Efficient Maximum Likelihood Joint Estimation of Angles and Times of Arrival of Multiple Paths

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***Abstract*—This paper presents and analyses two computationally attractive Maximum Likelihood (ML) estimators for joint Angle of Arrival (AoA) and Time of Arrival (ToA) using a Single Input Multiple Output (SIMO) link in an OFDM communication setting. We consider a rich multipath channel, which is the case of an indoor environment, where the received signal is a sum of scaled and delayed versions of the original transmit OFDM symbol. The first algorithm is a modification of the two dimensional Iterative Quadratic ML (2D-IQML) algorithm, where an additional constraint is added for joint ToA and AoA estimation. We show that 2D-IQML gives biased estimates of ToAs/AoAs and performs poorly at low SNR due to noise induced bias. The 2D-IQML cost function can be ”denoised” by eliminating the noise contribution: the resulting algorithm, two dimensional Denoised IQML (2D-DIQML), gives consistent estimates and outperforms 2D-IQML. Furthermore, 2D-DIQML is asymptotically globally convergent and hence insensitive to the initialisation. Also, we show that the 2D-DIQML algorithm behaves asymptotically at any SNR as the 2D-IQML algorithm behaves at high SNR. A simulation example has been presented to show the asymptotic behaviour of both algorithms at low SNR. Finally, joint AoA/ToA estimates could bring very useful information for localisation purposes, especially in a rich multipath channel, that could allow single anchor-based localisation.**

I. Introduction

Localisation has been one challenging topic over the past 60 years. In fact, many techniques have been developed in order to reliably position a wireless emitter. The first classical approach involves estimating the angle-of-arrival (AoA), received signal strenth (RSS), time-of-arrival (ToA), time-difference-of-arrival (TDoA), phase-of-arrival (PoA), etc.., of an emitter with respect to multiple base stations, in order to localise through triangulation or trilateration methods [1]. In favor of estimating signal parameters (ToA, AoA, etc..), the Maximum Likelihood (ML) technique was one of the first to be investigated [2]. However, it did not receive much attention due to the high computational load of the multivariate nonlinear minimisation problem involved, since it requires a $(pq + r)$-dimensional search, where p is the number of signal parameters of interest, q is the number of signals, and r are additional parameters that are part of the model and have to be estimated jointly with signal parameters. These r additional parameters could be antenna calibration parameters (see [3]) or synchronisation parameters (see [4]). To cope with this issue, a tradeoff has been done between complexity and performance, hence suboptimal techniques with reduced complexity have dominated the field. The most famous ones are: Minimum Variance Distortionless Response (MVDR) by Capon [5], followed by Multiple Signal Classification (MUSIC) developed in [6] and [7], independently. Also, less complex algorithms were implemented to replace the 1-D search of MUSIC by a polynomial root finding process [8], or a least squares fit [9].

The performance of the algorithms stated above are inferior to the ML technique. In addition, these suboptimal algorithms can not resolve *coherent* sources, which is the case of a specular multipath channel. Therefore, if a single user was transmitting a signal in such a channel, then all the above techniques (except for ML) could not properly estimate the signal parameters.

Many work has been done on presenting computationally attractive solutions for computing the ML estimator, such as ML by alternating projections [10]. This technique transforms the $q-$dimensional ML search into multiple 1-dimensional searches that terminate upon convergence. Another popular technique is the Iterative Quadratic ML (IQML) developed by Bresler and Macovski in [11], where the ML cost function at each iteration is seen as quadratic in the vector of parameters of interest, and thus closed form expressions could be derived, instead of applying multiple 1-D searches as done in [10]. Performance and complexity analysis of IQML were provided in the context of blind FIR channel estimation in [12]. Other techniques, that involve joint angle and time of arrival estimation for narrowband signals, are found in [13] and [14]. The former, also, transforms the multidimensional problem into sets of 1-D searches, whereas the latter is a two dimensional IQML, i.e. closed form expressions could be derived due to the quadratic nature of the cost function. As for UWB signals, the reader is referred to [28].

In this paper, we revise 2D-IQML and introduce a constraint that was not mentioned in [14]. This constraint would allow efficient joint estimation of AoAs/ToAs. The 2D-IQML algorithm needs only 1 iteration to converge to the true AoA/ToAs at high SNR, given a good initialisation, which is also provided in this paper. However, 2D-IQML gives biased estimates of the AoAs/ToAs and performs poorly at low SNR due to noise induced bias. The 2D-IQML cost function can be ”denoised” by eliminating the noise contribution: the resulting algorithm, Denoised 2D-IQML (2D-DIQML), gives consistent estimates and outperforms 2D-IQML. Attempts of ”denoising” the cost function of IQML were presented in [15] and [16]. We introduce a more judicious choice of the denoising parameter that leaves the Hessian of the problem positive semidefinite. The 2D-DIQML is asymptotically globally convergent and hence insensitive to the initialisation, as will be shown in this paper. However, its asymptotic performance does not reach the ML performance. The vast difference between 2D-IQML and 2D-DIQML could be seen at low SNR. We have shown that, indeed, *the 2D-DIQML algorithm behaves asymptotically at any SNR as the 2D-IQML algorithm behaves at high SNR*.

The algorithms considered here are generic and could also be applied to frequency estimation for 2D sinusoids in noise [14], joint AoA/ToA estimation [17] for multi-carrier signals impinging on a uniform linear antenna array, joint AoA/AoD (Angle of Departure) in a MIMO radar setting [18], joint angle and frequency estimation of a SIMO or MISO link [19], or even blind equalization of multiple FIR channels [20]. In this paper, we focus on joint AoA/ToA estimation of an OFDM transmitted symbol using a SIMO link, in an indoor environment (through a specular multipath channel), where coherency of multiple paths is possible. Furthermore, joint AoA/ToA estimates could bring very useful information for localisation purposes, especially in a rich multipath channel that could allow single anchor-based localisation [21]. This is due to the fact that each location has a unique ToA/AoA vector fingerprint, under a condition mentioned in [22]. Therefore, it is possible to form a database that maps ToA/AoA information to location, so that this database could be readily used in an online phase, where ToA/AoA are estimated at an unknown location, followed by a matching criteria to find the best matching location of the estimated ToA/AoA [23].

