

STRUCTURED ESTIMATION OF SPARSE CHANNELS IN QUASI-SYNCHRONOUS DS-CDMA

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ABSTRACT

We explore the channel estimation problem in the case of quasi-synchronous users in a DS-CDMA system. Knowledge of the transmit (TX) filter is assumed, and the anti-aliasing low-pass front end receive (RX) filter is designed for critical sampling at the Nyquist rate for the TX filter. It is shown that when the sampling frequency is larger than the Nyquist frequency, the discrete-time representation of the channel is not unique. However, all representations can be treated in a similar fashion once the Nyquist rate is satisfied. On the other hand, fractionally sampling the channel leads to a scenario in which the cut-off frequency can be approached arbitrarily close to the Nyquist rate. In the case of sparse channels, sampling the channel at any rate leads to a small number of non-zero coefficients in the finite-impulse response (FIR) representation of the channel. The structured channel estimation algorithm presented in this paper exploits the sparseness of this model. Results are compared with those of other recently proposed structured methods.

1. INTRODUCTION

The burst oriented nature of certain communication scenarios is particularly suited for quasi-synchronous (QS) applications. Timing advance or delay can be integrated as an upper layer process leading to a synchronous block structure of the received signal. An example is that of the TDD version of the UMTS proposal [1], based upon a hybrid TDMA/CDMA slotted structure very similar to that of GSM [2]. Uplink users arrive at the base-station quasi-synchronously due to the slotted structure of the TDD mode. The small inter-user misalignment and the multipath nature of the propagation channel, however, lead to spreading of the signal block beyond the strict slot duration. The overlap in successive blocks is avoided by introducing inter-slot guard periods. The TDD proposal [1] suggests joint maximum likelihood (ML) channel estimation based upon training sequences (chips) included as mid-ambles in the signal frame [3].

During the course of transmission of a block, the channel is considered to be time invariant. If the fading rate is slow enough, channel state information can be carried over from one block to the next, improving the channel estimate which is now averaged over training chip sequences of successive slots. Nevertheless, the number of training chips per block needs to be sufficient to estimate jointly the channels of all QS users for independent block to block

channel estimation and data detection [4]. The joint least-squares channel estimation (which corresponds to ML in white noise) imposes a lower limit on the number of training chips, which is a function of the delay spread (including the transmit and receive filters) and the number of active users. This naturally imposes a limit on the number of concurrent users. An estimator of this kind has been referred to as an *unstructured* channel estimator in the literature [5] since no information (structure) on the signal apart from the training information is exploited in the estimation algorithm. It is also shown in [5] and [6] that partial knowledge of the channel in terms of the TX/RX filter can be exploited to build a *structured* estimator which results in improved performance owing to the reduced number of parameters to be estimated. More users can therefore be accommodated for a training sequence of a given length.

If the overall channel vector is represented as the product of a TX/RX filter convolution matrix and the actual propagation channel (usually a set of delayed echos), then depending upon the discrete time representation of the latter, the former can be ill-conditioned (fat instead of tall). A solution to this problem has been proposed in [5] as the replacement of the TX/RX matrix by its *significant* left singular vectors. Alternatively, for the case of a sparse channel, if in addition to the TX/RX filters, the position of the delays is also known (estimated separately), the number of parameters to be estimated is simply the number of significant paths, resulting in an estimator which exploits a finer structure, and gives a lower mean-square channel estimation error.

In this paper, we discuss the discrete time representation of the channel impulse response as a function of the sampling rate and present a structured channel estimator which exploits the sparseness of the fractionally sampled channel impulse response instead of the singular value decomposition (SVD) based algorithm of [5]. It is shown that the SVD destroys the locality property imposed by the few non-zero samples of the channel response while significant gains can be obtained by exploiting this sparseness.

