ON THE EFFECT OF RANDOM SNAPSHOT TIMING JITTER ON THE COVARIANCE MATRIX FOR JADE ESTIMATION

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ABSTRACT

In this paper, we focus on joint multipath angle and delay estimation (JADE) in an OFDM communication setting. We analyse the effect of Gaussian random snapshot (OFDM symbol) timing jitter on the spatio-frequency sample covariance matrix containing delay and direction information. This sample covariance matrix is an input to the JADE and many other algorithms for signal parameter estimation. The analysis suggests a simple way to compensate for the jitter in the sample covariance matrix. We also present two simple methods for estimating the jitter variance, allowing its compensation. These techniques attempt to restore the low rank nature or other structure in the signal contribution. We then finally present some simulations for the resulting estimation quality of the multipath delays (ToAs) and angles (AoAs) of the incoming signals.

Index Terms— *Snapshot timing jitter, perturbed sample covariance matrix, JADE, ToA , DoA, matrix rank minimization*

1. INTRODUCTION

Localisation has been one challenging topic over the past 60 years. In fact, many techniques have been developed in order to reliably position a wireless emitter. The first classical approach involves estimating the angle-of-arrival (AoA), received signal strenth (RSS), time-of-arrival (ToA), time-difference-of-arrival (TDoA), phase-of-arrival (PoA), etc.., of an emitter with respect to multiple base stations, in order to localise through triangulation or trilateration methods [1]. In favor of estimating signal parameters (i.e. AoAs, ToAs, etc..), one of the first algorithms proposed was Maximum-Likelihood (ML) [2], which is computationally exhaustive as it requires a pq-dimensional search, (q being the number of sources and p being the number of signal parameters of interest). Nevertheless, subspace algorithms were proposed to cope with the aforementioned issue, such as MUSIC [3] and JADE [4]. These algorithms are much simpler and less complex than ML, as they require a p-dimensional search. Recently, fingerprinting techniques are finding their way through commercial use, where signal parameters are measured and stored, for each location, in an offline phase, and are readily used in the online phase via a matching criteria to lookup the database and match the received signal with its corresponding location. Wax *et al.* [5] introduced a fingerprinting technique based on multipath characteristic subspaces containing ToA and AoA information, whereas Oktem and Slock [6] have made use of other multipath characteristics such as Power, ToA, and Doppler. In any of the above mentioned techniques, the sample covariance matrix of the recieved signal is the key essence towards estimating signal parameters.

In this paper, we address a practical issue faced by Wi-Fi 802.11 a/b/g/n systems, where received OFDM symbols are sampled, and sent to a DFT bank. We strongly note that the sampler (ADC) is not continuously ON, due to power and other constraints imposed by the 802.11 standard. Therefore, there is no guarantee that all symbols are sampled at the same starting instant, so it should be seen as symbol-varying. This problem is to be distinguished from the one mentioned in [7], where authors study the effect of sampling jitter on one symbol, i.e. each sample collected at the output of the ADC is randomly shifted from it's nominal sampling instant due to noise caused by ADC and other equipments. In this paper, we call *snapshot timing jitter*, or simply *jitter* for short, a random timing shift introduced to all the samples collected from a certain symbol (or snapshot). We study this effect on the sample spatio-frequency covariance matrix containing ToA and AoA information and try to compensate for this jitter effect, in the presence of multipath. After jitter compensation, one can apply JADE (or 2D-MUSIC) so as to efficiently estimate the number of incoming multipath components and their respective ToAs and AoAs.

As will be clarified throughout the paper, the jitter perturbs the sample covariance matrix in a sense that the rank of this matrix (which is a good estimate of the number of incoming signals) increases. This rank-excess problem is, in fact, one problem in compressed sensing, where the true signal subspace is sparsely contained in the subspace of the covariance matrix. Such problems are tackled through optimisation problems, where the objective function is to minimise the rank of the matrix under some affine constraints. Unfortunately, solutions to this problem are quite complex and are known to be NP-hard [8]. We shall not proceed in that direction, but instead, make use of the signal structure and correlation sequences to propose simpler solutions.

The paper is divided as follows: Section 2 presents the system model. In Section 3, we introduce a function to analyse the effect of the jitter. We propose , in Section 4, two jitter estimation algorithms, which allow its compensation from the sample covariance matrix. In Section 5, we present our simulation results and conclude in Section 6.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. $E\{.\}$ is the statistical expectation. $\otimes$ and $\odot$ are the *Kronecker* and *Hadamard* products, respectively. For any $M \times M$ matrix $\mathbf{X}$, $vec(\mathbf{X})$ is the vector operator which returns an $M^2 \times 1$ vector by stacking the columns of $\mathbf{X}$, starting from the first to the last column, $\|\mathbf{X}\|_2^2$ is the *Frobenius* norm of $\mathbf{X}$, and $\mathbf{X}^{\langle i,j\rangle}$ is the $(i,j)^{th}$ entry of $\mathbf{X}$. $|z|$ is the magnitude of $z \in \mathbb{C}$.

