CHANNEL-BASED BLIND MULTICHANNEL IDENTIFICATION WITHOUT ORDER OVERESTIMATION PROBLEMS

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ABSTRACT We investigate a new approach for blind identification of multiple FIR channels that appears to be robust to the channel length overestimation. The linear prediction approach proposed in [1] constitutes a robust approach since it provides a consistent channel estimate if the channel order is overestimated. However, we focus here on methods that are parameterized by the channel directly such as the (signal) subspace fitting technique. To make the optimization criterion in these approaches well defined, a constraint on the channel coefficients has to be added. Typically, the unit norm constraint is used. It is the use of this constraint that leads to order overestimation problems. In our approach we replace this constraint by a unit norm for only the first vector coefficient of the vector channel. Our simulations demonstrate that the channel estimate obtained in this way is robust to order overestimation. Furthermore, if the exact quantities are used in the optimization criterion, the proposed channel estimate is the correct channel (up to the usual scaling factor) even if the order is overestimated. Hence, our channel estimate is consistent even with order overestimation.

1. INTRODUCTION.

In digital communications and especially in mobile radio communications systems, symbols are transmitted through unknown channels. The goal of blind identification is to identify the unknown channel using the received signal only. Oversampling the received signal leads to a Single Input Multiple Outputs (SIMO) vector channel representation. The multiple FIR channels we obtain in this representation are in that case due to oversampling of a single received signal, but can also be obtained from multiple received signals from an array of antennas (in the context of mobile digital communications [1],[2],[4]) or from a combination of both. To further develop the case of oversampling, consider linear digital modulation over a linear channel with additive noise so that the received signal $y(t)$ has the following form

$$y(t) \;=\; \sum_k h(t-kT)a(k) \;+\; v(t) \qquad (1)$$

In (1) $a(k)$ are the transmitted symbols, T is the symbol period and $h(t)$ is the channel impulse response. The channel is assumed to be FIR with length NT. If the received signal is oversampled at the rate $\frac{m}{T}$ (or if m different samples of the received signal are captured by m sensors every T seconds, or a combination of both), the discrete input-output relationship can be written as:

$$\mathrm{y}(k) = \sum_{i=0}^{N-1} \mathrm{h}(i)a(k-i) + \mathrm{v}(k) = \mathrm{H}\, A_N(k) + \mathrm{v}(k) \qquad (2)$$

where $\mathrm{y}(k) = [y_1^H(k)\cdots y_m^H(k)]^H$, $\mathrm{h}(i) = \left[h_1^H(i)\cdots h_m^H(i)\right]^H$, $\mathrm{v}(k) = [v_1^H(k)\cdots v_m^H(k)]^H$, $\mathrm{H} = [\mathrm{h}(N-1)\cdots\mathrm{h}(0)]$, $A_N(k) = \left[a(k-N+1)^H \cdots a(k)^H\right]^H$ and superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1}\mathrm{h}(i)z^{-i} = [\mathrm{H}_1^H(z)\cdots\mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function, and $\mathrm{h} = \left[\ \mathrm{h}^H(N-1)\cdots\mathrm{h}^H(0)\ \right]^H$. Consider additive independent white Gaussian circular noise $\mathrm{v}(k)$ with $r_{\mathrm{vv}}(k-i) = \mathrm{E}\,\mathrm{v}(k)\mathrm{v}(i)^H = \sigma_v^2 I_m\,\delta_{ki}$. Assume we receive M samples:

$$\mathrm{Y}_M(k) \;=\; \mathcal{T}_M(\mathrm{H}\,)\,A_{M+N-1}(k) + \mathrm{V}_M(k) \qquad (3)$$

where $\mathrm{Y}_M(k) = [\mathrm{y}^H(k-M+1)\cdots\mathrm{y}^H(k)]^H$ and similarly for $\mathrm{V}_M(k)$, and $\mathcal{T}_M(\mathrm{H}\,)$ is a block Toepliz matrix with M block rows and $[\mathrm{H}\ \ 0_{m\times(M-1)}]$ as first block row. We shall simplify the notation in (3) with $k = M-1$ to

