AN OUTER BOUND ON THE MIMO BC WITH EVOLVING CURRENT CSIT AND PERFECT DELAYED CSIT

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ABSTRACT

We derive a DoF outer bound for the two-user multiple-input multiple-output broadcast channel (MIMO BC) with good-quality delayed channel state information at the transmitter (perfect quality delayed CSIT), and with current CSIT of evolving quality (evolving current CSIT).

1. INTRODUCTION

1.1. MIMO BC model

He here consider the multiple-input multiple-output broadcast channel (MIMO BC), and specifically the symmetric variant where the transmitter has M antennas, and each of the receivers has N receive antennas. The corresponding channel model takes the form

$$\boldsymbol{y}_t^{(1)} = \boldsymbol{H}_t^{(1)}\boldsymbol{x}_t + \boldsymbol{z}_t^{(1)} \quad (1a)$$

$$\boldsymbol{y}_t^{(2)} = \boldsymbol{H}_t^{(2)}\boldsymbol{x}_t + \boldsymbol{z}_t^{(2)} \quad (1b)$$

where $\boldsymbol{H}_t^{(1)} \in \mathbb{C}^{N\times M}, \boldsymbol{H}_t^{(2)} \in \mathbb{C}^{N\times M}$ respectively represent the first and second receiver channels at time t, where $\boldsymbol{z}_t^{(1)}, \boldsymbol{z}_t^{(2)}$ represent unit power AWGN noise at the two receivers, where $\boldsymbol{x}_t \in \mathbb{C}^{M\times 1}$ is the input signal with power constraint $\mathbb{E}[||\boldsymbol{x}_t||^2] \leq P$.

1.2. Performance effect of feedback

We are interested in analyzing the relationship between performance and feedback quality and timeliness. Towards this we focus on the high signal-to-noise ration (high SNR) setting, and offer a degrees-of-freedom (DoF) analysis. Specifically, for an achievable rate pair (R_1, R_2) for the first and second user respectively, the corresponding degrees-of-freedom (DoF) pair (d_1, d_2) is given by

$$d_i = \lim_{P\to\infty} \frac{R_i}{\log P}, \; i = 1, 2 \quad (2)$$

and the corresponding DoF region is then the set of all achievable DoF pairs.

It is well known that feedback greatly affects the above DoF performance. Particularly for the current two-user MIMO BC setting here, it is known (cf. [1]) that perfect channel state information at the transmitter (perfect CSIT), allows for a DoF optimal region of

$$\Big\{ d_1 \leq \min\{M, N\}, \; d_2 \leq \min\{M, N\}, \quad (3)$$

$$d_1 + d_2 \leq \min\{M, 2N\} \Big\} \quad (4)$$

while having no CSIT only offers (cf. [2, 3])

$$\{d_1 + d_2 \leq \min\{M, N\}\}. \quad (5)$$

This gap has sparked substantial interest in finding ways to analyze and optimize the performance of multiuser systems in the presence of feedback that is both of imperfect quality as well as delayed. Such studies include [4, 5, 5–13]. This work here seeks to extend works like [5] and [6] and [7], by considering the idea of evolving current CSIT in the MIMO setting.

1.3. General framework for the channel and evolving feedback

As in the work in [7], we consider communication of an infinite duration n, during which we encounter a random fading process

$$\{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}\}_{t=1}^n \quad (6)$$

as well as encounter a general feedback process that provides CSIT estimates

$$\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t,t'=1}^n \quad (7)$$

(of channel $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$) at any time $t' = [1, \cdots, n]$. As a result, for any specific channel $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$ at a specific time t, there is a set of all available estimates $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'}$. This set can be split into *predicted estimates*

$$\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'<t}$$

that are offered before time t, into the *current estimate*

$$\hat{\boldsymbol{H}}_{t,t}^{(1)}, \hat{\boldsymbol{H}}_{t,t}^{(2)}$$

that is offered exact when $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$ materializes (i.e., at time t), and into the set of *delayed estimates*

$$\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'>t}$$

which are offered at any point after time t. As in [7], we consider to measure feedback quality, by the precision

$$\{(\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}), (\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)})\}_{t,t'=1}^{n} \quad (8)$$

of the estimates at any time t' about any channel $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$.

