A Traffic Light Extension to Cell Transmission Model for Estimating Urban Traffic Jam

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***Abstract*—Urban traffic congestions have become a financial and societal burden in many cities. Efficient traffic management solutions mitigating such congestions require a reliable modeling and estimation of traffic jams. In urban traffic, the modeling challenges are related to flow collisions and gridlocks created at intersections. In this paper, we propose a *traffic light extended Cell Transmission Model (CTM)*, where the influence of flow collisions and gridlocks are modeled by a single *CK* parameter. Our approach only requires adapting *CK* for each intersection type/geometry instead of a complex mathematical formulae proposed in related works. We formalize the description of our urban CTM, and evaluate its capability to model traffic volumes and jams against the microscopic traffic simulator SUMO (Simulation of Urban MObility). Results show that the *CK* parameter is able to closely reproduce the impact of collisions and gridlocks on traffic jam, making the proposed urban CTM suitable to predict traffic congestions in urban environments.**

***Keywords*—Urban Traffic Model, Flow Model, Traffic Jam, Macroscopic flow, Cell Transmission Model**

I. INTRODUCTION

Traffic congestion in large urban cities has become a severe economical and societal problem, in particular in China where the rapid grows of private vehicles and an undersized transportation capacity led to large scale chaotic traffic. The modeling of the source and evolution of traffic in large scale cities is a first step to understand the extend of the challenge and develop traffic management solutions.

Yet, large scale congestions remain difficult to observe or measure in a scientific precise manner [1]. Traffic flows may be mathematically represented either at microscopic, mesoscopic or macroscopic scales [2]. Microscopic models represent the precise interaction between vehicles, whereas mesoscopic or macroscopic models rather provide aggregated values such as average density or speed. Due to the scalability limitations of microscopic models, mesoscopic or macroscopic models are preferred for large scale cities or urban highways.

Modeling traffic flows in urban environments is more challenging than highways, as it is subject to significant flow interactions, such as intersection, traffic lights, or pedestrian crossings. In [3], the authors enhanced a macroscopic Cell Transmission Model (CTM) for traffic lights, but only considered a simple round robin cycle. Authors in [9] and [10] described the impact of more complex traffic light assignments on traffic jams. They observed that traffic jams could be created either from flow collisions or gridlocks. Fig. 1(a), depicts an example of flow collisions caused by the traffic light signal not being able to discriminate left-turn and opposite forward vehicles. Flow collisions may be mitigated by channelizing each flow as illustrated on Fig. 1(b). Yet, when the capacity of a given channel is exceeded, traffic will spill over upstream flows and generate gridlocks.

Both collisions and gridlocks play a significant role on the traffic flows and should be carefully considered when modeling traffic jams. The mathematical models proposed in [9,10] provide a detailed description of the impact of collisions and gridlocks but remain difficult to implement in large scale complex urban scenarios, as it is bound to each specific intersection type and channeling geometry. Moreover, gridlocks are mostly created by human behavior, for instance when drivers choose the wrong channel and block traffic, and cannot be reproduced solely from intersection types or geometry.

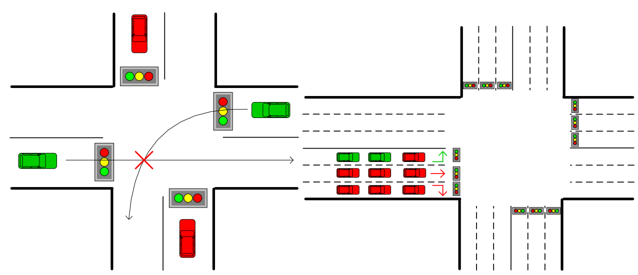


Fig. 1 (a) Collision on road with single lane; (b) Gridlock on road with multiple lanes

In this paper, our objective is to reproduce mathematically traffic jams and their propagation in large scale urban traffic. We propose a novel Traffic Light Extension to the CTM, which simplifies the modeling of collision and gridlock of [9,10] by the introduction of a single parameter *CG*. As collisions and gridlocks are generated by the interaction between vehicles at a microscopic level, we compare our approach against the microscopic simulator SUMO (Simulation of Urban MObility). We take two kinds of real traffic lights assignments into consideration, so that the traffic flows vary extremely similar to reality. Our simulation results show that although simpler compared to [9,10], our urban CTM is capable of reproducing traffic jams and their propagation over multiple intersections that a microscopic model could describe, yet supporting a significantly higher scalability.