$$\mathrm{Y} \;=\; \mathcal{T}(\mathrm{H}\,)\,A + \mathrm{V}\ . \qquad (4)$$

We assume that $mM > M+N-1$ in which case the channel convolution matrix $\mathcal{T}(\mathrm{H}\,)$ has more rows than columns. If the $\mathrm{H}_i(z)$, $i = 1,\ldots,m$ have no zeros in common, then $\mathcal{T}(\mathrm{H}\,)$ has full column rank (which we will henceforth assume). For obvious reasons, the column space of $\mathcal{T}(\mathrm{H}\,)$ is called the signal subspace and its orthogonal complement the noise subspace. The signal subspace is parameterized linearly by h.

2. LINEAR PREDICTION.

The linear prediction approach was introduced in the contribution [1] . This approach has for advantage the robustness to channel order overestimation. Now let $\mathbf{P}(z) = \sum_{i=0}^{L}\mathrm{p}(i)\,z^{-i}$ with $\mathrm{p}(0) = I_m$ be the MMSE multivariate prediction error filter of order L for the noise-free received signal $\mathrm{y}(k)$. If $L \geq \underline{L} = \lceil\frac{N-1}{m-1}\rceil$, then it can be shown [4] that

$$\mathbf{P}(z)\mathbf{H}(z) \;=\; \mathrm{h}(0)\ . \qquad (5)$$

From (5) it is clear that $\mathbf{H}(z)$ and $\mathbf{P}(z), \mathrm{h}(0)$ are equivalent parameterizations, and linear prediction is well known to be robust to order overestimation.

3. BLIND METHODS PARAMETRIZED BY h FOR DETERMINISTIC SYMBOLS.

3.1. Subchannel Response Matching (SRM)

The SRM technique was introduced in [5] and corresponds also to Liu and Xu's deterministic approach. To begin with, consider first the case of two channels: $m = 2$. One can observe that for noise-free signals, we have $\mathrm{H}_2(z)y_1(k) - \mathrm{H}_1(z)y_2(k) = 0$, which can be written in a matrix form as $[\mathrm{H}_2(z)\ \ {-\mathrm{H}_1(z)}]\,\mathrm{y}(k) = \mathbf{H}^{\perp\dagger}(z)\mathrm{y}(k) = 0$. Stacking these zeros into a vector for the signal $\{\mathrm{y}(k)\}_{k=0\cdots M-1}$, we get an expression of the form $\mathcal{Y}\,\mathrm{h} = 0$ for some structured matrix $\mathcal{Y}$. Under the constraint $\|\mathrm{h}\|_2 = 1$, we find $\mathrm{h} = V_{min}(\mathcal{Y}^H\mathcal{Y})$, the eigenvector of $\mathcal{Y}^H\mathcal{Y}$ corresponding to its minimum eigenvalue.. For $m > 2$, blocking equalizers $\mathbf{H}^{\perp\dagger}(z)$ can be constructed by considering the channels in pairs. The choice of $\mathbf{H}^{\perp\dagger}(z)$ is far from unique. To begin with, the number of pairs to be considered, which is the number of rows in $\mathbf{H}^{\perp\dagger}(z)$, is not unique. The minimum number is $m-1$ whereas the maximum number is $\frac{m(m-1)}{2}$. We shall call $\mathbf{H}^{\perp\dagger}(z)$ balanced

