Toward Multi-Layer Partial Frequency Reuse in Future Mobile Communication Systems

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***Abstract*—Frequency reuse improves system capacity but sacrifices cell-edge user equipments (UEs) due to inter-cell interference. To solve this problem, fractional frequency reuse (FFR) was proposed, which ameliorates cell-edge UEs by using a larger reuse factor for them than for cell-center UEs. In this paper, we investigate a common type of FFR schemes, called partial frequency reuse (PFR), and find that cell-edge UEs are not necessarily the worst ones. Instead, UEs on the edge of the inner area may be the worst in many cases. To improve max-min fairness without degrading system capacity, we study several multi-layer PFR schemes. By extensive simulations, we propose a 4-layer scheme with 7-portions of spectrum, which could improve both max-min fairness and system capacity, as long as the inner radius, the reuse factors of different layers, and the ratio of middle and outer areas are carefully designed.**

I. Introduction

Since the first and second generations of mobile communication systems, frequency reuse has been considered as a key technique to improve system capacity [1]. However, full reuse of the spectrum, i.e. reuse-1, results in serious inter-cell interference, which seriously degrades the performance of cell-edge user equipments (UEs). In near future communication systems, the existence of plenty of broadband UEs demands large system capacity, so per-cell reuse (such as reuse-3) seems to have a too small spectrum utilization (i.e. the portion of spectrum used by one cell divided by the whole spectrum). Therefore, frequency reuse in the fourth generation (4G) and beyond systems tends to consider compound scheme with an average reuse factor between 1 and 3, i.e. fractional frequency reuse (FFR) [2]–[4].

Seen from the inter-cell interference coordination (ICIC) point of view, FFR is one category called static ICIC. This type of schemes has a long resource allocation (RA) period, e.g. days, so they basically decide the RA strategy by geographic area the UEs belonging to, not by instantaneous channel states or traffic conditions [3]. Another category, dynamic ICIC, has a period of milliseconds and the RA strategy is decided by the dynamic change of the channel states and the traffics. Between them, there is the third category, called semi-static ICIC, whose RA decision is made by the UEs' geographic locations and their traffics but not by their channel states. Among the three categories, static ICIC is the easiest to realize during network planning, so various FFR schemes become the most common methods for ICIC and have been widely studied in the past decade [3], [4].

The main idea of FFR is to divide each cell into inner and outer areas. The inner area is usually a disc around the base station (BS), while the outer area is usually a ring centered on the BS with a large inner radius [9]. Hence, the outer area should be carefully treated to decrease inter-cell interference. In the literature, there are plenty of FFR schemes, including partial frequency reuse (PFR), soft frequency reuse (SFR), flexible fractional frequency reuse (FFFR), and sectorized fractional frequency reuse (SFFR). PFR, also called strict FFR or FFR with full isolation, completely isolates the spectrum for inner and outer areas, so inner areas and outer areas never use the same portion of spectrum. Instead of complete isolation, in SFR and FFFR, the spectrum used by the inner area of one cell could be also used by the outer area of its adjacent cells and vise versa [3], [4]. Since the spectra are completely isolated in PFR, there is no gain by changing the transmission power ratio between the inner and the outer areas, so PFR is probably the easiest to realize in all the above schemes.

Traditional PFR scheme divides the whole spectrum into 4 portions. All the inner layers, i.e. the gray area in each cell shown in Fig. 1(a), use the same portion of spectrum, i.e. with a reuse factor equaling 1. The outer layers use 3 portions of the spectrum with reuse factor equaling 3, i.e. outer layers of adjacent cells do not use the same portion of spectrum. Compared with reuse-1, traditional PFR decreases inter-cell interference on cell-edge UEs by using a larger reuse factor, hence it should be able to improve the performance of cell-edge UEs. Compared with reuse-3, traditional PFR might have a larger system capacity because it has a larger (or even much larger) spectrum utilization.

