Precoder Feedback versus Channel Feedback in Massive MIMO under User Cooperation

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***Abstract*—In multiuser massive MIMO systems, it is not clear whether users should feed back the channel or the precoder when they can exchange the channel state information (CSI). This paper compares the precoder feedback scheme with the channel feedback scheme. It is found that when there are sufficient number of bits for CSI exchange, the precoder feedback scheme can reduce the interference leakage to $1/(K-1)$ of the channel feedback scheme, where K is the number of users. Moreover, the interference leakage under the precoder feedback scheme decreases faster than the channel feedback scheme when the number of feedback bits increases.**

I. Introduction

Massive MIMO is a promising technique for next generation communication systems. Using a large number of antenna elements, the transmitter can design precoders that concentrate the signal to each target user, realizing both signal power enhancement for target users and interference suppression for undesired users. However, to achieve these benefits, the transmitter needs to know the channels of all the users, which is very challenging due to the high channel dimensions. Although there is a body of literature on precoding strategies in time-division duplex (TDD) systems, where the downlink CSI can be easily obtained from uplink transmissions via channel reciprocity, the current market is still dominated by frequency division multiplexing (FDD) systems, where channel reciprocity does not hold. Thus, tackling the challenges of feedback and precoding in FDD massive MIMO systems is very important.

To reduce the CSI feedback to the base station (BS), various channel quantization techniques, such as Grassmannian quantization and random vector quantization (RVQ), have been proposed in the literature [1]–[7]. However, these methods cannot be directly applied to massive MIMO scenarios, due to the fact that large codebooks (scale exponentially to the number of feedback bits) are required for quantizing the very large channel vectors. To reduce the channel dimension for instantaneous feedback, a two-layer precoding structure has been proposed in [8] and [9], where the MIMO precoder is decomposed into an outer precoder that adapts to the channel statistics and an inner precoder that adapts to the instantaneous *equivalent* channel which lies in a subspace with smaller dimension. However, the performance of the two-layer precoding highly depends on the low-rank property of the covariance matrix of the MIMO channel, which may not always be satisfied. Recently, a user cooperative feedback and precoding technique for multiuser MIMO systems enabled by device-to-device (D2D) communications was proposed in [10]. This technique assumes that perfect downlink CSI is available to each user. In the user cooperative precoding scheme, the users first collect the CSI from each other via D2D communications, and then select a precoder and feed it back to the BS based on the knowledge of global CSI. Numerical results show that the precoding assisted by user cooperation can significantly reduce the feedback loading while achieving similar sum rate performance of traditional precoding without user cooperation.

A fundamental difference between the user cooperative precoding and the traditional schemes under limited feedback is that the user cooperative scheme feeds back the precoder rather than the quantized channel vector. It is intuitive that, when the users know the perfect global CSI, it is better to feedback the precoder computed at the user side instead of using the precoder computed at the BS based on the imperfect CSI feedback from the users. However, the situation becomes complicated when the global CSI available at the user side is imperfect. Note that even with D2D communication capability, the users do not have infinite rate to exchange their CSI perfectly. As a result, the users may have only partial knowledge of other users' channels. Therefore, an essential question of practical interest is: *Should the users feedback the precoder or the channel when they can cooperate?*

This paper attempts to shed some light on this question by carrying out analytical comparisons between the precoder feedback scheme under user cooperation and the traditional precoding scheme based on channel feedback, where there are only a finite number of bits for the feedback to the BS and for the CSI exchange among users. Specifically, interference leakage is analyzed for both schemes: in the channel feedback scheme, a zero-forcing (ZF) precoder is computed at the BS, whereas in the precoder feedback scheme, a precoding vector from a finite rate codebook is chosen by each user to minimize the interference leakage to the other users. The two user scenario is first analyzed, where a simple explicit expression is derived to characterize the interference leakage in terms of the number of bits B_f to feedback and the number of bits B_c for CSI exchange under user cooperation. The analysis is then extended to the K user scenario. We find that under sufficiently large B_c, the precoder feedback scheme can reduce the interference leakage to $1/(K-1)$ of the channel feedback scheme. In addition, the interference leakage from the precoder feedback scheme decreases faster when the number of bits $B_{\rm f}$ increases. Specifically, it scales as $\mathcal{O}(2^{-B_{\rm f}/(K-1)})$ for the precoder feedback scheme versus $\mathcal{O}(2^{-B_{\rm f}/(N_t-1)})$ for the channel feedback. These results demonstrate that the precoder feedback scheme is preferable when the users can cooperate via sufficiently high data rate in the D2D communications.