The remainder of the paper is organized as follows. Section II describes related flow models. The core of our contribution can be found in Section III, which describes the Traffic Light Extension to CTM in details. Section IV provides validation results and discussions, while we conclude the paper in Section V.

II. Related Works

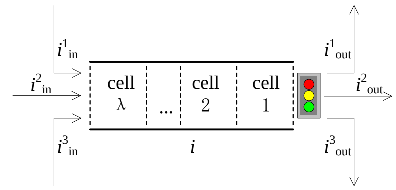
The traffic flow model can be classified into three classes [2]. The first one contains microscopic flow models, which describes the detailed interactions of specific cars with their environment. The second one includes macroscopic flow models, which do not consider mobility parameters of a specific car but instead quantities of macroscopic meanings such as flow, speed, or density. The third class relates to mesoscopic flow models, which propose to describe traffic flows at an intermediate level of details. In this paper, we use macroscopic flows models, first as we need scalability, and second as we are more concerned on the overall traffic density of urban scenarios rather than mobility parameters of individual vehicles.

In [4], Daganzo published the first formulation of the macroscopic CTM for freeway traffic, which has also been successfully applied to several different research areas [5,6]. Sumalee et al. proposed a first-order macroscopic dynamic traffic model, namely the stochastic cell transmission model (SCTM), to model traffic flow density on freeway segments with stochastic demand and supply [7]. Jabari et al. described a stochastic model of traffic flows that operates in discrete space and continuous time [8]. Considering urban traffic, authors in [9] extended CTM to urban traffic flows. In [10], Chen et al. adjusted CTM parameters to better fit to traffic volumes and geometry. Both [9,10] introduced complex mathematical models for gridlock and collisions that we propose to simplify in this paper.

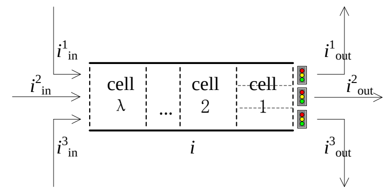
III. Traffic Light Extension to CTM

A. Problem Statement

As shown in Fig. 2(a) and Fig. 2(b), we divide each road segment into λ cells, whose length should satisfy $v_i \cdot T \le l_i$. The cell length must be longer than the free flow travel distance within a time interval. Unlike highways, several kinds of intersection exist in urban environment (e.g. T-shape, Cross-shape, etc..). Consequently, for each road segment, we consider a flows flowing into it from upstream road segments as well as b flows flowing out from it into downstream road segments. In Fig. 2, we have $a = b = 3$.



(a) Mixed flow traffic light



(b) Channeled traffic lights

Fig. 2 Different kinds of traffic lights assignments

We adjust traffic light assignments according to the cells. As shown in Fig. 2(a) for example, there is only one traffic light on the road with only one queue, and vehicles with different directions all merge into it. When the light turns green, vehicles are all permitted to go through the intersection. Consequently, there is no gridlock but flow collisions between left-turn and opposite through vehicles as modeled in [10]. In Fig. 2(b), the cell 1 is channelized into three sub cells, and there is a separate light for each direction. Vehicles on the lane whose light is green are permitted to go, whereas others on the other two lanes still have to wait. In this case, there is no collision but instead gridlock, either when traffic jam spill over upstream cells (cell 2... λ) or if a driver selected a wrong cell. The collisions and gridlocks cannot be avoided and have a significant role on road traffic. In our model, we use a parameter CG to reflect them.

