Distributed Sensing and Transmission of Sporadic Random Samples

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***Abstract*—This work considers distributed sensing and transmission of sporadic random samples. Lower bounds are derived for the reconstruction error of a single normally or uniformly-distributed vector imperfectly measured by a network of sensors and transmitted with finite energy to a common receiver via an additive white Gaussian noise asynchronous multiple-access channel. Transmission makes use of a perfect causal feedback link to the encoder connected to each sensor. A retransmission protocol inspired by the classical scheme in [1] applied to the transmission of single and bi-variate analog samples analyzed in [2] and [3] is extended to the more general network scenario, for which asymptotic upper-bounds on the reconstruction error are provided. Both the upper and lower-bounds show that collaboration can be achieved through energy accumulation under certain circumstances.**

I. Introduction

The objective of this work is to provide asymptotic performance measures and a realizable, simple transmission strategy for large one-hop sensor networks. We model systems where each sensor measures signals with a finite and small number of source dimensions, in comparison to the number of channel dimensions. This is motivated by applications such as remote sensing using broadband wireless infrastructure (e.g. 4G cellular networks) where sensors take sporadic samples of a random event, feed them back to the network via base stations and subsequently return to an idle state to conserve power. As a result, we do not consider coding of sequences of samples, but rather exploit spatial expansion and correlation between a network of sensors with independent observation noise. Since the applications target 4G wireless networks, it is reasonable to assume a feedback-based transmission strategy, and both the asymptotic results as well as the transmission strategy studied here will exploit feedback. The latter allows for simple and energy-efficient strategies, even if feedback is not required for optimality.

The main results of this work are firstly the derivation of lower-bounds governing the reconstruction error of a single random vector imperfectly measured by a network of sensors and transmitted to a common receiver via an additive white Gaussian noise asynchronous multiple-access channel with a perfect causal feedback link to the encoder connected to each sensor. The bounds are expressed both for a uniform random-vector source with uniformly-distributed observation noise and for a Gaussian source with Gaussian observation noise. Secondly, we extend a retransmission protocol inspired by the classical scheme in [1] applied to the transmission of single and bi-variate analog samples analyzed in [2] and [3] to the more general network with M noisy observations of a common random sample. We restrict the second analysis to uniform one-dimensional sources. The simple two-round transmission scheme combines uniform quantization and orthogonal modulation, for which we provide asymptotic upper-bounds on the reconstruction error as a function of the total received energy and observation noise level. Both the upper and lower-bounds show that a trade-off exists between the source SNR and channel SNR indicating the extent to which collaboration to be achieved through energy accumulation.

With respect to multiple-source systems, the authors in [4] and [5] derive a threshold signal-to-noise ratio (SNR) through the correlation between the sources so that below this threshold, minimum distortion is attained by uncoded transmission in a Gaussian multiple access channel with and without feedback, respectively. In these works, the authors consider transmission of a bi-variate normal source and the distortion can be characterized by two regimes as a function of the relationship between the channel SNR and the source SNR. In the high-correlation regime the distortion is reduced through collaboration in the received energy from the multiple-access channel and amounts to essentially the distortion of a single-source with a factor 4 in energy efficiency. [6] and [7] can be given as further examples where collaboration has the effect of linearly increasing the reconstruction fidelity of the source with the network size. In [6], however, the system parameters are chosen so that the trade-off between source and channel collaboration is not immediately evident.

The outline of the paper is as follows: in the following subsection II-A, we give a description of the general model to explain the problem addressed. It is followed by the derivation of the information-theoretic bounds on the reconstruction error for the two different distribution outlined above. In Section III, we provide an M-sensor adaptation of Yamamoto's protocol for a uniformly-distributed source with uniform observation error along with analysis of its asymptotic performance. We then draw conclusions based on the results of the two analyses.

