maximum number of paths arriving at the same time but with different angles is less than K_N, i.e. $\max_i Q_i \leq K_N$, and the maximum number of paths arriving at the same angles but with different times is less than K_M, i.e. $\max_i P_i \leq K_M$, then $\bar{\mathbf{R}}_{\gamma\gamma}$ is of rank q.

Proof: Using (20), $\bar{\mathbf{R}}_{\gamma\gamma}$ could be written as

$$\bar{\mathbf{R}}_{\gamma\gamma} = \mathbf{D} \mathbf{Q} \mathbf{D}^\dagger \quad (21)$$

where $\mathbf{D}$ is a $q \times q K_M K_N$ matrix given by

$$\mathbf{D} = [\mathbf{T}, \mathbf{D}_\theta \mathbf{T}, \ldots \mathbf{D}_\theta^{K_N-1} \mathbf{T}] \quad (22a)$$

and

$$\mathbf{T} = [\mathbf{I}_q, \mathbf{D}_\tau, \ldots, \mathbf{D}_\tau^{K_M-1}] \quad (22b)$$

and $\mathbf{Q}$ is a block diagonal $q K_M K_N \times q K_M K_N$ matrix expressed as

$$\mathbf{Q} = \frac{1}{K_M K_N} \mathbf{I}_{K_M K_N} \otimes \mathbf{R}_{\gamma\gamma} \quad (23)$$

Equation (21) can be expressed as follows

$$\bar{\mathbf{R}}_{\gamma\gamma} = \mathbf{W} \mathbf{W}^\dagger \quad (24)$$

with

$$\mathbf{W} = [\mathbf{T}_c, \mathbf{D}_\theta \mathbf{T}_c, \ldots, \mathbf{D}_\theta^{K_N-1} \mathbf{T}_c] \quad (25a)$$

and

$$\mathbf{T}_c = [\mathbf{C}, \mathbf{D}_\tau \mathbf{C}, \ldots, \mathbf{D}_\tau^{K_M-1} \mathbf{C}] \quad (25b)$$

where $\mathbf{C}$ is the square root of $\frac{1}{K_M K_N} \mathbf{R}_{\gamma\gamma}$:

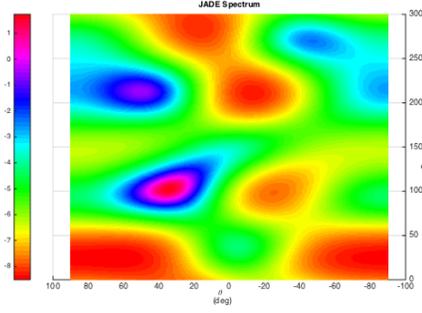$$\frac{1}{K_M K_N} \mathbf{R}_{\gamma\gamma} = \mathbf{C} \mathbf{C}^\dagger \quad (26)$$
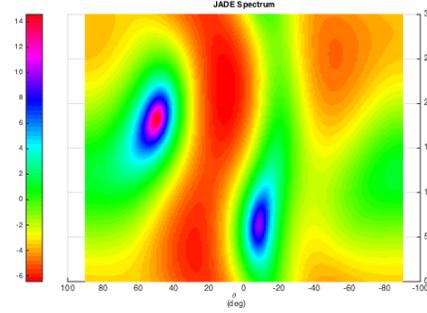
Fig. 2: Spatial Smoothing with $N_p = 2$.

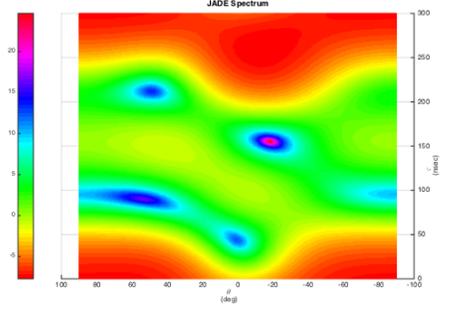**Fig. 3**: Frequency Smoothing with $M_p = 2$.

Fig. 4: Spatio-Frequential Smoothing with $N_p = 2$ and $M_p = 3$.

The rank of $\bar{\mathbf{R}}_{\gamma\gamma}$ is equal to the rank of $\mathbf{W}$. Now, using the fact that the rank of a matrix is unchanged under column permutations, then we can write the following:

$$\text{rank } \mathbf{W} = \text{rank} \begin{pmatrix} c_{11}\mathbf{v}_1 \otimes \mathbf{t}_1 & \cdots & c_{1q}\mathbf{v}_1 \otimes \mathbf{t}_1 \\ \vdots & \ddots & \vdots \\ c_{q1}\mathbf{v}_q \otimes \mathbf{t}_q & \cdots & c_{qq}\mathbf{v}_q \otimes \mathbf{t}_q \end{pmatrix} \quad (27)$$

where c_{ij} is the $(i,j)^{th}$ entry of $\mathbf{C}$. Vectors $\mathbf{v}_i$ and $\mathbf{t}_i$ $(i = 1 \ldots q)$ are of sizes $1 \times K_N$ and $1 \times K_M$, respectively, given as

$$\mathbf{v}_i = [1, e^{-j\pi sin(\theta_i)}, \ldots, e^{-j\pi(K_N-1)sin(\theta_i)}] \quad (28a)$$

$$\mathbf{t}_i = [1, e^{-j2\pi\triangle_f \tau_i}, \ldots, e^{-j2\pi\triangle_f(K_M-1)\tau_i}] \quad (28b)$$