2. SIGNAL MODEL

We consider K users in a DS-CDMA system (see fig. 1). The channel as seen from the m th sensor is

$$h_{k,m}(t) = \int_0^{\Delta T_c} p(t - \tau) c_{k,m}(\tau) d\tau, \quad (1)$$

where, $p(t)$ is the combined TX/RX filter (assumed to be the same for all K users), and $c_{k,m}(t)$ is the continuous time propagation

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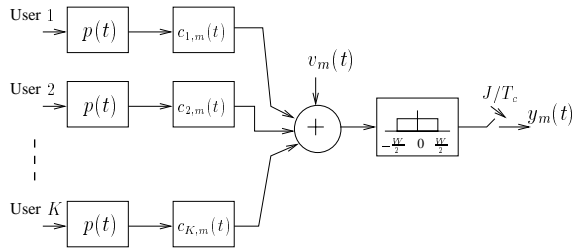


Figure 1: Baseband signal model for the RX signal at m th sensor.

channel impulse response between the k th user and the m th sensor. T_c denotes the chip period, and ΔT_c is the maximum duration of $c_{k,m}(t)$, i.e., the delay spread of the propagation channel. Δ is a positive integer. The TX filter, $p(t)$ is a band-limited pulse shaping filter (e.g., a root-raised cosine, with an excess bandwidth α as shown in fig. 2), while the RX is an anti-aliasing, ideal low-pass filter with a cut-off frequency corresponding to the sampling frequency, W . Hence, to satisfy the anti-aliasing condition imposed by the well-known sampling theorem, the bandwidth W , of the low-pass RX filter can lie anywhere beyond $f_{\text{nyq}} = (1 + \alpha)/T_c$, which is the Nyquist frequency, and corresponds to critically sampling the received signal to avoid aliasing.

Let us consider sampling, at a rate W . The oversampled discrete representation for the overall channel would be

$$h_{k,m}(t) = \sum_{l=0}^{L-1} p(t - \frac{l}{W})c_{k,m}(l). \quad (2)$$

The $c_{k,m}(l)$ are the discrete representation of $c_{k,m}(t)$, corresponding to a sampled version (at rate W) of an ideally low-pass filtered version of $c_{k,m}(t)$. It must be noted that in principle, if $W > f_{\text{nyq}}$, as shown in fig. 2, the discrete time representation, $c_{k,m}(l)$, of the propagation channel is not unique. One can essentially add any signal to it that lies within the shaded portion (between f_{nyq} and W) to alter the coefficients $c_{k,m}(l)$, since the components corresponding to those frequencies will be removed by the TX filter. This reflects the redundancy introduced in the sampled channel coefficients due to excessive oversampling. Alternatively, one can adjust the sampling frequency W to have the cut-off arbitrarily close to the TX pulse bandwidth. This can be achieved

- either by oversampling by a factor β and then down-sampling by a factor γ , with $\beta > \gamma$, so that $\frac{\beta}{\gamma} \rightarrow 1 + \alpha$ (this results in a uniformly sampled signal)
- or by non-uniform sampling, in the event of which one still needs to satisfy the following extension to the sampling theorem (see e.g., [7])

Theorem 1 *A signal with limited spectral support can be reconstructed from its non-uniform samples as long as the average sampling rate exceeds the Nyquist rate.*

Furthermore, the representation of the overall channel in terms of sampled versions of TX/RX filter and the actual channel is justified by the following result.

Theorem 2 *The sampled version of the convolution of two band-limited signals can be represented by the convolution of sampled versions of the two signals, once the sampling rate equals or exceeds the Nyquist rate for at least one of the two signals.*

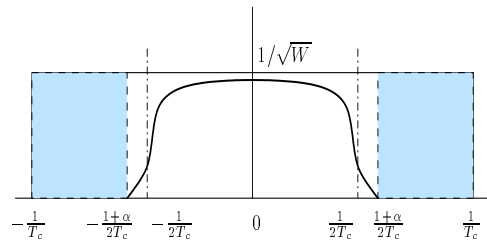


Figure 2: RX filter with cut-off frequency $\frac{1}{T_c}$, and $\frac{1+\alpha}{2T_c}$.