II. System Model and Lower-bounds on Distortion

A. Model description

Let us begin with the description of the system shown in Figure 1. The construction of the sources is given by the following linear expression.

$$\mathbf{V}_j = \rho \mathbf{U} + \sqrt{1-\rho^2}\mathbf{U}'_j \tag{1}$$

Here we denote the M auxiliary random vectors representing

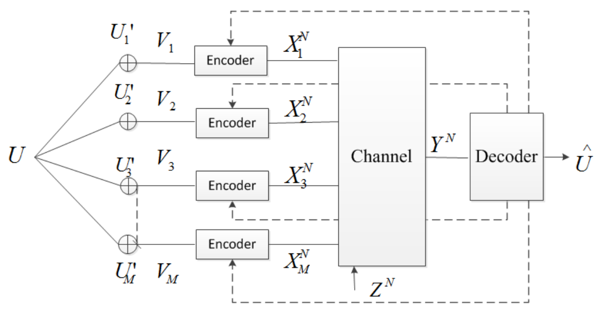

Fig. 1. Pictorial representation of the described system

the observation noise in each sensor by $\mathbf{U}'_j$ and the observation of the mutual source $\mathbf{U}$ by $\mathbf{V}_j$, both of dimension K, for $j = 1, 2, ..., M$. Each realization of $\mathbf{V}_j$ is mapped into $\mathbf{X} \triangleq (X_1, \ldots, X_N)$ which is then sent across the channel corrupted by a white complex circularly symmetric Gaussian noise sequence $\mathbf{Z}$, and is received as the output signal $\mathbf{Y}$. The receiver constructs an estimate $\widehat{\mathbf{U}}$ of $\mathbf{U}$ given $\mathbf{Y}$. The transmitted sequence $\mathbf{X}$ is encoded as $X_i = f_{i,j}(\mathbf{V}_j, Y_1, \cdots, Y_{i-1})$, where the function $f_{i,j}$ is an arbitrary mapping for the j^{th} sensor in dimension i and depends on perfect knowledge of past observations. The latter models an ideal causal feedback path from the receiver. The dimension of the channel input is denoted by N and can be assumed to be large, whereas K is assumed to be finite and small.

We consider two cases for the distribution of $\mathbf{U}$. In the first case, both the $\mathbf{U}'_j$ and $\mathbf{U}$ are uniformly distributed with zero mean and unit variance, i.e. defined in the range $(-\sqrt{3}, \sqrt{3})$. Depending on the level of correlation, $\mathbf{V}_j$ defined by (1) has a contaminated uniform distribution. We will consider the case where $\mathbf{V}_j$, $\mathbf{U}$ and $\mathbf{U}'_j$ are standard normally distributed which is equivalent to having the parameters $\mathcal{N}(0, 1)$. The output signal and the power constraints are given in the following by

$$Y_i = \sum_{j=1}^{M} X_{i,j} e^{i\phi_{i,j}} + Z_i, \quad \frac{1}{K} \sum_{i=1}^{N} E[|X_{i,j}|^2] \leq \mathcal{E}_j \tag{2}$$

for $j = 1, 2, ..., M$ and $i = 1, ..., N$, respectively. The criteria for source-channel code design is chosen as the squared-error distortion measure, which is $d(u_i, \hat{u}_i) = (u_i - \hat{u}_i)^2$ for $i = 1, 2, \cdots K$, and the average distortion is defined as $D = \frac{1}{K}\mathrm{E}\left[\sum_{k=1}^{K} d(u_i, \hat{u})\right]$. $\phi_j = \{\phi_{j,i}; i = 1, ..., N\}$ denotes the random phase sequences which are assumed to be i.i.d. uniform over $[0, 2\pi)$ and unknown to the transmitter and receiver. The latter models an asynchronous network and the fact that a coherent reception model is unrealistic for sporadic information transfer. These assumptions are implicitly relaxed in the lower bounds on the distortion discussed in the following section but are applied in the coding strategy considered in Section III.