2. SYSTEM MODEL

Consider an OFDM symbol $s(t)$ composed of M subcarriers and centered at a carrier frequency f_c, impinging an antenna array of N antennas via q multipath components, each arriving at different AoAs $\{\theta_i\}_{i=1}^{q}$ and ToAs $\{\tau_i\}_{i=1}^{q}$. In baseband, we could write the l^{th} received OFDM symbol at the n^{th} antenna as:

$$r_n^{(l)}(t) = \sum_{i=1}^{q} \gamma_i^{(l)} a_n(\theta_i) s(t-\tau_i) + n_n^{(l)}(t) \quad (1)$$

where

$$s(t) = \begin{cases} \sum_{m=0}^{M-1} b_m e^{j2\pi m \triangle_f t} & \text{if } t \in [0,T] \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

where $T = \frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, b_m is the modulated symbol onto the m^{th} subcarrier, $a_n(\theta)$ is the n^{th} antenna response to an incoming signal at angle θ. The form of $a_n(\theta)$ depends on the array geometry. $\gamma_i^{(l)}$ is the complex coefficient of the i^{th} multipath component; Note that under the slow fading assumption, $\gamma_i^{(l)}$ changes from symbol to symbol and not within a symbol, thus the superscript (l) without any time dependency. $n_n^{(l)}(t)$ is additive Gaussian noise of zero mean and variance σ^2, assumed to be white over space, time, and symbols; we also assume that the noise is independent from the signal and the multipath coefficients.

We are now ready to address the *jitter* issue. Plugging (2) in (1) and sampling $r_n^{(l)}(t)$ at regular intervals of $k \triangleq k\frac{T}{M} + \delta^{(l)} + t_0$, we get $r_{n,k}^{(l)} \triangleq r_n^{(l)}(k\frac{T}{M} + \delta^{(l)} + t_0)$ as:

$$r_{n,k}^{(l)} = \sum_{i=1}^{q} \sum_{m=0}^{M-1} b_m e^{j2\pi\frac{km}{M}} e^{j2\pi m \triangle_f(\delta^{(l)}+t_0-\tau_i)} \gamma_i^{(l)} a_n(\theta_i) + n_{n,k}^{(l)} \quad (3)$$

Collecting M samples, we can apply an M-point DFT, so observing the m^{th} subcarrier at the n^{th} antenna, and denoting $\overline{\delta}^{(l)} = t_0 + \delta^{(l)}$ we get:

$$\begin{aligned} R_{n,m}^{(l)} &= \sum_{k=0}^{M-1} r_{n,k}^{(l)} e^{-j2\pi m\frac{k}{M}} \\ &= b_m e^{j2\pi m \triangle_f \overline{\delta}^{(l)}} \sum_{i=1}^{q} \gamma_i^{(l)} a_n(\theta_i) e^{-j2\pi m \triangle_f \tau_i} + N_{n,m}^{(l)} \end{aligned} \quad (4)$$

We claim that the transmitted OFDM symbol $s(t)$ is a preamble field of the Wi-Fi 802.11 frame, thus prior knowledge of the modulated symbols $\{b_m\}_{m=0}^{M-1}$ is a valid assupmtion, since this stream of symbols (each at it's corresponding sub-carrier) are repeated in each OFDM symbol placed at the beginning of the Wi-Fi frame for channel estimation and frequency offset purposes. Therefore, at each OFDM symbol reception, we compensate for all such symbols (multiplying by $\frac{b_m^*}{|b_m|^2}$) and hence omit b_m from (4). Re-writing (4) in a compact matrix form, we have:

$$\tilde{\mathbf{x}}^{(l)} = \mathbf{C}^{(l)} \mathbf{A} \boldsymbol{\gamma}^{(l)} + \mathbf{n}^{(l)}, l = 1 \dots L \quad (5)$$

where $\tilde{\mathbf{x}}^{(l)}$ and $\mathbf{n}^{(l)}$ are $MN \times 1$ vectors

$$\tilde{\mathbf{x}}^{(l)} = vec\{\mathbf{R}\}, \qquad \mathbf{R}^{\langle m,n\rangle} = R_{n,m}^{(l)} \quad (6)$$

$$\mathbf{n}^{(l)} = vec\{\mathbf{N}\}, \qquad \mathbf{N}^{\langle m,n\rangle} = N_{n,m}^{(l)} \quad (7)$$