II. System Model

A. Channel Model

Consider a K user single cell massive MIMO network, where the BS is equipped with N_t antennas and $N_t \geq K$ for the sake of simplifying the discussion. Denote the downlink channel of user k as $\mathbf{h}_k \in \mathbb{C}^{N_t}$. The channel is assumed to be block fading, with independent fading from block to block. The entries of the channel vectors are independent and identically distributed (i.i.d.) and follow a complex Gaussian distribution with zero mean and unit variance denoted by $\mathcal{CN}(0,1)$. Moreover, the channel vectors $\mathbf{h}_k$ are independent across users. The received signal of user k is given by

$$y_k = \sqrt{\frac{P}{K}}\mathbf{h}_k^{\rm H}\mathbf{w}_k s_k + \sqrt{\frac{P}{K}}\sum_{j\neq k}\mathbf{h}_k^{\rm H}\mathbf{w}_j s_j + n_k$$

where s_k, with $\mathbb{E}\{|s_k|^2\}=1$, denotes the symbol for user k, $\mathbf{w}_k \in \mathbb{C}^{N_t}$ is the precoder for user k, $n_k \sim \mathcal{CN}(0,1)$ is the additive Gaussian noise, and P is the total transmission power. Here, equal power allocation among users is assumed.

The system is operating in FDD mode, and the knowledge of CSI at the BS requires explicit feedback from the users. During each channel coherent time block, each user can feedback $B_{\rm f}$ bits to the BS. On the user side, perfect CSI $\mathbf{h}_k$ is available to each user k, but the knowledge of global CSI needs explicit signaling via D2D communications between users. Assume that each user has $B_{\rm c}$ bits to quantize its channel and share it with all the other users. Fig. 1 illustrates a two-user example on the signaling structure of the precoder feedback scheme under user cooperation.

B. Feedback and Precoding Schemes

With CSI exchange among users, the users have two options to feedback to the BS: the channel $\mathbf{h}_k$ or the precoder $\mathbf{w}_k$.

1) Channel Feedback Scheme: Each user has a *channel codebook* $\mathcal{C}_k$ that contains $2^{B_{\rm f}}$ N_t-dimensional unit norm vectors.[1] In this work, RVQ is assumed for performance analysis. Under RVQ, the codebook can be generated from a sequence of random vectors that are independently and isotropically distributed on the N_t-dimensional sphere. To feed back the CSI, each user quantizes its channel to the vector that is closest to its channel vector, measured by the inner product. Thus, the index of the quantization vector

$$\hat{\mathbf{g}}_k = \arg\max_{\mathbf{u}\in\mathcal{C}_k} |\mathbf{h}_k^{\rm H}\mathbf{u}| \tag{1}$$

[1]The fact that each user has a different codebook can avoid multiple users quantizing their channels into the same vector.

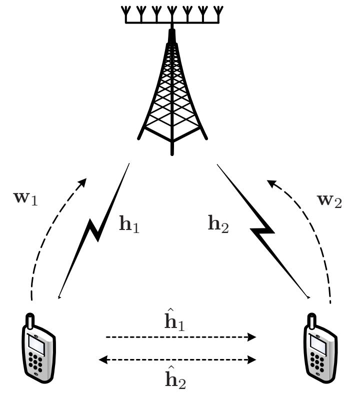

Figure 1. An example on the signaling structure of the precoder feedback scheme under user cooperations.

is fed back to the BS. Note that as ZF precoding is employed, only the channel direction is conveyed to the BS, and the knowledge of channel magnitude $\|\mathbf{h}_k\|$ is not required.