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if $\text{tr}\,\{\mathbf{H}^{\perp\dagger}(z)\mathbf{H}^{\perp}(z)\} = \alpha\mathbf{H}^{\dagger}(z)\mathbf{H}(z)$ for some real scalar α and $\mathbf{H}^{\dagger}(z) = \mathbf{H}^{H}(1/z^*)$. People usually take the maximum number of rows, which corresponds to a balanced $\mathbf{H}^{\perp\dagger}(z)$. The minimum number of rows in $\mathbf{H}^{\perp\dagger}(z)$ to be balanced is m. We get for instance

$$\mathbf{H}^{\perp\dagger}_{min}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\mathrm{H}_m(z) & 0 & \cdots & \mathrm{H}_1(z) \end{bmatrix} \quad (6)$$

$$\mathbf{H}^{\perp\dagger}_{bal}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & \cdots & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \mathrm{H}_m(z) & 0 & \cdots & 0 & -\mathrm{H}_1(z) \end{bmatrix} \quad (7)$$

Continuing with this $\mathbf{H}^{\perp\dagger}_{bal}(z)$, its i^{th} row can be written as

$$\mathbf{H}^{\perp\dagger}_{bal,i}(z) = \mathbf{H}^{T}(z)\,\mathcal{P}_i,\ \mathcal{P}_i = \mathcal{C}\mathcal{P}_{i-1}\mathcal{C}^{H},$$

$$\mathcal{P}_1 \begin{bmatrix} 0 & 1 & 0 & \cdots \\ -1 & 0 & \cdots & \\ 0 & \vdots & \ddots & \\ \vdots & & & \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & 0 & 1 & 0 \end{bmatrix}. \quad (8)$$

For this $\mathbf{H}^{\perp\dagger}_{bal}(z)$, the SRM criterion $\min_{\mathrm{h}} \left\|\mathcal{T}(\mathrm{h}^{\perp})\mathrm{Y}\right\|_2^2$ can be written as the minimization w.r.t. h of

$$\begin{aligned} &\text{tr}\,\{\mathcal{T}_M(\mathrm{h}^{\perp})\mathrm{Y}\mathrm{Y}^{H}\mathcal{T}(\mathrm{h}^{\perp})^{H}\} \\ &= \text{tr}\,\{\mathrm{h}^{\perp}\left(\textstyle\sum_{k=N-1}^{M-1}\mathrm{Y}_N(k)\mathrm{Y}_N^{H}(k)\right)\mathrm{h}^{\perp H}\} \\ &= (M-N+1)\ \text{tr}\,\{\mathrm{h}^{\perp}\widehat{R}_{YY}\mathrm{h}^{\perp H}\} \end{aligned} \quad (9)$$

where the i^{th} row of $\mathrm{h}^{\perp}$ is

$$\mathrm{h}^{\perp}_i = \mathrm{h}^{T}\mathcal{S}_i\,,\ \mathcal{S}_i = I_N \otimes \mathcal{P}_i \quad (10)$$

Hence the SRM criterion in (9) becomes

$$\min_{\mathrm{h}} \mathrm{h}^{H} B\,\mathrm{h}\ ,\ \text{where } B = \sum_{i=1}^{m} \mathcal{S}_i \widehat{R}^{*}_{YY}\mathcal{S}_i^{T}\,. \quad (11)$$

If the exact R_{YY} is used, then the noise contribution to the criterion (11) is $2\sigma_v^2\,\|\mathrm{h}\|^2$ (and here the motivation for chosing a balanced $\mathbf{H}^{\perp}(z)$ becomes apparent). Hence the minimization of (11) subject to $\|\mathrm{h}\| = 1$ leads to the consistent SRM estimate $\mathrm{h} = V_{min}(B)$, at least if the order is chosen correctly. Since $\sigma_v^2 = \lambda_{min}(R_{YY})$, the minimum eigenvalue of R_{YY}, the noise contribution can be eliminated by replacing $\widehat{R}_{YY}$ by $\widehat{R}_{YY} - \lambda_{min}(\widehat{R}_{YY})I$ or, even better, by replacing B by $A = B - \lambda_{min}(B)I$ (the former choice doesn't make B singular with a finite amount of data). With this modification, the criterion in (11) becomes (asymptotically) insensitive to the noise contribution and any normalization of h will lead to a consistent estimate.