This paper is organised as follows: Section 2 presents the system model, general assumptions, and the problem formulation. In Section 3, the deterministic Maximum Likelihood (DML) estimator to our problem is derived. Parametrisation of the noise subspace is presented in Section 4. In Section 5, we revise the 2D-IQML proposed in [14], where we add an additional constraint for proper estimation of signal parameters, and we show that asymptotically, the 2D-IQML algorithm performs poorly at low SNR. An original "denoising" criteria is presented in Section 6, where we also show that *the 2D-DIQML algorithm behaves asymptotically at any SNR as the 2D-IQML algorithm behaves at high SNR*. This aforementioned statement is also observed through simulations in Section 7. Finally, we conclude our paper in Section 8.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation. $\otimes$ represents the *Kronecker* product. For any $N \times M$ matrix $\mathbf{X}$, $vec(\mathbf{X})$ is the vector operator which returns an $NM \times 1$ vector by stacking the columns of $\mathbf{X}$, starting from the first to the last column, $\|\mathbf{X}\|_2$ is the *Frobenius* norm of $\mathbf{X}$, and $\mathbf{X}^{\langle i,j \rangle}$ is the $(i,j)^{th}$ entry of $\mathbf{X}$. $|z|$ is the magnitude of $z \in \mathbb{C}$. The vector $\mathbf{e}_1$ is a vector of all-zeros except the first entry set to 1. The matrix $\mathbf{J}$ is the backward identity matrix, i.e. a matrix of all-zeros except for its anti-diagonal elements that are set to 1. The matrix $\mathbf{I}_N$ is the identity matrix of dimensions $N \times N$.

II. System Model

A. Analytic Formulation

Consider an OFDM symbol $s(t)$ composed of M subcarriers and centered at a carrier frequency f_c, impinging an antenna array of N antennas via q multipath components, each arriving at different AoAs $\{\theta_i\}_{i=1}^q$ and ToAs $\{\tau_i\}_{i=1}^q$. In baseband, we could write the l^{th} received OFDM symbol at the n^{th} antenna as:

$$r_n^{(l)}(t) = \sum_{i=1}^{q} \gamma_i^{(l)} a_n(\theta_i) s(t-\tau_i) + n_n^{(l)}(t) \tag{1}$$

where

$$s(t) = \begin{cases} \sum\limits_{m=0}^{M-1} b_m e^{j2\pi m \triangle_f t} & \text{if } t \in [0,T] \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

where $T = \frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, b_m is the modulated symbol onto the m^{th} subcarrier, $a_n(\theta)$ is the n^{th} antenna response to an incoming signal at angle θ. The form of $a_n(\theta)$ depends on the array geometry. $\gamma_i^{(l)}$ is the complex coefficient of the i^{th} multipath component. The term $n_n^{(l)}(t)$ is background noise. Plugging (2) in (1) and sampling $r_n^{(l)}(t)$ at regular intervals of $k \triangleq k\frac{T}{M}$, we get $r_{n,k}^{(l)} \triangleq r_n^{(l)}(k\frac{T}{M})$ as:

$$r_{n,k}^{(l)} = \sum_{i=1}^{q} \sum_{m=0}^{M-1} b_m e^{j2\pi \frac{km}{M}} e^{-j2\pi m \triangle_f \tau_i} \gamma_i^{(l)} a_n(\theta_i) + n_{n,k}^{(l)} \tag{3}$$

Collecting M samples, we can apply an M-point DFT, so observing the m^{th} subcarrier at the n^{th} antenna, we get:

$$\begin{aligned} R_{n,m}^{(l)} &= \sum_{k=0}^{M-1} r_{n,k}^{(l)} e^{-j2\pi m \frac{k}{M}} \\ &= b_m \sum_{i=1}^{q} \gamma_i^{(l)} a_n(\theta_i) e^{-j2\pi m \triangle_f \tau_i} + N_{n,m}^{(l)} \end{aligned} \tag{4}$$

We claim that the transmitted OFDM symbol $s(t)$ is a preamble field of the Wi-Fi 802.11 frame, thus prior knowledge of the modulated symbols $\{b_m\}_{m=0}^{M-1}$ is a valid assumption, since this stream of symbols (each at its corresponding sub-carrier) are repeated in each OFDM symbol placed at the beginning of the Wi-Fi frame for channel estimation and frequency offset purposes. Therefore, at each OFDM symbol reception, we compensate for all such symbols (multiplying by $\frac{b_m^*}{|b_m|^2}$) and hence omit b_m from (4). Re-writing (4) in a compact matrix form, we have:

$$\mathbf{x}(l) = \mathbf{H}\boldsymbol{\gamma}(l) + \mathbf{n}(l), \quad l = 1 \dots L \tag{5}$$

where $\mathbf{x}(l)$ and $\mathbf{n}(l)$ are $MN \times 1$ vectors

$$\mathbf{x}(l) = vec\{\mathbf{R}\}, \qquad \mathbf{R}^{\langle m,n \rangle} = R_{n,m}^{(l)} \tag{6}$$

$$\mathbf{n}(l) = vec\{\mathbf{N}\}, \qquad \mathbf{N}^{\langle m,n \rangle} = N_{n,m}^{(l)} \tag{7}$$

