Achieving the DoF Limits of the SISO X Channel with Imperfect-Quality CSIT

Jingjing Zhang
Mobile Communications Department
EURECOM, Sophia Antipolis, France
Email: jingjing.zhang@eurecom.fr

Dirk Slock
Mobile Communications Department
EURECOM, Sophia Antipolis, France
Email: dirk.slock@eurecom.fr

Petros Elia
Mobile Communications Department
EURECOM, Sophia Antipolis, France
Email: petros.elia@eurecom.fr

***Abstract*—In the setting of the two-user single-input single-output X channel, recent works have explored the degrees-of-freedom (DoF) limits in the presence of perfect channel state information at the transmitter (CSIT), as well as in the presence of perfect-quality delayed CSIT. Our work shows that the same DoF-optimal performance — previously associated to perfect-quality current CSIT — can in fact be achieved with current CSIT that is of imperfect quality. The work also shows that the DoF performance previously associated to perfect-quality delayed CSIT, can in fact be achieved in the presence of imperfect-quality delayed CSIT. These follow from the presented sum-DoF lower bound that bridges the gap — as a function of the quality of delayed CSIT — between the cases of having no feedback and having delayed feedback, and then another bound that bridges the DoF gap — as a function of the quality of current CSIT — between delayed and perfect current CSIT. The bounds are based on novel precoding schemes that are presented here and which employ imperfect-quality current and/or delayed feedback to align interference in space and in time.**

***Index Terms*—degrees of freedom, SISO X channel, CSIT.**

I. Introduction

We consider the two-user Gaussian single-input single-output (SISO) X channel (XC), with two single-antenna transmitters and two single-antenna receivers, where each transmitter has an independent message for each of the two receivers. The corresponding channel model takes the form

$$\begin{aligned} y_t &= h_t^{(1)} x_t^{(1)} + h_t^{(2)} x_t^{(2)} + m_t \\ z_t &= g_t^{(1)} x_t^{(1)} + g_t^{(2)} x_t^{(2)} + n_t \end{aligned} \quad (1)$$

where at any time t, $h_t^{(i)}$, $g_t^{(i)}$ denote the scalar fading coefficients of the channel from transmitter i to receiver 1 and 2 respectively, where m_t, n_t denote the unit-power AWGN noise at the two receivers, and where $x_t^{(i)}$, $i = 1, 2$ denotes the transmitted signals at transmitter i, satisfying a power constraint $\mathbb{E}(|x_t^{(i)}|^2) \leq P$. Naturally each $x_t^{(i)}$ may include some private information – originating from transmitter i – intended for receiver 1, and some private information intended for receiver 2.

In this setting, for a quadruple of achievable rates $R_{ij},\ i, j = 1, 2$ corresponding to communication from transmitter i to receiver j, we adopt the high-SNR degrees-of-freedom (DoF) approximation $d_{ij} = \lim_{P\to\infty} \frac{R_{ij}}{\log P},\ \ i, j = 1, 2$ to describe the limits of performance over the XC, particularly focusing on the sum DoF measure $d_\Sigma := d_{11} + d_{21} + d_{12} + d_{22}$.

In this context, the challenge originates from the fact that each transmitter is both an interferer as well as an intended transmitter to both receivers. Crucial in addressing this challenge is the role of feedback — and specifically of channel state information at the transmitters (CSIT) — which can allow for separation, at each receiver, of the intended and the interfering signals. In particular, while the optimal sum DoF without CSIT has been shown to be $d_\Sigma = 1$ (cf. [1]), the DoF increases to $d_\Sigma = \frac{6}{5}$ in the presence of perfect-quality delayed CSIT (see [2] which proved that this performance is optimal over all linear schemes), and the DoF further increases to an optimal sum-DoF of $d_\Sigma = \frac{4}{3}$ (see [3]) in the presence of perfect-quality and instantaneously available CSIT (perfect current CSIT).

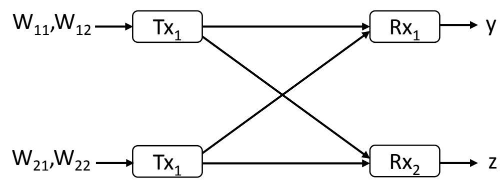

Fig. 1. 2-user SISO X channel.

A. Feedback quality model

Motivated by practical settings of limited feedback links, we here consider the case where feedback can be of imperfect-quality, and potentially also delayed. Towards this, let $\hat{h}_t^{(i)}, \hat{g}_t^{(i)}$ denote the current CSIT estimates of channels $h_t^{(i)}, g_t^{(i)}$ respectively, and let

$$\tilde{h}_t^{(i)} = h_t^{(i)} - \hat{h}_t^{i}, \quad \tilde{g}_t^{(i)} = g_t^{(i)} - \hat{g}_t^{(i)} \quad (2)$$

be the estimation errors, modeled here as having i.i.d Gaussian entries.

Similarly for delayed CSIT, along the same lines as in [4], [5], let $\check{h}_t^{(i)}, \check{g}_t^{(i)}$ denote the delayed estimates of channels $h_t^{(i)}, g_t^{(i)}$, where these estimates are obtained sometime after the channel elapses, and let

$$\ddot{h}_t^{(i)} = h_t^{(i)} - \check{h}_t^{(i)}, \quad \ddot{g}_t^{(i)} = g_t^{(i)} - \check{g}_t^{(i)} \quad (3)$$

be the associated CSIT errors, modeled here as having i.i.d Gaussian entries.