B. Derivation of the Bounds

In order to obtain a bound on the fidelity of estimating the random vector $\mathbf{U}$, we obtain upper and lower bounds on a cut-set mutual information functional $I(\mathbf{U}; \mathbf{Y}|\{\mathbf{V}_j\}_S)$ based on a subset $S \subseteq 1, 2, \cdots, M$ and its complement S^c. $\{\mathbf{V}_j\}_S$ denotes the subset of $\mathbf{V}_j$'s for $j \in S$. The derivations of the two bounds are given in the Appendix for both uniform and normal distributions. The bounds are summarized as

$$\begin{aligned} I(\mathbf{U}; \mathbf{Y}|\{\mathbf{V}_j\}_S) \geq -h(\{\mathbf{V}_j\}_S) + h(\{\mathbf{V}_j\}_S|\mathbf{U}) \\ + h(\mathbf{U}) - h(\mathbf{U} - \hat{\mathbf{U}}). \end{aligned} \tag{3}$$

$$I(\mathbf{U}; \mathbf{Y}|\{\mathbf{V}_j\}_S) \leq N \log\left(1 + \frac{K\sum_{j\in S^c} \mathcal{E}_j}{NN_0}\right). \tag{4}$$

Combination of these two bounds allows us to express the distortion level for estimating the mutual random vector $\mathbf{U}$ as

$$D \geq \max_{|S|} C_D (1 - \rho^2)^{|S|} \left(1 + \frac{K\sum_{j\in S^c} \mathcal{E}_j}{NN_0}\right)^{-\frac{2N}{K}} \tag{5}$$

with C_D being a constant which varies based on the distribution type and defined as

$$C_D = \begin{cases} (\frac{6}{\pi e})^{|S|+1} & \text{for } U \sim \mathcal{U}(-\sqrt{3}, \sqrt{3}) \\ 1 & \text{for } U \sim \mathcal{N}(0, 1). \end{cases}$$

The general bound given above by (5) includes two parameters; the correlation coefficient ρ and the energy term and is valid for all $0 \leq |S| \leq M$.

In the source-channel coding scheme proposed in the following section which targets broadband networks and small amounts of analog information, we are mostly interested in the case where $N \gg K$, or where the channel bandwidth is significantly higher than the source bandwidth. For $N \to \infty$ and $\mathcal{E}_j = \mathcal{E} \ \ \forall j$, (5) becomes

$$D \geq \max_{|S|} C_D (1 - \rho^2)^{|S|} \exp\left(-\frac{2(M - |S|)\mathcal{E}}{N_0}\right). \tag{6}$$

which can easily be simplified to

$$D \geq \begin{cases} C_D (1 - \rho^2)^M & 1 - \rho^2 \geq e^{-\frac{2\mathcal{E}}{N_0}} \\ C_D \exp\left(-\frac{2M\mathcal{E}}{N_0}\right) & 1 - \rho^2 \leq e^{-\frac{2\mathcal{E}}{N_0}}. \end{cases} \tag{7}$$

The above result brings to light the effect of collaboration between the sensors which is achieved either through the spatial expansion in the channel or in the source. To see this, we note that the condition $1 - \rho^2 \geq e^{-\frac{2\mathcal{E}}{N_0}}$ is equivalent to saying that the distortion in each sensor node induced by the observation process is more significant than the lowest distortion offered by the channel when estimating $\mathbf{V}_j$ (which is $D_c \geq e^{-\frac{2\mathcal{E}}{N_0}}$) in the absence of the signals from the other sensors. Note that this is the classical point-to-point optimal distortion derived in [8]. Under this condition the channel is used to convey the $\mathbf{V}_j$ independently and the estimation of $\mathbf{U}$ results in a distortion of at least $(1 - \rho^2)^M$, which through spatial expansion reduces the point-to-point distortion at the sources exponentially in M. A comparable trade-off regarding the collaboration effect due to the source or channel can be seen in [4], [5] for the case $K = N$. As mentioned in the introduction, another example is the Gaussian sensor network application [6, sections VI and VII] (again for $K = N$) or the CEO problem studied in [7], where estimation fidelity decays linearly with the size of the network in a manner similar to (5).