With the quantized CSI feedback, the BS computes the precoder $\mathbf{w}_k$ for user k as the normalized kth column of the precoding matrix $\mathbf{W} = \hat{\mathbf{G}}(\hat{\mathbf{G}}^{\rm H}\hat{\mathbf{G}})^{-1}$, where $\hat{\mathbf{G}}$ is a $N_t \times K$ matrix with the kth column given by the channel quantization vector $\hat{\mathbf{g}}_k$.

2) Precoder Feedback Scheme: Each user has two codebooks: the *channel codebook* $\mathcal{C}_k^{\rm c}$ that contains $2^{B_{\rm c}}$ N_t-dimensional unit norm vectors for CSI exchange among users, and the *precoder codebook* $\mathcal{C}_k^{\rm w}$ that contains $2^{B_{\rm f}}$ N_t-dimensional unit norm vectors for precoder feedback to the BS. RVQ is assumed for both of the codebooks, i.e., they consist of independently and isotropically distributed random vectors on the sphere in N_t-dimensional space.

To exchange the CSI, each user quantizes the direction of the channel vector to the nearest quantization vector $\hat{\mathbf{g}}_k^{\rm c}$ measured by the inner product (similar to (1)), and evaluates the channel magnitude $\gamma_k = \|\mathbf{h}_k\|^2$. Both the quantized channel direction $\hat{\mathbf{g}}_k^{\rm c}$ and the channel magnitude γ_k are shared to the other users via D2D communications.[2] The partial CSI available at user $j \neq k$ is thus given by $\hat{\mathbf{h}}_k = \sqrt{\gamma_k}\hat{\mathbf{g}}_k^{\rm c}$.

With the partial global CSI from the other users, each user chooses a precoder under the minimum interference leakage criterion, in which a vector $\mathbf{w}_k$ from the precoder codebook $\mathcal{C}_k^{\rm w}$ is chosen to minimize the total interference leakage:

$$\mathbf{w}_k^{\rm c} = \arg\min_{\mathbf{w}\in\mathcal{C}_k^{\rm w}} \sum_{j\neq k} |\hat{\mathbf{h}}_j^{\rm H}\mathbf{w}|^2. \tag{2}$$

Note that the minimum interference leakage criterion (2) is the counterpart of the ZF precoding computed at the BS under the channel feedback scheme, with the difference in that the precoder in (2) is chosen from a finite set, whereas the ZF precoder is computed in a continuous space.

Intuitively, in the undesired regime $B_{\rm c} \leq B_{\rm f}$, the channel quantization vector $\hat{\mathbf{g}}_k^{\rm c}$ shared among the users is, at least, as

[2]In this work, the channel magnitude γ_k (a scaler) is assumed to be exchanged perfectly without any cost in D2D communications.

poor as $\hat{\mathbf{g}}_k$ in (1) that is the information fed back to the BS. As a result, the precoder feedback scheme would perform worse, since the precoder $\mathbf{w}_k$ in (2) is chosen from a finite set and additional interference is induced. In this paper, we focus on the preferred regime $B_{\rm c} > B_{\rm f}$ to see how the performance scales with $B_{\rm c}$ and $B_{\rm f}$, and whether there is a performance advantage of the precoder feedback scheme under finite $B_{\rm c}$.

III. Interference Analysis

This section analyzes the performance in terms of interference leakage defined as $I_k = \rho \sum_{j \neq k} |\mathbf{h}_j^{\rm H} \mathbf{w}_k|^2$, where $\rho = P/K$ denotes the power allocation. Before going through the derivations, we first state the main result as follows.

Main results: Under large N_t and $B_{\rm f}$, the average interference leakage under the precoder feedback scheme is roughly upper bounded by

$$\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_k\right\} \lesssim \rho 2^{-\frac{B_{\rm f}}{K-1}} + \rho(K-1)2^{-\frac{B_{\rm c}}{N_t-1}}$$

where the expectation $\mathbb{E}_{\mathcal{H},\mathcal{C}}\{\cdot\}$ is taken over the distributions of the channels and the codebooks.