3.2. Signal Subspace Fitting (SSF)

The covariance matrix of the received signal can be decomposed into signal and noise subspace contributions:

$$\begin{aligned} R_{YY} = E\mathrm{Y}\mathrm{Y}^{H} &= \sum_{i=1}^{M+N-1} \lambda_i V_i V_i^{H} + \sum_{i=M+N}^{mM} \lambda_i V_i V_i^{H} \\ &= \mathcal{V}_S\Lambda_S\mathcal{V}_S^{H} + \mathcal{V}_N\Lambda_N\mathcal{V}_N^{H} \end{aligned} \quad (12)$$

Hence, the following signal subspace fitting problem can be formulated:

$$\min_{\mathrm{h},T} \|\mathcal{T}(\mathrm{H}\,) - \mathcal{V}_S T\|_F \quad (13)$$

where the Frobenius norm of a matrix X can be defined in terms of the trace operator $\|X\|_F^2 = \text{tr}\,\{X^H X\}$. It can be shown (see[2]) that this leads to the following problem:

$$\min_{\|\mathrm{h}\|_2=1} \text{tr}\,\{\mathcal{T}(\mathrm{H}\,)^{H} P^{\perp}_{\mathcal{V}_S}\mathcal{T}(\mathrm{H}\,)\} = \min_{\|\mathrm{h}\|_2=1} \mathrm{h}^{H} B\,\mathrm{h} \quad (14)$$

where $P^{\perp}_X = I - P_X = I - X(X^H X)^{+}X^H$ and $^{+}$ denotes Moore-Penrose pseudo-inverse. B can be determined from $P^{\perp}_{V_S} = P_{V_N}$. Under the constraint $\|\mathrm{h}\|_2 = 1$ the solution is $\mathrm{h} = V_{min}(B)$.

3.3. Noise Subspace Fitting (NSF)

Similarly, $\mathcal{V}_N$ spans the noise subspace and $\mathcal{T}^{H}(\mathrm{h}^{\perp})$ spans most of it. Hence, the following noise subspace fitting can be introduced:

$$\min_{\mathrm{h},T} \left\|\mathcal{T}^{H}(\mathrm{h}^{\perp}) - \mathcal{V}_N T\right\|_F \quad (15)$$

After optimization w.r.t. T, we obtain $\min_{\|\mathrm{h}\|_2=1}$ of

$$\text{tr}\,\{\mathcal{T}(\mathrm{h}^{\perp})P^{\perp}_{\mathcal{V}_N}\mathcal{T}^{H}(\mathrm{h}^{\perp})\} = \text{tr}\,\{\mathrm{h}^{\perp}C\mathrm{h}^{\perp H}\} = \mathrm{h}^{H}B\mathrm{h} \quad (16)$$

where C can be determined from $P^{\perp}_{\mathcal{V}_N} = P_{\mathcal{V}_S}$ and $B = \sum_{i=1}^{m}\mathcal{S}_i C\mathcal{S}_i^{H}$.

3.4. Deterministic Maximum Likelihood (DML)

The DML criterion can be written as $\min_{\mathrm{h}} \mathrm{Y}^{H}P^{\perp}_{\mathcal{T}(\mathrm{H}\,)}\mathrm{Y}$. Since $P^{\perp}_{\mathcal{T}(\mathrm{H}\,)} \approx P_{\mathcal{T}^{H}(\mathrm{h}^{\perp})}$ (where the approximation error has negligible influence asymptotically), we get $\min_{\|\mathrm{h}\|=1}$ of

$$\mathrm{Y}^{H}P^{\perp}_{\mathcal{T}(\mathrm{H}\,)}\mathrm{Y} = \mathrm{h}^{H}(\mathcal{Y}^{H}\left[\mathcal{T}(\mathrm{h}^{\perp})\mathcal{T}^{H}(\mathrm{h}^{\perp})\right]^{+}\mathcal{Y})\mathrm{h} \quad (17)$$

where $\mathcal{T}(\mathrm{h}^{\perp})\mathrm{Y} = \mathcal{Y}\mathrm{h}$ for some $\mathcal{Y}$. The iterative quadratic (IQ) strategy considers the quadratic "numerator" of the criterion, and for $\mathrm{h}^{\perp}$ in the "denominator" the value from the previous iteration is used.