III. Achievable Scheme for a network with Uniform sources

The two-way protocol introduced in [2] for a single source and its extension to dual-source studied in detail in [3] is generalized to large networks where the same approach is applied to a scheme with M sources for $M \geq 2$. The protocol consists of two phases; a data and a control phase, which constitute one round and can be repeated twice based on the outcome of the control phase making use of ACK/NACK signals. It should be noted that the scheme could be generalized to more than two rounds. We fix the total energy which is used by the protocol and for the sake of simplicity the energy used in one round is allocated equally among the sources V_j for $j = 1, 2, ..., M$. For the data phase of the first round the aggregate energy is denoted by $\mathcal{E}_{\mathrm{D},1}$ where $\mathcal{E}_{\mathrm{D},1} = \sum_{j=1}^{M} \mathcal{E}_{\mathrm{D},1,j}$. We assume that the source sample of the j^{th} source which is uniformly quantized is subsequently encoded into 2^{B_j} messages with dimension N where B_j's are equal to the same value B. The chosen method is 2^B-ary orthogonal modulation with non-coherent reception. In the data phase, the j^{th}source sends its message $m_j = Q(V_j)$ to the receiver with energy $\mathcal{E}_{\mathrm{D},1,j}$. The aggregated source messages are denoted by $\mathbf{m}$ which is a vector of the messages $(m_1, m_2, ..., m_M)$ with dimension M. Note that all messages from different sources are mutually orthogonal. The receiver decodes $\hat{m}_j$ and feeds it back. The output signal based on the N dimensional observation of the j^{th} source is given by $\mathbf{Y}_{d_j} = \sqrt{\mathcal{E}_{\mathrm{D},1,j}} \mathrm{e}^{j\Phi_j} \mathbf{S}_{m_j} + \mathbf{Z}_{d_j}$. We assume the random phases Φ_j to be distributed uniformly on $[0, 2\pi)$, the channel noise $\mathbf{Z}_{d_j}$ to have zero mean and equal auto-correlation $N_0 \mathbf{I}_{N \times N}$ for $j = 1, 2, ..., M$ and $\mathbf{S}_{m_j}$ are the N-dimensional messages, with $m = 1, 2, \cdots, 2^B$. At the receiver end, we consider the following exhaustive search. To decode the first message m_1, there are 2^B possibilities whereas $m_{j>1}$ is constrained to $2^B(\frac{2\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}})$ since it cannot fall outside of the interval $V_1 + \sqrt{1-\rho^2}(U'_j - U'_1)$ as depicted in Figure 2. Using the notation from [9, Chapter 12], the M possible detection metrics $\mathbf{U}_{\mathbf{m}'} = \sum_{j=1}^{M} |\mathbf{S}_{m'_j}^H \mathbf{Y}_j|^2$, are computed. Assuming the message $(m_1, m_2, ..., m_M)$ is transmitted, the decision metrics can be written as

$$\sum_{j=1}^{M} |\mathbf{S}_{m'_j}^H \mathbf{Y}_j|^2 = \sum_{j:m_j=m'_j} |\sqrt{\mathcal{E}_{D,1,j}} + N_j|^2 + \sum_{j:m_j \neq m'_j} |N_j|^2. \tag{8}$$

According to (8), the receiver chooses $\hat{\mathbf{m}} = \mathrm{argmax}_{\hat{\mathbf{m}}'} \mathbf{U}_{\mathbf{m}'}$. After the estimation and feedback of $\hat{m}_j$ to each encoder, the data phase of the first round ends and the encoders enter the control phase to inform the receiver about the correctness of its decision by sending ACK/NACK signals regarding its own message to the decoder. During the control phase the receiver observes $\mathbf{Y}_c$ with $\mathbf{Y}_{c_j} = \sqrt{\mathcal{E}_{\mathrm{C},1,j}} \mathbf{A}_j \mathrm{e}^{j\Phi_j} \mathbf{S}_{c_j} + \mathbf{Z}_{c_j}$ for j^{th} source where $\mathbf{A}_j$ takes the value 0 for a signal ACK and 1 for a NACK and $\mathcal{E}_{\mathrm{C},1,j}$ here denotes the energy of the control phase in the first round on one source. So the encoders inform the receiver whether or not its decision was correct via a signal $\sqrt{\mathcal{E}_{\mathrm{C},1,j}} \mathbf{S}_{c_j}$ of energy $\sqrt{\mathcal{E}_{\mathrm{C},1,j}}$ if the decision is incorrect and $\mathbf{0}$ if the decision was correct. A NACK is chosen for the j^{th} source if $e_j = I\left(|y_{c,j}|^2 > \lambda \mathcal{E}_{\mathrm{C},1,j}\right) = 1$, with $y_{c,j} = \mathbf{Y}_{c,j}^H \mathbf{S}_{c,j}$, where $I(\cdot)$ is the indicator function and λ is a threshold to be optimized and included within the interval $[0, 1)$. $\Pr(E_{e \to c,1} | k \;\; in \;\; error)$ denotes the total probability of uncorrectable error given that k sources are in error in the first round. Using the recent bound introduced in [10, eq. 12] $\Pr(E_{e \to c,1} | k \;\; in \;\; error)$ is bounded as