$\mathbf{H}$ is an $MN \times q$ matrix given as

$$\mathbf{H} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1) \dots \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)] \tag{8}$$

where $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ are $N \times 1$ and $M \times 1$, respectively. The n^{th} entry of $\mathbf{a}(\theta)$, denoted $\mathbf{a}_n(\theta)$, is the response of the n^{th} antenna to a signal arriving at angle θ with respect to the antenna array. We shall assume a Uniform Linear Array (ULA), thus $\mathbf{a}_n(\theta) = e^{-jd2\pi f_c(n-1)sin(\theta)}$, where d is the distance between 2 adjacent antennas. Similarly, the m^{th} entry of $\mathbf{c}(\tau)$, denoted $\mathbf{c}_m(\tau) = e^{-j2\pi\tau(m-1)\triangle_f}$, is the response of the m^{th} subcarrier to a signal arriving with time delay τ. The $q \times 1$ vector $\boldsymbol{\gamma}(l)$ is composed of the multipath coefficients

$$\boldsymbol{\gamma}(l) = [\gamma_1^{(l)} \dots \gamma_q^{(l)}]^T \tag{9}$$

B. Assumptions and Problem Statement

We assume the following:

- **A1**: $\mathbf{H}$ is full column rank.
- **A2**: The multipath coefficients, $\boldsymbol{\gamma}(l)$, are fixed within a snapshot, and may vary from one snapshot to another.
- **A3**: The number of multipath components q is known.
- **A4**: The vector $\mathbf{n}(l)$ is additive Gaussian noise of zero mean and variance $\sigma^2\mathbf{I}$, assumed to be white over space, frequencies, and symbols; we also assume that the noise is independent from the multipath coefficients.

Condition **A1** is valid as long as:

- **A1.1**: $q < MN$.
- **A1.2**: Let q^τ be the number of distinct ToAs, i.e. $\tau^1, \dots, \tau^{q^\tau}$; and let the following integers $P_1, \dots, P_{q^\tau}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\tau} P_i = q$. This condition states that $\max_i P_i < N$.
- **A1.3**: Similarly as **A1.2**, let q^θ be the number of distinct AoAs, i.e. $\theta^1, \dots, \theta^{q^\theta}$; and let the following integers $Q_1, \dots, Q_{q^\theta}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\theta} Q_i = q$. This condition states that $\max_i Q_i < M$.

Condition **A2** is a valid assumption since the time it takes for an indoor channel to change significantly is of the order of milliseconds [24], whereas the OFDM symbol duration of a snapshot T is of the order of microseconds.

Techniques for estimating the number of sources could be done through hypothesis testing [25] or via information theoretic criteria [26]. However, we assume knowledge of the number of sources, i.e. q is known.

Any further assumptions will be mentioned. Now, we address our problem: *Given* $\{\mathbf{x}(l)\}_{l=1}^L$ *and* q*, estimate the signal parameters* $\{(\theta_i, \tau_i)\}_{i=1}^q$.

III. A Deterministic ML estimator

In a deterministic approach, the signal parameters $\{(\theta_i, \tau_i)\}_{i=1}^q$ and multipath components $\{\boldsymbol{\gamma}(l)\}_{l=1}^L$ are not sample functions of random processes. Instead, these quantities are modelled as unknown deterministic sequences, and are jointly estimated through the criterion:

$$[\hat{\mathbf{H}}, \hat{\boldsymbol{\gamma}}(1), \dots, \hat{\boldsymbol{\gamma}}(L)] = \underset{\mathbf{H}, \boldsymbol{\gamma}(1), \dots, \boldsymbol{\gamma}(L)}{\arg\min} \sum_{l=1}^{L} \|\mathbf{x}(l) - \mathbf{H}\boldsymbol{\gamma}(l)\|^2 \tag{10}$$

Minimising with respect to $\{\boldsymbol{\gamma}(l)\}_{l=1}^L$, we obtain:

$$\hat{\boldsymbol{\gamma}}(l) = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{x}(l), \quad l = 1 \dots L \tag{11}$$

Treating $\{\boldsymbol{\gamma}(l)\}_{l=1}^L$ as nuissance parameters, we substitute its estimate obtained by (11) in (10) to get:

$$\hat{\mathbf{H}} = \underset{\mathbf{H}}{\arg\min} \sum_{l=1}^{L} \left\| \mathscr{P}_{\mathbf{H}}^{\perp}\mathbf{x}(l) \right\|^2 = \underset{\mathbf{H}}{\arg\min}\, \mathrm{tr}\left\{ \mathscr{P}_{\mathbf{H}}^{\perp}\hat{\mathbf{R}}_{xx} \right\} \tag{12}$$

where $\mathscr{P}_{\mathbf{H}}^{\perp} = \mathbf{I}_{MN} - \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ is the orthogonal projection onto the noise subspace. The matrix $\hat{\mathbf{R}}_{xx}$ is the sample covariance matrix obtained by $\hat{\mathbf{R}}_{xx} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}(l)^H$. Equation (12) represents the DML criteria.

IV. Parameterisation of the Noise Subspace

The Determisitic ML (DML) criterion in (12) is highly nonlinear, as it requires a $2q$-dimensional search, and its direct optimisation would require cumbersome optimisation techniques. The key to a computationally attractive solution of the DML problem is a parameterisation of the noise subspace, as done in this section. Consider the two following polynomials:

$$A(z) = \sum_{i=0}^{q} a_i z^{q-i} = \prod_{i=1}^{q}(z - z_{\tau_i}) \tag{13a}$$

and

$$B(z) = \sum_{i=0}^{q-1} b_i z^{q-1-i} = \sum_{i=1}^{q} z_{\theta_i} \prod_{k=1, k\neq i}^{q} \frac{(z - z_{\tau_k})}{(z_{\tau_i} - z_{\tau_k})} \tag{13b}$$

where $z_{\tau_i} = e^{-j2\pi\tau_i\Delta f}$ and $z_{\theta_i} = e^{-jd2\pi f_c sin(\theta_i)}$. Note that $A(z_{\tau_i}) = 0$ and $B(z_{\tau_i}) = z_{\theta_i}$. The coefficient $a_0 = 1$ so that $A(z)$ is monic. Furthermore, $\mathbf{W}(\mathbf{f})$ is a $\big((2N-1)(M-q) + N - 1\big) \times MN$ matrix given as