However, traditional PFR does not fully solve the problem. The smaller the inner radius is, the smaller is the system capacity. However, by increasing the inner radius, the UEs on the edge of inner layer become the poorest. Therefore, to improve max-min fairness and keep a high system capacity, we should consider to improve the performance of the worst UEs, and our idea is to divide the whole cell into more than 2 layers and set them to different reuse factors. In this paper, we study one 3-layer scheme and four 4-layer schemes. They set different reuse factors and achieve different performance. Some of them are with high system capacity, while some are with high performance for cell-edge UEs. In the end, we find a 4-layer PFR scheme achieving both higher system capacity and max-min fairness than traditional PFR.

The rest of the paper is organized as follows: Section

II provides a summary of existing studies on multi-layer schemes; Section III gives the system model and explains how the following performance evaluations are done; Section IV describes several multi-layer PFR schemes one by one; Section V evaluates the performance of all the proposed schemes and also the traditional scheme; in the end, the paper is concluded in Section VI.

II. Related Work

Although FFR has been widely studied in recent years [3]–[8], researches on multi-layer PFR scheme are still quite limited.

D. Liang and W. Wang described a 3-layer scheme [10], in which the outer layer used one portion of the spectrum, the middle layer used the remaining, and the inner layer used the full spectrum. In their scheme, each cell used twice the whole spectrum. Instead of working on reuse pattern, such as the optimal inner radius, their scheme focused on power allocation for inner, middle and outer layers. Z. Xie and B. Walke proposed an enhanced FFR scheme with 3 layers [11], which used reuse-1 and low power for inner layer, reuse-3 and moderate power for middle layer, and reuse-9 and high power for outer layer. Simulations showed that the scheme performed better than traditional schemes in many aspects. C. Kosta *et al.* proposed a 4-layer sectorized SFR 3/7 scheme [12], in which each layer was interfered by equivalently two circles in the 6 adjacent cells. Simulations showed that the scheme could improve spectral efficiency in heterogeneous cellular scenario. R. Ghaffar and R. Knopp proposed a 3-layer scheme [13], which divided each cell into 3 layers and the whole spectrum into 4 portions. That scheme is described in Subsection IV.A. It led to 33% improvement of the average spectral efficiency, but increased inter-cell interference. F. B. Mugdim described a 3-layer scheme [14] which used different reuse factors for different layers. The inner layer used reuse-1, the middle layer used reuse-3, while the outer layer used reuse-7. The borders between adjacent layers were adjusted flexibly. E. Haro *et al.* proposed a 3-layer scheme [15] which divided the whole spectrum into 3 portions, so each layer used exactly one portion. Moreover, the power for inner, middle, and outer layers gradually increased. Simulations showed that the scheme had a lower cell throughput than reuse-1 but improved the cell edge UEs. A survey [4] on FFR-related techniques was provided, which referred to [13]–[15] as the category of FFR with multiple user classes.

To sum up, the above studies indicate that multi-layer PFR might be a good choice for static ICIC, so this paper provides extensive simulations on several multi-layer PFR schemes and compares them with our proposed schemes. In the end, we find that one of our proposed schemes out-performs many of the above schemes in terms of max-min fairness and system capacity.

III. System Model

When a cell is divided into multiple layers, we assume that the size of each portion of spectrum corresponds to the area of each layer. Combining with the assumption that UEs are uniformly distributed in each cell, each UE gets the same bandwidth. Capacity of each UE is calculated by

$$C = B\log(1+\frac{S}{N_0+I}), \tag{1}$$

where B is the assigned bandwidth to this UE, S is the received power of the useful uplink signal, I is the power of the total interference, and N_0 is the variance of additive white Gaussian noise (AWGN).

Since, in reality, UEs' traffic is not obviously correlated with their positions in the cell, we simply assume that the resource blocks (RBs) are assigned to the UEs in a generally random manner, so interference from adjacent cells should be calculated averagely. Taking the traditional 2-layer PFR scheme in Fig. 1(a) as an example, average capacity of cell-edge UEs can be calculated as