In our context, motivated by the approach in [6]–[9], we consider

$$\alpha = -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\tilde{h}_t^{(i)}||^2]}{\log P} = -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\tilde{g}_t^{(i)}||^2]}{\log P}$$

to be the *current CSIT quality exponent* describing the quality of current CSIT, equally for both $i = 1, 2$, and we similarly consider

$$\beta = -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\ddot{h}_t^{(i)}||^2]}{\log P} = -\lim_{P\to\infty} \frac{\log \mathbb{E}[||\ddot{g}_t^{(i)}||^2]}{\log P}$$

to be the *delayed-CSIT quality exponent*. In our DoF setting, and following arguments directly from [10], we can safely consider that the two quality exponents are bounded as

$$0 \le \alpha \le \beta \le 1.$$

Having $\alpha = 1$ corresponds to the case of perfect CSIT, for which case — as stated above — the optimal sum DoF was established to be equal to $d_\Sigma = \frac{4}{3}$, while having $\beta = 1$ ($\alpha = 0$), corresponds to the case of perfect-quality delayed CSIT, for which case the optimal linear sum DoF was established in [2] to be $d_\Sigma = \frac{6}{5}$.

II. DoF performance with imperfect-quality current and delayed CSIT

Motivated by works such as that in [11] which presented a distributed interference management technique which can obtain the optimal DoF with local and perfect current CSIT with a certain *fractional* delay, and by works on imperfect current and delayed CSIT over the broadcast channel [6]–[9], we here explore the role of feedback in moving between these extremal points ($\alpha = 1$ and $\beta = 1, \alpha = 0$) by considering different values of α and β.

Theorem 1: For the two-user XC with perfect-quality delayed CSIT, and with imperfect current CSIT of quality exponent α, the optimal sum DoF is lower bounded as

$$d_\Sigma \ge \min\Big(\frac{4}{3}, \frac{6}{5} + \frac{2\alpha(2-3\alpha)}{5(4-7\alpha)}\Big). \quad (4)$$

As a result, the optimal sum DoF $d_\Sigma = \frac{4}{3}$ can be achieved with imperfect current CSIT of quality that need not exceed $\alpha = \frac{4}{9}$.

Proof: The result follows by analyzing the performance of the communication scheme which will be presented in the next section. ■

We note that the above expression, evaluated at $\alpha = 0$, yields the aforementioned sum DoF $d_\Sigma = \frac{6}{5}$.

We now shift attention to the case of imperfect-quality delayed feedback, and of no (or very limited) current feedback, corresponding to having $\beta < 1$ and $\alpha = 0$. As argued in [9], interest in imperfect-quality *delayed* CSIT relates to the fact that β is more indicative of the quality of the *entirety* of feedback (timely plus delayed), and hence, any attempt to limit the total amount of feedback — that is communicated during a certain communication process — must focus on reducing β, rather than just focusing on reducing α. We proceed with the associated result.

Theorem 2: For the two-user XC with no current CSIT and with imperfect delayed CSIT of quality exponent β, the optimal sum DoF is lower bounded as

$$d_\Sigma \ge \min\Big(\frac{6}{5}, 1 + \frac{\beta}{3}\Big). \quad (5)$$

As a result, the (linear-) optimal sum-DoF $d_\Sigma = \frac{6}{5}$, previously associated to perfect-quality delayed feedback, can in fact be achieved with imperfect-quality delayed CSIT of quality that need not exceed $\beta = \frac{3}{5}$.

The proof is based on a construction of an interference management scheme that utilizes imperfect-quality delayed feedback, and which — due to lack of space — is left to be presented in the journal version.

We proceed with the description of the scheme corresponding to the first theorem.

III. Schemes for XC with imperfect current CSIT

The schemes are designed to have S phases, where the sth phase ($s = 1, \cdots, S$) consists of T_s channel uses. In describing the schemes, we will use a double time index s, t to correspond to the t^{th} time slot, $t = 1, \cdots, T_s$, of phase s.

The general structure of the transmitted signals at any timeslot t of phase s, will be

$$\boldsymbol{x}_{s,t} = \begin{bmatrix} u_{s,t}^{(1)} a_{s,t}^{(1)} + {u'}_{s,t}^{(1)} {a'}_{s,t}^{(1)} + v_{s,t}^{(1)} b_{s,t}^{(1)} + {v'}_{s,t}^{(1)} {b'}_{s,t}^{(1)} \\ c_{s,t} + u_{s,t}^{(2)} a_{s,t}^{(2)} + {u'}_{s,t}^{(2)} {a'}_{s,t}^{(2)} + v_{s,t}^{(2)} b_{s,t}^{(2)} + {v'}_{s,t}^{(2)} {b'}_{s,t}^{(2)} \end{bmatrix}$$

where, depending on the instance, some of these symbols will be deactivated resulting in a simpler transmitted signal. In the above, $a_{s,t}^{(i)}, {a'}_{s,t}^{(i)}$ will denote independent information symbols, from transmitter i to receiver 1, while symbols $b_s^{(i)}, {b'}_s^{(i)}$ are intended for receiver 2, again from transmitter i. In addition, $c_{s,t}$ will represent a common information symbol generally intended for both receivers. Furthermore $u_{s,t}^{(i)}, {u'}_{s,t}^{(i)}, v_{s,t}^{(i)}, {v'}_{s,t}^{(i)}$ are unit-norm 'precoding' scalars which — when combined in time and space — help align the interference from the distributed transmitters at the unintended receivers.