$$\mathbf{W}(\mathbf{f}) = \left[\begin{array}{c} \mathbf{I}_N \otimes \mathbf{A} \\ \hline [\mathbf{I}_{N-1}|\mathbf{0}] \otimes \mathbf{B} - [\mathbf{0}|\mathbf{I}_{N-1}] \otimes \mathbf{I}_{M,q-1} \end{array}\right] \tag{14}$$

where $\mathbf{A}$ is $(M-q) \times M$

$$\mathbf{A} = \begin{bmatrix} a_q & \cdots & a_1 & a_0 & & 0 \\ & \ddots & & \ddots & \ddots & \\ 0 & & a_q & \cdots & a_1 & a_0 \end{bmatrix} \tag{15a}$$

and $\mathbf{B}$ is $(M-q+1) \times M$

$$\mathbf{B} = \begin{bmatrix} b_{q-1} & \cdots & b_1 & b_0 & & 0 \\ & \ddots & & \ddots & \ddots & \\ 0 & & b_{q-1} & \cdots & b_1 & b_0 \end{bmatrix} \tag{15b}$$

Also, $\mathbf{f}$ is $2(q+1) \times 1$ given as

$$\mathbf{f}^T = \begin{bmatrix} \mathbf{a}^T & \mathbf{b}^T & 1 \end{bmatrix} \tag{15c}$$

$$\mathbf{a}^T = [a_0 \quad \cdots \quad a_q] \tag{15d}$$

$$\mathbf{b}^T = [b_0 \quad \cdots \quad b_{q-1}] \tag{15e}$$

Finally, $\mathbf{I}_{M,q-1}$ is $(M-q+1) \times M$ defined by

$$\mathbf{I}_{M,q-1} = [\mathbf{I}_{M-q+1}|\overbrace{\mathbf{0} \cdots \mathbf{0}}^{q-1}] \tag{15f}$$

Theorem: $\mathbf{W}(\mathbf{f})$ has row rank $MN - q$ if $q \leq \frac{M+1}{2}$ and $\mathbf{H}$ has full column rank.

Proof: See [14].

Under assumption **A1** and $q \leq \frac{M+1}{2}$, the rows of the matrix $\mathbf{W}(\mathbf{f})$ (equivalently, the columns of $\mathbf{W}^H(\mathbf{f})$) span the noise subspace, i.e. $\mathbf{W}(\mathbf{f})\mathbf{H} = \mathbf{0}$ and thus we can write $\mathscr{P}_{\mathbf{H}}^{\perp} = \mathscr{P}_{\mathbf{W}^H(\mathbf{f})}$.

Note that this parameterisation resolves maximally $\frac{M+1}{2}$ paths. It is worth mentioning that if $N > M$, one would want to resolve $\frac{N+1}{2}$ paths (and not $\frac{M+1}{2}$ paths), so a simple modification of the model in (5) is done by interchanging $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ in (8), then constructing matrices $\mathbf{A}$ and $\mathbf{B}$ (equivalently, the polynomials $A(z)$ and $B(z)$) of N and $N-1$ coefficients, respectively. In general, we could find a noise parameterisation that could allow the resolvability of $\frac{max(M,N)+1}{2}$.

V. 2D-Iterative Quadratic ML (2D-IQML)

We rewrite the DML cost function in (12) as follows

$$\begin{aligned}\hat{\mathbf{H}} &= \arg\min_{\mathbf{H}} \sum_{l=1}^{L} \left\| \mathscr{P}_{\mathbf{W}^H(\mathbf{f})}\mathbf{x}(l) \right\|^2 \\ &= \arg\min_{\mathbf{H}} \mathrm{tr}\Big\{ \mathscr{P}_{\mathbf{H}}^{\perp}\hat{\mathbf{R}}_{xx} \Big\} \\ &= \arg\min_{\mathbf{f}} \sum_{l=1}^{L} \mathbf{x}^H(l)\mathbf{W}^H(\mathbf{f})\Big(\mathbf{W}(\mathbf{f})\mathbf{W}^H(\mathbf{f})\Big)^{\dagger}\mathbf{W}(\mathbf{f})\mathbf{x}(l)\end{aligned} \tag{16}$$

where the Moore-Penrose pseudoinverse has to be introduced since $\mathbf{W}(\mathbf{f})\mathbf{W}^H(\mathbf{f})$ is singular for $q < \frac{M+1}{2}$, and non-singular for $q = \frac{M+1}{2}$ if M is odd. Note that $\mathbf{W}(\mathbf{f})\mathbf{x}(l) = \mathcal{X}_l\mathbf{f}$, where $\mathcal{X}_l$ is an $((2N-1)(M-q)+N-1) \times (2q+2)$ matrix formed of elements of $\mathbf{x}(l)$. Finally, (16) boils down to the following

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{f}} \mathbf{f}^H\mathcal{Q}\mathbf{f} \tag{17a}$$

where

$$\mathcal{Q} = \sum_{l=1}^{L} \mathcal{X}_l^H\Big(\mathbf{W}(\mathbf{f})\mathbf{W}^H(\mathbf{f})\Big)^{\dagger}\mathcal{X}_l \tag{17b}$$

The cost function in (17) could be solved in an iterative fashion as

$$\hat{\mathbf{f}}_{(n)} = \arg\min_{\mathbf{f}} \mathbf{f}^H\mathcal{Q}_{(n-1)}\mathbf{f} \tag{18a}$$

where

$$\mathcal{Q}_{(n-1)} = \sum_{l=1}^{L} \mathcal{X}_l^H\Big(\mathbf{W}(\hat{\mathbf{f}}_{(n-1)})\mathbf{W}^H(\hat{\mathbf{f}}_{(n-1)})\Big)^{\dagger}\mathcal{X}_l \tag{18b}$$