$$\begin{aligned} &C_{outer\_edge} = \\ &\frac{1}{(\pi R^2-\pi x^2)^6}\int_0^{2\pi}\int_x^R \cdots \int_0^{2\pi}\int_x^R \\ &\frac{B_T}{\pi R^2}\cdot\frac{R^2}{3R^2-2x^2}\log\big(1+\frac{\frac{GP_t}{R^2}}{N_0+\sum_{j=1}^{6}\frac{GP_t}{d_j^2+r_j^2-2d_jr_j\cos\theta_j}}\big) \\ &r_1dr_1d\theta_1...r_6dr_6d\theta_6, \end{aligned} \tag{2}$$

where B_T is the total system bandwidth, x is the inner radius, R is the cell radius, P_t is the transmission power, d_j is the distance between the central interfered BS and the BS the interfering UE belongs to, r_j is the distance between the interfering UE and its own BS, θ_j is the angle between d_j and r_j, $G=\frac{G_tG_r\lambda^2}{4\pi^2}$ is a constant related with the transmission antenna gain G_t, the reception antenna gain G_r, and the light wavelength λ. Note that the factor $\frac{R^2}{3R^2-2x^2}$ is obtained by assuming that each UE, no matter in the inner layer or the outer layer, gets the same amount of resource.

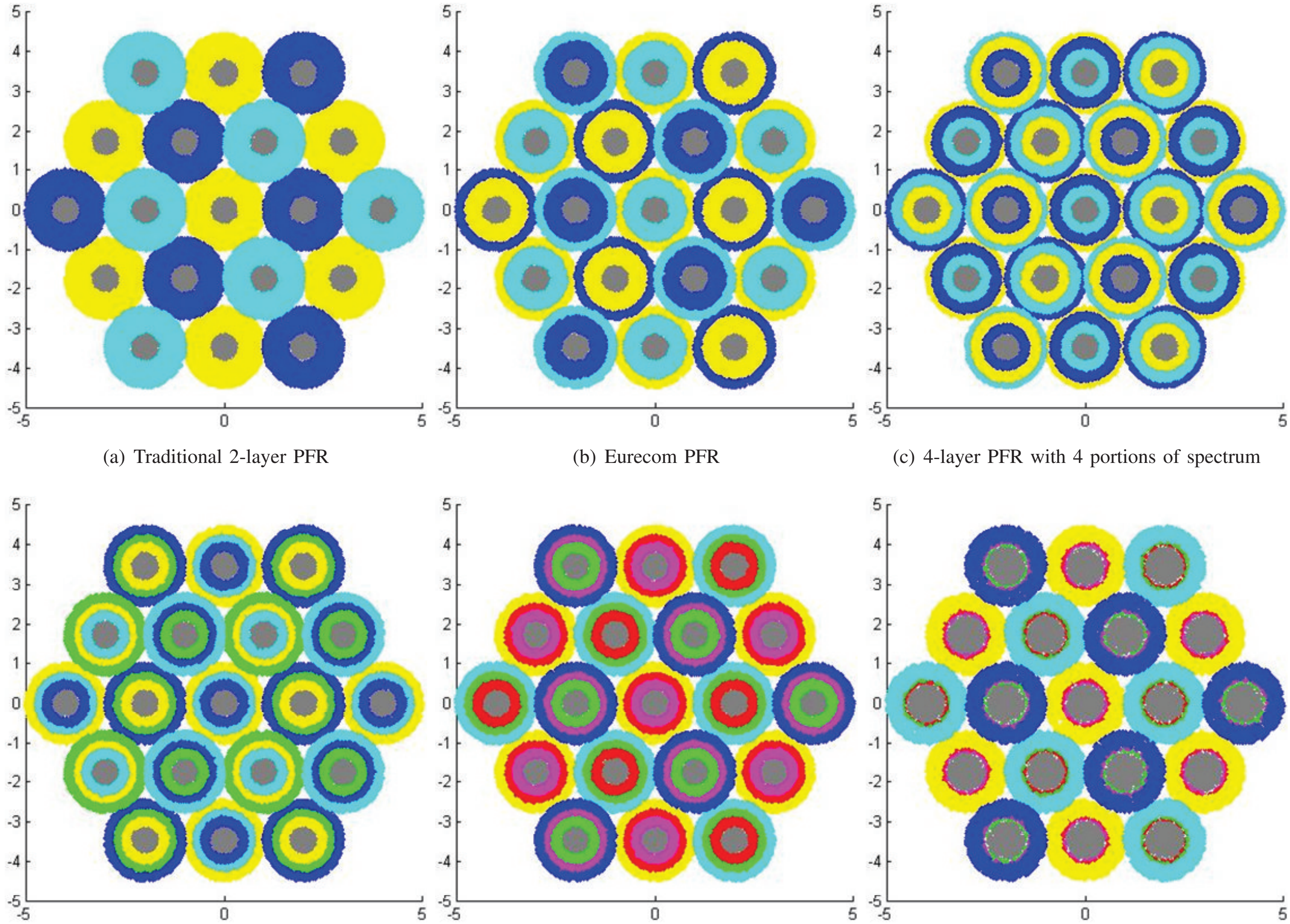
Similarly, capacity of the UEs on the edge of the inner layer can be represented by