It is known that the interference leakage of the channel feedback scheme is lower bounded by $\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_k\right\} > \rho(K-1)2^{-\frac{B_{\rm f}}{N_t-1}}$ from [3]. Our result thus shows that for sufficiently large $B_{\rm c}$, the interference leakage from the precoder feedback scheme is dominated by the first term, which is lower than $1/(K-1)$ of the channel feedback scheme and decreases faster as $B_{\rm f}$ increases (for $K < N_t$).

A. Characterization of the Interference Leakage

We first characterize the interference leakage in terms of the precoding vectors and the quantization errors.

Lemma 1 (Characterization of the Interference Leakage): The mean of the interference leakage $I_1 = \rho \sum_{j \neq 1} |\mathbf{h}_j^{\rm H} \mathbf{w}_1|^2$ can be characterized as

$$\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_1\right\} = \rho N_t \sum_{j \neq 1} \mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{(1-Z_j)\left|\hat{\mathbf{g}}_j^{\rm H}\mathbf{w}_1\right|^2 + Z_j\left|\mathbf{s}_j^{\rm H}\mathbf{w}_1\right|^2\right\} \tag{3}$$

where $Z_j \triangleq 1 - |\hat{\mathbf{g}}_j^{\rm H}\mathbf{g}_j|^2$ with $\mathbf{g}_j = \mathbf{h}_j/\|\mathbf{h}_j\|$ and $\mathbf{s}_j \triangleq (\mathbf{I} - \hat{\mathbf{g}}_j\hat{\mathbf{g}}_j^{\rm H})\mathbf{g}_j/\sqrt{Z_j}$.[3]

Note that the quantity Z_j captures the channel quantization error in magnitude, and $\mathbf{s}_j$ captures the difference in direction between the quantized vector $\hat{\mathbf{g}}_j$ and the true channel $\mathbf{g}_j$ in the (N_t-1)-dimensional space.[4] Thus, Lemma 1 illustrates that the average interference is a sum of the interference leakage due to precoding and the residual interference due to channel quantization errors.

Intuitive comparisons between precoder feedback and channel feedback can be made from (3). In the channel feedback scheme, the first term in (3) gives $\left|\hat{\mathbf{g}}_j^{\rm H}\mathbf{w}_1\right|^2 = 0$ due to the ZF precoding at BS. The second term characterizes the interference leakage due to quantization error, which is in terms of $B_{\rm f}$. The results in [3] show that it is roughly $N_t\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{Z_j\left|\mathbf{s}_j^{\rm H}\mathbf{w}_1\right|^2\right\} \approx 2^{-\frac{B_{\rm f}}{N_t-1}}$.

In the precoder feedback scheme, the first term $\left|\hat{\mathbf{g}}_j^{\rm H}\mathbf{w}_1^{\rm c}\right|^2 \neq 0$, since $\mathbf{w}_1^{\rm c} \in \mathcal{C}_1^{\rm w}$ is chosen from a finite number of vectors. By contrast, the second term is affected by the quantization error in terms of $B_{\rm c}$ for D2D CSI exchange. Specifically, the channel quantization error bounds can be given by [3] $\frac{N_t-1}{N_t}2^{-\frac{B_{\rm c}}{N_t-1}} < \mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{Z_j\right\} < 2^{-\frac{B_{\rm c}}{N_t-1}}$. Moreover, $\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{\left|\mathbf{s}_j^{\rm H}\mathbf{w}_1^{\rm c}\right|^2\right\} = 1/(N_t-1)$, and $\left|\mathbf{s}_j^{\rm H}\mathbf{w}_1\right|^2$ is independent of Z_j. Hence, $N_t\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{Z_j\left|\mathbf{s}_j^{\rm H}\mathbf{w}_1\right|^2\right\} < \frac{N_t}{N_t-1}2^{-\frac{B_{\rm c}}{N_t-1}}$.