B. Proportion of Turning Movement

As most macroscopic models, we do not differentiate traffic flows by their destinations, but introduce a turning ratio vector $\vec{\beta} = (\beta_1, ..., \beta_b)$ providing the turning probabilities to any possible directions when a vehicle reaches an intersection. In our model, we assume that U-turns are forbidden. We assume that $\vec{\beta}$ satisfies Eq. (1), where β_j represents turning ratio to direction j.

$$\left|\vec{\beta}\right| = \sum_{j=1}^{j=b} \beta_j = 1 \qquad (1)$$

C. Formalization

The total density of cell i is composed of b individual traffic densities, corresponding to flows exiting the cell in left, ahead, right or any other direction.

$$\rho_i(k) = \sum_{j=1}^{j=b} \rho_i^j(k) \qquad (2)$$

Take left inflow and left outflow for example. The left outflow of i can be calculated as Eq. (3), similar for the other outflows.

$$\begin{aligned} qout_i^j(k+1) = \min\{ & v_i\rho_i(k)\beta_1, v_i\rho_i^j, Q_{i_{out}^j}, \\ & w_{i_{out}^j}(\mu_{i_{out}^j} - \rho_{i_{out}^j}(k))\} \cdot TL_i^j(k) \cdot (1 - CG_i^j(k)) \end{aligned} \tag{3}$$

Comparing to basic CTM, we divide the flows into several sub-flows by different directions, therefore, for each direction j, we to compute its out flow and inflow. In Eq. (3), i_{out}^j represents the cell which is the out of cell i with direction j, $TL_i^j(k)$ is the traffic light state of direction j at the intersection at time k. $CG_i^j(k)$ represents the collision or gridlock. It takes values in $[0,1]$. The bigger is $CG_i^j(k)$, the more serious are collisions or gridlocks.

Based on the relationship among road segments, we could observe that the j outflow of cell i_{in}^j is the j inflow of cell i. There are also flows generated at i which will turn at direction j and flows disappeared at i with original direction j. Consequently, $\rho_i^j(k+1)$ can be calculated via Eq. (4).

$$\begin{aligned} \rho_i^j(k+1) &= \rho_i^j(k) + \rho_i^j(k)_{generated} - \rho_i^j(k)_{disappeared} \\ &\quad + \frac{T}{l_i}[qin_i(k+1)\beta_j - qout_i^j(k+1)] \\ &= \rho_i^j(k) + \rho_i^j(k)_{generated} - \rho_i^j(k)_{disappeared} \\ &\quad + \frac{T}{l_i}[qout_{i_{in}^j}(k+1)\beta_j - qout_i^j(k+1)] \end{aligned} \tag{4}$$

In Eq. (4), $\rho_i^j(k)_{generated}$ and $\rho_i^j(k)_{disappeared}$ represent the generated density and disappeared density at cell i with direction j, respectively. Consequently, we could get the total $\rho_i(k+1)$ as Eq. (2). In this paper, we assume that $\rho_i^j(k)_{generated} = \rho_i^j(k)_{disappeared} = 0$.

IV. Numerical Example

A. Experiment Parameters

Figure 3 depicts the 9×9 two-way grid network employed in the validation for both our model and SUMO. We label each intersection with a integer from 1 to 81. The length of each road segment is 240m which is divided into 3 cells. In the following, we artificially block the first cell of the road segment (32, 41). It then blocks traffic in all lanes of (32, 41), therefore creating a traffic jam. The observation time interval is 5 seconds. Other parameters are listed in Table I.

Table I. Basic Parameters in Both Simulation and Model

Lanes	Length of Vehicle	minGap	Jam Density	Initial Density	Free Speed
3	5 m	2.5 m	133.3 vehs/km	62.5 vehs/km	13.889 m/s

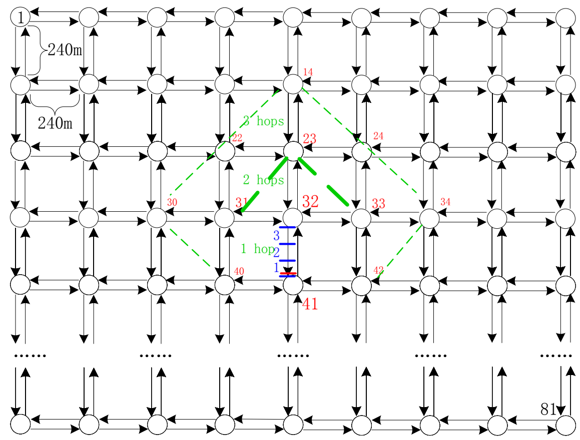


Fig. 3 9×9 two-way grid network

In this grid network, there are 288 road segments for a total amount of $62.5\times288\times240/1000=4320$ vehicles. There are 28 T-shape intersections, 4 corners, and 49 Cross intersections. U-turns are forbidden. The proportion of turning movement is $\vec{\beta}=(0.2,0.3,0.5)$ for Cross intersections. T-shape intersections have equal turning probabilities. We also integrate two kinds of traffic light assignments for Cross intersections, which values are shown in Table II and Table III for intersection 32.