The vector $\hat{\mathbf{f}}_{(n)}$ is the estimated vector of $\mathbf{f}$ at iteration (n). A good initialisation would be to set $\mathbf{W}(\hat{\mathbf{f}}_{(0)})\mathbf{W}^H(\hat{\mathbf{f}}_{(0)}) = \mathbf{I}$. If the constraint $\mathbf{e}_1^T\mathbf{f} = 1$ was posed to solve (18a), then at any iteration (n), the vector $\hat{\mathbf{f}}_{(n)}$ would estimate the coefficients in $\mathbf{a} = [a_o \dots a_q]^T$ properly, but the rest of its entries corresponding to the coefficients in $\mathbf{b} = [b_o \dots b_{q-1}]^T$ would be zero because there is no constraint posed on $\mathbf{f}$ in order to take the structure of $\mathbf{b} = [b_o \dots b_{q-1}]^T$ into account.
To cope with the aforementioned issue, we add the contraint $(\mathbf{J}\mathbf{e}_1)^T\mathbf{f} = 1$. Note that this constraint is reasonable since, indeed, the last entry of $\mathbf{f}$ is 1. In short, we aim to solve (18) subject to:

$$\mathbf{e}_1^T\mathbf{f} = 1 \tag{19a}$$

and

$$\mathbf{e}_1^T\mathbf{J}\mathbf{f} = 1 \tag{19b}$$

We write the Lagrangian function as

$$L(\mathbf{f}, \mu_1, \mu_2) = \mathbf{f}^H\mathcal{Q}_{(n-1)}\mathbf{f} - \mu_1(\mathbf{e}_1^T\mathbf{f} - 1) - \mu_2(\mathbf{e}_1^T\mathbf{J}\mathbf{f} - 1) \tag{20}$$

where μ_1 and μ_2 are constants. Setting the derivative of $L(\mathbf{f}, \mu_1, \mu_2)$ with respect to $\mathbf{f}$ to 0, we get

$$\frac{\partial}{\partial \mathbf{f}}L(\mathbf{f}, \mu_1, \mu_2) = 2\mathcal{Q}_{(n-1)}\mathbf{f} - \mu_1\mathbf{e}_1 - \mu_2\mathbf{J}\mathbf{e}_1 = 0 \tag{21}$$

So, with some straightforward manipulations, we have

$$\mathbf{f} = \mu_1^{'}\mathcal{Q}_{(n-1)}^{-1}\mathbf{e}_1 + \mu_2^{'}\mathcal{Q}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1 \tag{22}$$

where $\mu_i^{'} = \frac{\mu_i}{2}$. Plugging (22) in (19a) and (19b), we have the following set of equations

$$\begin{bmatrix} \alpha & \gamma \\ \gamma^* & \beta \end{bmatrix} \begin{bmatrix} \mu_1^{'} \\ \mu_2^{'} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{23}$$

where α, β, and γ are given as:

$$\alpha = \mathbf{e}_1^T\mathcal{Q}_{(n-1)}^{-1}\mathbf{e}_1 \tag{24a}$$

$$\beta = \mathbf{e}_1^T\mathbf{J}\mathcal{Q}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1 \tag{24b}$$

$$\gamma = \mathbf{e}_1^T\mathcal{Q}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1 \tag{24c}$$

Finally, solving (23) with respect to $\mu_1^{'}$ and $\mu_2^{'}$, we get the following

$$\hat{\mathbf{f}}_{(n)} = \frac{(\beta - \gamma)\mathcal{Q}_{(n-1)}^{-1}\mathbf{e}_1 + (\alpha - \gamma^*)\mathcal{Q}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1}{\alpha\beta - |\gamma|^2} \tag{25}$$

The 2D-IQML could be summarised as follows:

- Step1. Given $\Big\{\mathbf{x}(l)\Big\}_{l=1}^{L}$, form $\Big\{\mathcal{X}_l\Big\}_{l=1}^{L}$.
- Step2. Initialise $\mathcal{Q}_{(0)} = \sum_{l=1}^{L}\mathcal{X}_l^H\mathcal{X}_l$.
- Step3. Iterate over (n) to compute $\hat{\mathbf{f}}_{(n)}$, using (24) and (25). Stop when $\|\hat{\mathbf{f}}_{(n)} - \hat{\mathbf{f}}_{(n-1)}\| < \xi$ (Pre-defined Threshold).
- Step4. Form the polynomials $A(z)$ and $B(z)$ using the estimate of $\hat{\mathbf{f}}_{(n)}$ obtained in the last iteration of *Step3* and equations (13), (15c), (15d), (15e).
- Step5. Find the q roots of $A(z_{\hat{\tau}_i}) = 0$, which give estimates of the ToAs as $\Big\{z_{\hat{\tau}_i} = e^{-j2\pi\hat{\tau}_i\Delta_f}\Big\}_{i=1}^{q}$.
- Step6. Compute $B(z_{\hat{\tau}_i}) = z_{\hat{\theta}_i}$, which give estimates of the q AoAs as $\Big\{z_{\hat{\theta}_i} = e^{-jd2\pi f_c sin(\hat{\theta}_i)}\Big\}_{i=1}^{q}$.