$$\begin{aligned} &C_{inner\_edge} = \\ &\frac{1}{(\pi x^2)^{18}}\int_0^{2\pi}\int_0^x \cdots \int_0^{2\pi}\int_0^x \\ &\frac{B_T}{\pi R^2}\cdot\frac{R^2}{3R^2-2x^2}\log\big(1+\frac{\frac{GP_t}{R^2}}{N_0+\sum_{j=1}^{18}\frac{GP_t}{d_j^2+r_j^2-2d_jr_j\cos\theta_j}}\big) \\ &r_1dr_1d\theta_1...r_{18}dr_{18}d\theta_{18}. \end{aligned} \tag{3}$$

To calculate the average capacity of the outer layer, we need further integrate on the central interfered cell based on (2), given by

$$\begin{aligned} &\overline{C_{outer}} = \frac{1}{(\pi R^2-\pi x^2)^7}\int_0^{2\pi}\int_x^R \cdots \int_0^{2\pi}\int_x^R \\ &\frac{B_T}{\pi R^2}\cdot\frac{R^2}{3R^2-2x^2}\log\big(1+\frac{\frac{GP_t}{s^2}}{N_0+\sum_{j=1}^{6}\frac{GP_t}{d_j^2+r_j^2-2d_jr_j\cos\theta_j}}\big) \\ &r_1dr_1d\theta_1...r_6dr_6d\theta_6sdsd\theta. \end{aligned} \tag{4}$$

Similarly, the average capacity of the inner layer can be



(a) Traditional 2-layer PFR

(b) Eurecom PFR

(c) 4-layer PFR with 4 portions of spectrum

(d) 4-layer PFR with 5 portions of spectrum

(e) 4-layer PFR with 7 portions of spectrum (equal areas for middle and outer layers)

(f) Near-optimal 4-layer PFR with 7 portions of spectrum (ratio between middle and outer layers = 1/5, inner radius = 0.53×cell radius)

Fig. 1. Cell division pattern of various schemes.

calculated as

$$\overline{C_{inner}} = \frac{1}{(\pi x^2)^{19}} \int_0^{2\pi} \int_0^{x} \cdots \int_0^{2\pi} \int_0^{x} \frac{B_T}{\pi R^2} \cdot \frac{R^2}{3R^2 - 2x^2} \log\left(1 + \frac{\frac{GP_t}{s^2}}{N_0 + \sum_{j=1}^{18} \frac{GP_t}{d_j^2 + r_j^2 - 2 d_j r_j \cos\theta_j}}\right) r_1 dr_1 d\theta_1 \ldots r_{18} dr_{18} d\theta_{18} s ds d\theta. \tag{5}$$

Apparently, the above integrals are too complex to easily reach any closed-form result. We resort to Monte Carlo integration techniques to compute them. The performance presented in Section V are based on such Monte Carlo integration.

IV. Multi-Layer PFR

A. 3-Layer PFR with 4 Portions of Spectrum

In this subsection, we describe in details the scheme proposed in [13]. In the following, this scheme is called Eurecom PFR, in order to distinguish it with our proposals in this paper. We describe this scheme in details because it was considered as a revised scheme for the traditional 2-layer scheme and it will be compared with our proposals later.