Any meaningful effort to capture the performance effects of feedback, must by stochastic in nature, considering the manner with which estimation-errors fluctuate. We here place soft and common restrictions on the estimation errors, and only assume that these errors have zero-mean circularly-symmetric complex Gaussian entries, that are *spatially* uncorrelated. Furthermore we ask that the current estimation errors $\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t}^{(1)}$ (resp. $\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t}^{(2)}$) are statistically independent of the channel estimates up to that time $\{\hat{\boldsymbol{H}}_{t,t'}^{(1)}\}_{t'<t}$ (resp. $\{\hat{\boldsymbol{H}}_{t,t'}^{(2)}\}_{t'<t}$).

Furthermore we adhere to the common convention (see [4, 9, 14]) of assuming perfect and global knowledge of channel state information at the receivers (perfect global CSIR), where the receivers know all channel states and all estimates. We will also adopt the common convention (see [9, 14, 15]) of assuming that the current estimation error is statistically independent of current and past estimates. A discussion on this can be found in [7] which argues that this assumption fits well with many channel models, spanning from the fast fading channel (i.i.d. in time), to the correlated channel model as this is considered in [15], to the quasi-static block fading model where the CSIT estimates are successively refined while the channel remains static. Additionally we consider the entries of each estimation error matrix $\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t'}^{(i)}$ to be i.i.d. Gaussian [1].

1.4. Notation

We use $(\bullet)^{\mathsf{T}}$ and $(\bullet)^{\mathsf{H}}$ to denote the transpose and conjugate transpose of a matrix respectively, $||\bullet||_F$ to denote the Frobenius norm of a matrix, $||\bullet||$ to denote the Euclidean norm, and $|\bullet|$ to denote the magnitude of a scalar. $o(\bullet)$ comes from the standard Landau notation, where $f(x) = o(g(x))$ implies $\lim_{x\to\infty} f(x)/g(x) = 0$.

[1]We here make it clear that we are simply referring to the MN entries in each such specific matrix $\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t'}^{(i)}$, and that we certainly do not suggest that the error entries are i.i.d. in time or across users.

Adopting the notation of [7] to the MIMO setting, and after the extra symmetry assumption that the CSIT statistics are the same for the two users, we consider the following

$$\alpha_t \triangleq -\lim_{P\to\infty} \frac{\mathbb{E}[||\boldsymbol{H}_t^{(i)} - \hat{\boldsymbol{H}}_{t,t}^{(i)}||_F^2]}{\log P} \quad (9)$$

$i = 1, 2$, where α_t represents the *current quality exponent* for the CSIT for channel at time t. Furthermore we will use the notation

$$\bar{\alpha} \triangleq \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \alpha_t \quad (10)$$

to denote the average of the quality exponents.

Remark 1 *It can be seen (see [16]) that in the DoF setting of interest we can limit our attention to the range*

$$0 \le \alpha_t \le 1. \quad (11)$$

Additionally, we note that $\alpha_t = \alpha_t = 1$ implies CSIT of (essentially) perfect quality and timing for the channel at time t (i.e., for $\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}$), while $\alpha_t = 1\ \forall t$ implies that feedback - which has sufficiently good quality - comes after the materialization of the channel, i.e., arrives at some point $t' > t$.

Finally we define $\langle\bullet\rangle' \triangleq \min\{\bullet, M\}$,

$$\Omega_{[n]} \triangleq \{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}, \hat{\boldsymbol{H}}_{t,t'}^{(1)}, \hat{\boldsymbol{H}}_{t,t'}^{(2)}, \}_{t=1}^{n}{}_{t'=1}^{n}$$

and $\boldsymbol{y}_{[n]}^{(1)} \triangleq \{\boldsymbol{y}_t^{(1)}\}_{t=1}^{n}, \quad \boldsymbol{y}_{[n]}^{(2)} \triangleq \{\boldsymbol{y}_t^{(2)}\}_{t=1}^{n}$.

