NetDoFs of the MISO Broadcast Channel with Delayed CSIT Feedback for Finite Rate of Innovation Channel Models

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***Abstract*—Channel State Information at the Transmitter (CSIT) is of utmost importance in multi-user wireless networks, in which transmission rates at high SNR are characterized by Degrees of Freedom (DoF, the rate prelog). In recent years, a number of ingenious techniques have been proposed to deal with delayed and imperfect CSIT. However, we show that the precise impact of these techniques in these scenarios depends heavily on the channel model. We introduce the use of linear Finite Rate of Information (FRoI) signals to model time-selective channel coefficients, a model which turns out to be well matched to DoF analysis. Both the block fading model and the stationary bandlimited channel model are special cases of the FRoI channel model (CM). However, the fact that FRoI CMs model stationary channel evolutions allows to exploit one more dimension: arbitrary time shifts. In this way, the FroI CM allows to maintain the DoF unaffected in the presence of CSIT feedback (FB) delay, by increasing the FB rate. We call this Foresighted Channel Feedback (FCFB). We then consider netDoF, by accounting also for the DoF consumed in training overhead and feedback. We work out the details for the MISO broadcast channel (BC), including optimization of the number of users, and exhibit unmatched netDoF performance compared to existing approaches.**

I. Introduction

In this paper, Tx and Rx denote transmit/transmitter/ transmitting/transmission and receive/receiver/receiving/reception. Interference is undoubtedly the main limiting factor in multi-user wireless communication systems. Tx side or Rx side zero-forcing (ZF) beamforming (BF) or joint Tx/Rx ZF BF (signal space interference alignment (IA)) allow to obtain significant Degrees of Freedom (DoFs) (= multiplexing factor, or rate prelog). These technique require very good Channel State Information at TX and Rx (CSIT/CSIR). Especially CSIT is problematic since it requires feedback (FB) which involves delay, which may be substantial if FB Tx is slot based. It therefore came as a surprise that with totally outdated delayed CSIT, the MAT scheme [1] is still able to produce significant DoF gains for multi-antenna transmission. Using a sophisticated variation of the MAT scheme, [2] was able to propose an improved scheme for the case where the FB delay is less than the channel coherence time. It was generally believed that any delay in the feedback necessarily causes a DoF loss. However, Lee and Heath in [3] proposed a scheme that achieves N_t (sum) DoF in the block fading underdetermined MISO BC with N_t transmit antennas and $K = N_t + 1$ users if the feedback delay is small enough ($\leq \frac{T_c}{K}$). We introduce FRoI channel models and exploit their approximately stationary character to propose a simple ZF scheme based on Foresighted Channel FB (FCFB). The DoF of FCFB ZF are also insensitive to FB delay. We then analyze the netDoF of these and other recent schemes.

II. Some Channel Model State of the Art

One category of popular channel models is the (first-order) autoregressive (Gauss-Markov) channel model, see e.g. [4]. However, these models (at finite and especially low order) do not allow perfect prediction and hence do not lead to interesting DoF results. These models are called regular in [5]. The two classical (nonregular) channel models that allow permanent perfect CSIT for Doppler rate perfect channel feedback are block fading and bandlimited (BL) stationary channels. The block fading model dates back to the time of GSM where it was quite an appropriate model for the case of frequency hopping. However, though this model is very convenient for very tractable analysis (e.g. for sinngle-user MIMO [6]), it is inappropriate for DoF analysis which works at infinite SNR and requires exact channel models. Now, whereas exact channel models do not exist, channel models for DoF analysis should at least be good approximations. Indeed, mobile speeds and Doppler shifts are finite. This leads to a strictly BL Jakes Doppler spectrum. However, in the Jakes model, the mobile terminal has a certain speed without ever moving (attenuation, directions of arrival, path delays, speed vector etc. are all constant forever). In reality, the channel evolution constantly evolves from one temporarily BL Doppler spectrum to another, leading to a possibly overall stationary process but that is not BL.

Another aspect is that there is a difference between channel modeling for CSIR only and for CSIT. In the CSIR case, causality is not much of an issue and channel estimation can be done in a non-causal fashion. Hence block processing and

† EURECOM's research is partially supported by its industrial members: ORANGE, BMW Group, Swisscom, SFR, ST Microelectronics, Symantec, SAP, Monaco Telecom, iABG, and also by the EU FP7 NoE NEWCOM#. The research of Orange Labs is partially supported by the EU FP7 project METIS.