3.5. Determination of $\mathbf{H}(z)$ from $\overline{\mathbf{P}}(z) = \mathrm{h}^{\perp H}(0)\mathbf{P}(z)$

This technique was proposed in [4]. Consider the full rank $m\times(m-1)$ matrix $\mathrm{h}^{\perp}(0)$ defined such that $\mathrm{h}^{\perp H}(0)\mathrm{h}(0) = 0$, then (5) implies that $\overline{\mathbf{P}}(z) = \mathrm{h}^{\perp H}(0)\mathbf{P}(z)$ is a $(m-1)\times m$ polynomial that satisfies $\overline{\mathbf{P}}(z)\mathbf{H}(z) = 0$. $\overline{\mathbf{P}}(z)$ or equivalently $\mathbf{P}(z)$ and $\mathrm{h}(0)$ can be estimated using linear prediction or IQDML. If $\overline{\mathbf{P}}(z)$ is estimated in a way that is robust to order overestimation, then the order of $\mathbf{H}(z)$ is known and $\mathbf{H}(z)$ can be estimated straightforwardly from $\overline{\mathbf{P}}(z)$. If not, then we can consider the following problem

$$\min_{\mathrm{h}} \frac{1}{2\pi j}\oint \mathbf{H}^{\dagger}(z)\overline{\mathbf{P}}^{\dagger}(z)\overline{\mathbf{P}}(z)\mathbf{H}(z)\frac{dz}{z} \quad (18)$$

which is again of the form $\min_{\mathrm{h}} \mathrm{h}^{H}A\mathrm{h}$.

4. THE CONSTRAINT $\|\mathrm{h}(0)\| = 1$.

4.1. The Basic Approach

The approach considered here consists of writing $\mathbf{H}(z)$ as $\mathbf{Q}(z)\mathrm{h}(0)$ or $\mathrm{h} = \left[\mathrm{h}^{H}(N-1)\cdots\mathrm{h}^{H}(1)\,\mathrm{h}^{H}(0)\right]^{H}$ as $\mathrm{Q}\mathrm{h}(0) = \left[\mathrm{q}^{H}(N-1)\cdots\mathrm{q}^{H}(1)\ I_m\right]^{H}\mathrm{h}(0)$ where the square monic $\mathbf{Q}(z)$ is somewhat analogous to the linear prediction polynomial $\mathbf{P}(z)$. The key idea is to "anchor" the impulse response at its first coefficient. This is one of the properties that leads to robustness to channel overestimation. Our approach can be used in the commonly used channel-based techniques in blind channel identification described above. One can observe that the common paradigm of the previously described methods is the problem formulation $\mathrm{h}^H B \mathrm{h}$, which is solved typically under the unit norm channel constraint $\|\mathrm{h}\|_2 = 1$. In the noiseless case, B is singular with a nullspace of dimension one in the methods described above. In the presence of noise, we can make B singular by replacing B by $A = B - \lambda_{min}(B)I$. The minimization of $\mathrm{h}^H B \mathrm{h}$ or $\mathrm{h}^H A \mathrm{h}$ with $\|\mathrm{h}\|_2 = 1$ leads to $\mathrm{h} = V_{min}(B) = V_{min}(A)$. The corresponding minimum value of the criteron $\mathrm{h}^H A \mathrm{h}$ is zero. When on the other hand we minimize $\mathrm{h}^H A \mathrm{h}$ subject to $\|\mathrm{h}(0)\|_2 = 1$, it is clear that $\mathrm{h} = \alpha V_{min}(A)$ with α chosen such that $\|\mathrm{h}(0)\|_2 = 1$ makes $\mathrm{h}^H A \mathrm{h}$ zero and hence this $\mathrm{h} = \alpha V_{min}(A)$ is the minimizer of $\mathrm{h}^H A \mathrm{h}$ subject to $\|\mathrm{h}(0)\|_2 = 1$. In other words, by replacing B by A, the problems $\min_{\|\mathrm{h}\|_2=1} \mathrm{h}^H B \mathrm{h}$ and $\min_{\|\mathrm{h}(0)\|_2=1} \mathrm{h}^H A \mathrm{h}$ lead to solutions for h that are proportional and hence equivalent. The problem $\min_{\|\mathrm{h}(0)\|_2=1} \mathrm{h}^H A \mathrm{h}$ can be reformulated as