$$\Pr(E_{e \to c,1} | k \;\; in \;\; error) \leq \frac{1}{2} \exp\left(-\frac{k(\sqrt{\lambda}-1)^2 \mathcal{E}_{\mathrm{C},1}}{M N_0}\right) \tag{9}$$

where $\mathcal{E}_{\mathrm{C},1} = \sum_{j=1}^{M} \mathcal{E}_{\mathrm{C},1,j}$ is the aggregate energy in the control phase of the first round. In the case of at least one NACK out of M control signals is received, the protocol goes on one more round for retransmission, otherwise it is terminated. And the second data phase starts after the sources are instructed by the destination in order to do the retransmission using the energy per source $\mathcal{E}_{\mathrm{D},2,j}$.

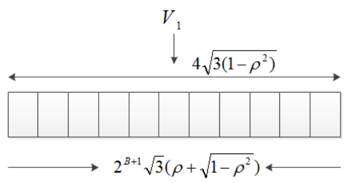

Fig. 2. Pictorial representation of detection

The union bound on $P_e(\mathbf{m})$, the probability of error at the end of the first round, is given by (10) on the top of the next page. To bound $P_e(\mathbf{m})$, the decision variables defined by (8) are used to bound the conditional probability through (10). $P_2(k)$ is defined in [9, eq:12-1-24] through the following equality

$$P_2(k) = \frac{1}{2^{2k-1}} e^{-\gamma} C_k \tag{11}$$

with C_k defined as $C_k = \sum_{n=0}^{k-1} \left(\frac{1}{n!} \sum_{l=0}^{k-1-n} \binom{2k-1}{l}\right) \gamma^n$ where γ denotes the SNR. The second round decision variables are represented by $\mathbf{U}_{\mathbf{m}'}^{(2)} = \mathbf{U}_{\mathbf{m}'} + \sum_{j=1}^{M} |\mathbf{S}_{m_j}^H \mathbf{Y}_j^{(2)}|^2$. We note that this is analogous to soft or Chase-combining in conventional hybrid automatic-repeat-request (HARQ) protocols. As in the first round, the receiver chooses $\hat{\mathbf{m}} = \mathrm{argmax}_{\hat{\mathbf{m}}'} \mathbf{U}_{\mathbf{m}'}^{(2)}$. Using the estimator $\hat{u} = \frac{1}{M} \sum_{i=1}^{M} \hat{v}_j / \rho$, the protocol terminates with the following distortion at the end of the second round, $D = D_q(1-P_e) + \sum_{k=1}^{M-1} D_{e,k} P_{e,k} + D_{e,M} P_{e,M}$, where k corresponds to the number of the sources in error. The error probability corresponding to the case where at least one out of M sources being correct is represented in the above expression by $P_{e,k}$ which is the case of k sources being in error including the uncorrectable error after the first round, or k being in error at the end of the second round. $P_{e,M}$ represents all of the M sources being in error after the first or second round. Note that $P_2(M)$ and $P_2(2M)$ shape together $P_{e,M}$. In the same way, $P_{e,k}$ is the sum of $P_2(2k)$ and $P_2(k)$. Accordingly, the corresponding distortion terms are denoted by $D_{e,k}$ and $D_{e,M}$ respectively. When $\mathbf{m}$ is decoded correctly, the reconstruction error D_q is caused solely by the quantization process and

$$P_e(\mathbf{m}) \leq \sum_{m_j \neq m'_j} \Pr(\mathbf{U}_{m_j} < \mathbf{U}_{m'j}|\mathbf{m}) \leq P_2(M)2^B \left[2^{B+1}\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}\right]^{M-1} + \sum_{k=1}^{M-1}\binom{M}{k}P_2(k)\left[2^{B+1}\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}\right]^k \quad (10)$$