In the following part, we focus on quantifying the first term in (3) for the interference leakage under the precoder feedback scheme with user cooperation.

B. Interference Upper Bound in the Two-user Case

In two-user case, the precoder $\mathbf{w}_1$ only depends on $\hat{\mathbf{g}}_2$, and the interference leakage from user 1 is just the interference at user 2. The first term in (3) on the interference due to discrete precoding can be characterized in the following lemma.

Lemma 2 (Interference due to Discrete Precoding): The random variable $\left|\hat{\mathbf{g}}_2^{\rm H}\mathbf{w}_1^{\rm c}\right|^2$ follows a beta distribution $\mathcal{B}(1,(N_t-1)2^{B_{\rm f}})$ and its mean is given by $\left(1+(N_t-1)2^{B_{\rm f}}\right)^{-1}$.

Using this result, the interference upper bound under the precoder feedback scheme can be derived in the following theorem.

Theorem 1 (Interference Upper Bound for Two Users): The mean of the interference leakage $I_1^{\rm c} = \rho|\mathbf{h}_2^{\rm H}\mathbf{w}_1^{\rm c}|^2$ is upper bounded by

$$\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_1^{\rm c}\right\} \leq \frac{\rho N_t}{N_t-1}\left[2^{-B_{\rm f}} + \left(1 - \frac{N_t-1}{N_t}2^{-B_{\rm f}}\right)2^{-\frac{B_{\rm c}}{N_t-1}}\right].$$

The following corollary characterizes the case under perfect CSI exchange among users.

Corollary 1 (Interference Upper Bound under Perfect CSI Exchange): With perfect CSI exchange, i.e., $B_{\rm c} = \infty$, the interference is upper bounded by $\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_1^{\rm c}\right\} \leq \frac{\rho N_t}{N_t-1}2^{-B_{\rm f}}$.

As a comparison, the mean of the interference $I_1 = \rho|\mathbf{h}_2^{\rm H}\mathbf{w}_1|^2$ under the channel feedback scheme with ZF precoding at the BS can be bounded as [3]:

$$\rho 2^{-\frac{B_{\rm f}}{N_t-1}} < \mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_1\right\} < \frac{\rho N_t}{N_t-1}2^{-\frac{B_{\rm f}}{N_t-1}}. \tag{4}$$

With imperfect CSI exchange, Theorem 1 shows that for $B_{\rm c} \gg B_{\rm f}$, the interference under the precoder feedback scheme is smaller, and it decreases faster than the channel feedback scheme when the number of feedback bits $B_{\rm f}$ increases. On the other hand, when $B_{\rm c}$ is small, the interference under precoder feedback scheme is dominated by the residual interference due to channel quantization errors for CSI exchange among users.

Note that in the case of $N_t = K = 2$, the two schemes yield the same the interference upper bound when the CSI exchange is perfect.

[3]Due to the page limit, the proofs of the results are omitted here. Details can be found in [11].

[4]One can verify that $\mathbf{s}_j$ has unit norm and is orthogonal to $\hat{\mathbf{g}}_j$.

C. Interference Leakage in the K-user Case

In $K > 2$ user case, Lemma 2 does not hold because the precoding vector $\mathbf{w}_1^{\mathrm{c}}$ depends on more than one channel vectors, i.e., $\{\hat{\mathbf{h}}_j\}_{j\neq 1}$. The exact distribution of $\sum_{j\neq 1} |\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{w}_1^{\mathrm{c}}|^2$ is intractable. We resolve this challenge by using large system approximations and the extreme value theory in the scenario where both N_t and $2^{B_{\mathrm{f}}}$ are large.