$$\min_{\|\mathrm{h}(0)\|_2=1} \mathrm{h}^H(0) \left(\min_{\mathrm{Q}(0)=I_m} \mathrm{Q}^H A\, \mathrm{Q} \right) \mathrm{h}(0)\,. \tag{19}$$

Let's rewrite A and h in the partitioned forms: $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $\mathrm{h} = \begin{bmatrix} \widetilde{\mathrm{h}} \\ \mathrm{h}(0) \end{bmatrix}$ where A_{11} is $m(N-1) \times m(N-1)$ and A_{22} is $m \times m$. The solution of (19) can be found to be:

$$\begin{aligned} \mathrm{h}(0) &= V_{min}\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right), \\ \widetilde{\mathrm{h}} &= \left[\ \mathrm{h}^H(N-1) \cdots \mathrm{h}^H(1)\ \right]^H = -A_{11}^{-1}A_{12}\mathrm{h}(0)\,. \end{aligned} \tag{20}$$

The interlacing property for the eigenvalues of nested matrices and the fact that the nullspace of A has dimension one leads to A_{11} being nonsingular.

4.2. Robustness to Order Overestimation

As one can remark from the previous discussion, the anchoring of the impulse response by itself does not lead to robustness to order overestimation. To analyze the problem structure when the channel order is overestimated, consider the SRM method (the conclusions below hold for the other methods also). Below, let $\mathbf{H}^{\perp}(z)$ be the $\mathbf{H}^{\perp}_{bal}(z)$ in (7). In the frequency domain, the SRM criterion is

$$\frac{1}{M}\frac{1}{2\pi j}\oint \|\widehat{\mathbf{H}}^{\perp\dagger}(z)\,\mathbf{y}(z)\|^2 \frac{dz}{z}$$
$$\overset{M\to\infty}{\longrightarrow}\ \mathrm{tr}\left\{\frac{1}{2\pi j}\oint \widehat{\mathbf{H}}^{\perp\dagger}(z)\,S_{\mathrm{yy}}(z)\widehat{\mathbf{H}}^{\perp}(z)\frac{dz}{z}\right\} = \widehat{\mathrm{h}}^T B^* \widehat{\mathrm{h}}^* =$$
$$\frac{1}{2\pi j}\oint \widehat{\mathbf{H}}^T(z)\left(2\sigma_v^2 I_m + \sum_{i=1}^{m} \mathcal{P}_i \mathbf{H}(z)\mathbf{H}^\dagger(z)\mathcal{P}_i^T\right)\widehat{\mathbf{H}}^{T\dagger}(z)\frac{dz}{z}$$