This scheme divides each cell into 3 layers and divides the whole spectrum into 4 portions. The 3 layers are called inner, middle, and outer layers, respectively. The inner layer uses reuse-1 with 1 portion of spectrum, while the middle and the outer layers both have a reuse factor equaling 3 using the other 3 portions. Therefore, the middle and outer layers together have a reuse factor equaling 3/2 with the same 3 portions of spectrum, as shown in Fig. 1(b). To minimize inter-cell interference, middle layers of adjacent cells do not use the same portion of spectrum, neither outer layers of adjacent cells.

Since the same portion of spectrum is used for both middle and outer layers (in adjacent cells), we set the areas of middle and outer layers equaling to each other, which guarantees hard fairness, i.e. each UE is assigned the same bandwidth.

Mapping the inner and middle layers in this 3-layer scheme to the inner layer in the traditional 2-layer scheme, we find that this 3-layer scheme could improve the performance of the UEs on the edge of the original inner layer (i.e. the UEs on the edge of the middle layer in Fig. 1(b)), which is exactly the objective to propose this new scheme. By contrast, the drawback of this scheme is the degradation of cell-edge UEs, which is caused by the decrease of the reuse factor of outer layer in the traditional scheme (i.e. 3) to the compound reuse factor of the middle and outer layers in this scheme (i.e. 3/2). In a word, this change increases the inter-cell interference for cell-edge UEs.

B. 4-Layer PFR with 4 Portions of Spectrum

Compared with traditional PFR, Eurecom PFR has a larger spectrum utilization and improves the performance of UEs on the edge of the middle layer. This idea motivates us to further increase spectrum utilization, so a 4-layer PFR with 4 portions of spectrum is studied in this subsection.

In this scheme, each cell is divided into 4 layers: inner layer, middle-1 layer, middle-2 layer, and outer layer. The whole spectrum is divided into 4 portions, and each layer uses exactly one portion. The difference between cells is the order of portions used for middle and outer layers. As shown in Fig. 1(c), the middle and outer layers of the central cell uses the portions of spectrum in azure, navy-blue, and yellow, respectively. Three cells around it uses navy-blue, yellow, and azure. While another three cells around it uses yellow, azure, and navy-blue. In this way, middle-1 layers of adjacent cells never use the same portion of spectrum, neither middle-2 layers nor outer layers.

Compared with Eurecom PFR, this scheme has a larger spectrum utilization, i.e. each cell uses the whole spectrum indicating a reuse-1 scheme for the whole cell. For each of the middle and outer layers, it has a reuse factor equaling 3, but it could not obviously improve any UE's performance in any layer. Taking the outer layer as an example, we call it 'reuse-3' for checking only the outer layers. In fact, each adjacent cell uses the same portion of spectrum, but in different layers, so inter-cell interference is not decreased at all.

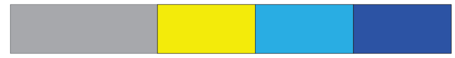
Note that this scheme still belongs to the category of PFR. This special case happens only when there are 3 or more layers. Imagine that the middle and outer layers could be merged as one single layer which makes it a reuse-1 scheme with spectrum division between inner and outer layers. The inner and the outer layers are completely isolated, which is the key feature of PFR. Therefore, this scheme is not an SFR scheme, although each cell uses the complete spectrum.

C. 4-Layer PFR with 5 Portions of Spectrum

The study on the above scheme motivates us to set the compound reuse factor of the whole cell larger than 1, because inter-cell interference might be seriously large otherwise. Therefore, in this subsection, we consider a new scheme which divides each cell into 4 layers but the whole spectrum into 5 portions, as shown in Fig. 1(d).

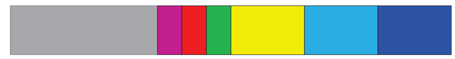
In this scheme, the inner layers of all the cells reuse the same portion, while the two middle layers and the outer layer reuse the rest 4 portions. The reuse factors of a middle or outer layer is 4, while the compound reuse factor for middle and outer layers is 4/3.