$\mathbf{A}$ is an $MN \times q$ matrix given as

$$\mathbf{A} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1) \dots \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)] \quad (8)$$

where $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ are $N \times 1$ and $M \times 1$, respectively. The n^{th} entry of $\mathbf{a}(\theta)$, denoted $\mathbf{a}_n(\theta)$, is the response of the n^{th} antenna to a signal arriving at angle θ with respect to the antenna array. Similarly, the m^{th} entry of $\mathbf{c}(\tau)$, denoted $\mathbf{c}_m(\tau) = e^{-j2\pi\tau(m-1)\triangle_f}$, is the response of the m^{th} subcarrier to a signal arriving with time delay τ. The $q \times 1$ vector $\boldsymbol{\gamma}^{(l)}$ is composed of the multipath coefficients

$$\boldsymbol{\gamma}^{(l)} = [\gamma_1^{(l)} \dots \gamma_q^{(l)}]^T \quad (9)$$

$\mathbf{C}^{(l)}$ is an $MN \times MN$ diagonal matrix, which can be seen as a function of the block jitter, i.e. $\delta^{(l)}$ and is given by

$$\mathbf{C}^{(l)} = \mathbf{I}_N \otimes diag\{\mathbf{c}(\overline{\delta}^{(l)})\} \quad (10)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix.
We shall also assume a perfect model with no snapshot jitter effect

$$\mathbf{x}^{(l)} = \mathbf{C}_0 \mathbf{A} \boldsymbol{\gamma}^{(l)} + \mathbf{n}^{(l)}, l = 1 \dots L \quad (11)$$

where $\mathbf{C}_0 = \mathbf{I}_N \otimes diag\{\mathbf{c}(t_0)\}$.
(5) is referred to as the perturbed model with respect to the perfect model (11). It is straightforward to see that in (11), the matrix $\mathbf{C}_0$ shifts all time delays by a constant value, t_0, thus relative delays of paths is still preserved. In the next section, we model $\delta^{(l)}$ to be a realisation of a prior known distribution, namely Gaussian, and observe the impact of this jitter on the sample covariance matrix.

3. EFFECT OF JITTER ON THE SAMPLE COVARIANCE MATRIX

In this section, we study the effect of the jitter which is modelled as a Gaussian random variable on the sample covariance matrix. The covariance matrix of $\tilde{\mathbf{x}}^{(l)}$ given in the perturbed model in (5) is referred to as the *perturbed covariance matrix* and is given by

$$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = E_l\{\tilde{\mathbf{x}}^{(l)} \tilde{\mathbf{x}}^{(l)^H}\} \quad (12)$$

where $E_l\{.\}$ is the expectation operator over (l). In what follows, we seek a relation between $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ and $\mathbf{R}_{\mathbf{xx}}$, where $\mathbf{R}_{\mathbf{xx}} = E_l\{\mathbf{x}^{(l)}\mathbf{x}^{(l)^H}\}$, and $\mathbf{x}^{(l)}$ is given by (11).
Recall that the noise $\mathbf{n}^{(l)}$ is of zero mean, covariance $\sigma^2 \mathbf{I}_{MN}$, and independent from $\mathbf{C}^{(l)}$, $\mathbf{A}$, and $\boldsymbol{\gamma}^{(l)}$, thus it is easy to show that

$$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = E_l\{\mathbf{C}^{(l)} \mathbf{A} \boldsymbol{\gamma}^{(l)} \boldsymbol{\gamma}^{(l)^H} \mathbf{A}^H \mathbf{C}^{(l)^H}\} + \sigma^2 \mathbf{I}_{MN} \quad (13)$$

which could also be written as

$$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = E_l\{\sum_{i=1}^{q}\sum_{j=1}^{q} \mathbf{C}^{(l)} \mathbf{A}^{\langle :,i\rangle} \gamma_i^{(l)} \gamma_j^{*(l)} \mathbf{A}^{\langle :,j\rangle^H} \mathbf{C}^{(l)^H}\} + \sigma^2 \mathbf{I}_{MN} \quad (14)$$

where $\mathbf{A}^{\langle :,i\rangle} = [\mathbf{a}(\theta_i) \otimes \mathbf{c}(\tau_i)]$ is the i^{th} column of $\mathbf{A}$. Using the linearity property of the expectation operator and assuming that the jitter $\overline{\delta}^{(l)}$ is independent from multipath $\boldsymbol{\gamma}^{(l)}$, the $(m,n)^{th}$ entry of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, i.e. $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}{}^{\langle m,n\rangle}$, is given by

$$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}{}^{\langle m,n\rangle} = E_l\{e^{-j2\pi\triangle_f(m_1-n_1)\delta}\}.\mathbf{R}_{\mathbf{xx}}{}^{\langle m,n\rangle} \quad (15)$$

where

$$\mathbf{R_{xx}}^{\langle m,n\rangle} = \sum_{i=1}^{q}\sum_{j=1}^{q} \mathbf{R}_{\gamma\gamma}{}^{\langle i,j\rangle} c_{m_1}(\tau_i+t_0)c^*_{n_1}(\tau_j+t_0) \\ a_{m_2}(\theta_i)a^*_{n_2}(\theta_j) + \sigma^2 1_{(m,n)} \tag{16}$$

and

$$\mathbf{R}_{\gamma\gamma}{}^{\langle i,j\rangle} = E_l\{\gamma_i^{(l)}\gamma_j^{*(l)}\} \tag{17}$$