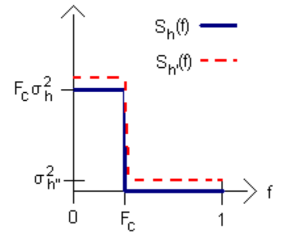

Fig. 1. A bandlimited (BL) Doppler spectrum and its noisy version.

associated channel models as in [4] and references therein are acceptable. In the CSIT case however, the CSI needs to be fed back for adaptation of the Tx. Due to the feedback delay, the channel estimation in the CSIT scenario is necessarily causal (case of prediction). Hence different channel models are required.

III. The Bandlimited (BL) Doppler Spectrum Case

In an optimal approach, all channel coefficients (in the channel impulse response) need to be treated jointly. However, if no deterministic relations exist between the channel coefficients, then for the purpose of DoF analysis, we may consider the case of i.i.d. channel coefficients. In what follows we consider one such generic channel coefficient h. Its temporal evolution is a stationary discrete-time process, at the sampling rate (channel uses) of the communications channel. We assume this sampling rate to be normalized to 1. We assume the Doppler spectrum $S_h(f)$, the spectrum of the process h, to be bandlimited to F_c, which is the total Doppler Bandwidth (as the channel coefficients are complex, the position of the Doppler spectrum w.r.t. the carrier frequency is less crucial, so we can assume the Doppler support to be $[0, F_c]$ as in Fig. 1; also, $S_h(f)$ is periodic in frequency f with period 1). We denote the coherence time as $T_c = 1/F_c$. Due to the (deterministic) estimation of the channel in the downlink, and its imperfect feedback to the Tx, the Tx has a noisy version h' with additive estimation noise h'' (note that the use of a prior channel distribution in a Bayesian approach can be postponed until the prediction operation to follow). The noisy spectrum is $S_{h'}(f) = S_h(f) + S_{h''}(f) = S_h(f) + \sigma^2_{h''}$ assuming independent white noise h''. Let T_{fb} be the delay with which the channel estimate h' arrives at the Tx for (instantaneous) adaptation of the transmitter. That means that the Tx has to perform channel prediction over a horizon of T_{fb}. Assuming a Gaussian channel and estimation noise, linear minimum mean squared error (LMMSE) prediction is optimal (if MMSE is the optimality criterion). Prediction over a horizon of T_{fb} samples will become prediction by one sample if we downsample the channel estimate signal by a factor T_{fb}. Downsampling in the time domain leads to a expansion of the spectrum support by a factor T_{fb} (of course, prediction from a subsampled version is of a degraded quality in the noisy case). Considering Fig. 1, as long as $F_c T_{fb} < 1$ (or $T_{fb} < T_c$), the downsampled channel signal remains bandlimited. Let $S(f)$ denote the downsampled version of $S_{h'}(f)$. Then we get for the (infinite order) one sample ahead prediction MSE

$$\sigma^2_{\tilde{h}'} = e^{\int_0^1 \ln S(f)\, df} \sim \sigma_{h''}^{2(1-F_c T_{fb})} \,. \tag{1}$$

A similar behavior is obtained for the T_{fb} ahead prediction error from the original unsubsampled process. The prediction error $\tilde{h}'$ considered in (1) is actually the error in estimating h' from its past. However, what we are really interested in is estimating h from the past of h', with prediction error $\tilde{h}$. Now, since h'' is white noise, we get in fact $\sigma^2_{\tilde{h}} = \sigma^2_{\tilde{h}'} - \sigma^2_{h''}$. When $T_{fb} > 0$, the dominating term at high SNR is still $\sigma^2_{\tilde{h}'}$ though.

Let $P(f) = 1 - \sum_{n=1}^{\infty} p_n e^{-j2\pi f n}$ be the (one sample ahead) prediction error filter for h'. The p_k are the prediction coefficients, both for h' or h. As infinite order prediction succeeds in whitening the prediction error, we have that

$$S_{h'}(f) = \frac{\sigma^2_{\tilde{h}'}}{|P(f)|^2} \tag{2}$$

which is the Kolmogorov representation, an infinite order autoregressive (AR(∞)) model. Since $|P(f)|$ is a scaled version of $1/\sqrt{S_{h'}(f)}$, it can easily be seen that $P(f)$ is a high-pass filter, and converges to an ideal high-pass filter as the SNR increases [7]. This has led a number of researchers (see [7] and references therein) to construct predictors for bandlimited signals simply by approximating ideal high-pass filters. These FIR filters are typically chosen to be linear phase and are made monic ($p_0 = 1$) by dividing the filter by its first coefficient. However, the prediction error filter $P(f)$ is not only monic but also minimum-phase.