Communication takes place under an average power constraint P on both transmitters. We use the following notation to describe the allocated power on different symbols

$$P_s^{(a)} \triangleq \mathbb{E}|a_{s,t}^{(i)}|^2,\ P_s^{(a')} \triangleq \mathbb{E}|{a'}_{s,t}^{(i)}|^2,\ P_s^{(b)} \triangleq \mathbb{E}|b_{s,t}^{(i)}|^2,\ P_s^{(b')} \triangleq \mathbb{E}|{b'}_{s,t}^{(i)}|^2$$

and note that this holds equally for both transmitters, $i = 1, 2$. Furthermore, we use $r_s^{(a)}$ to mean that, during phase s, each symbol $a_{s,t}^{(i)}, t = 1, \cdots, T_s$, carries $r_s^{(a)} \log P + o(\log P)$ bits. Similarly, we use $r_s^{(a')}, r_s^{(b)}, r_s^{(b')}, r_s^{(c)}$ to describe the prelog factor of the number of bits carried by ${a'}_{s,t}^{(i)}, b_{s,t}^{(i)}, {b'}_{s,t}^{(i)}, c_{s,t}$ respectively.

Remark 1: We note that typically, a receiver encounters interference originating from two transmitters. The general idea behind our scheme is that a receiver uses linear combinations of received signals to remove as much interference as possible from one transmitter, and then have the other transmitter help out — with precoding that employs imperfect-current and delayed feedback — in removing the remaining interference. This will be achieved with a proper choice of precoding scalars that are functions of imperfect current and delayed CSIT. Given that this CSIT can be of imperfect quality, the interference may not be fully removed immediately, thus forcing power-and-rate regulation of the information symbols, as well as a multiphase scheme that uses proactive encoding which handles interference at later stages of the communication process, and which then allows for retrospective decoding of the original private information.

A. Coding

The phase durations $T_1, T_2, \cdots, T_S$ are chosen to be integers such that

$$\begin{aligned} &T_2 = T_1\xi, \quad T_s = T_{s-1}\mu, \ \forall s \in \{3,4,\cdots,S-1\}, \\ &T_S = T_{S-1}\gamma = T_1\xi\mu^{S-3}\gamma \end{aligned} \tag{6}$$

where $\xi = \frac{8(1-\alpha)}{3(4-7\alpha)}, \mu = \frac{2\alpha}{4-7\alpha}, \gamma = \frac{\alpha}{2(1-\alpha)}$.

We proceed with the description of the phases.

a) Phase 1*:* Phase 1 consists of $\frac{T_1}{3}$ sub-phases, with each sub-phase consisting of three consecutive time slots. We will focus on the first such sub-phase (i.e., the first 3 time slots of the first phase), corresponding to time $(1,1),(1,2),(1,3)$. The rest of the sub-phases will simply be a repetition of this first sub-phase, with each sub-phase corresponding to new information symbols. In this first sub-phase, the transmitted signals are

$$\boldsymbol{x}_{1,1}=\begin{bmatrix} a_{1,1}^{(1)} \\ a_{1,1}^{(2)}+a'^{(2)}_{1,1}\end{bmatrix}, \boldsymbol{x}_{1,2}=\begin{bmatrix} b_{1,2}^{(1)} \\ b_{1,2}^{(2)}+b'^{(2)}_{1,2}\end{bmatrix}, \boldsymbol{x}_{1,3}=\begin{bmatrix} a_{1,1}^{(1)}+b_{1,2}^{(1)} \\ u_{1,3}^{(2)}a_{1,1}^{(2)}+v_{1,3}^{(2)}b_{1,2}^{(2)}\end{bmatrix}$$

where the power and normalized rates are set as

$$\begin{aligned} &P_1^{(a)} \doteq P_1^{(b)} \doteq P, \ P_1^{(a')} \doteq P_1^{(b')} \doteq P^{1-\alpha} \\ &r_1^{(a)} = r_1^{(b)} = 1, \ r_1^{(a')} = r_1^{(b')} = 1-\alpha \end{aligned} \tag{7}$$

and where

$$u_{1,3}^{(2)} = \frac{g_{1,1}^{(2)}\hat{g}_{1,3}^{(1)}}{g_{1,1}^{(1)}\hat{g}_{1,3}^{(2)}}, \qquad v_{1,3}^{(2)} = \frac{h_{1,2}^{(2)}\hat{h}_{1,3}^{(1)}}{h_{1,2}^{(1)}\hat{h}_{1,3}^{(2)}}.$$

To gain insight into the workings of the scheme, we note that, for example, $u_{1,3}^{(2)}$ is chosen to assist receiver 2 remove the interference from transmitter 2 using delayed estimates $g_{1,t}^{(1)}, g_{1,t}^{(2)}$ as well as using current imperfect estimates of the two channels leading to receiver 2 from the two transmitters. We note that the above expression reflects our assumption that delayed CSIT here is of perfect quality.

Excluding the noise term for the sake of brevity, the received signals at receiver 1 take the form

$$\begin{aligned} y_{1,1} &= h_{1,1}^{(1)}a_{1,1}^{(1)} + h_{1,1}^{(2)}(a_{1,1}^{(2)} + a'^{(2)}_{1,1}) \\ y_{1,2} &= h_{1,2}^{(1)}b_{1,2}^{(1)} + h_{1,2}^{(2)}(b_{1,2}^{(2)} + b'^{(2)}_{1,2}) \\ y_{1,3} &= h_{1,3}^{(1)}(a_{1,1}^{(1)} + b_{1,2}^{(1)}) + h_{1,3}^{(2)}(u_{1,3}^{(2)}a_{1,1}^{(2)} + v_{1,3}^{(2)}b_{1,2}^{(2)}). \end{aligned} \tag{8}$$