The first iteration of 2D-IQML could be seen as a Subchannel Response Matching (SRM) [27]. Note that, in a first iteration of 2D-IQML, we minimise:

$$\frac{1}{L}\sum_{l=1}^{L}\mathbf{f}^H\mathcal{X}_l^H\mathcal{X}_l\mathbf{f} \simeq E_l\Big\{\mathbf{f}^H\mathcal{X}_l^H\mathcal{X}_l\mathbf{f}\Big\} = E_l\Big\{\mathbf{f}^H\mathcal{G}_l^H\mathcal{G}_l\mathbf{f}\Big\} + \sigma^2\mathrm{tr}\Big\{\mathbf{W}^H(\mathbf{f})\mathbf{W}(\mathbf{f})\Big\} \tag{26}$$

where $\mathbf{g}(l) = \mathbf{H}\gamma(l)$ and $\mathbf{W}(\mathbf{f})\mathbf{g}(l) = \mathcal{G}_l\mathbf{f}$, with $\mathcal{G}_l$ being a matrix formed by elements of $\mathbf{g}(l)$. (26) tells us that a balanced $\mathbf{f}$ yields asymptotically unbiased and consistent estimates, whereas unbalanced $\mathbf{f}$ yield biased and inconsistent estimates. One should also note that different parameterisations of the noise subspace give different estimates of $\mathbf{f}$. This initialisation could be seen as a non-weighted version of 2D-IQML. Furthermore, it is easy to see that the optimal value of $\mathbf{f}$, denoted hereby $\mathbf{f}^o$, is the one that nulls $E_l\{\mathbf{f}^H\mathcal{G}_l^H\mathcal{G}_l\mathbf{f}\}$. Therefore, in a noiseless scenario, a first iteration of 2D-IQML gives the true value $\mathbf{f}^o$. In general, at sufficiently high SNR, 2D-IQML performs well; however, at low SNR, the 2D-IQML estimate is biased. Indeed, consider the asymptotic situation in which the number of subcarriers M grow to infinity. By the law of large numbers, the 2D-IQML criterion becomes essentially equivalent to its expected value, viz.

$$\begin{aligned}&\frac{1}{M}\mathbf{f}^H\mathcal{X}_l^H\mathcal{R}^\dagger\mathcal{X}_l\mathbf{f}\\ &= \mathrm{tr}\Big\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger\mathbf{W}(\mathbf{f})E\{\mathbf{x}(l)\mathbf{x}^H(l)\}\Big\} + \mathcal{O}(\frac{1}{\sqrt{M}})\\ &= \frac{1}{M}\mathbf{f}^H\mathcal{G}_l^H\mathcal{R}^\dagger\mathcal{G}_l\mathbf{f} + \frac{\sigma^2}{M}\mathrm{tr}\Big\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger\mathbf{W}(\mathbf{f})\Big\} + \mathcal{O}(\frac{1}{\sqrt{M}})\end{aligned} \tag{27}$$

where $\mathcal{R} \triangleq \mathcal{R}(\mathbf{f}) = \mathbf{W}(\mathbf{f})\mathbf{W}^H(\mathbf{f})$.
Recall that the minimiser of $\mathbf{f}^H\mathcal{G}_l^H\mathcal{R}^\dagger\mathcal{G}_l\mathbf{f}$ is $\mathbf{f}^o$. Therefore, at high SNR, the 2D-IQML estimate $\mathbf{f}$ differs from the optimal $\mathbf{f}^o$ by an asymptotically vanishing estimation error, because $\frac{\sigma^2}{M}\mathrm{tr}\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger\mathbf{W}(\mathbf{f})\}$ is negligible. However, this is not the case at low SNR, simply because $\mathbf{f}^o$ is not the minimiser of $\frac{\sigma^2}{M}\mathrm{tr}\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger\mathbf{W}(\mathbf{f})\}$, even if $\mathcal{R} \triangleq \mathcal{R}(\mathbf{f}^o)$. More explicitely,

$$\begin{aligned}&\min_{\mathbf{f}}\Big\{\mathrm{tr}\{\mathbf{W}^H(\mathbf{f})\mathcal{R}(\mathbf{f}^o)^\dagger\mathbf{W}(\mathbf{f})\}\Big\}\\ &< \mathrm{tr}\Big\{\mathscr{P}_{\mathbf{W}^H(\mathbf{f}^o)}\Big\} = MN - q\end{aligned} \tag{28}$$

Finally, we can say from (28) that $\frac{\sigma^2}{M}\mathrm{tr}\Big\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger\mathbf{W}(\mathbf{f})\Big\}$ is minimised at $\mathbf{f}^1 \neq \mathbf{f}^o$, so the 2D-IQML criteria is minimised at $\mathbf{f}^2 \neq \mathbf{f}^o$. Hence, due to presence of noise, $\mathbf{f}^o$ is not asymptotically near a stationary point of the algorithm and 2D-IQML performs poorly for any initialisation.

We propose here a method to "denoise" the 2D-IQML criterion in a sense that it will correct the 2D-IQML bias and provide a consistent esimate of the vector $\mathbf{f}$.

VI. 2D-Denoised IQML (2D-DIQML)

A. Asymptotic Number of Subcarriers (Large M)

The asymptotic noise contribution to the DML criterion is $\sigma^2\mathrm{tr}\{\mathscr{P}_{\mathbf{W}^H(\mathbf{f})}\}$ (see (27)). The denoising strategy consists of removing this asymptotic noise term, or more precisely, an estimate of it i.e. $\hat{\sigma}^2\mathrm{tr}\{\mathscr{P}_{\mathbf{W}^H(\mathbf{f})}\}$ from the DML criterion, which becomes

$$\begin{aligned}&\min_{\mathbf{f}}\sum_{l=1}^{L}\left\{\mathrm{tr}\Big\{\mathscr{P}_{\mathbf{W}^H(\mathbf{f})}\big(\mathbf{x}(l)\mathbf{x}^H(l) - \hat{\sigma}^2\mathbf{I}_{MN}\big)\Big\}\right\} \Leftrightarrow\\ &\min_{\mathbf{f}}\sum_{l=1}^{L}\left\{\mathbf{f}^H\mathcal{X}_l^H\mathcal{R}^\dagger(\mathbf{f})\mathcal{X}_l\mathbf{f} - \hat{\sigma}^2\mathrm{tr}\Big\{\mathbf{W}^H(\mathbf{f})\mathcal{R}^\dagger(\mathbf{f})\mathbf{W}(\mathbf{f})\Big\}\right\}\end{aligned} \tag{29}$$

subject to (19a) and (19b).
Note that this operation does not change the optimizer of the DML criterion as $\hat{\sigma}^2\mathrm{tr}\{\mathscr{P}_{\mathbf{W}^H(\mathbf{f})}\} = \hat{\sigma}^2(MN - q)$ is constant with respect to $\mathbf{f}$. We take $\hat{\sigma}^2$ to be a consistent estimate of the noise variance. The denoised DML criterion is now solved in the 2D-IQML way, i.e.