$1_{(m,n)}$ is the dirac-delta function which returns 1 if $m = n$ and zero otherwise. The indices m_1, n_1, m_2, and n_2 are given as

$$\begin{cases} m_1 = m - \lfloor \frac{m-1}{M} \rfloor M \\ n_1 = n - \lfloor \frac{n-1}{M} \rfloor M \\ m_2 = \lfloor \frac{m-1}{M} \rfloor + 1 \\ n_2 = \lfloor \frac{n-1}{M} \rfloor + 1 \end{cases} \tag{18}$$

$\lfloor x \rfloor$ denotes the floor operator of a real number x. Rewriting (17) is a compact form, we have

$$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = \mathbf{\Upsilon} \odot \mathbf{R_{xx}} \tag{19}$$

$\mathbf{\Upsilon}$ is an $MN \times MN$ matrix given by $\mathbf{\Upsilon} = \mathbf{J}_N \otimes \mathbf{T}$, where $\mathbf{J}_N$ is the all-ones square matrix of size N and $\mathbf{T}$ is an $M \times M$ Toeplitz matrix given by $\mathbf{T}^{\langle m,n\rangle} = E_l\{e^{-j2\pi\triangle_f(m-n)\delta^{(l)}}\}$. Now, we follow the assumption that all jitter realisations $\{\overline{\delta}^{(l)}\}_{l=1}^{L}$ are drawn from a Gaussian distribution of mean t_0 and a variance σ_δ^2, but since $\overline{\delta}^{(l)} = \delta^{(l)} + t_0$ and t_0 was explicitly included in the perfect covariance matrix $\mathbf{R_{xx}}$ in (16), we shall base our work on the centered Gaussian δ which has zero mean and variance σ_δ^2, i.e.

$$\delta \sim \mathcal{N}(0, \sigma_\delta^2) \tag{20}$$

For any real number α, we have the following

$$E\{e^{j\alpha\delta}\} = \frac{1}{\sqrt{2\pi\sigma_\delta^2}} \int_{-\infty}^{+\infty} e^{j\alpha\delta} e^{-\frac{\delta^2}{2\sigma_\delta^2}} d\delta = e^{-\frac{\alpha^2\sigma_\delta^2}{2}} \tag{21}$$

Equation (21) tells us that $\mathbf{T}$ is now a symmetric Toeplitz matrix. Denoting $\alpha_k = 2\pi \triangle_f |k|$ for any integer k, we can now say that

$$\mathbf{T}^{\langle m,n\rangle} = H(\alpha_{|m-n|}) \tag{22}$$

where

$$H(\alpha_k) = e^{-\frac{\alpha_k^2\sigma_\delta^2}{2}} \tag{23}$$

The number of largest eigenvalues of the sample covariance matrix is a good estimate of the number of multipath components in the case of non-coherent multipath components and sufficiently high Signal-to-Noise-Ratio (SNR), since $rank(\mathbf{A}\mathbf{R}_{\gamma\gamma}\mathbf{A}^H) = q$. In our case, where jitter is present, the sample covariance matrix is element-wise multiplied by $\mathbf{\Upsilon}$, thus the number of largest eigenvalues of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ depends on the rank of $\mathbf{\Upsilon} \odot \mathbf{A}\mathbf{R}_{\gamma\gamma}\mathbf{A}^H$. Using the fact that the rank of a Hadamard product of two matrices is bounded above by the product of their ranks [9] and that $rank(\mathbf{\Upsilon}) = M$, then unfortunately, the rank of the Perturbed covariance matrix would be

$$rank(\mathbf{\Upsilon} \odot \mathbf{A}\mathbf{R}_{\gamma\gamma}\mathbf{A}^H) \leq qM \tag{24}$$

Equation (24) tells us that there is an ambiguity in detecting the number of sources, since the rank of $\mathbf{\Upsilon} \odot \mathbf{A}\mathbf{R}_{\gamma\gamma}\mathbf{A}^H$ is upper bounded by qM. Before we continue the analysis, since the matrices involved are of size $MN \times MN$, we shall denote $\mathbf{B}_{\langle i,j\rangle}$, for $\mathbf{B}$ of size $MN \times MN$, to be

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_{\langle 1,1\rangle} & \cdots & \mathbf{B}_{\langle 1,N\rangle} \\ \vdots & \ddots & \vdots \\ \mathbf{B}_{\langle N,1\rangle} & \cdots & \mathbf{B}_{\langle N,N\rangle} \end{pmatrix} \tag{25}$$

where each $\mathbf{B}_{\langle i,j\rangle}$ block matrix is of size $M \times M$, or in other words

$$\mathbf{B}^{\langle m,n\rangle} = \mathbf{B}^{\langle m_1+m_2M, n_1+n_2M\rangle} = \mathbf{B}^{\langle m_1,n_1\rangle}_{\langle m_2,n_2\rangle} \tag{26}$$