Upon receiving the above, receiver 1 removes the unintended symbol $b_{1,2}^{(1)}$ from transmitter 1, using the following linear combination, to get

$$\begin{aligned} &y_{1,3}/h_{1,3}^{(1)} - y_{1,2}/h_{1,2}^{(1)} \\ &= \underbrace{a_{1,1}^{(1)}}_{P} + \underbrace{\frac{h_{1,3}^{(2)}}{h_{1,3}^{(1)}}u_{1,3}^{(2)}a_{1,1}^{(2)}}_{P} + \underbrace{\overbrace{(\frac{h_{1,3}^{(2)}}{h_{1,3}^{(1)}}v_{1,3}^{(2)} - \frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}})b_{1,2}^{(2)} - \frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}}b'^{(2)}_{1,2}}^{i_{1,1}}}_{P^{(1-\alpha)}} \end{aligned}$$

where under each term we noted the order of the summand's average power, where $i_{1,1}$ denotes the interference from transmitter 2 onto receiver 1 during this first sub-phase of the first phase, and where the power of this interference is bounded as

$$\begin{aligned} \mathbb{E}|i_{1,1}|^2 &= \mathbb{E}\left|\frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}}(\frac{h_{1,3}^{(2)}}{h_{1,3}^{(1)}}\frac{\hat{h}_{1,3}^{(1)}}{\hat{h}_{1,3}^{(2)}} - 1)b_{1,2}^{(2)}\right|^2 + \mathbb{E}\left|\frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}}b'^{(2)}_{1,2}\right|^2 \\ &\doteq \mathbb{E}\left|\frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}}\frac{\tilde{h}_{1,3}^{(2)}\hat{h}_{1,3}^{(1)} - \hat{h}_{1,3}^{(2)}\tilde{h}_{1,3}^{(1)}}{h_{1,3}^{(1)}\hat{h}_{1,3}^{(2)}}b_{1,2}^{(2)}\right|^2 + P^{1-\alpha} \doteq P^{1-\alpha}. \end{aligned} \tag{9}$$

In the above, precoding with $v_{1,3}^{(2)}$, managed to bring down the residual interference of $b_{1,2}^{(2)}$ (due to imperfections in current CSIT), to the levels of the interference imposed by $b'^{(2)}_{1,2}$.

Receiver 2, which follows a parallel course of action, now experiences interference $\theta_{1,1}$ from transmitter 2, where this interference is similarly bounded above by $P^{1-\alpha}$. At the end of this first sub-phase (3 time slots), transmitter 2 uses its partial knowledge of current CSIT to reconstruct $\{i_{1,1}, \theta_{1,1}\}$, and to quantize each term to get

$$\bar{i}_{1,1} = i_{1,1} - \tilde{i}_{1,1}, \quad \bar{\theta}_{1,1} = \theta_{1,1} - \tilde{\theta}_{1,1}.$$

Allowing for $(1-\alpha)\log P$ bits per interference term, allows in turn for $\mathbb{E}(|\tilde{i}_{1,1}|^2) \doteq \mathbb{E}(|\tilde{\theta}_{1,1}|^2) \doteq 1$.

At this point, this same procedure described here for the first sub-phase of the first phase, is repeated for the remaining $\frac{T_1}{3} - 1$ sub-phases, corresponding though to new information. This process results in the accumulation of a total of $\frac{2T_1}{3}(1-\alpha)\log P$ quantization bits representing residual interference. These bits will be distributed evenly into the common symbols $\{c_{2,t}\}_{t=1}^{T_2}$ of the second phase.

2) Phase s, $2 \le s \le S-1$*:* Phase s consists of $\frac{T_s}{4}$ sub-phases, each consisting of 4 consecutive channel uses. As before, we describe the first sub-phase, corresponding to time $(s,1),(s,2),(s,3),(s,4)$, where the transmitted signals are

$$\boldsymbol{x}_{s,1} = \begin{bmatrix} a_{s,1}^{(1)} + a'^{(1)}_{s,1} \\ c_{s,1} + a_{s,1}^{(2)} \end{bmatrix}, \quad \boldsymbol{x}_{s,2} = \begin{bmatrix} b_{s,2}^{(1)} + b'^{(1)}_{s,2} \\ c_{s,2} + b_{s,2}^{(2)} \end{bmatrix}$$

$$\boldsymbol{x}_{s,3} = \begin{bmatrix} a_{s,1}^{(1)} + a'^{(1)}_{s,1} + b_{s,2}^{(1)} + b'^{(1)}_{s,2} \\ c_{s,3} + u_{s,3}^{(2)}(a_{s,1}^{(2)} + a'^{(2)}_{s,3}) + v_{s,3}^{(2)}(b_{s,2}^{(2)} + b'^{(2)}_{s,3}) \end{bmatrix}$$

$$\boldsymbol{x}_{s,4} = \begin{bmatrix} u_{s,4}^{(1)}a_{s,1}^{(1)} + v_{s,4}^{(1)}b_{s,2}^{(1)} \\ c_{s,4} + a'^{(2)}_{s,3} + b'^{(2)}_{s,3} \end{bmatrix}$$

where

$$\begin{aligned} u_{s,3}^{(2)} &= \frac{g_{s,1}^{(2)}\hat{g}_{s,3}^{(1)}}{g_{s,1}^{(1)}\hat{g}_{s,3}^{(2)}}, \quad u_{s,4}^{(1)} = \frac{g_{s,1}^{(1)}}{g_{s,1}^{(2)}}(\frac{g_{s,3}^{(1)}\hat{g}_{s,3}^{(2)}}{g_{s,3}^{(2)}\hat{g}_{s,3}^{(1)}} - 1)\frac{\hat{g}_{s,4}^{(2)}}{\hat{g}_{s,4}^{(1)}} \\ v_{s,3}^{(2)} &= \frac{h_{s,2}^{(2)}\hat{h}_{s,3}^{(1)}}{h_{s,2}^{(1)}\hat{h}_{s,3}^{(2)}}, \quad v_{s,4}^{(1)} = \frac{h_{s,2}^{(1)}}{h_{s,2}^{(2)}}(\frac{h_{s,3}^{(1)}\hat{h}_{s,3}^{(2)}}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}} - 1)\frac{\hat{h}_{s,4}^{(2)}}{\hat{h}_{s,4}^{(1)}} \end{aligned} \tag{10}$$