Compared with the scheme in the previous subsection, each middle or outer layer is only interfered by 4 adjacent cells in this scheme, which decreases inter-cell interference. Compared with Eurecom PFR, this scheme increases spectrum utilization, so it might increase the capacity, especially for inner and middle-1 layers, but outer layer encounters more inter-cell interference, so the capacity of outer layers and that of the cell-edge UEs might be worse than before. Moreover, the UEs on the edge of middle-2 layers are also interfered by adjacent cells more than Eurecom PFR, so the capacity of middle-2 layers could be also worse.



(a) Traditional PFR with 2 layers

(b) 4-layer PFR with 7 portions of spectrum



(c) 4-layer PFR with 7 portions of spectrum, unequal areas for middle and outer layers

Fig. 2. Division of the whole spectrum in various schemes.

D. 4-Layer PFR with 7 Portions of Spectrum

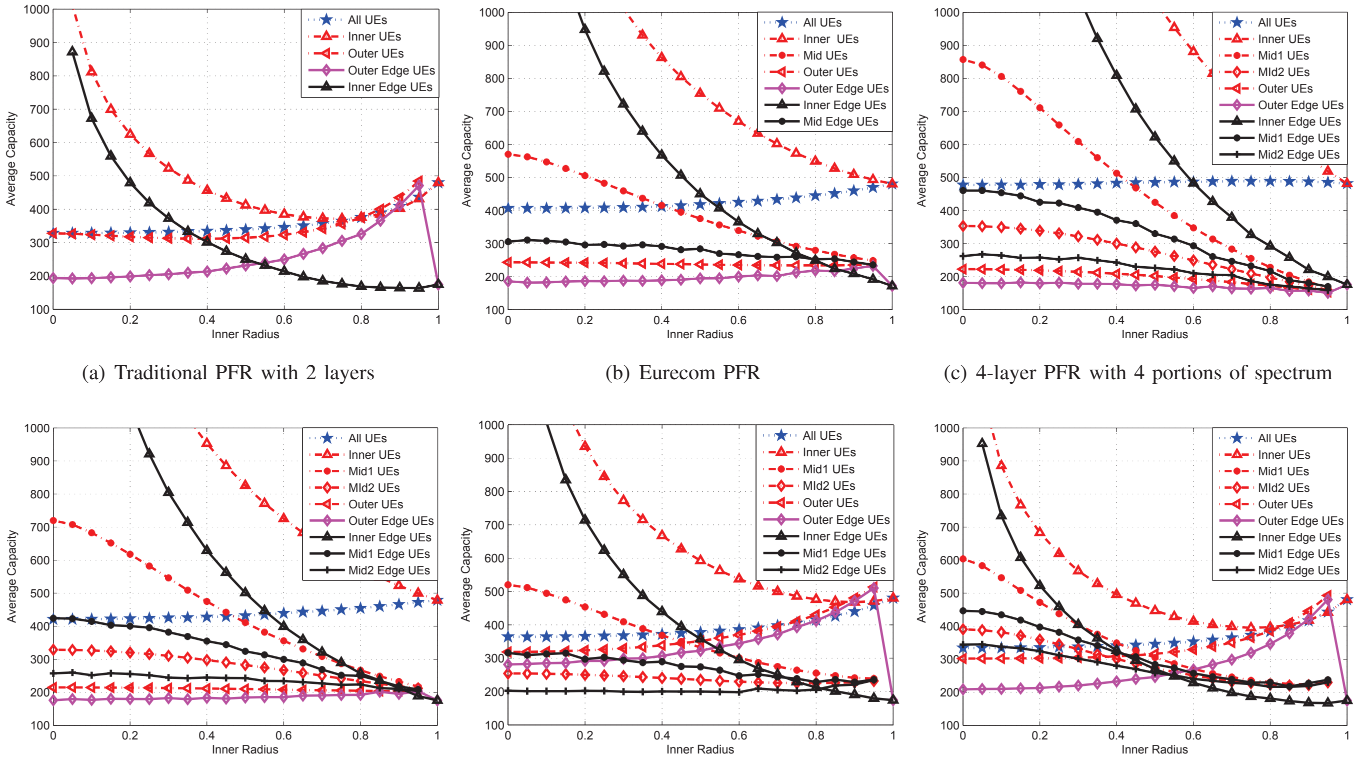
The spectrum utilization of the scheme in the previous subsection is already quite large, but the inter-cell interference results in low capacity for middle-2 and outer layers.