2. DOF OUTER BOUND FOR THE SYMMETRIC MIMO BC WITH EVOLVING CURRENT CSIT, PERFECT DELAYED CSIT AND PERFECT GLOBAL CSIR

We proceed with the main DoF outer bound, that holds for a large family of possibly correlated channel processes $\{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}\}_{t=1}^{n}$ and general feedback processes of quality defined by the statistics of $\{(\boldsymbol{H}_t^{(1)} - \hat{\boldsymbol{H}}_{t,t'}^{(1)}), (\boldsymbol{H}_t^{(2)} - \hat{\boldsymbol{H}}_{t,t'}^{(2)})\}_{t=1,t'=1}^{n}$.

Lemma 1 *The DoF region of the two-user MIMO BC is up-*

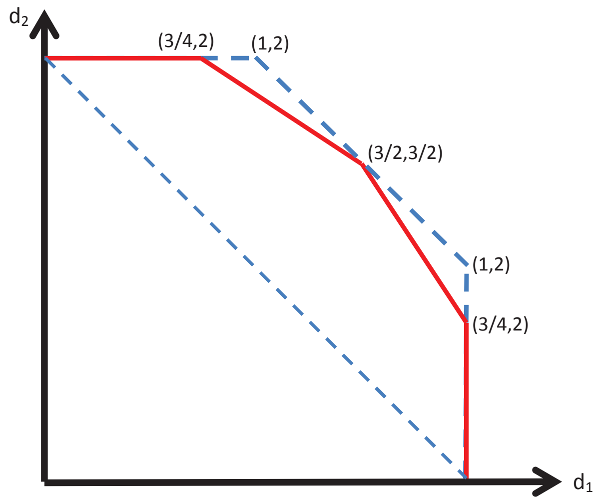

Fig. 1. DoF bound (solid line) for the two-user MIMO BC with $M = 3, N = 2$, where $\alpha_t = 0$ for $t = 0$ modulo 4 and $\alpha_t = 1$ for $t = 1, 2, 3$ modulo 4. Inner dotted line corresponds to no CSIT, and outer dotted line corresponds to perfect CSIT. Note that, despite the absence of perfect CSIT, the sum-DoF achieved is the same as that for the perfect CSIT case.

per bounded as

$$d_1 \leq \min\{M, N\} \quad (12)$$

$$d_2 \leq \min\{M, N\} \quad (13)$$

$$d_1 + d_2 \leq \min\{M, 2N\} \quad (14)$$

$$\frac{d_1}{\min\{M, N\}} + \frac{d_2}{\min\{M, 2N\}} \leq 1 + \frac{\min\{M, 2N\} - \min\{M, N\}}{\min\{M, 2N\}} \bar{\alpha} \quad (15)$$

$$\frac{d_1}{\min\{M, 2N\}} + \frac{d_2}{\min\{M, N\}} \leq 1 + \frac{\min\{M, 2N\} - \min\{M, N\}}{\min\{M, 2N\}} \bar{\alpha}. \quad (16)$$

2.0.1. Proof of outer bound for the symmetric MIMO BC

The proof draws from [9], [11] and [7].

We first design a degraded version of the MIMO BC, by giving the observations and messages of receiver 1 to receiver 2. This allows for

$$nR_1 \leq I(W_1; \boldsymbol{y}_{[n]}^{(1)} | \Omega_{[n]}) + n\epsilon \quad (17)$$

$$nR_2 \leq I(W_2; \boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} | W_1, \Omega_{[n]}) + n\epsilon \quad (18)$$

due to Fano's inequality, due to the basic chain-rule of mutual information, and due to the fact that messages from different users are independent. This now gives that

$$nR_1 \leq h(\boldsymbol{y}_{[n]}^{(1)} \,|\, \Omega_{[n]}) - h(\boldsymbol{y}_{[n]}^{(1)} \,|\, W_1, \Omega_{[n]}) + n\epsilon \quad (19)$$

$$nR_2 \leq h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, \Omega_{[n]}) - h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, W_2, \Omega_{[n]}) + n\epsilon \quad (20)$$