Specifically, given a quantized channel realization $\{\hat{\mathbf{h}}_j\}_{j\neq 1}$ and a sequence of i.i.d. unit norm isotropic random vectors $\mathbf{w}_1^{\mathrm{c}}, \mathbf{w}_2^{\mathrm{c}}, \dots, \mathbf{w}_N^{\mathrm{c}}$ independent of $\{\hat{\mathbf{h}}_j\}_{j\neq 1}$, we first approximate the random variables $\widetilde{Y}_i \triangleq \sum_{j\neq 1} |\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{w}_i^{\mathrm{c}}|^2$ as independent chi-square random variables (multiplied by a scale factor $\frac{1}{2}$) with degrees of freedom $2(K-1)$. Note that such approximation becomes exact in large N_t.

The following lemma gives the asymptotic distribution of $\widetilde{Y}_i$ under the large N_t regime.

Lemma 3 (Asymptotic Chi-square Distribution): Let $X_1, X_2, \dots, X_N$ be a sequence of i.i.d. random variables that follows chi-square distribution $\chi^2(2(K-1))$. Then, $(\widetilde{Y}_1, \widetilde{Y}_2, \dots, \widetilde{Y}_N)$ converges to $\frac{1}{2}(X_1, X_2, \dots, X_N)$ in distribution, as $N_t \to \infty$.

Lemma 3 shows that as N_t becomes large, the variables $\widetilde{Y}_i$ and $\widetilde{Y}_j$ tend to become independent and $\frac{1}{2}\chi^2(2(K-1))$ chi-square distributed.

Consider the minimum interference leakage precoding criterion in (2), and note that the precoding vector $\mathbf{w}_k^{\mathrm{c}}$ is chosen from a set of i.i.d. isotropic vectors in $\mathcal{C}_k^{\mathrm{w}}$. Thus, the resultant interference leakage $\sum_{j\neq 1} |\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{w}_k^{\mathrm{c}}|^2$ is approximately the minimum of $2^{B_{\mathrm{f}}}$ i.i.d. chi-square distributed (with a constant factor $\frac{1}{2}$) random variables $\widetilde{Y}_i = \sum_{j\neq 1} |\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{w}_i^{\mathrm{c}}|^2$, $i = 1, 2, \dots, 2^{B_{\mathrm{f}}}$. As the codebook size $N = 2^{B_{\mathrm{f}}}$ is usually very large, one can apply extreme value theory to approximate the distribution of $\min_i \widetilde{Y}_i$ in order to yield simple expressions.

Let $\hat{I}_k \triangleq \sum_{j\neq k} |\hat{\mathbf{h}}_j^{\mathrm{H}}\mathbf{w}_k^{\mathrm{c}}|^2$, where $\mathbf{w}_k^{\mathrm{c}}$ is chosen from the precoder codebook $\mathcal{C}_k^{\mathrm{w}}$ under minimum interference leakage criterion (2). Thus, $\hat{I}_k = \min_i \widetilde{Y}_i$. The asymptotic property of $\hat{I}_k$ can be characterized in the following lemma.

Lemma 4 (Asymptotic Distribution of $\hat{I}_k$): Let $N = |\mathcal{C}_k^{\mathrm{w}}|$. Then,

$$\lim_{N\to\infty} \lim_{N_t\to\infty} \mathbb{P}\left\{\hat{I}_k < \phi_N y\right\} = 1 - \exp(-y^{K-1}), \qquad x \geq 0$$

where

$$\phi_N = \sup\left\{x : \frac{1}{\Gamma(K-1)} \int_0^x t^{K-2} e^{-t} dt \leq \frac{1}{N}\right\} \tag{5}$$

in which $\Gamma(x)$ denotes the Gamma function. Moreover, for small K, ϕ_N can be approximated by $\phi_N \approx \Gamma(K)^{-\frac{1}{K-1}} N^{-\frac{1}{K-1}}$.

Lemma 4 suggests that for large $N = 2^{B_{\mathrm{f}}}$ and large N_t, the interference leakage $\hat{I}_k$ due to finite precoding can be approximated by a random variable $\phi_N W_{K-1}$ in distribution, where W_{K-1} is Weibull distributed with cumulative distribution function (CDF) given by $f_W(x; K-1) = 1 - \exp(-x^{K-1})$, $x \geq 0$, and mean $\mathbb{E}\{W_{K-1}\} = \Gamma\left(\frac{K}{K-1}\right)$.