To prove that, for $q \leq K_M K_N$, $\max_i Q_i \leq K_N$, and $\max_i P_i \leq K_M$, the matrix $\mathbf{W}$ is of rank q, we should prove the following:
(a) $\mathbf{W}$ does not have an all-zero row, i.e. for a given row i, there exists at least one j such that $c_{ij} \neq 0$.
(b) The vectors $\{\mathbf{v}_i \otimes \mathbf{t}_i\}_{i=1}^q$ are linearly independent.
The proof of **(a)** is found in [18]. As for **(b)**, let $\mathbf{H}$ be a $K_M K_N \times q$ matrix of columns $\{\mathbf{v}_i \otimes \mathbf{t}_i\}_{i=1}^q$. The matrix $\mathbf{H}$ is full column rank under the following three conditions:
(b.1) $q \leq K_M K_N$. (Similar to **A1.1**)
(b.2) $\max_i Q_i \leq K_N$. (Similar to **A1.3**)
(b.3) $\max_i P_i \leq K_M$. (Similar to **A1.4**) ∎
Conditions **(b.1)** till **(b.3)** are sufficient to attain full rank of $\bar{\mathbf{R}}_{\gamma\gamma}$. But, in order for subspace methods to work properly, one should also have that $\bar{\mathbf{A}}$ (see (15)) is full column rank. Note that $\bar{\mathbf{A}}$ has dimensions $N_p M_p \times q$. This is valid when $q \leq M_p N_p$ (Similar to **A1.1**), $\max_i Q_i \leq N_p$ (Similar to **A1.3**), and $\max_i P_i \leq M_p$ (Similar to **A1.4**). In general, one must have:
(c.1) $q \leq \min\{K_M K_N, M_p N_p\}$.
(c.2) $\max_i Q_i \leq \min\{K_N, N_p\}$.
(c.3) $\max_i P_i \leq \min\{K_M, M_p\}$.
Finally, the advantage of *spatio-frequential* smoothing is that it offers $K_M K_N$ subarrays to smooth over, in contrast to *spatial* and *frequential* smoothing that naturally provide K_N and K_M subarrays, respectively. Therefore, one could be able to resolve more coherent sources, as given in **(b.1)**. This advantage is, also, presented through simulations.

5. SIMULATIONS

Simulation results are presented to show the advantage of *spatio-frequential* smoothing over the conventional *spatial* and *frequential*

smoothing. Simulations have been done with $N = 3$ antennas and $M = 4$ subcarriers at SNR = 20dB. The subcarrier spacing is chosen $\triangle_f = 3.125$ MHz. We have fixed $q = 4$ paths, where their corresponding angles and times of arrival are $(\theta_1, \tau_1) = (0°, 40 \text{ nsec})$, $(\theta_2, \tau_2) = (60°, 100 \text{ nsec})$, $(\theta_3, \tau_3) = (-20°, 150 \text{ nsec})$ and $(\theta_4, \tau_4) = (50°, 200 \text{ nsec})$. The complex attenuation vector γ is fixed to a constant arbitrary value. Finally, $L = 3$ snapshots were collected.

Figure 2 shows the JADE spectrum after preprocessing only by spatial smoothing, i.e. $M = M_p = 4$ and $N_p = 2$. Indeed, there is an ambiguity in detecting the 4 peaks corresponding to the true angles and times of arrival due to insufficient number of subarrays to smooth over, i.e. only $K_N = 2 < q$ spatial subarrays are available. The same argument is done when one applies only frequency smoothing, i.e. $N = N_p = 3$ and $M_p = 2$. In that case, one will have $K_M = 3 < q$ subarrays to smooth over. As a result, false peaks appear in figure 3.

To this end, we could see that we need at least $q = 4$ subarrays to smooth over. This is done by preprocessing through spatio-frequential smoothing. Choosing $N_p = 2$ and $M_p = 3$ would lead to $K_N K_M = 4$ subarrays in total. After smoothing over space and frequencies, one could observe 4 clear peaks corresponding to the true angles and times of arrival of the 4 paths in figure 4.

6. CONCLUSION

We have presented a 2D smoothing preprocessing technique, applied to a *Spatial-Frequential* array, to "decorrelate" multipath components. Then, any 2D subspace algorithm could be applied to estimate the times and angles of arrivals of the different paths. The 2D smoothing technique presented here, naturally, offers more subarrays to smooth over and, therefore, one could be able to resolve more coherent paths.

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