Consider the following function

$$F^{\mathbf{B}}(k) = \frac{1}{2(M-k)N^2} \sum_{\substack{|m_1-n_1|=k \\ m_2=1\ldots N \\ n_2=1\ldots N}} |\mathbf{B}^{\langle m_1,n_1\rangle}_{\langle m_2,n_2\rangle}| \tag{27}$$

where $k = 1 \ldots M-1$. For a given k, the function $F^{\mathbf{B}}(k)$ averages up the magnitudes of the k^{th} sub and super diagonals of the block matrices $\mathbf{B}_{\langle m_2,n_2\rangle}$. Note that the number of entries found in the k^{th} sub and super diagonals of any $\mathbf{B}_{\langle m_2,n_2\rangle}$ block matrix of $\mathbf{B}$ is $2(M-k)$. Therefore, the total number of entries found in the k^{th} sub and super diagonals over the values of m_2 and n_2 is $2(M-k)N^2$, which is the normalisation factor in (27). It is now easy to see that

$$F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k) = H(\alpha_k)F^{\mathbf{R_{xx}}}(k) \tag{28}$$

4. JITTER ESTIMATION/COMPENSATION

4.1. Simple Least Squares Estimate

In this section, we estimate $H(\alpha_k)$ in (28). Taking log on both sides, we get:

$$log(F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)) = -\frac{\alpha_k^2\sigma_\delta^2}{2} + log(F^{\mathbf{R_{xx}}}(k)) \tag{29}$$

Again $F^{\mathbf{R_{xx}}}(k)$ is approximated to be almost invariant. It does fluctuate over different values of k but for the sake of simplicity, we assume it constant, i.e. $F^{\mathbf{R_{xx}}}(k) = F^{\mathbf{R_{xx}}}$. Under this assumption, we re-write (29) as

$$\mathbf{f} = \mathbf{Pg} \tag{30}$$

where $\mathbf{f}$ is an $(M-1) \times 1$ vector given by

$$\mathbf{f} = [log(F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(1)), \ldots, log(F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(M-1))]^T \tag{31}$$

and $\mathbf{P}$ is an $(M-1) \times 2$ matrix

$$\mathbf{P}^{\langle :,1\rangle} = [-\frac{\alpha_1^2}{2}, \ldots, -\frac{\alpha_{M-1}^2}{2}]^T \tag{32a}$$

$$\mathbf{P}^{\langle :,2\rangle} = [1, \ldots, 1]^T \tag{32b}$$

$\mathbf{g}$ is given by a 2×1 vector

$$\mathbf{g} = [\sigma_\delta^2, F^{\mathbf{R_{xx}}}]^T \tag{33}$$

We can now apply Least Squares (LS) to estimate $\mathbf{g}$ as

$$\hat{\mathbf{g}} = \arg\min_{\mathbf{g}} \|\mathbf{f} - \mathbf{Pg}\|_2^2 \tag{34}$$

The solution of (34) is

$$\hat{\mathbf{g}} = (\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{f} \tag{35}$$

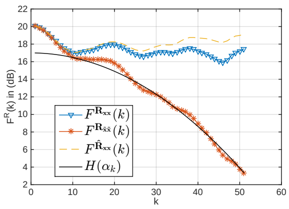

Fig. 1: Behaviour of $F^{\mathbf{R_{xx}}}(k)$, $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$, $F^{\hat{\mathbf{R}}_{\mathbf{xx}}}(k)$, and $H(\alpha_k)$ over k.

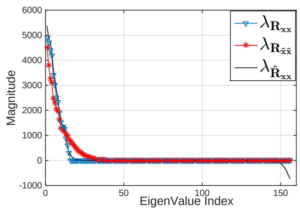

Fig. 2: Eigenvalues of $\mathbf{R_{xx}}$,$\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, and $\hat{\mathbf{R}}_{\mathbf{xx}}$.

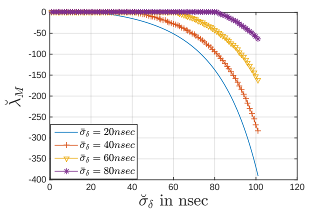

Fig. 3: $\breve{\lambda}_N$ v.s.$\breve{\sigma}_\delta$ for different $\bar{\sigma}_\delta$.

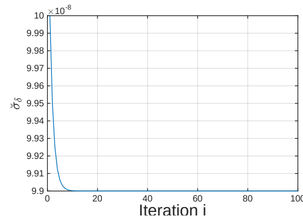

Fig. 4: $\breve{\sigma}_\delta$ v.s. iteration index for $\bar{\sigma}_\delta = 95nsec$.