With these results, the mean interference leakage under the precoder feedback scheme can be derived in the following theorem.

Theorem 2 (Interference Leakage for K Users): The mean of the interference leakage $I_k^{\mathrm{c}} = \rho \sum_{j\neq k} |\mathbf{h}_j^{\mathrm{H}}\mathbf{w}_k^{\mathrm{c}}|^2$ under K-user networks can be approximated by

$$\begin{aligned}\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_k^{\mathrm{c}}\right\} &\approx \rho\Gamma\left(\frac{K}{K-1}\right)\phi_N \\ &\quad + \rho\left[\frac{N_t(K-1)}{N_t-1} - \frac{N_t-1}{N_t}\Gamma\left(\frac{K}{K-1}\right)\phi_N\right] 2^{-\frac{B_{\mathrm{c}}}{N_t-1}}\end{aligned} \tag{6}$$

where ϕ_N is given in (5) with $N = 2^{B_{\mathrm{f}}}$. Moreover, for small K,

$$\begin{aligned}\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_k^{\mathrm{c}}\right\} &\approx \rho\Phi(K) 2^{-\frac{B_{\mathrm{f}}}{K-1}} \\ &\quad + \rho\left[\frac{N_t(K-1)}{N_t-1} - \frac{N_t-1}{N_t}\Phi(K)2^{-\frac{B_{\mathrm{f}}}{K-1}}\right] 2^{-\frac{B_{\mathrm{c}}}{N_t-1}}\end{aligned} \tag{7}$$

where $\Phi(K) \triangleq \Gamma(\frac{K}{K-1})\Gamma(K)^{-\frac{1}{K-1}}$.

One can numerically verify that the term $\Phi(K)$ is decreasing in K and $\Phi(K) \leq 1$ for $K \geq 2$. Therefore, under sufficiently large B_{c} and N_t, the interference leakage $\mathbb{E}_{\mathcal{H},\mathcal{C}}\left\{I_k^{\mathrm{c}}\right\}$ is roughly upper bounded by $\rho 2^{-\frac{B_{\mathrm{f}}}{K-1}} + \rho(K-1)2^{-\frac{B_{\mathrm{c}}}{N_t-1}}$, which is significantly smaller than that of the channel feedback scheme $\rho(K-1)2^{-\frac{B_{\mathrm{f}}}{N_t-1}}$. On the other hand, in the undesired small B_{c} regimes, the second term in (7) dominates, which represents the residual interference due to poor quantization for CSI exchange among users.

IV. Numerical Results

We consider a single cell downlink massive MIMO system with $N_t = 20$ antennas at the BS serving $K = 4$ single antenna users. The channel model is given in Section II-A. The total (noise normalized) transmission power is $P = 20$ dB. Equal power allocation is applied to serve the K users. The following schemes are evaluated:

- **CSI-FB:** The users feedback the quantized CSI in B_{f} bits and the BS computes the precoder using ZF.
- **PRC-FB:** The users first exchange the CSI quantized by B_{c} bits for each vector, and then feedback the precoder in B_{f} bits based on minimum interference leakage (2).
- **PRC-FB Ideal:** The users first exchange the CSI perfectly, and then feedback the precoder in B_{f} bits based on minimum interference leakage.

Fig. 2 compares the mean interference leakage $\mathbb{E}\{I_1\}$ of the CSI-FB and PRC-FB schemes versus the number of bits B_{c} for channel quantization and CSI exchange under the PRC-FB scheme. Each user has $B_{\mathrm{f}} = 6$ bits for the feedback. The analytical expression (7) closely approximates the interference leakage results for the PRC-FB scheme. As shown, when B_{c} increases, the interference leakage $\mathbb{E}\{I_1\}$ decreases, since the users have better CSI knowledge to suppress the interference under the PRC-FB scheme. In the regimes where $B_{\mathrm{c}} \geq 7$ bits,