Table II. Simple Traffic Light Assignment

Roads \ Interval	1	2
(31, 32)	Green	Red
(33, 32)	Green	Red
(23, 32)	Red	Green
(41, 32)	Red	Green

The time interval is 35 seconds for all scenarios. It is obvious that collisions and gridlock with the simple assignment are re serious than with a more complex one.

We describe on Table II the three scenarios we developed to validate our CTM. We first perform a dry-run simulation to reach steady state, then use it as the initialization of our model. Moreover, the traffic lights assignments in the model are the same as in SUMO. Therefore, our CTM and SUMO are strictly synchronous. Please note that in the next section, the terms simulation and model refer to SUMO and our CTM respectively.

Table III. Complex Traffic Light Assignment

Roads \ Interval	1	2	3	4
(31, 32) straight	Green	Red	Red	Red
(31, 32) left, right	Red	Green	Red	Red
(33, 32) straight	Green	Red	Red	Red
(33, 32) left, right	Red	Green	Red	Red
(23, 32) straight	Red	Red	Green	Red
(23, 32) left, right	Red	Red	Red	Green
(41, 32) straight	Red	Red	Green	Red
(41, 32) left, right	Red	Red	Red	Green

Table IV. Four Different Scenarios

Symbol	Traffic Light Assignment	Traffic Jam
I	Simple	No
II	Simple	Yes
III	Complex	No

B. Results

We first study the detailed number of vehicles in the cells of road segment (32, 41). As shown in Fig. 4, the results of this model excellently agree with that of simulation. Due to the same initialization and traffic lights assignments, the traffic state of this model closely fits that of the simulation.

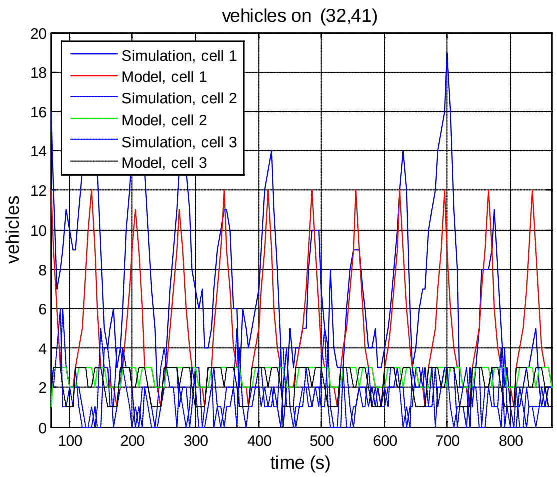


Fig. 4 The number of vehicles on (32, 41) in Scenario I

Then, we consider the traffic flows within *k*-hops of road (32, 41). For instance, for $k=1$, we have a single road segment (32, 41). For $k=2$, we have the 3 road segments (31, 32), (23, 32), (33, 32). For $k=3$, the 9 road segments are depicted on Fig. 3. As shown in Fig. 5, after the traffic flows come into a steady state, the number of vehicles on all the roads in our scenario also becomes steady. As density of traffic flows is not very high, collisions and gridlocks are not significant. The influence of the *CG* parameter in scenario I is therefore important.

For scenario II, as shown in Fig. 6, the block happens in cell 1 of road (32, 41) at $t=70s$, blocking all 3 lanes. The cell 1 becomes jammed firstly after about $t=150s$. Then the cell 2 and cell 3 become jammed. Because the jam density is 133.33ves/km, and the length of each cell is about 80 meters, each road segment could have about 33 vehicles at most. However, in SUMO, the width of intersection is not zero, so that the length of cell 1 and cell 3 are shorter than 80 meters. This is the reason for that the results of the model have a little difference from that of the simulation.