source observation error. Let us denote the estimation error by e, so that its variance $E[u-\hat{u}|l\ in\ error]^2$ for $l=0$ yields the quantization distortion with the following expansion,

$$D_q = E\left[\frac{1}{\rho M}\sum_{j=1}^{M}\left(\sqrt{1-\rho^2}u'_j + e_{q,j}\right)\right]^2 \quad (12)$$

where $e_q = \frac{1}{\rho M}\sum_{j=1}^{M} e_{q,j}$ denotes the estimation error. In order to bound D_q, each tail of the distribution is considered as one quantization bin with a size proportional to $1-\rho^2$ and the interior part which is composed by 2^B-2 bins is uniformly quantized. The squared distortion when k out of M sources are decoded in error at the end of the second round is calculated through $D_{e,k} = E\left[u-\hat{u}|k\ in\ error\right]^2$ for $k=1,2,...,M-1$ by using the chosen estimator expanded as

$$D_{e,k} = E\left[\frac{1}{\rho M}\left(\sum_{\substack{j\ s.t.\\ \hat{v}_j\neq v_j}}(\rho u-\hat{v}_j) + \sum_{\substack{j\ s.t.\\ \hat{v}_j=v_j}}(\rho u-\hat{v}_j)\right)\right]^2 \quad (13)$$

and bounded considering the furthest distances between u and its estimate for the cases when $\hat{v}_j$ is correctly and incorrectly decoded. Note that, D_q and $D_{e,k}$ $(1<k<M)$ are in the exponential order of 2^{-2B} while $D_{e,M}$ is upper bounded by an order of 1. Using the above defined error probabilities, the distortion can be written in the form given by (14) on the top of the next page, where K_1 is $O(1)$, K_2, K_3 are $O((\mathcal{E}_{\mathrm{D},1})^{M-1})$, K_4, K_5 are $O((\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2})^{M-1})$ whereas $K_6(k)$ and $K_7(k)$ are $O((\mathcal{E}_{\mathrm{D},1})^{k-1})$, $K_8(k)$ and $K_9(k)$ correspond to $O((\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2})^{k-1})$ with $\epsilon(\rho)\in[0,1)$.

In order to have a vanishing $P_e(\mathbf{m})$ in the first round, we set the relations of the energies as $\mathcal{E}_{C,1} = \frac{\mathcal{E}_{D,2}}{2(1-\sqrt{\lambda})^2}$ and $\mathcal{E}_{D,2} = (2-\mu)\mathcal{E}_{D,1}$ where μ is an arbitrary constant satisfying $\mu\in(0,2)$, so that the average energy used by the protocol

$$\begin{aligned}\mathcal{E} \leq \mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{C},1}P_e(\mathbf{m}) + \mathcal{E}_{\mathrm{D},2}[P_e(\mathbf{m})(1-\Pr(E_{e\to c,1})) \\ + (1-P_e(\mathbf{m}))\Pr(E_{c\to e,1})]\end{aligned} \quad (15)$$

can be made arbitrarily close to $\mathcal{E}_{\mathrm{D},1}$ guaranteed by vanishing union error probability. Here, $\Pr(E_{c\to e,1})$ represents the total mis-detected acknowledged error probability in the first round which is equivalent to $\exp\{-\frac{\lambda\mathcal{E}_{\mathrm{C},1}}{N_0}\}$. When the condition of $2^{B+1}\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}} < \theta$ is satisfied, we obtain the asymptotic bound with respect to B on distortion as

$$D \leq \alpha(\mathcal{E}_{\mathrm{D},1},\rho,M)\exp\left\{-\frac{\mathcal{E}_{\mathrm{D},1}(1-\mu/3)}{N_0}\right\} + \beta(\mathcal{E}_{\mathrm{D},1},\rho,M) \quad (16)$$