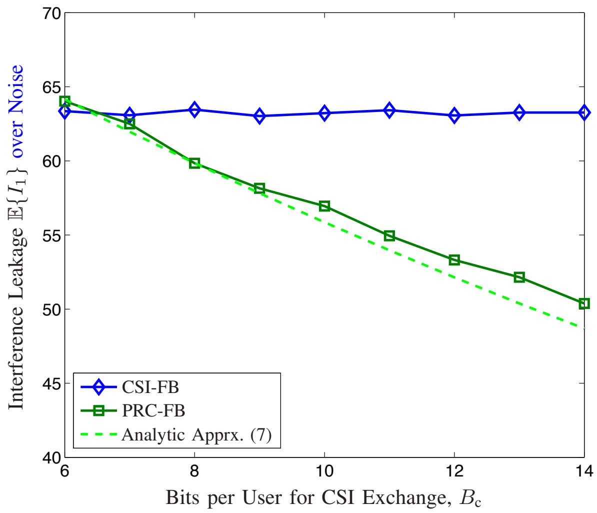

Figure 2. Comparison on the interference leakage over the numbers of bits B_c for channel quantization and CSI exchange under the PRC-FB scheme, where each user has $B_f = 6$ bits for the feedback.

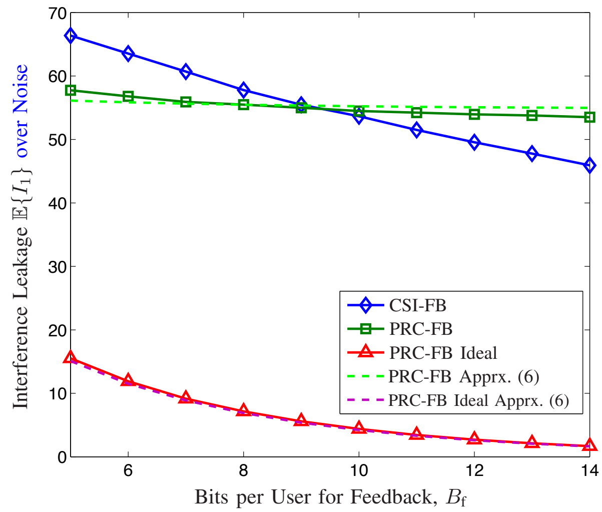

Figure 3. Comparison on the interference leakage over the numbers of bits B_f used for the feedback, with $B_c = 10$ bits for the PRC-FB scheme.

the PRC-FB scheme achieves smaller interference leakage than the CSI-FB scheme.

Fig. 3 evaluates the case under perfect CSI exchange among users. It compares the mean interference leakage $\mathbb{E}\{I_k\}$ versus the number of bits B_f per user for the feedback, with $B_c = 10$ bits for the PRC-FB scheme. Again, the analytical expression (6) closely approximates the interference leakage for both PRC-FB and PRC-FB Ideal schemes. When B_f increases, the interference leakage for all of the schemes decrease, while the PRC-FB Ideal achieves much lower interference leakage (lower than $1/3$) than the other two schemes. Note that, under insufficient B_c (10 bits versus $N_t = 20$), the interference leakage of the PRC-FB scheme may decrease slowly as B_f increases, because the interference is mainly due to the poor quantization for CSI exchange among users.

V. Conclusions

This paper derived analytical results to compare the interference leakage between the precoder feedback scheme and the channel feedback scheme in massive MIMO systems where the users are allowed to exchange some CSI with each other. The average interference leakage for the precoder feedback scheme was first derived for two-user networks, and then extended to K-user networks under massive antenna and large codebook approximations. We have shown analytically how the quantization errors on the CSI exchange among users affect the residual interference under the precoder feedback scheme. We have found that for sufficient number of bits for CSI exchange among users, the precoder feedback scheme can reduce the interference leakage to $1/(K-1)$ of channel feedback scheme under limited feedback to the BS. In addition, the interference leakage of the precoder scheme decreases faster than the channel feedback scheme when the number of feedback bits increases. Numerical results confirm the analytical results and demonstrates the performance advantage of the precoder feedback scheme.

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