where $B = A + 2\sigma_v^2 I$ is block Toeplitz. $\sum \mathcal{P}_i \mathbf{H}(z)\mathbf{H}^\dagger(z)\mathcal{P}_i^T$ has rank $m-1$ (for any choice of $\mathbf{H}^{\perp}(z)$). If the length of $\widehat{\mathrm{h}}$ is N' (whereas the length of h is N), then A and A_{11} are singular of degree $N'-N+1$ and $N'-N$ respectively. The choice of the minimum-norm soluton for $\widetilde{\mathrm{h}}$ (corresponding to the use of the Moore-Penrose pseudo-inverse A_{11}^{+} for A_{11}) would still lead to order overestimation problems. Now consider $A_{11} = UDU^H$, the UDL triangular factorization of A_{11} (of block size $N'-1$). A_{11} being Hermitian and block Toeplitz, the UDL factorization can be computed efficiently by the modular multichannel Schur algorithm. Since A_{11} is singular and block Toeplitz, U and D become block Toeplitz after N blocks. So the last diagonal block in D appears $N'-N$ times and its diagonal consists of $m-1$ nonzero elements followed by a zero. To solve the order overestimation problem, these (repeated) singular diagonal blocks in D should be replaced by zero. Furthermore, the elements of U above its diagonal should be put equal to zero in columns that correspond to the resulting zeros in D. In the finite data case, A_{11} and hence D will not be exactly singular. However, small elements in D should be forced to zero. This approach for $\mathbf{Q}(z)$ is the dual of the approach outlined in [2] for $\mathbf{P}(z)$, to make the prediction approach robust to order overestimation.

5. SIMULATION RESULTS.

In the simulations presented here, the performance measure is the Normalized MSE (NMSE) which is computed over 300 Monte Carlo runs as

$$\text{NMSE } = \frac{1}{300}\sum_{i=1}^{300} \|\widehat{\mathrm{h}}^{(i)} - \mathrm{h}\|^2/\|\mathrm{h}\|^2.$$

We use a random complex channel H with $N = 3$ and $m = 3$ which is given by :

$$\begin{bmatrix} 0.0591-0.3600j & 0.3516+1.2460j & 1.1650+0.8717j \\ 1.7971-0.1356j & -0.6965-0.6390j & 0.6268-1.4462j \\ 0.2641-1.3493j & 1.6961+0.5774j & 0.0751-0.7012j \end{bmatrix}$$

the symbols are i.i.d. BPSK, and the data length is $M = 100$. The SNR is defined as $(\|\mathrm{h}\|^2\sigma_a^2)/(m\sigma_v^2)$.

5.1. SRM

In Figure 1, we compare the performance of the SRM estimates obtained with the normalizations $\|\mathrm{h}\| = 1$ and $\|\mathrm{h}(0)\| = 1$ assuming the correct channel order. We also compare with the normalized Cramer-Rao bound (CRB) [4]. The two normalizations give comparable performance, especially at high SNR. Note that when $\|\mathrm{h}\| = 1$ is used, the subtraction of the noise contribution has no influence. In Figure 2, the problem of channel order overdetermination is illustrated: we apply SRM assuming the channel order is $N' = 4 > 3 = N$. We can notice that the use of $\|\mathrm{h}(0)\| = 1$ leads to only a moderate increase (one element in D zeroed out) or even a decrease (m elements in D zeroed out) in the NMSE, whereas the channel order overestimation problem of the $\|\mathrm{h}\| = 1$ approach is clear.

5.2. SSF

When the channel length is assumed to be $N' > N$, then the assumed signal subspace will be of increased dimension $M+N'-1$. In the SSF method, the channel estimate will be obtained by searching for orthogonality with a noise subspace of reduced dimension $(m-1)M-N'+1$. This does not pose any fundamental problem since expressing orthogonality to as few as $m-1$ noise subspace vectors leads to identifiability [4]. (Note that for the NSF method on the other hand, even if one works with a potentially overdimensioned channel length, one should not overestimate the signal subspace dimension).