In order to emphasize the significant term, it is isolated in the above given bound. Here, α and β denote functions of $\mathcal{E}_{\mathrm{D},1}$, ρ and M which arose from the terms K_3, K_5 and corresponding distortion in (14) and from the terms K_7, K_9 and corresponding distortions with lower order terms, respectively. Clearly, k does not play a role on the exponential behaviour of the bound (16). Note that in (14) k equals to M for the terms with the factors from K_2 to K_5. Because of the quantizer construction, $1-\rho^2$ is considered as in the same order of 2^{-2B} and consequently should be chosen to behave as $\exp\left\{-\frac{\mathcal{E}_{\mathrm{D},1}}{N_0}\right\}$. As a result we obtain the same collaboration effect as in (7) albeit with a factor 2 gap in energy efficiency. The latter may be due to simplifying steps in the outer-bound. Furthermore, we notice that condition for exploiting collaboration between the sources is based on relationship between the observation error variance $(1-\rho^2)$ and the *aggregate energy* as opposed to the individual source energies.

IV. Conclusion

This paper covered an adaptation of two-way low-latency feedback protocol for minimal distortion studied in [2] and [3] to a large network with multiple sources. Specifically, we have provided lower-bounds on the reconstruction error of arbitrary multi-sensor transmission strategies which can serve in a subsequent step to determine the optimality of particular multiple-accces and encoding strategies. To this end, we have proposed one such collaborative strategy exploiting correlation between sensors. Asymptotic upper-bounds on the reconstruction error have been provided for the proposed protocol. Both the upper and lower-bounds show that collaboration can be achieved through energy accumulation and bring to light a trade-off in source and channel SNR allowing it to occur. Future work will consider more general distributed sensing and transmission strategies for multi-dimensional sources aiming at energy-efficiency and low-latency protocols as well as tightening the lower-bounds (6).

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VI. Appendix

For the first expansion of $I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S)$ based on the sources, we have two different derivations for the two distribution types.

$$\begin{aligned}I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S) &= h(\mathbf{U}|\{\mathbf{V}_j\}_S) - h(\mathbf{U}|\mathbf{Y},\{\mathbf{V}_j\}_S) \\ &= -I(\mathbf{U};\{\mathbf{V}_j\}_S) + h(\mathbf{U}) - h(\mathbf{U}-\hat{\mathbf{U}}|\mathbf{Y},\{\mathbf{V}_j\}_S) \\ &\geq -h(\{\mathbf{V}_j\}_S) + h(\{\mathbf{V}_j\}_S|\mathbf{U}) + h(\mathbf{U}) - h(\mathbf{U}-\hat{\mathbf{U}}).\end{aligned} \quad (17)$$

$$D \leq K_1 D_q + D_{e,M}\left(K_2\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}e^{(B+1)\ln 2} + K_3\epsilon(\rho)\right)^{M-1} e^{(B-2M+2)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}+2\mathcal{E}_{\mathrm{C},1}(\sqrt{\lambda}-1)^2}{2N_0}}$$
$$+D_{e,M}\left(K_4\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}e^{(B+1)\ln 2} + K_5\epsilon(\rho)\right)^{M-1} e^{(B-4M+2)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}}$$
$$+\sum_{k=1}^{M-1} D_{e,k}\binom{M}{k}\left(K_6(k)\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}e^{(B+1)\ln 2} + K_7(k)\epsilon(\rho)\right)^{k} e^{-\frac{k(\mathcal{E}_{\mathrm{D},1}+2\mathcal{E}_{\mathrm{C},1}(\sqrt{\lambda}-1)^2)}{2MN_0}}$$
$$+\sum_{k=1}^{M-1} D_{e,k}\binom{M}{k}\left(K_8(k)\frac{\sqrt{1-\rho^2}}{\rho+\sqrt{1-\rho^2}}e^{(B+1)\ln 2} + K_9(k)\epsilon(\rho)\right)^{k} e^{-\frac{k(\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2})}{2MN_0}} \quad (14)$$