and where the rates and power are allocated as follows

$$\begin{aligned} &P_s^{(c)} \doteq P, \ P_s^{(a)} = P_s^{(b)} \doteq P^{2\alpha}, \ P_s^{(a')} = P_s^{(b')} \doteq P^{\alpha} \\ &r_s^{(c_{s,1})} = r_s^{(c_{s,2})} = r_s^{(c_{s,3})} = 1-2\alpha, r_s^{(c_{s,3})} = 1-\alpha, \\ &r_s^{(a)} = r_s^{(b)} = 2\alpha, r_s^{(a')} = r_s^{(b')} = \alpha. \end{aligned} \tag{11}$$

We focus on the noiseless version of the signals of the first receiver, which take the form

$$y_{s,1} = h_{s,1}^{(2)}c_{s,1} + h_{s,1}^{(1)}(a_{s,1}^{(1)} + {a'}_{s,1}^{(1)}) + h_{s,1}^{(2)}a_{s,1}^{(2)}$$
$$y_{s,2} = h_{s,2}^{(2)}c_{s,2} + h_{s,2}^{(1)}(b_{s,2}^{(1)} + {b'}_{s,2}^{(1)}) + h_{s,2}^{(2)}b_{s,2}^{(2)}$$
$$y_{s,3} = h_{s,3}^{(2)}c_{s,3} + h_{s,3}^{(1)}(a_{s,1}^{(1)} + {a'}_{s,1}^{(1)} + b_{s,2}^{(1)} + {b'}_{s,2}^{(1)}) + h_{s,3}^{(2)}\Big(u_{s,3}^{(2)}(a_{s,1}^{(2)} + {a'}_{s,3}^{(2)}) + v_{s,3}^{(2)}(b_{s,2}^{(2)} + {b'}_{s,3}^{(2)})\Big)$$
$$y_{s,4} = h_{s,4}^{(2)}c_{s,4} + h_{s,4}^{(1)}(u_{s,4}^{(1)}a_{s,1}^{(1)} + v_{s,4}^{(1)}b_{s,2}^{(1)}) + h_{s,4}^{(2)}({a'}_{s,3}^{(2)} + {b'}_{s,3}^{(2)}).$$

At this point, receiver 1 decodes $c_{s,1}, c_{s,2}, c_{s,3}, c_{s,4}$ by treating the other signals as noise[1]. After removal of these common symbols, receiver one has ${y'}_{s,t} = y_{s,t} - h_{s,t}^{(2)}c_{s,t}, t = 1, \cdots, 4$, where

$$\begin{aligned}
{y'}_{s,1} &= h_{s,1}^{(1)}(a_{s,1}^{(1)} + {a'}_{s,1}^{(1)}) + h_{s,1}^{(2)}a_{s,1}^{(2)} \\
{y'}_{s,2} &= h_{s,2}^{(1)}(b_{s,2}^{(1)} + {b'}_{s,2}^{(1)}) + h_{s,2}^{(2)}b_{s,2}^{(2)} \\
{y'}_{s,3} &= h_{s,3}^{(1)}(a_{s,1}^{(1)} + {a'}_{s,1}^{(1)} + b_{s,2}^{(1)} + {b'}_{s,2}^{(1)}) \\
&\quad + h_{s,3}^{(2)}\Big(u_{s,3}^{(2)}(a_{s,1}^{(2)} + {a'}_{s,3}^{(2)}) + v_{s,3}^{(2)}(b_{s,2}^{(2)} + {b'}_{s,3}^{(2)})\Big) \\
{y'}_{s,4} &= h_{s,4}^{(1)}(u_{s,4}^{(1)}a_{s,1}^{(1)} + v_{s,4}^{(1)}b_{s,2}^{(1)}) + h_{s,4}^{(2)}({a'}_{s,3}^{(2)} + {b'}_{s,3}^{(2)}).
\end{aligned} \quad (12)$$

b) Interference alignment and power reducing: Receiver 1 considers the following linear combination

$$\frac{{y'}_{s,3}}{h_{s,3}^{(1)}} - \frac{{y'}_{s,2}}{h_{s,2}^{(1)}} = \sigma_{s,1} + \overbrace{\underbrace{(\frac{h_{s,3}^{(2)}}{h_{s,3}^{(1)}}v_{s,3}^{(2)} - \frac{h_{s,2}^{(2)}}{h_{s,2}^{(1)}})b_{s,2}^{(2)} + \frac{h_{s,3}^{(2)}}{h_{s,3}^{(1)}}v_{s,3}^{(2)}{b'}_{s,3}^{(2)}}_{P^\alpha}}^{i_{s,1}}$$

as a means of canceling the unintended information from transmitter 1. In the above, we use $\sigma_{s,1}$ to simply denote the part of the received signal — during this sub-phase — that consists of desired symbols, while we also recall that $i_{s,1}$ denotes the interference from transmitter 2. The interference relating to $b_{s,2}^{(1)} + {b'}_{s,2}^{(1)}$ has been removed by the actions of transmitter 1. Choosing $v_{s,3}^{(2)} = \frac{h_{s,2}^{(2)}\hat{h}_{s,3}^{(1)}}{h_{s,2}^{(1)}\hat{h}_{s,3}^{(2)}}$, guarantees that