To further decrease inter-cell interference for these layers, we propose a 4-layer PFR scheme with 7 portions of spectrum in this subsection. For the 7 portions of spectrum, inner layers reuse 1 portion, middle layers reuse 3 portions, while outer layers reuse the rest 3 portions, as shown in Fig. 1(e). Therefore, inner layers have a reuse factor equaling 1, middle layers have a reuse factor equaling 3/2, while outer layers have a reuse factor equaling 3. In this way, the reuse factors of inner, middle, and outer layers gradually increase, which guarantees sustainable inter-cell interference for middle and outer layers.

Compared with the traditional 2-layer PFR scheme in Fig. 1(a), the spectrum utilization increases because middle and outer layers together use 3 portions out of 6, so the performance of inner and outer layers should be improved. Meanwhile, the reuse factor of middle layers is smaller, which decreases their performance, leading to a tradeoff between middle and outer layers. To sum up, this scheme tends to increase both system capacity and max-min fairness. UEs with the worst performance might be those on the edges of middle-2 layers.

E. 4-Layer PFR with 7 Portions of Spectrum, unequal areas for middle and outer layers

Even if we divide the cells into the same number of layers and the spectrum into the same number of portions, the performance could be also largely different if the areas of different layers change. In the literature, there are plenty of studies searching for the optimal inner radius for the traditional 2-layer PFR [5]–[8]. For the 4-layer scheme in this subsection, the corresponding operation is to change the areas of the inner, middle, and outer layers.



(a) Traditional PFR with 2 layers

(b) Eurecom PFR

(c) 4-layer PFR with 4 portions of spectrum

(d) 4-layer PFR with 5 portions of spectrum

(e) 4-layer PFR with 7 portions of spectrum

(f) Near-optimal 4-layer PFR with 7 portions of spectrum (ratio between middle and outer layers = 1/5, inner radius = 0.53×cell radius)

Fig. 3. Performance comparison of various schemes.

To further increase max-min fairness of the scheme in the previous subsection, we now consider the case where the areas of middle and outer layers are unequal, as shown in Fig. 1(f), such that the UEs on the edge of middle-2 layers can be improved.

Moreover, in Fig. 2, we demonstrate the division of the spectrum in the traditional scheme, the 4-layer PFR with 7 portions of spectrum in the previous subsection, and the scheme in this subsection.

V. Performance Evaluation

In this section, we evaluate the performance of all the above schemes in a common scenario described below.

Without loss of generality we focus on the average total capacity of a single central cell. By assuming that the interference from farer BSs is negligible, we evaluate the performance of such a cell surrounded by 18 cells, as shown in Fig. 1. Cell radii are all 1 km, while the distances between adjacent BSs are 2 km. In each cell, there are 100000 static UEs uniformly distributed, and the BSs are deployed in the center of each cell. The main simulation parameters are listed in Table I.

Each UE could be interfered by any UE with equal probability from a layer with the same color in an adjacent cell, so the power of the total interference to each UE is obtained by the sum of the interference from each randomly chosen UE in the corresponding layer of each adjacent cell. In this way,

TABLE I. Main Parameters in Simulations

Parameter	Value
Number of cells	19
Cell radius	1 km
Inter-BS distance	2 km
Number of UEs per cell	100000
Transmission power	1 W
Pathloss exponent	2
AWGN variance	70 dBm
Total spectrum	20 MHz
Edge width of a layer	0.002 km

the average capacity of a number of UEs represents the real average capacity of such a layer under the total interference from corresponding layers of adjacent cells. The edge of a cell is defined as 0.002×cell radius, i.e. a 2 meters circle. Similarly, the edge of a layer is also a 2-meter circle on the edge of that layer.

For each scheme above, we show the average capacity of all the UEs, the average capacity of UEs in each layer, and the average capacity of UEs on the edge of each layer (including cell-edge UEs in the outer layer). We set the x-axis to be the inner radius because many studies [5]–[8] showed that inner radius is a key factor affecting the performance.