For the case where $\mathbf{U}$ and whole set of $\mathbf{U}_j$'s are uniformly distributed, the above expansion (17) becomes

$$I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S) \geq -\frac{|S|K}{2}\log 2\pi e + |S|K\log(2\sqrt{3(1-\rho^2)})$$
$$+ K\log 2\sqrt{3} - \frac{K}{2}\log(2\pi e D)$$
$$\geq K\log\left(\frac{(2\sqrt{3})^{|S|+1}(1-\rho^2)^{\frac{|S|}{2}}}{(2\pi e)^{\frac{|S|+1}{2}}D^{1/2}}\right) \quad (18)$$

whereas the same expansion becomes

$$I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S) \geq -\frac{|S|K}{2}\log(2\pi e)$$
$$+\frac{|S|K}{2}\log(1-\rho^2)2\pi e + \frac{K}{2}\log(2\pi e) - \frac{K}{2}\log(2\pi e D)$$
$$= \frac{K}{2}\log\left(\frac{(1-\rho^2)^{|S|}}{D}\right) \quad (19)$$

for the Gaussian case where $|S|$ denotes the size of the set $\mathbf{V}_j$ and using the following bound on entropy $h(\mathbf{U}-\hat{\mathbf{U}})$

$$h(\mathbf{U}-\hat{\mathbf{U}}) \leq \sum_{j=1}^{K} h(U_j-\hat{U}_j) \leq \frac{K}{2}\log\left(\frac{2\pi e}{K}\sum_{j=1}^{K}\mathbb{E}[(U_j-\hat{U}_j)^2]\right)$$
$$\leq K\log\left(\sqrt{2\pi e D}\right). \quad (20)$$

The second expansion of (17) based on the output signals is given by

$$I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S) \overset{(a)}{\leq} I(\mathbf{U};\mathbf{Y}|\{\mathbf{V}_j\}_S,\Phi)$$
$$= h(\mathbf{Y}|\{\mathbf{V}_j\}_S,\Phi) - h(\mathbf{Y}|\mathbf{U},\{\mathbf{V}_j\}_S,\Phi)$$
$$= \sum_{i=1}^{N} h(Y_i|Y^{i-1},\{\mathbf{V}_j\}_S,\Phi) - \sum_{i=1}^{N} h(Y_i|Y^{i-1},\{\mathbf{V}_j\}_S,\mathbf{U},\Phi)$$
$$\leq \sum_{i=1}^{N} h(Y_i|Y^{i-1},\{\mathbf{V}_j\}_S,\{\mathbf{X}_j e^{i\phi_j}\}_S,\Phi)$$
$$-\sum_{i=1}^{N} h(Y_i|Y^{i-1},\mathbf{U},\{\mathbf{X}_j e^{i\phi_j}\}_S,\{\mathbf{X}_j e^{i\phi_j}\}_{S^c},\Phi)$$
$$= \sum_{i=1}^{N} h(Y_i - \sum_{j\in S} X_{i,j}e^{i\phi_{i,j}}|Y^{i-1},\{\mathbf{V}_j\}_S,\{\mathbf{X}_j e^{i\phi_j}\}_S,\Phi)$$
$$-\sum_{i=1}^{N} h(Y_i - \sum_{j\in S} X_{i,j}e^{i\phi_{i,j}} - \sum_{j\in S^c} X_{i,j}e^{i\phi_{i,j}}|Y^{i-1},\mathbf{U},$$
$$\{\mathbf{X}_j e^{i\phi_j}\}_S,\{\mathbf{X}_j e^{i\phi_j}\}_{S^c},\Phi)$$
$$= \sum_{i=1}^{N} h(\sum_{j\in S^c} X_{i,j}e^{i\phi_{i,j}} + Z_i|Y^{i-1},\{\mathbf{V}_j\}_S,\{\mathbf{X}_j e^{i\phi_j}\}_S,\Phi)$$
$$-\sum_{i=1}^{N} h(Z_i)$$
$$\leq \sum_{i=1}^{N}\log\left(1+\frac{\sum_{j\in S^c}\mathcal{E}_{i,j}}{N_0}\right)$$
$$\leq N\log\left(1+\frac{\sum_{i=1}^{N}\sum_{j\in S^c}\mathcal{E}_j}{NN_0}\right)$$
$$\leq N\log\left(1+\frac{K\sum_{j\in S^c}\mathcal{E}_j}{NN_0}\right). \quad (21)$$

where (a) is a result of the fact that $\mathbf{U}$ and Φ are independent.

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