It must be mentioned that in the instance of a sparse channel, as is often the case in mobile communication scenarios, only a few of the $c_{k,m}(l)$ are non-zero. The overall channel in (2) can now be sampled at any rate J/T_c to obtain

$$h_{k,m}(nT_c + j\frac{T_c}{J}) = \sum_{l=0}^{L-1} p(nT_c + j\frac{T_c}{J} - \frac{l}{W})c_{k,m}(l), \quad (3)$$

where, $j = 1, \dots, J$, and $n = 0, 1, \dots, \Psi$, where, $\Psi = \Delta + \Phi$, and ΦT_c , the effective duration of the chip pulse shaping filter $p(t)$. The above equation can be written as

$$h_{k,m}(nT_c + j\frac{T_c}{J}) = \mathbf{p}_j^T(n) \mathbf{c}_{k,m}, \quad (4)$$

and where,

$$\mathbf{p}_j(n) = \left[p(nT_c + j\frac{T_c}{J}), \dots, p(nT_c + j\frac{T_c}{J} - \frac{L-1}{W}) \right]^T,$$

and, $\mathbf{c}_{k,m} = [c_{k,m}(0), \dots, c_{k,m}(L-1)]^T$, where, L is the effective FIR length of the low-pass filtered and sampled channel impulse response.

Now, the overall chip-rate channel for the k th user and the m th sensor can be written as

$$\mathbf{h}_{k,m} \triangleq \left[h_{k,m}(0), h_{k,m}(\frac{T_c}{J}), \dots, h_{k,m}(\frac{(J-1)T_c}{J}), h_{k,m}(T_c), h_{k,m}(T_c + \frac{T_c}{J}), \dots, h_{k,m}(\Psi T_c + \frac{(J-1)T_c}{J}) \right]^T, \quad (5)$$

and, $\mathbf{h}_k \triangleq [\mathbf{h}_{k,1}^T, \mathbf{h}_{k,2}^T, \dots, \mathbf{h}_{k,M}^T]^T$ is the $MJ\Psi \times 1$ overall channel vector as seen by the M sensors.

$$\mathbf{P} = [\mathbf{p}_0(0), \mathbf{p}_1(0), \dots, \mathbf{p}_{J-1}(0), \mathbf{p}_0(1), \dots, \mathbf{p}_{J-1}(\Psi-1)]^T \quad (6)$$

is the $J\Psi \times L$ pulse shaping matrix. We can now write the overall channel as

$$\mathbf{h}_k = (\mathbf{I}_M \otimes \mathbf{P}) \mathbf{c}_k, \quad (7)$$

where, $\mathbf{c}_k = [\mathbf{c}_{k,1}^T, \dots, \mathbf{c}_{k,M}^T]^T$.

Now, let us suppose that the k th user's signal received at the m th antenna at time $nT_c + jT_c/J$ is $x_{k,m}(nT_c + jT_c/J)$. Stacking together the oversampled signal in a vector we can write the received signal over the n th chip period as

$$\begin{aligned} \mathbf{x}_{k,m}(n) &= \left[x_{k,m}(nT_c), x_{k,m}(nT_c + \frac{T_c}{J}), \dots, \right. \\ &\quad \left. \dots, x_{k,m}(nT_c + (J-1)\frac{T_c}{J}) \right]^T \\ &= (\mathbf{b}_k^T(n) \otimes \mathbf{I}_J) \mathbf{P} \mathbf{c}_{k,m}, \end{aligned} \quad (8)$$

where,

$$\mathbf{b}_k(n) = [b_k(n), b_k(n-1), \dots, b_k(n-\Psi+1)]^T$$

is the chip sequence vector for the k th user at chip period n . Since $\Psi - 1$ is the overall delay spread including the effect of the TX/RX filter and the actual propagation channel $c_{k,m}(t)$, we need to consider a total of N_{ts} training chips per user, leading to $N = N_{ts} - \Psi + 1$ chips of the known received signal. Stacking the k th user's training chips in a $N \times \Psi$ Hankel matrix, $\mathbf{B}_k = [b_k(\Psi), b_k(\Psi+1), \dots, b_k(N_{ts})]^T$, and assuming that all users have the same channel length, Δ , we can write the $MNJ \times 1$ discrete time received signal (sampled at rate J/T_c) corresponding to the training duration at the M sensors as

$$\begin{aligned} \mathbf{Y}_{ts} &= \{\mathbf{I}_M \otimes (\mathbf{B}_{ts} \otimes \mathbf{I}_J)\} \tilde{\mathbf{h}} + \mathbf{V}_{ts} \\ &= \{\mathbf{I}_M \otimes (\mathbf{B}_{ts} \otimes \mathbf{I}_J) \tilde{\mathbf{P}}\} \tilde{\mathbf{c}} + \mathbf{V}_{ts}, \end{aligned} \quad (9)$$