$$\begin{aligned}
\mathbb{E}|i_{s,1}|^2 &= \mathbb{E}\left|(\frac{h_{s,3}^{(2)}}{h_{s,3}^{(1)}}v_{s,3}^{(2)} - \frac{h_{s,2}^{(2)}}{h_{s,2}^{(1)}})b_{s,2}^{(2)}\right|^2 + \mathbb{E}\left|\frac{h_{s,3}^{(2)}}{h_{s,3}^{(1)}}v_{s,3}^{(2)}{b'}_{s,3}^{(2)}\right|^2 \\
&\doteq \mathbb{E}\left|\frac{h_{s,2}^{(2)}}{h_{s,2}^{(1)}}\frac{(\tilde{h}_{s,3}^{(2)}\hat{h}_{s,3}^{(1)} - \tilde{h}_{s,3}^{(1)}\hat{h}_{s,3}^{(2)})}{h_{s,3}^{(1)}\hat{h}_{s,3}^{(2)}}b_{s,2}^{(2)}\right|^2 + P^\alpha \doteq P^\alpha
\end{aligned} \quad (13)$$

and using ${y'}_{s,2}$ and ${y'}_{s,3}$, receiver 1 removes the interference from transmitter 1 and also reduces interference from transmitter 2. Similar arguments apply also for the interference $\theta_{s,1}$ at receiver 2 originating from transmitter 2.

We also consider

$$\frac{{y'}_{s,3}}{h_{s,3}^{(2)}v_{s,3}^{(2)}} - \frac{{y'}_{s,2}}{h_{s,2}^{(2)}} - \frac{{y'}_{s,4}}{h_{s,4}^{(2)}} = \sigma_{s,2} + \underbrace{(\eta - \frac{h_{s,4}^{(1)}}{h_{s,4}^{(2)}}v_{s,4}^{(1)})b_{s,2}^{(1)}}_{P^0} + \underbrace{\eta{b'}_{s,2}^{(1)}}_{P^0}$$

where $\sigma_{s,2}$ is the desired signal, and where $\eta = \frac{h_{s,3}^{(1)}}{h_{s,3}^{(2)}v_{s,3}^{(2)}} - \frac{h_{s,2}^{(1)}}{h_{s,2}^{(2)}}$. With proper choice of precoding scalars, we have

$$\begin{aligned}
\mathbb{E}\left|(\eta - \frac{h_{s,4}^{(1)}}{h_{s,4}^{(2)}}v_{s,4}^{(1)})b_{s,2}^{(1)}\right|^2 &= \mathbb{E}\left|\frac{h_{s,2}^{(1)}}{h_{s,2}^{(2)}}(\frac{h_{s,3}^{(1)}\hat{h}_{s,3}^{(2)}}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}} - 1)(1 - \frac{h_{s,3}^{(1)}\hat{h}_{s,3}^{(2)}}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}})b_{s,2}^{(1)}\right|^2 \\
&= \mathbb{E}\left|\frac{h_{s,2}^{(1)}}{h_{s,2}^{(2)}}\frac{(\tilde{h}_{s,3}^{(1)}\hat{h}_{s,3}^{(2)} - \tilde{h}_{s,3}^{(2)}\hat{h}_{s,3}^{(1)})}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}}\frac{(\tilde{h}_{s,3}^{(2)}\hat{h}_{s,3}^{(1)} - \tilde{h}_{s,3}^{(1)}\hat{h}_{s,3}^{(2)})}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}}b_{s,2}^{(1)}\right|^2 \doteq P^0
\end{aligned}$$

and

$$\mathbb{E}\left|\eta{b'}_{s,2}^{(1)}\right|^2 = \mathbb{E}\left|\frac{h_{s,2}^{(1)}}{h_{s,2}^{(2)}}\frac{(\tilde{h}_{s,3}^{(1)}\hat{h}_{s,3}^{(2)} - \tilde{h}_{s,3}^{(2)}\hat{h}_{s,3}^{(1)})}{h_{s,3}^{(2)}\hat{h}_{s,3}^{(1)}}{b'}_{s,2}^{(1)}\right|^2 \doteq P^0$$

Using ${y'}_{s,2}$ and ${y'}_{s,4}$, receiver 1 removes the interference relating to $b_{s,2}^{(2)} + {b'}_{s,3}^{(2)}$ in ${y'}_{s,3}$, and the power of $b_{s,2}^{(1)}$ and ${b'}_{s,2}^{(2)}$ is reduced below the noise level.

Similarly receiver 1 can also get

$$\sigma_{s,3} = \sigma_{s,2} - \frac{h_{s,3}^{(1)}}{h_{s,3}^{(2)}v_{s,3}^{(2)}}\sigma_{s,1} = -\frac{h_{s,4}^{(1)}}{h_{s,4}^{(2)}}u_{s,4}^{(1)}a_{s,1}^{(1)} - {a'}_{s,3}^{(2)}$$

which will be used later to decode.

c) Quantizing and retransmitting the interference: Up to this point, we have removed the unintended signals from transmitter 1, and now focus on the interference originating from transmitter 2. After the first sub-phase of phase s, transmitter 1 reconstructs $i_{s,1}, \theta_{s,1}$ using its knowledge of delayed CSIT, and quantizes these into

$$\bar{i}_{s,1} = i_{s,1} - \tilde{i}_{s,1}, \quad \bar{\theta}_{s,1} = \theta_{s,1} - \tilde{\theta}_{s,1}$$

requiring a total of $2\alpha\log P$ bits, allowing for bounded noise

$$\mathbb{E}(|\tilde{i}_{s,1}|^2) \doteq \mathbb{E}(|\tilde{\theta}_{s,1}|^2) \doteq 1.$$

Consequently during phase s, a total of $\frac{\alpha}{2}T_s\log P$ bits are accumulated, and will be distributed evenly into the common symbol sets $\{c_{s+1,t}\}_{t=1}^{T_{s+1}}$ of the next phase.