$$\hat{\mathbf{f}}_{(n)} = \arg\min_{\mathbf{f}}\mathbf{f}^H\Big\{\mathcal{Q}_{(n-1)} - \hat{\sigma}^2\mathcal{D}\Big\}\mathbf{f} \tag{30}$$

subject to (19a) and (19b).
The matrix $\mathcal{D}$ is such that $\mathbf{f}^{''H}\mathcal{D}\mathbf{f}^{'} = \mathrm{tr}\big\{\mathbf{W}^H(\mathbf{f}^{''})\mathcal{R}^\dagger(\mathbf{f})\mathbf{W}(\mathbf{f}^{'})\big\}$. Asymptotically in the number of subcarriers, 2D-DIQML is globally convergent. Indeed, asymptotically it is essentially equivalent to the denoised criterion

$$\frac{1}{M}\mathbf{f}^H\Big\{\mathcal{Q}_{(n-1)} - \hat{\sigma}^2\mathcal{D}\Big\}\mathbf{f} = \frac{1}{M}\mathbf{f}^H\mathcal{G}_l^H\mathcal{R}^\dagger\mathcal{G}_l\mathbf{f} + \mathcal{O}(\frac{1}{\sqrt{M}}) \tag{31}$$

if $\sigma^2 - \hat{\sigma}^2 = \mathcal{O}(\frac{1}{\sqrt{M}})$. Notice, again, that the $\mathbf{f}^o$ minimises the first term on the right hand side of (31). Therefore, one iteration of 2D-DIQML yields an estimate of the form $\hat{\mathbf{f}} = \rho\mathbf{f}^o + \mathcal{O}(\frac{1}{\sqrt{M}})$, for some scaling factor ρ. So, *the 2D-DIQML algorithm behaves asymptotically at any SNR as the 2D-IQML algorithm behaves at high SNR.*

B. Finite Number of Subcarriers

The choice of $\hat{\sigma}^2$ turns out to be crucial. In practice, with large but finite number of subcarriers M, and the true noise variance, the central matrix $\mathcal{Q} - \sigma^2\mathcal{D}$ in (30) is indefinite, thus the minimisation problem is no longer well posed. Simulations show that the performance of 2D-DIQML in that case is very poor. The central matrix $\mathcal{Q} - \hat{\sigma}^2\mathcal{D}$ should be constrained to be positive semi-definite.

For the consistent estimate of σ^2, we choose here a certain λ that renders $\mathcal{Q} - \lambda\mathcal{D}$ exactly positive semi-definite with one singularity. The 2D-DIQML criterion becomes

$$\hat{\mathbf{f}}_{(n)} = \arg\min_{\mathbf{f},\lambda}\mathbf{f}^H\Big\{\mathcal{Q}_{(n-1)} - \lambda\mathcal{D}\Big\}\mathbf{f} \tag{32}$$

subject to (19a), (19b), and $\mathcal{Q}_{(n-1)} - \lambda\mathcal{D}$ being positive semi-definite.
The solution of λ is $\lambda = \lambda_{min}\Big(\mathcal{Q}_{(n-1)}, \mathcal{D}\Big)$, the minimal generalised eigenvalue of $\mathcal{Q}_{(n-1)}$ and $\mathcal{D}$. After solving for λ, we get $\mathbf{f}$ at iteration (n) as

$$\hat{\mathbf{f}}_{(n)} = \frac{(\beta^{'} - \gamma^{'})\mathcal{S}_{(n-1)}^{-1}\mathbf{e}_1 + (\alpha^{'} - \gamma^{'*})\mathcal{S}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1}{\alpha^{'}\beta^{'} - |\gamma^{'}|^2} \tag{33}$$

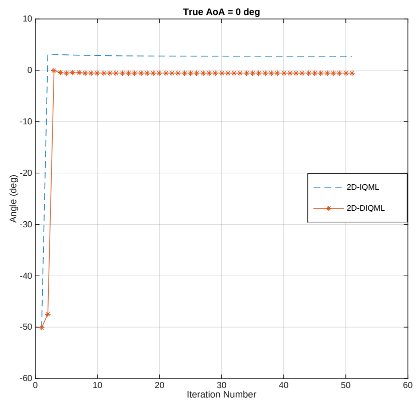

Fig. 1. 2D-IQML vs. 2D-DIQML on AoA estimation of 1st Path, where true AoA = 0 deg at SNR = -5dB

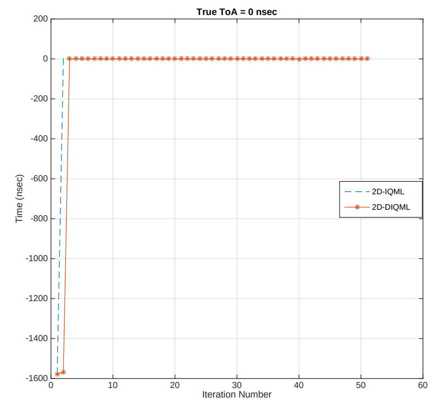

Fig. 2. 2D-IQML vs. 2D-DIQML on ToA estimation of 1st Path, where true ToA = 0 nsec at SNR = -5dB

where

$$\mathcal{S}_{(n-1)} = \mathcal{Q}_{(n-1)} - \lambda\mathcal{D} \tag{34a}$$

$$\alpha^{'} = \mathbf{e}_1^T \mathcal{S}_{(n-1)}^{-1}\mathbf{e}_1 \tag{34b}$$

$$\beta^{'} = \mathbf{e}_1^T \mathbf{J}\mathcal{S}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1 \tag{34c}$$

$$\gamma^{'} = \mathbf{e}_1^T \mathcal{S}_{(n-1)}^{-1}\mathbf{J}\mathbf{e}_1 \tag{34d}$$