where, $\mathbf{B}_{ts} = [\mathbf{B}_1, \dots, \mathbf{B}_K]$ is the $N \times K\Psi$ training chip sequence matrix, $\tilde{\mathbf{P}} = \mathbf{I}_K \otimes \mathbf{P}$ is the $JK\Psi \times KL$ matrix, $\tilde{\mathbf{c}} = [c_1^T, \dots, c_K^T]^T$ is the $KL M \times 1$ concatenation of channel vectors of the K users, and $\tilde{\mathbf{h}} = \{\mathbf{I}_M \otimes \tilde{\mathbf{P}}\} \tilde{\mathbf{c}}$ is the $JKM\Psi \times 1$ overall channel vector for all K users and across all M sensors. Let us further denote $\mathbf{B}_{ts} \otimes \mathbf{I}_J$ by \mathbf{B} in order to simplify notation. \mathbf{V}_{ts} represents the vector of the additive white channel noise.

3. STRUCTURED CHANNEL ESTIMATION

Let $N_{ts} \geq 2\Psi - 1$ be the number of training chips per user. The unstructured least-squares estimate of the multiuser channel $\tilde{\mathbf{h}}$ can be obtained as the solution to the problem

$$\hat{\tilde{\mathbf{h}}} = \arg \min_{\tilde{\mathbf{h}}} |\mathbf{Y}_{ts} - (\mathbf{I}_M \otimes \mathbf{B}) \tilde{\mathbf{h}}|^2, \quad (10)$$

resulting in

$$\hat{\tilde{\mathbf{h}}} = \left\{ (\mathbf{I}_M \otimes \mathbf{B})^H (\mathbf{I}_M \otimes \mathbf{B}) \right\}^{-1} (\mathbf{I}_M \otimes \mathbf{B})^H \mathbf{Y}_{ts}. \quad (11)$$

Alternatively, taking into account the structure of the problem in terms of the knowledge of the pulse shaping matrix $\tilde{\mathbf{P}}$, we can obtain an estimate of the propagation channel as

$$\hat{\tilde{\mathbf{c}}} = \arg \min_{\tilde{\mathbf{c}}} |\mathbf{Y}_{ts} - (\mathbf{I}_M \otimes \mathbf{B} \tilde{\mathbf{P}}) \tilde{\mathbf{c}}|^2, \quad (12)$$

resulting in

$$\begin{aligned} \hat{\tilde{\mathbf{c}}} &= \left\{ \mathbf{I}_M \otimes \left(\tilde{\mathbf{P}}^H \mathbf{B}^H \mathbf{B} \tilde{\mathbf{P}} \right) \right\}^{-1} (\mathbf{I}_M \otimes \mathbf{B} \tilde{\mathbf{P}})^H \mathbf{Y}_{ts} \\ &= \left\{ \mathbf{I}_M \otimes \left(\tilde{\mathbf{P}}^H (\mathbf{B}_{ts}^H \mathbf{B}_{ts} \otimes \mathbf{I}_J) \tilde{\mathbf{P}} \right) \right\}^{-1} (\mathbf{I}_M \otimes \mathbf{B} \tilde{\mathbf{P}})^H \mathbf{Y}_{ts} \end{aligned} \quad (13)$$

3.1. Rank Deficiency in the Pulse Shaping Matrix

As pointed out in [5], if the sampling rate for the channel, i.e., W is large, then, so is L , the FIR length of the channel impulse response (fine temporal resolution). Under these conditions the pulse shaping matrix, $\tilde{\mathbf{P}}$ can be fat rather than tall, and $\tilde{\mathbf{P}}^H (\mathbf{B}_{ts}^H \mathbf{B}_{ts} \otimes \mathbf{I}_J) \tilde{\mathbf{P}}$ becomes rank deficient. The solution proposed in [5] comprises of

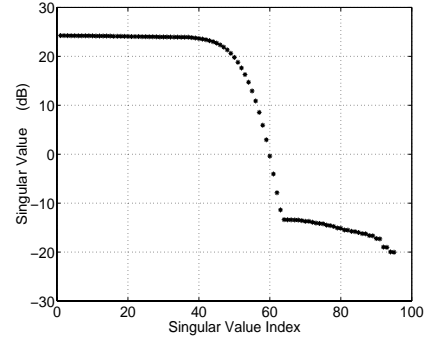


Figure 3: Singular value distribution of $\tilde{\mathbf{P}}$ with $W = 2/T_c$.