A. The Noiseless BL Case: two-time scale model

Now consider the noiseless case, $\sigma^2_{h''} = 0$. Then clearly the prediction errors become zero, $\sigma^2_{\tilde{h}} = \sigma^2_{\tilde{h}'} = 0$. Hence the signal can be perfectly predicted from its past. For simplicity let T_c be an integer. Let h_k denote the channel coefficient at discrete time k and consider one sample ahead prediction, then $h_k = \sum_{n=1}^{\infty} p_n\, h_{k-n}$. Note that the prediction error filter $P(f)$, which is an ideal high-pass filter, can be chosen to be independent of the actual Doppler spectrum $S_h(f)$ within its support, and can be chosen to be only a function of the Doppler spread $F_c = \frac{1}{T_c}$. Let us denote this spectrum independent prediction error filter as $P_{T_c}(f)$. As we have perfect prediction, we can repeat the one sample ahead prediction recursively to perfectly predict multiple samples ahead. Can this be repeated indefinitely? No because when we hit prediction horizon T_c, T_c-ahead prediction being here (in terms of zero prediction error) equivalent to 1-ahead prediction on a T_c times downsampled signal, downsampling (and hence stretching its support) $S_h(f)$ by a factor T_c makes it non-singular at all frequencies (non bandlimited). Note also that due to the perfect predictibility over the horizon $\{1, \ldots, T_c - 1\}$, linear estimation in terms of the complete past is equivalent to linear estimation in terms of a T_c times downsampled version of the past, since the samples in between can be filled up causally from a downsampled version. At prediction horizon T_c now, from a T_c times downsampled past, we are dealing with standard 1-ahead linear prediction of a non bandlimited stationary process, which under some regularity conditions can be considered as an AR(∞) process (Kolmogorov model). Let the infinite order prediction error filter for the T_c times downsampled process be $A(f)$. The reasoning above allows us to formulate the following theorem.

Theorem 1: **Two-Time Scale BL Model** The prediction error filter for a stationary process h_k bandlimited to $1/T_c$ (T_c integer) can be modeled as

$$P(f) = P_{T_c}(f)\, A(T_c f) \tag{3}$$

where $P_{T_c}(f)$ is the prediction error filter for a BL process with flat Doppler spectrum and $A(f)$ is the prediction error filter for the downsampled h_{kT_c}.

Let $G(f) = 1/P_{T_c}(f) = \sum_{n=0}^{\infty} g_n\, e^{-j2\pi f n}$ which is like $P_{T_c}(f)$ again a minimum-phase monic causal filter. Note that $G(f)$ behaves like an ideal low-pass filter with bandwith $1/T_c$, hence the T_c times downsampled version of its impulse response is a delta function: $g_{nT_c} = g_0\, \delta_{n0}$. Then the stationary BL process h_k can be generated as

$$h_k \;=\; g_k \;*\; h_k^{\downarrow\uparrow} \tag{4}$$

where $h_k^{\downarrow\uparrow}$ is the T_c times downsampled and then T_c times upsampled (inserting $T_c - 1$ zeros between consecutive samples) version of h_k and $*$ denotes convolution. The block fading model is similar to (4) with g_k now a rectangle: $g_k = 1$, $k = 0, 1, \ldots, T_c - 1$ and zeros elsewhere. With this similarity, the block fading and BL stationary case have in common that for every consecutive coherence period T_c, if the first sample (and the past) is known, then the remaining $T_c - 1$ samples of the current coherence period are known [8].

B. Back to the Noisy BL Case

The prediction of a BL process is not a stable operation [9] as can be seen from (1) where $\sigma^2_{\tilde{h}'}$ grows more rapidly than linear in $\sigma^2_{\tilde{h}''}$ (assuming $\sigma^2_{\tilde{h}''}$ is small). This is related to the fact that the (noiseless) prediction coefficients p_k are of infinite length and are not rapidly decaying. In [10], it was shown (for CSIR purposes) that the stationary BL model and the block fading model become equivalent as $F_c \to 0$. Such equivalence in the limit will also result for CSIT purposes here. But we want to go beyond the limit of very small Doppler spread.