3) Phase S: Phase S has $\frac{T_S}{3}$ sub-phases, each consisting of 3 consecutive time slots. Focusing on $(S,1), (S,2), (S,3)$, we have

$$\boldsymbol{x}_{S,1} = \begin{bmatrix} a_{S,1}^{(1)} \\ c_{S,1} + a_{S,1}^{(1)} \end{bmatrix}, \ \boldsymbol{x}_{S,2} = \begin{bmatrix} b_{S,2}^{(1)} \\ c_{S,2} + b_{S,2}^{(2)} \end{bmatrix},$$
$$\boldsymbol{x}_{S,3} = \begin{bmatrix} a_{S,1}^{(1)} + b_{S,2}^{(1)} \\ c_{S,3} + u_{S,3}^{(2)}a_{S,1}^{(2)} + v_{S,3}^{(2)}b_{S,2}^{(2)} \end{bmatrix}$$

where

$$u_{S,3}^{(2)} = \frac{g_{S,1}^{(2)}\hat{g}_{S,3}^{(1)}}{g_{S,1}^{(1)}\hat{g}_{S,3}^{(2)}}, \qquad v_{S,3}^{(2)} = \frac{h_{S,2}^{(2)}\hat{h}_{S,3}^{(1)}}{h_{S,2}^{(1)}\hat{h}_{S,3}^{(2)}}$$

and where the power and rate are set to

$$\begin{aligned}
&P_S^{(c)} \doteq P, P_S^{(a)} \doteq P^\alpha, P_S^{(b)} \doteq P^\alpha \\
&r_S^{(c)} = 1 - \alpha, r_S^{(a)} = r_S^{(b)} = \alpha.
\end{aligned} \quad (14)$$

As before, receiver 1 decodes and removes the common symbols to get

$$\begin{aligned}
{y'}_{S,1} &= h_{S,1}^{(1)}a_{S,1}^{(1)} + h_{S,1}^{(2)}a_{S,1}^{(2)} \\
{y'}_{S,2} &= h_{S,2}^{(1)}b_{S,2}^{(1)} + h_{S,1}^{(2)}b_{S,2}^{(2)} \\
{y'}_{S,3} &= h_{S,3}^{(1)}(a_{S,1}^{(1)} + b_{S,2}^{(1)}) + h_{S,3}^{(2)}(u_{S,3}^{(2)}a_{S,1}^{(2)} + v_{S,3}^{(2)}b_{S,2}^{(2)}).
\end{aligned} \quad (15)$$

[1]For the case of $c_{s,4}$, note that this is achieved by proper choice of $u_{s,4}^{(1)}$.

which are then linearly combined to get

$$\frac{y'_{S,3}}{h^{(1)}_{S,3}} - \frac{y'_{S,2}}{h^{(1)}_{S,2}} = \underbrace{a^{(1)}_{S,1} + \frac{h^{(2)}_{S,3}}{h^{(1)}_{S,3}} u^{(2)}_{S,3} a^{(2)}_{S,1}}_{P^\alpha} + \underbrace{(\frac{h^{(2)}_{S,3}}{h^{(1)}_{S,3}} v^{(2)}_{S,3} - \frac{h^{(2)}_{S,2}}{h^{(1)}_{S,2}}) b^{(2)}_{S,2}}_{P^0}$$

where

$$\mathbb{E}\left|(\frac{h^{(2)}_{S,3}}{h^{(1)}_{S,3}} v^{(2)}_{S,3} - \frac{h^{(2)}_{S,2}}{h^{(1)}_{S,2}}) b^{(2)}_{S,2}\right|^2 = \mathbb{E}\left|\frac{\tilde{h}^{(2)}_{S,3}\hat{h}^{(1)}_{S,3} - \tilde{h}^{(1)}_{S,3}\hat{h}^{(1)}_{S,3}}{h^{(1)}_{S,3}\hat{h}^{(1)}_{S,3}} b^{(2)}_{S,2}\right|^2 \doteq P^0$$

B. Decoding

1) Phase S: Receiver 1 first decodes $c_{S,t}$ by treating all other signals as noise, and then removes $h^{(2)}_{S,t} c_{S,t}$ from $y_{S,t}$. At this point, for each sub-phase, the receiver experiences an equivalent 2×2 MIMO channel of the form (with bounded noise)

$$\begin{bmatrix} y'_{S,1} \\ \frac{y'_{S,3}}{h^{(1)}_{S,3}} - \frac{y'_{S,2}}{h^{(1)}_{S,2}} \end{bmatrix} = \begin{bmatrix} h^{(1)}_{S,1} & h^{(1)}_{S,2} \\ 1 & \frac{h^{(2)}_{S,3}}{h^{(1)}_{S,3}} u^{(2)}_{S,3} \end{bmatrix} \begin{bmatrix} a^{(1)}_{S,1} \\ a^{(2)}_{S,1} \end{bmatrix}$$

allowing receiver 1 to decode $a^{(1)}_{S,1}, a^{(2)}_{S,1}$. Now receiver 1 can go back one phase and reconstruct $\{\bar{i}_{S-1,t}\}_{t=1}^{T_{S-1}}$ using knowledge of the common symbols of this last phase. Similar actions are performed by receiver 2.