computing the SVD of \mathbf{P} as $\mathbf{P} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where the $J\Psi \times q$ matrix \mathbf{U} consists of the left orthonormal singular vectors, $\mathbf{\Sigma}$ is the diagonal matrix of q positive singular values, and \mathbf{V} is the $q \times L$ matrix of right orthonormal singular vectors (q is the effective rank of \mathbf{P}), and replacing $\tilde{\mathbf{P}}$ by $\tilde{\mathbf{U}}$ (where, $\tilde{\mathbf{U}} = \mathbf{I}_K \otimes \mathbf{U}$) in (13), whenever $\tilde{\mathbf{P}}$ is numerically ill-conditioned. The column span of \mathbf{U} is the same as that of $\tilde{\mathbf{P}}$. Therefore, we can write $\tilde{\mathbf{h}} = (\mathbf{I}_M \otimes \tilde{\mathbf{P}}) \tilde{\mathbf{c}} = (\mathbf{I}_M \otimes \tilde{\mathbf{U}}) \tilde{\mathbf{g}}$, where, $\tilde{\mathbf{g}} = \{\mathbf{I}_M \otimes (\mathbf{I}_K \otimes \mathbf{\Sigma} \mathbf{V}^H)\} \tilde{\mathbf{c}}$, resulting in

$$\hat{\tilde{\mathbf{g}}} = \left\{ \mathbf{I}_M \otimes \left(\tilde{\mathbf{U}}^H (\mathbf{B}_{ts}^H \mathbf{B}_{ts} \otimes \mathbf{I}_J) \tilde{\mathbf{U}} \right)^{-1} \right\} (\mathbf{I}_M \otimes \mathbf{B} \tilde{\mathbf{U}})^H \mathbf{Y}_{ts} \quad (14)$$

Fig. 3 shows an example of the singular value spread of the matrix $\tilde{\mathbf{P}}$, with a channel sampling rate of $W = 2/T_c$, as shown in fig. 2, with $J = 2$ samples per chip, and a channel with $\Delta = 24$, so that L is quite large. It can be seen that there is a large concentration of singular values at the two limits. However, there is no clear transition point in between (quite a few singular values are smeared out in the transition region), with the result that there is no clear selection criterion for q . We shall discuss this in more detail while presenting other numerical examples.

One more drawback of the above approach is that in the case of sparse channels, i.e., when very few of the $c_{k,m}(l)$ are non-zero, the SVD destroys the locality property in the matrix $\tilde{\mathbf{P}}$, i.e., while each column of $\tilde{\mathbf{P}}$ was associated with a particular $c_{k,m}(l)$ (of which very few are non-zero), the singular vectors in \mathbf{U} are not, with the consequence that a certain delay contributes in all positions in addition to its own position. In other words, \mathbf{P} and $\tilde{\mathbf{P}}$ are banded (so that h_k is sparse if c_k is), while \mathbf{U} is not. No gains can therefore be obtained if the channel is known to have a sparse rather than a full FIR impulse response.

3.2. Estimation of Fractionally Sampled Channel

Alternatively, W can be made to approach the Nyquist frequency, f_{nyq} , as closely as possible (in one of the two ways as discussed above), in the event of which the $c_{k,m}(l)$ become unique and L is smaller than the case of integer sampling, i.e., $W = n/T_c$, $n = 2, 3, \dots$. As mentioned before, sampling at $W = (1 + \alpha)/T_c$, $\alpha = 1/\sigma$, $\sigma = 1, 2, 3, \dots$ is realizable by non-uniform sampling, e.g., with an initial sampling rate of $W_i = 2/T_c$, by taking all odd samples and one out of σ (periodically) of the even samples. As can be verified, the average sampling rate still satisfies the Nyquist rate, even though some of the temporal resolution is lost. Sparseness can now be integrated

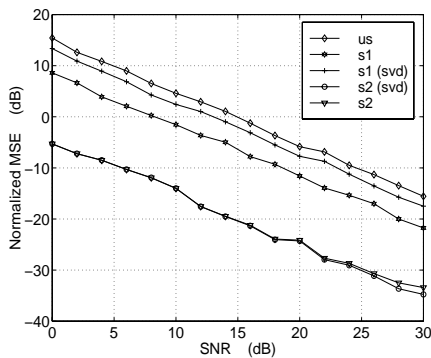


Figure 4: Normalized mean square estimation error (MSE) for structured and unstructured channel estimation methods.

in the model as the deletion of the columns of $\tilde{\mathbf{P}}$ (corresponding to fractional down-sampling), that multiply the insignificant (nearly zero) elements in $\tilde{\mathbf{c}}$.