and that

$$\begin{aligned}
&\frac{nR_1}{\langle N \rangle'} + \frac{nR_2}{\langle 2N \rangle'} - \left(\frac{n}{\langle N \rangle'} + \frac{n}{\langle 2N \rangle'}\right)\epsilon \\
&\leq \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)} \,|\, \Omega_{[n]}) + \frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)} \,|\, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, W_2, \Omega_{[n]}) \\
&= \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)} \,|\, \Omega_{[n]}) + \frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)} \,|\, W_1, \Omega_{[n]}) + no(\log P) \qquad (21) \\
&\leq n \log P + no(\log P) + \frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)} \,|\, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_{[n]}^{(1)} \,|\, W_1, \Omega_{[n]}) + no(\log P) \qquad (22) \\
&= \sum_{t=1}^{n} \Bigg(\frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_t^{(1)}, \boldsymbol{y}_t^{(2)} \,|\, \boldsymbol{y}_{[t-1]}^{(1)}, \boldsymbol{y}_{[t-1]}^{(2)}, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_t^{(1)} \,|\, \boldsymbol{y}_{[t-1]}^{(1)}, W_1, \Omega_{[n]}) \Bigg) + n \log P \\
&+ no(\log P) \qquad (23) \\
&\leq \sum_{t=1}^{n} \Bigg(\frac{1}{\langle 2N \rangle'} h(\boldsymbol{y}_t^{(1)}, \boldsymbol{y}_t^{(2)} \,|\, \boldsymbol{y}_{[t-1]}^{(1)}, \boldsymbol{y}_{[t-1]}^{(2)}, W_1, \Omega_{[n]}) \\
&- \frac{1}{\langle N \rangle'} h(\boldsymbol{y}_t^{(1)} \,|\, \boldsymbol{y}_{[t-1]}^{(1)}, \boldsymbol{y}_{[t-1]}^{(2)}, W_1, \Omega_{[n]}) \Bigg) + n \log P \qquad (24) \\
&\leq \frac{1}{\langle 2N \rangle' \langle N \rangle'} \sum_{t=1}^{n} \Big((\langle 2N \rangle' - \langle N \rangle') \langle N \rangle' \alpha_t \log P \Big) \\
&+ n \log P + no(\log P) \qquad (25) \\
&= \frac{n}{\langle 2N \rangle' \langle N \rangle'} \big((\langle 2N \rangle' - \langle N \rangle') \langle N \rangle' \bar{\alpha} \log P \big) \\
&+ n \log P + no(\log P) \\
&= \bar{\alpha} \frac{n(\langle 2N \rangle' - \langle N \rangle')}{\langle 2N \rangle'} \log P + n \log P + no(\log P). \qquad (26)
\end{aligned}$$

In the above, (21) is due to the fact that knowledge of $\{W_1, W_2, \Omega_{[n]}\}$ allows for reconstruction of $\boldsymbol{y}_{[n]}^{(1)}, \boldsymbol{y}_{[n]}^{(2)}$ up to noise level, while (22) is due to the fact that $h(\boldsymbol{y}_{[n]}^{(1)} \,|\, \Omega_{[n]}) \leq \langle N \rangle' \log P + o(\log P)$. Additionally (23) is due to the chain rule of differential entropy, (24) is due to the fact that conditioning reduces differential entropy, and (25) is directly from [11, Proposition 4] after setting $U = \{\boldsymbol{y}_{[t-1]}^{(1)}, \boldsymbol{y}_{[t-1]}^{(2)}, W_1, \Omega_{[n]}\} \setminus \{\boldsymbol{H}_t^{(1)}, \boldsymbol{H}_t^{(2)}, \hat{\boldsymbol{H}}_{t,t}^{(1)}, \hat{\boldsymbol{H}}_{t,t}^{(2)}\}$ [2]. Finally the above implicitly assumed perfect knowledge of delayed CSIT.

The above gives the bound in (15). Interchanging the roles of the users gives the bound in (16). The bounds in (12),(13) are basic single-user constraints, while the bound in (14) corresponds to an assumption of user cooperation. This concludes the proof.

3. CONCLUSIONS

The work, extending on recent work on the MISO BC, considered the symmetric MIMO BC, where symmetry was in terms of the statistics of feedback across the two users, in terms of the number of receive antennas at the two users. In the presence of perfect delayed CSIT, the work made progress towards bounding the tradeoff between performance, and address issues of feedback timeliness and quality. This DoF bound captures different DoF bounds that have been considered in the literature for the setting of the two-user BC.

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