Asymptotically, 2D-DIQML could becomes

$$\begin{aligned} &\frac{1}{M}\mathbf{f}^H\big(\mathcal{X}_l^H\mathcal{R}^{\dagger}\mathcal{X}_l - \lambda\mathcal{D}\big)\mathbf{f} \\ &= \frac{1}{M}\mathbf{f}^H\mathcal{G}_l^H\mathcal{R}^{\dagger}\mathcal{G}_l\mathbf{f} + \frac{1}{M}\big(\sigma^2-\lambda\big)\mathbf{f}^H\mathcal{D}\mathbf{f} + \mathcal{O}(\frac{1}{\sqrt{M}}) \end{aligned} \tag{35}$$

Notice that, first, optimisation with repect to λ subject to the non-negativity constraint would give $\lambda = \sigma^2 + \mathcal{O}(\frac{1}{\sqrt{M}})$, regardless of any initialisation of $\mathbf{f}$. Hence, λ asymptotically nulls the noise contribution, and the optimal value of $\mathbf{f}$ is $\mathbf{f}^o$. Therefore, global convergence applies for $\mathbf{f}$ (to $\mathbf{f}^o$) and λ (to σ^2).

VII. Simulation Results

We have observed that, indeed, *the 2D-DIQML algorithm behaves asymptotically at any SNR as the 2D-IQML algorithm behaves at high SNR*. To that extent, we fix the following simulation parameters:

- $M = 64$ (Large M) subcarriers and $N = 3$ antennas.
- $\triangle_f$= 0.3125MHz and $d = \frac{\lambda}{2}$
- $q = 2$ coherent paths with:
    1) AoAs: $\theta_1 = 0$ and $\theta_2 = 30$ degrees.
    2) ToAs: $\tau_1 = 0$ and $\tau_2 = 100$ nsecs.
- $L = 10$ snapshots.
- $SNR = -5$ dB (Low SNR).

At high SNR, both algorithms perform equally the same, i.e. both give unbiased estimates of ToA/AoAs. Therefore, we have excluded this case from simulations. Nevertheless, it is

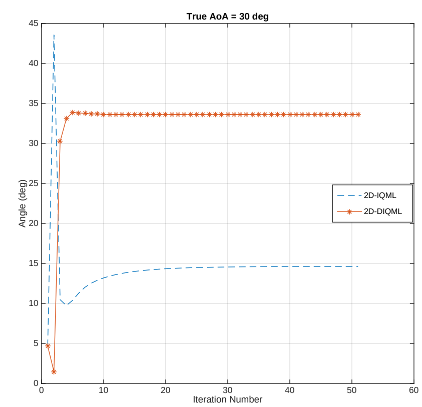

Fig. 3. 2D-IQML vs. 2D-DIQML on AoA estimation of 2nd Path, where true AoA = 30 deg at SNR = -5dB

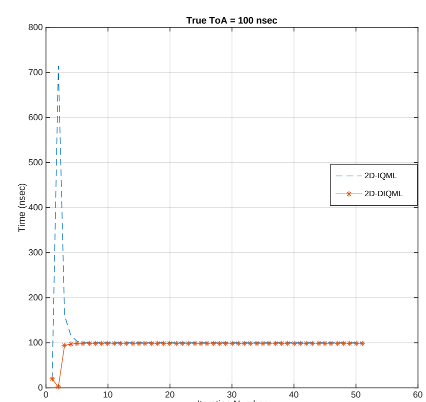

Fig. 4. 2D-IQML vs. 2D-DIQML on ToA estimation of 2nd Path, where true ToA = 100 deg at SNR = -5dB

of vast interest to see how both algorithms perform at low SNR and with a large number of subcarriers. As one can see, the estimated ToAs of both algorithms converge to the true ToA value (see figures 2 and 4). However, 2D-IQML AoA estimates are much more biased compared to 2D-DIQML AoA estimates. Indeed, as one could observe in figure 1, the AoA of the first path which was set to be 0 degrees, was estimated to be 4 degrees by 2D-IQML and 0 degrees by 2D-DIQML. Also, by taking a look at figure 3, the AoA of the second path which was set to be 30 degrees, was estimated to be 15 degrees by 2D-IQML and 33 degrees by 2D-DIQML. Finally, we can say that, at low SNR and high number of subcarriers, the 2D-IQML estimates are biased compared to the 2D-DIQML estimates.

VIII. Conclusion

We have presented two techniques to solve the highly nonlinear DML algorithm for joint times and angles of arrival: 2D-IQML and 2D-DIQML. Asymptotic performance analysis of both techniques were provided. It has been shown that 2D-IQML gives biased estimates of ToA/AoA and performs poorly at low SNR due to noise. An original "denoising" strategy is proposed, which constrains the Hessian of the cost function to be positive semi-definite. This "denoising" strategy is called 2D-DIQML that has been shown to be globally convergent. Furthermore, 2D-DIQML outperforms 2D-IQML because the former behaves asymptotically at any SNR as the latter behaves at high SNR. Finally, for localisation purposes, joint AoA and ToA information could be used to form a database, where a mapping is done between ToA/AoA vectors and location. Then, this database could be used in an online stage, where joint AoA/ToA estimation is done using the proposed algorithms, followed by a matching criteria that finds the best match in the database to obtain an estimate of the location of a wireless transmitter.

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