For 3 hops, as shown in Fig. 7, CG plays a significant important role on the traffic flows. Although the collision and gridlocks is not serious when density is low as shown in Fig. 5, it become quite serious when the jam happens and expands to a large region.

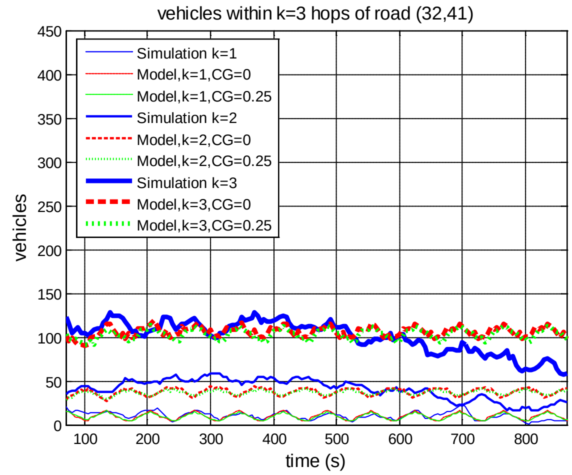


Fig. 5 Scenario I

Fig. 8 represents traffic volumes up to 3 blocks (hops) for more complex traffic lights assignments. While Fig. 8 is quite similar to Fig. 5 as both do not have traffic jam at steady state, we could find in Fig. 8 that, due to the complex traffic lights assignments which have 4 intervals for crossing intersections, the traffic flows vary more slowly than that in Fig. 5. Moreover, the error between simulation and model becomes larger, but the trend remains similar. An optimization of $\vec{\beta}$ could help reduce such deviation.

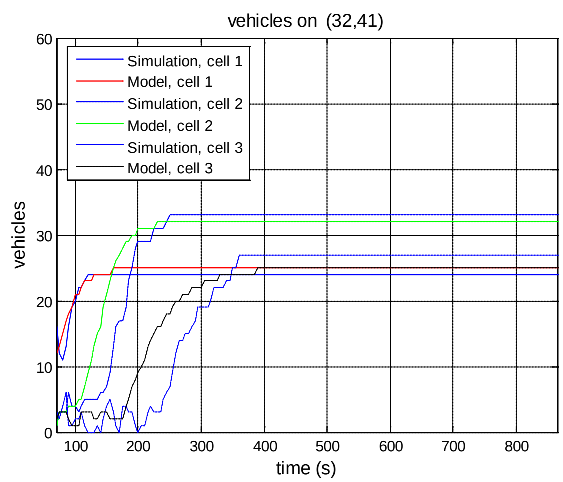


Fig. 6 The number of vehicles on (32, 41) in Scenario II

Finally, we compare the error between the results of the model and simulation. Let VM_i^t and VS_i^t denote the number of vehicles on road i at time t of the model and simulation, respectively. We define the *modeling error* as Eq. (5), where T is the total observation time intervals.

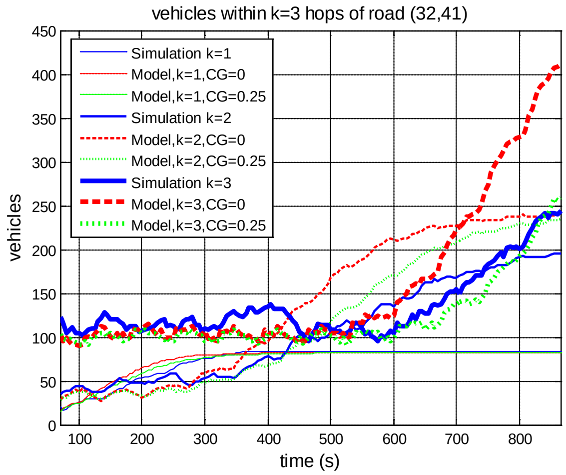


Fig. 7 Scenario II

$$Error = \frac{\sum_{i=1,t=1}^{i=288,t=T} (VM_i^t - VS_i^t)}{288 \cdot T} \qquad (5)$$

Table V. Modeling Errors for Different Scenarios

	I	I	II	II	III	III
CG	0	0.25	0	0.25	0	0.25
Error	-2.7233	-2.6862	-2.6864	-2.6786	-2.6532	-2.6453

The resulting errors are listed in Table V. These errors represent the average deviation between the simulated and modeled traffic volumes on each road segment. Considering an average number of vehicles per road segment of $4320/288 = 15$, these errors are acceptable.