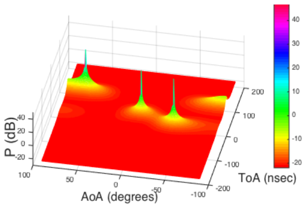

Fig. 5: 2D-MUSIC spectrum using $\mathbf{R_{xx}}$.

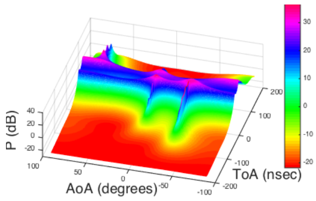

Fig. 6: 2D-MUSIC spectrum using $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$.

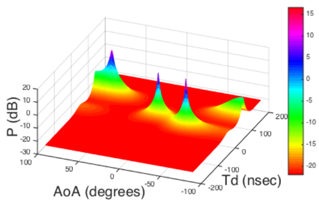

Fig. 7: 2D-MUSIC spectrum using $\hat{\mathbf{R}}_{\mathbf{xx}}$.

So, the first entry of $\hat{\mathbf{g}}$ gives an estimate of σ_δ^2, i.e. $\hat{\sigma}_\delta^2$. In the next section, we shall see, through simulations, that the function $F^{\mathbf{R_{xx}}}(k)$ that processes the entries of the perfect covariance matrix $\mathbf{R_{xx}}$ is somehow invariant over k, although it does fluctuate, and that the function $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$ follows the behavior of $H(\alpha_k) = e^{-\frac{\alpha_k^2 \sigma_\delta^2}{2}}$, and thus we could estimate $H(\alpha_k)$ by searching for the appropriate value σ_δ^2 which best fits $H(\alpha_k)$, or equivalently $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$. So, we do the following: (Algorithm 1)

Step 1. Compute the perturbed sample covariance matrix $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$.

Step 2. Compute its corresponding function $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$ using (27).

Step 3. Calculate $\hat{\sigma}_\delta^2$ using (31), (32a), (32b), (33), (35).

Step 4. Knowing $\hat{\sigma}_\delta^2$, compute $H(\alpha_k)$ by (23).

Step 5. Compensate $\mathbf{\Upsilon}$ in (19) by multiplying every entry of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ with its appropriate value $H^{-1}(\alpha_k)$, according to (22), and get an estimate of $\mathbf{R_{xx}}$, denoted hereby as $\hat{\mathbf{R}}_{\mathbf{xx}}$.

4.2. Jitter Estimation based on Negative Eigenvalues

Let $\bar{\sigma}_\delta^2$ denote the true value of σ_δ^2. For the sake of notation, we shall consider a single-antenna case, i.e. $N = 1$ (The following argument could be extended to the multi-antenna case). Now given $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, we shall try to retrieve $\mathbf{R_{xx}}$ by pre-Hadamard multiplication of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ in order to cancel out the matrix $\mathbf{T}$ (or $\mathbf{\Upsilon}$ for $N \geq 1$). In other words, span $\breve{\sigma}_\delta^2 \in [0, S]$ (where S is the upperbound of the search and $\bar{\sigma}_\delta^2 \in [0, S]$) till the smallest eigenvalue of the sample covariance matrix is negative, i.e.

$$\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = \breve{\mathbf{T}} \odot \mathbf{R_{xx}} \tag{36}$$

where $\breve{\mathbf{T}}^{\langle m,n \rangle} = e^{(m-n)^2 \alpha^2 \epsilon}$ and $\epsilon = \frac{\breve{\sigma}_\delta^2 - \bar{\sigma}_\delta^2}{2}$ and $\alpha = 2\pi\Delta_f$.

Let $\lambda_1 \geq \ldots \geq \lambda_M$ and $\breve{\lambda}_1 \geq \ldots \geq \breve{\lambda}_M$ denote the eigenvalues of $\mathbf{R_{xx}}$ and $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, respectively. Also, let $\mathbf{u}_1 \ldots \mathbf{u}_M$ and $\breve{\mathbf{u}}_1 \ldots \breve{\mathbf{u}}_M$ denote their corresponding eigenvectors.

We will base our discussion on 3 cases:

Case 1: If $\epsilon \leq 0$, then $\breve{\mathbf{T}}$ is a symmetric Toeplitz with negative exponents, and thus a positive definite matrix. $\mathbf{R_{xx}}$ is a positive semi-definite matrix due to the fact that it is a covariance matrix. It is well known that the Hadamard product of two positive semi-definite matrices is also positive semi-definite [9]. Therefore $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ is positive semi-definite, i.e. $\breve{\lambda}_M \geq 0$. Note that (24) tells us that there is no need that $\breve{\lambda}_1 \ldots \breve{\lambda}_q$ are the only large eigenvalues as the rank is no more q but could be much more than that, but $trace\{\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}\} = trace\{\mathbf{R_{xx}}\}$ which implies that the signal eigenvalues $\breve{\lambda}_1 \ldots \breve{\lambda}_q$ tend "to leak" onto the noise ones $\breve{\lambda}_{q+1} \ldots \breve{\lambda}_M$.