By comparing the simulation result in Fig. 3, we find the following features:

1) average capacity of all UEs for a given scheme almost

TABLE II. SUMMARY OF PERFORMANCE EVALUATION

Scheme	All UEs	Outer-edge UEs	Max-Min Fairness
Traditional 2-layer PFR	-	-	-
Eurecom PFR	↑	↓	↓
Fig. 1(c)	↑	↓	↓
Fig. 1(d)	↑	↓	↓
Fig. 1(e)	↑	↑	↓
Fig. 1(f)	↑	↑	↑

always increases with regard to inner radius. This indicates that reuse-1 actually corresponds to the maximum system capacity and explains the reason that FFR schemes are supposed to be better than traditional per-cell reuse schemes in 2G.

2) average capacity of cell-edge UEs generally increases with regard to inner radius, thanks to the increase of spectrum utilization. However, once the inner radius equals cell radius, cell-edge UEs drop to a very low performance. That is because the interference from adjacent cells suddenly becomes coming from the inner layer instead of the middle and outer layers as before. This explains why, to improve cell-edge UEs, FFR scheme should be considered instead of reuse-1.

3) schemes with large spectrum utilization, such as the schemes shown in Fig. 1(b), 1(c), 1(d), usually sacrifice some UEs' benefits. The above three schemes all lead to very low performance for cell-edge UEs, as shown in Fig. 3(b), 3(c), 3(d).

4) cell-edge UEs are not necessarily those with the worst achievable rate. When the inner radius is large, UEs on the edge of the inner layer could be worse than cell-edge UEs due to the difference of their reuse factors. Therefore, for most schemes, there is a performance tradeoff between cell-edge UEs and the UEs on the edge of the inner layer. We find that inner radius around $0.53\times$cell radius is a good tradeoff in traditional PFR scheme and our proposed scheme in Fig. 1(f).

5) When we divide each cell into inner, middle, and outer layers, we find that the inner layer should not be too small, otherwise the spectrum utilization is too small which degrades the performance of all the UEs. Meanwhile, we find that the outer layer should not be too small too, otherwise the UEs on the edge of the middle layer will be too far from its BS which degrades these UEs' performance. Therefore, the middle area should be quite thin, as shown in Fig. 3(f).

To sum up, we obtain a near-optimal 4-layer scheme, as shown in Fig. 3(f). The ratio between outer area and inner area is set to 5. The case with inner radius around $0.53\times$cell radius in Fig. 3(f) and the case in Fig. 3(a) both correspond to max-min fairness. By comparing them, we find that our proposed scheme could improve the max-min capacity from around 235 bps to around 253 bps (i.e. about 7.7% amelioration). Meanwhile, the average capacity of all the UEs increases from around 341 bps to around 347 bps (i.e. about 1.8% amelioration). Therefore, we conclude that our new scheme in Fig. 1(f) could improve both max-min fairness and system capacity.

In the end, we summarize the performance of all the schemes we evaluated, as shown in Table II. The traditional 2-layer PFR is used as a benchmark, and the other schemes are compared with it. We can see that all the other schemes improve the average capacity, but the last scheme is the only one that also improves both cell-edge UEs and max-min fairness.

VI. CONCLUSION

Through extensive analysis on frequency reuse techniques, we found out that multi-layer PFR represented a best candidate choice for 4G and beyond frequency reuse. We designed a series of multi-layer PFR schemes with the objective of improving the worst UEs' performance. Based on extensive simulations, we found a near-optimal 4-layer scheme, which achieved a good tradeoff between cell-edge UEs and the UEs on the edge of the inner or middle layers. In details, the inner radius was set to be around $0.53\times$cell radius, the ratio between the middle and outer areas was set to 1/5, and the reuse factors for inner, middle, and outer layers were set to 1, 1.5, and 3, respectively. Compared with the traditional 2-layer PFR scheme, this new scheme improved the worst UEs' capacity by 7.7% and the system capacity by 1.8%. A near future work is to extend this study to heterogeneous cellular network with dense femtocells.

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