V. Conclusion

In this paper, we proposed a simplified modeling of the effects of real traffic light assignments on flow collisions and gridlocks. We introduced a Traffic Light extension to a macroscopic CTM based on a single *CG* parameter jointly modeling collisions and gridlocks. We compared the results of our model with the microscopic modeling of gridlocks and collisions by traffic simulator SUMO. We observed that while complex traffic light assignments mitigate collisions and gridlocks, simple assignments make them prominent, especially in conjunction to a traffic jam. We also showed that deviation of our CTM compared to SUMO is acceptable.

We emphasize that our choice of a macroscopic CTM allows large scale predictions of urban traffic volumes/flow. Our model may also be used for connectivity or data dissemination in vehicular networks. The most suitable value of the *CG* parameter for specific intersection types/geometry is our future work.

Acknowledgment

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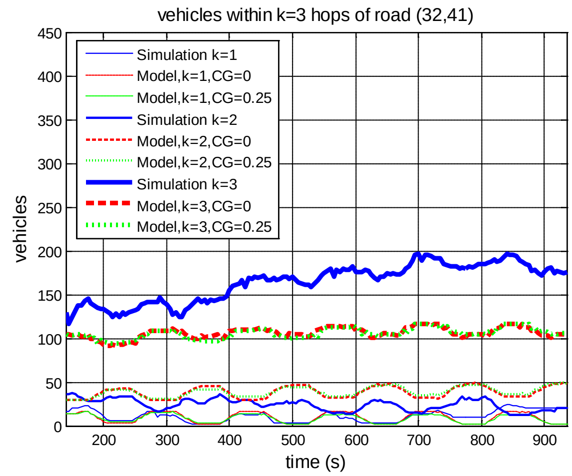

Fig. 8 Scenario III

References

[1]. P. Roberg-Orenstein, C. Abbess, and C. Wright, "Traffic jam simulation," Journal of Maps, vol. 3, no. 1, pp. 107–121, Jan. 2007.

[2]. J. Härri, "Vehicular mobility modeling for vanet," VANET Vehicular Applications and Inter-Networking Technologies, Hannes Hartenstein (Editor), Kenneth Laberteaux (Editor), ISBN: 978-0-470-74056-9, Chapter 5, pp. 107–156, Jan. 2010.

[3]. C. Wright and P. Roberg, "The conceptual structure of traffic jams," Transp Policy 5, pp. 23–35, 1998.

[4]. C. F. Daganzo, "The cell transmission model: a simple dynamic representation of highway traffic," Transportation Research Part B, vol. 28, no. 4, pp. 269–287, Aug. 1994.

[5]. H. K. Lo and W. Y. Szeto, "A cell-based variational inequality formulation of the dynamic user optimal problem," Transportation Research Part B, vol. 36, no. 5, pp. 421–443, Jun. 2002.

[6]. W. Lin and H. K. Lo, "A theoretical probe of a german experiment on stationary moving traffic jams," Transportation Research Part B, vol. 37, no. 3, pp. 251–261, Mar. 2003.

[7]. A. Sumalee, R. Zhong, T. Pan, and W. Szeto, "Stochastic cell transmission model (sctm): A stochastic dynamic traffic model for traffic state surveillance and assignment," Transportation Research Part B, vol. 45, no. 3, pp. 507–533, Mar. 2011.

[8]. S. E. Jabari and H. X. Liu, "A stochastic model of traffic flow: Theoretical foundations," Transportation Research Part B, vol. 46, no. 1, pp. 156–174, Jan. 2012.

[9]. J. Long, Z. Gao, X. Zhao, A. Lian, and P. Orenstein, "Urban traffic jam simulation based on the cell transmission model," Networks and Spatial Economics, vol. 11, no. 1, pp. 43–64, 2008.

[10]. C. Chen, J. Hu, and Y. Wang, "Cell-based simulation and estimation of urban traffic network," in Annual Conference on Intelligent Transportation Systems (ITS), 2010 13th International IEEE, Sep. 2010, pp. 7–12.