In Figure 3, we apply SSF assuming the channel order is $N' = 5 > 3 = N$ and we use the procedure described previously except that we don't force all m elements in the singular blocks of D to zero, but only take the last element in each excess block equal to zero. We find that using the constraint $\|\mathrm{h}(0)\| = 1$ leads to only a moderate increase in the NMSE, whereas the channel order overestimation problem using the constraint $\|\mathrm{h}\| = 1$ is clear. In Figure 4, the same channel order overestimation is assumed and we estimate h according to the previously mentioned procedure, now putting all m elements in each singular block of

D equal to zero. The curves of NMSEs corresponding to $\|h(0)\| = 1$ with order overestimation and $\|h\| = 1$ with the correct order become superimposed for SNR $\geq$ 10dB.

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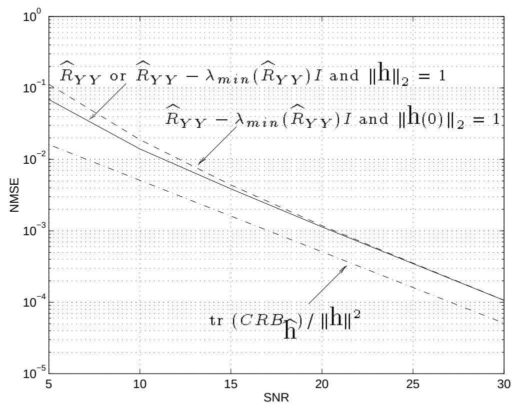

Figure 1. SRM: comparison of NMSEs obtained with the constraints $\|h\|_2 = 1$ and $\|h(0)\|_2 = 1$

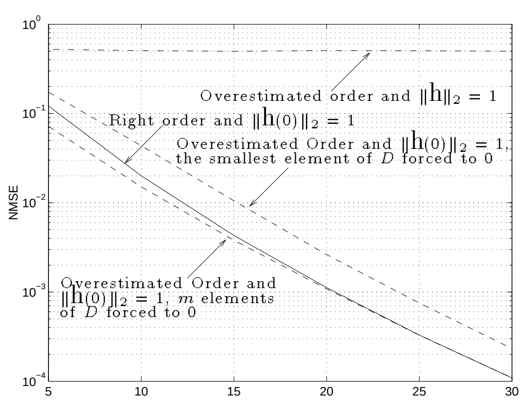

Figure 2. SRM: robustness to order overestimation with the constraints $\|h\|_2 = 1$ and $\|h(0)\|_2 = 1$

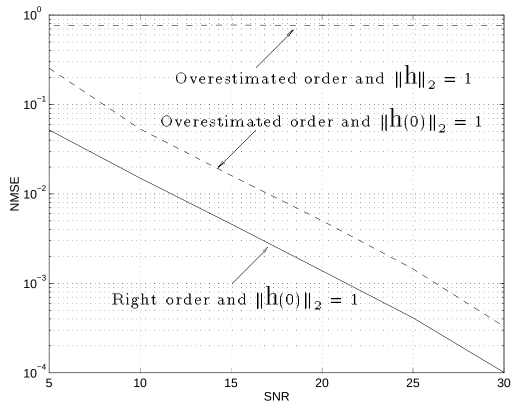

Figure 3. SSF: robustness to order overestimation with the constraints $\|h\|_2 = 1$ and $\|h(0)\|_2 = 1$ using an intermediate solution.

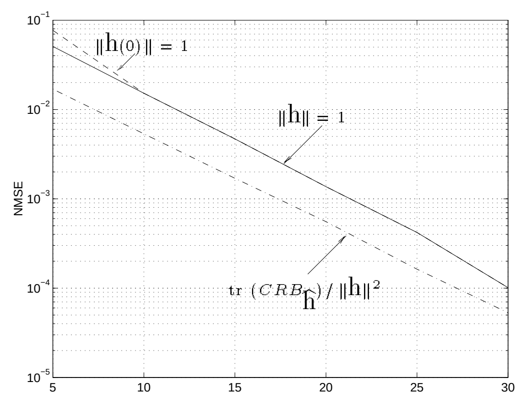

Figure 4. SSF: robustness to order overestimation with the constraint $\|h(0)\|_2 = 1$ using the proposed procedure.