2) Phase $s, s = S-1, \cdots, 2$. As before, receiver 1 first decodes $c_{s,t}$. Using the already decoded $\{c_{s+1,t}\}_{t=1}^{T_{s+1}}$, receiver 1 reconstructs $\{\bar{i}_{s,t}, \bar{\theta}_{s,t}\}_{t=1}^{T_s}$, and for each sub-phase, subtracts $\bar{i}_{s,1}$ to get $\sigma_{s,1}$, up to bounded noise. The same receiver also employs the estimate $\bar{\theta}_{s,1}$ as an extra observation. Thus now receiver 1 sees a 4×4 MIMO channel of the form

$$\begin{bmatrix} y'_{s,1} \\ \sigma_{s,1} \\ \sigma_{s,3} \\ \bar{\theta}_{s,1} \end{bmatrix} = \underbrace{\begin{bmatrix} h^{(1)}_{s,1} & h^{(2)}_{s,1} & h^{(1)}_{s,1} & 0 \\ 1 & \frac{h^{(2)}_{s,3}}{h^{(1)}_{s,3}} u^{(2)}_{s,3} & 1 & \frac{h^{(2)}_{s,3}}{h^{(1)}_{s,3}} u^{(2)}_{s,3} \\ -\frac{h^{(1)}_{s,4}}{h^{(2)}_{s,4}} u^{(1)}_{s,4} & 0 & 0 & -1 \\ 0 & \frac{g^{(2)}_{s,3}}{g^{(1)}_{s,3}} u^{(2)}_{s,3} - \frac{g^{(2)}_{s,1}}{g^{(1)}_{s,1}} & 0 & \frac{g^{(2)}_{s,3}}{g^{(1)}_{s,3}} u^{(2)}_{s,3} \end{bmatrix}}_{A} \begin{bmatrix} a^{(1)}_{s,1} \\ a^{(2)}_{s,1} \\ a'^{(1)}_{s,1} \\ a'^{(2)}_{s,3} \end{bmatrix}$$

where one can check that matrix has a full rank almost surely. With the above linear independence established, we now see that we have accumulated two observations of power $P^{2\alpha}$, and two observations of power P^α, while at the same time, there are two information symbols of power $P^{2\alpha}$ and of rate 2α and two information symbols of power P^α and of rate α. This suffices for receiver 1 to decode $a^{(1)}_{s,1}, a^{(2)}_{s,1}, a'^{(1)}_{s,1}, a'^{(2)}_{s,3}$. The process is the similar for receiver 2.

3) Phase 1: Similarly, receiver 1 first reconstructs $\{\bar{i}_{1,1}, \bar{\theta}_{1,1}\}_{t=1}^{T_1}$ from $\{c_{2,t}\}_{t=1}^{T_2}$ for each sub-phase to get a 3×3 MIMO channel of the form

$$\begin{bmatrix} y_{1,1} \\ \frac{y_{1,3}}{h^{(1)}_{1,3}} - \frac{y_{1,2}}{h^{(1)}_{1,2}} - \bar{i}_{1,1} \\ \bar{\theta}_{1,1} \end{bmatrix} = \begin{bmatrix} h^{(1)}_{1,1} & h^{(2)}_{1,1} & h^{(2)}_{1,1} \\ 1 & \frac{h^{(2)}_{1,3}}{h^{(1)}_{1,3}} u^{(2)}_{1,3} & 0 \\ 0 & \frac{g^{(2)}_{1,3}}{g^{(1)}_{1,3}} u^{(2)}_{1,3} - \frac{g^{(2)}_{1,1}}{g^{(1)}_{1,1}} & -\frac{g^{(2)}_{1,1}}{g^{(1)}_{1,1}} \end{bmatrix} \begin{bmatrix} a^{(1)}_{1,1} \\ a^{(2)}_{1,1} \\ a'^{(2)}_{1,1} \end{bmatrix}$$

where obviously the matrix has a full rank almost surely. Therefore, receiver can decode $a^{(1)}_{1,1}, a^{(2)}_{1,1}$ with rate 1 respectively, and $a'^{(2)}_{1,1}$ with rate $1-\alpha$. Receiver 2 acts the same.

C. DoF calculation

Adding up the private information transmitted over the different phases, we get that

$$d_\Sigma = (\frac{T_1}{3}(6-2\alpha) + \sum_{i=2}^{S-1} \frac{T_i}{4}(12\alpha) + \frac{T_S}{3} 4\alpha)/(\sum_{i=1}^{S} T_i)$$

$$= 3\alpha + (T_1(2 - \frac{11}{3}\alpha) - T_S \frac{5}{3}\alpha)/(\sum_{i=1}^{S} T_i)$$

Considering that $0 \leq \mu \leq 1$, for an asymptotically high S, we get

$$d_\Sigma = 3\alpha + \frac{\frac{T_2}{\xi}(2 - \frac{11}{3}\alpha) - \frac{5}{3} T_2 \mu^{S-3} \gamma\alpha}{\frac{T_2}{\xi} + T_2(\frac{1}{1-\mu} + \mu^{S-3}(\gamma - \frac{\mu}{1-\mu}))} = \frac{6}{5} + \frac{2\alpha(2-3\alpha)}{5(4-7\alpha)}$$

which proves the result, and which additionally shows that the optimal sum DoF $\frac{4}{3}$ is achievable even with $\alpha = \frac{4}{9}$.

IV. Conclusions

This work provided analysis and a novel scheme for the setting of the two-user SISO X channel with imperfect quality current and delayed CSIT, offering insight on how much delayed and current feedback quality suffices to achieve a certain target sum-DoF performance.

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