Case 2: If ϵ is in the neighborhood of 0, we can expand the entries of the matrix $\breve{\mathbf{T}}$ using Taylor series. Thus in the neighborhood of 0, we have

$$\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} = \mathbf{R_{xx}} + \epsilon\alpha^2 \mathbf{D} \odot \mathbf{R_{xx}} \tag{37}$$

where $\mathbf{D}^{\langle m,n \rangle} = (m-n)^2$. Now, with (37) in hand, and knowing that $\epsilon\alpha^2$ is small and $\mathbf{R_{xx}}$ and $\mathbf{D} \odot \mathbf{R_{xx}}$ are Hermitian, then following [10], we can approximate $\breve{\lambda}_M$ as a linear equation of λ_M, namely

$$\breve{\lambda}_M = \lambda_M + \epsilon\alpha^2 \mathbf{u}_M^H \mathbf{D} \odot \mathbf{R_{xx}} \mathbf{u}_M \tag{38}$$

Knowing that $D = (\mathbf{v} \odot \mathbf{v})\mathbf{1}^H + \mathbf{1}(\mathbf{v} \odot \mathbf{v})^H - 2\mathbf{v}\mathbf{v}^H$ where $\mathbf{1}$ is the all-ones $M \times 1$ vector and $\mathbf{v} = [1 \ldots M]^T$, we get

$$\begin{aligned}
\mathbf{u}_M^H \mathbf{D} \odot \mathbf{R_{xx}} \mathbf{u}_M &= \sum_{i=1}^{q} \lambda_i \mathbf{u}_M^H \{\mathbf{u}_i(\mathbf{u}_i \odot \mathbf{v} \odot \mathbf{v})^H \\
&\quad + (\mathbf{u}_i \odot \mathbf{v} \odot \mathbf{v})\mathbf{u}_i^H - 2(\mathbf{u}_i \odot \mathbf{v})(\mathbf{u}_i \odot \mathbf{v})^H\}\mathbf{u}_M \\
&= -2\sum_{i=1}^{q} \lambda_i |\mathbf{u}_M^H(\mathbf{u}_i \odot \mathbf{v})|^2 \leq 0
\end{aligned} \tag{39}$$

where we used the fact that eigenvectors of a Hermitian matrix are orthogonal, i.e. $\mathbf{u}_i^H \mathbf{u}_j = 1_{i,j}$. If we assume $\lambda_{q+1} = \ldots = \lambda_M = 0$, then $\breve{\lambda}_M \leq 0$ (using (38) and (39)), otherwise when $\lambda_{q+1} = \ldots = \lambda_M = \sigma^2$, then $\breve{\lambda}_M \leq \sigma^2$. Note that $\breve{\lambda}_M$ is also decreasing with respect to increasing ϵ at the neighborhood of 0.

Case 3: Intuitively speaking, the entries of the first row of $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ are the values of a correlation sequence, which should be bounded above in magnitude by the first element $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}^{\langle 1,1 \rangle}$. This is not valid anymore as ϵ grows large, because this growth amplifies the off-diagonal elements too much that they are no longer bounded in magnitude by the diagonal ones, and as a result $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ looses definitness and therefore negative eigenvalues are always present.

So, we propose the following: (Algorithm 2)

Step 1: Discretize the interval $[0, S]$ into uniform intervals of length η, set $\breve{\sigma}_\delta^2(0) = 0$ and $i = 1$.
Step 2: $\breve{\sigma}_\delta^2(i) = \breve{\sigma}_\delta^2(i-1) + \eta$.
Step 3: Compensate to get $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ in (36) and compute its' smallest eigenvalue $\breve{\lambda}_{min}$. Increment $i \leftarrow i + 1$
Step 4: If $\breve{\lambda}_{min} \geq \upsilon$ (pre-defined threshold), go back to *Step 2*.
Step 5: If $\breve{\lambda}_{min} < \upsilon$, choose $\breve{\sigma}_\delta^2(i) = \frac{\breve{\sigma}_\delta^2(i-1)+\breve{\sigma}_\delta^2(i-2)}{2}$ and do *Step 2* to get $\breve{\lambda}_{min}$. Increment $i \leftarrow i + 1$. If $\breve{\lambda}_{min} < \eta$, choose $\breve{\sigma}_\delta^2(i) = \frac{\breve{\sigma}_\delta^2(i-1)+\breve{\sigma}_\delta^2(i-3)}{2}$, else choose $\breve{\sigma}_\delta^2(i) = \frac{\breve{\sigma}_\delta^2(i-1)+\breve{\sigma}_\delta^2(i-2)}{2}$. Repeat *Step 5* until $|\breve{\sigma}_\delta^2(i) - \breve{\sigma}_\delta^2(i-1)| < \zeta$ (pre-defined threshold).