4. NUMERICAL EXAMPLES

We consider $K = 16$ users in a quasi-synchronous system where the users are block synchronous with a timing misalignment of up to a quarter of a chip. This scenario corresponds to 100% loading ($K =$ processing gain) on the uplink of the TDD version of the UMTS proposal [1] for third generation cellular systems. The transmitted block is assumed to contain a mid-amble of $N_{ts} = 256$ training chips. For the maximum channel lengths (ISI) presumed in the third generation systems, this number of training chips is insufficient to accommodate more than 8 uplink users. However, we consider scenarios where this number suffices for estimation of all K channels in the unstructured fashion. Fig. 4 shows the normalized mean-square error (NMSE) of the channel estimation algorithms based upon SVD and the fractionally spaced sampling. us refers to unstructured, s_1 to the case where only the channel delay spread Δ is assumed to be known, and s_2 to the case where timing delays of the few physical multipath components are known (estimated separately). As seen in this figure, there is no difference between the performance of the two s_2 methods, since the sparseness is taken into account by both SVD based and fractionally spaced methods. However, there is a significant performance gap between the two methods in the s_1 case when the channel is sampled at an integer rate ($W = 2/T_c$ here) followed by SVD, and the fractionally spaced sampling with sparseness exploited. Here, the TX pulse is a root-raised cosine with an excess bandwidth of $\alpha = 0.22$, and the sampling rate is $1.25/T_c$. For the s_1 SVD based method, the ratio of the maximum singular value to the q th one is taken to be $30dB$. As stated above (fig. 3), there is no clear selection criterion for q . Simulations show that there is a marked performance difference in setting the threshold to 10^2 as opposed to e.g., $35dB$, for which the NMSE essentially becomes the same level as for the unstructured case. The same phenomenon is observable if too few of the singular vectors constitute \mathbf{U} .

There is a slight flooring effect in all cases (understandably, more for the case of fractional sampling) due to numerical approximations, e.g., the fact that the pulse shaping matrix and the RX filter are both time-limited (and are hence only approximately band-limited).

The performance of the corresponding linear MMSE receiver,

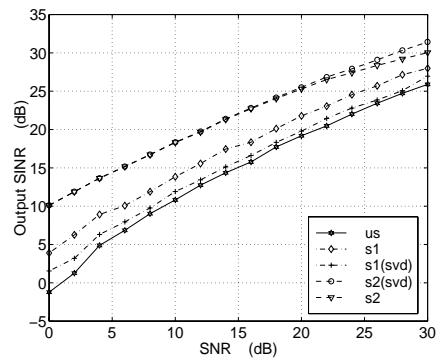


Figure 5: Output SINR performance of the MMSE receiver with channel estimates obtained from various methods.

built from the estimated channel and the true received signal covariance matrix) is shown in fig. 5 for the case of unstructured and the two structured methods. The same trend is observed in the performance of the linear receiver.

5. CONCLUSIONS

We compared structured methods for the case of training sequences based channel estimation in multiuser quasi-synchronous CDMA. It was shown that sampling the channel at integer multiples of the signal bandwidth leads to non-unique channel coefficients. In addition, the pulse-shaping matrix is ill-conditioned because of the redundancy introduced due to excessive sampling rate. Furthermore if significant singular vectors of this matrix are employed in the estimation algorithm instead of the matrix itself, the sparseness property of the channel is lost.

Alternatively, if the anti-aliasing low-pass front end receive (RX) filter is designed for critical (fractional) sampling at the Nyquist rate for the TX filter, a fractionally subsampled version of the pulse shaping matrix can be used to exploit/conserves sparseness of the channel, leading to improved performance for the channel estimator.

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