5. SIMULATION RESULTS

5.1. $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$ vs $F^{\mathbf{R}_{\mathbf{xx}}}(k)$ and Algorithm 1

We have devoted a special subsection just to show how the two functions $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$ and $F^{\mathbf{R}_{\mathbf{xx}}}(k)$ behave with respect to k. First, we shall set some simulation parameters in this sebsection to the following: SNR is fixed to 20dB, $N = 3$ antennas spaced at half a wavelength. $M = 52$ subcarriers spaced by $\triangle_f = 0.3125$ MHz. We collect $L = 10^2$ snapshots, with $q = 15$ uncorrelated multipath complex coefficients. The AoAs and ToAs are chosen randomly, and the jitter $\delta \sim \mathcal{N}(0, (25nsec)^2)$.
By observing figure 1, one could see that $F^{\mathbf{R}_{\mathbf{xx}}}(k)$ does fluctuate, but could be considered somehow relatively stable with respect to $F^{\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}}(k)$, which itself follows the aysmptotics of $H(\alpha_k)$ (Here $H(\alpha_k)$ is tuned to the true parameter $\sigma_\delta^2 = (25nsec)^2$. After steps 1 to 4 of Algorithm 1, we get an estimate $\hat{\sigma}_\delta^2 = (27.2nsec)^2$, followed by step 5 which is a compensation with $\hat{\sigma}_\delta^2$, thus an estimate $\hat{\mathbf{R}}_{xx}$ is now available. From $\hat{\mathbf{R}}_{xx}$, we plot $F^{\hat{\mathbf{R}}_{xx}}(k)$, which is close to $F^{\mathbf{R}_{\mathbf{xx}}}(k)$.
Note here that this is an over-compenstion ($\epsilon > 0$) which corresponds to case 2 or 3 in section 4.2. This could only mean that an over-compensation leads to negative eigenvalues of the estimated covariance matrix, as shown in figure 2. On the other hand, the largest eigenvalues corresponding to signal subspace are detected with the over-compensated matrix $\hat{\mathbf{R}}_{\mathbf{xx}}$; whereas the 15 largest eigenvalues of $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$ attenuate and "spill" onto the noise eigenvalues in a smooth fashion. Thus, one can not directly estimate q using $\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$.

5.2. Algorithm 2

Figure 3 plots the evolution of the $\breve{\lambda}_{min}$ of a sample covariance matrix $\breve{\mathbf{R}}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}$, (with same simulation parameters as before but $q = 3$). Indeed, there is a massive drop of $\breve{\lambda}_{min}$ towards negative values just after $\breve{\sigma}_\delta$ exceeds the true values $\hat{\sigma}_\delta$. Figure 4 shows the evolution of values of $\breve{\sigma}_\delta$ with iterations of Algorithm 2. Note here that the true value was $\bar{\sigma}_\delta \simeq 95nsec$, which was estimated to be $\hat{\sigma}_\delta = 99nsec$. This bias was due to setting a threshold $\upsilon = 0$ in the presence of noise. In other words, once $\breve{\sigma}_\delta$ exceeds $\bar{\sigma}_\delta$, then $\breve{\lambda}_M < \sigma^2$ and thus more iterations are needed so that $\breve{\lambda}_M$ falls below 0.

5.3. Estimating ToAs and AoAs by 2D-MUSIC

The previous simulation parameters are fixed but we change q to be equal to 3 and (θ_i, τ_i) (of units degrees and nsec, respectively) are fixed to $(0, -40)$, $(30, 0)$, and $(95, 70)$. Figure 5 detects the correct peaks by processing the perfect covariance matrix $\mathbf{R}_{\mathbf{xx}}$ by applying JADE, whereas in figure 6, the AoAs could be detected correctly but their respective ToAs leave alot of ambiguity due to: *(i)* Wrong estimation of q and *(ii)* ToAs are shifted differently at each snapshot by the value δ. After applying steps 1 through 5, and computing an estimate $\hat{\mathbf{R}}_{\mathbf{xx}}$, JADE is then applied to $\hat{\mathbf{R}}_{\mathbf{xx}}$, and therefore we can now resolve the true ToAs/AoAs through their corresponding peaks as depicted in figure 7 when compared to figure 6. The only difference between figures 5 and 7 is that the peaks become a bit broader in 7, due to the estimation errors of σ_δ^2.

6. CONCLUSION

In short, we introduced a new model by including *snapshot timing jitter*, that is missampling of the collected OFDM symbols. This *jitter* leads to an over-estimation of the number of incoming signals, which also leads to wrong estimation of AoAs/ToAs. The effect of *jitter* was analysed through a function, which pre-processes entries of the *perturbed sample covariance* matrix. We suggested two algorithms that allow estimation/compensation of the *jitter*. These algorithms are simple to implement and are not computationally prohobited. Simulations show promising results if the JADE-MUSIC algorithm, or any other algorithm that requires second-order moments, was used to estimate ToAs/AoAs of incoming signals.

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