In [2], the behavior of (1) is exploited to show the resulting DoF of the 2 user MISO BC. However, what is not mentioned there is that these results correspond to a channel model that needs to be in a range between two extreme models. The one extreme model is block fading over blocks of length T_{fb}, with stationary F_c-BL evolution of the value of the blocks, and channel feedback every T_{fb}. The other extreme is a genuine F_c-BL stationary channel model, but then the channel needs to be fed back every sample! In [8], it was shown that the DoFs of [2] can be reproduced very simply in the case of a block fading model, by the MAT-ZF scheme, a simple combination of MAT (during T_{fb}, while waiting for the channel FB) and ZF for the rest of the coherence period. In [11] it was shown in an alternative fashion that the channel FB rate could be reduced w.r.t. [2] by a factor T_c/T_{fb} (equivalent to FB every T_c instead of every T_{fb}). To reproduce these results for the stationary BL case is not easy though, and the scheme of [2] is quite intricate, involving, as in MAT, FB of (residual) interference (now necessarily digital, with superposition coding and sequential decoding). The models we introduce next allow to retain the simplicity of block fading models and even go beyond them.

IV. Linear Finite Rate of Innovation (FRoI) Channel Models (CM)

FRoI signal models were introduced in [12]. Innovation here could be a somewhat misleading term since historically (in Kalman filter parlance) the term "innovations" has been used to refer to the infinite order prediction errors. In [12] and here, the rate of innovation could be considered to be the DoF of signals (i.e. the source coding rate prelog). FRoI represents the time series case of sparse modeling. The FRoI signal models that have been considered in [12] could be in general non-linear. In other words, the FRoI represents the average number of parameters per time unit needed to describe the signal class and these parameters could enter the signal model in an arbitrary fashion. For instance, the signal could be a linear superposition of basis functions of which also the positions (delays, and in the channel modeling case e.g. also Doppler shifts) are parameterized. For the purpose of channel modeling and FB, with essentially stationary signals that need to be processed in a causal fashion, it would appear reasonable to stick to linear FRoI models, in which the parameters are just the linear combination coefficients of fixed, periodically appearing basis functions, commensurate with the Doppler bandwith. In the case of a single basis function, the FRoI channel model is similar to (4):

$$h_k = g_k \;*\; a_k^{\uparrow} \tag{5}$$

where $a_k^{\uparrow}$ is a T_c times upsampled discrete-time signal of which the non-zero samples (parameters) appear once every T_c sampling periods, and the basis function g_k is a causal FIR approximation to an ideal lowpass filter with bandwidth F_c. The length of the basis function g_k is intended to span several T_c. Again, the block fading model (with g_k of length T_c) is a special case. By making the filter longer however, a bandlimited characteristic can be better approximated. Obviously, the BL model (4) can be obtained by letting the filter length become infinite. Starting from a stationary sequence a_k, the process h_k generated by (5) is cyclostationary. By letting g_k better approximate a lowpass (or bandpass) filter, the cyclostationary process gets closer to stationary. In any case, at the start of each new coherence period T_c, knowing the past, the estimation of the sample h_k allows the estimation of the new parameter $a_k^{\uparrow}$ involved. And this in turn allows to determine the evolution of h_k for the next $T_c - 1$ samples. In the presence of noise, it is clearly desirable to have a first coefficient g_0 that is large (though any non-zero coefficient is sufficient for DoF analysis purposes). Due to the finite length and energy of the filter g_k, the effect of noise is limited and the prediction error variance over the coherence period will remain of the order of $\sigma^2_{\tilde{h}''}$, the noise level in the channel FB. We leave the subject of the optimization of the basis function p_k for further research, but an FIR predictor (for an ideal lowpass spectrum) is clearly a good candidate. As the sampling rate (and hence FB frequency) of BL signals increases, the horizon of perfect prediction increases proportionally, and becomes infinite as the continuous-time past signal becomes available [9]. Of course, for all real-world signals for which a BL model seems plausible (e.g. the speech signal), this does not work because real-world signals are only approximately stationary and bandlimited over a limited time horizon. From this point of view, linear FRoI models which are approximately bandlimited but with a finite memory might be better approximations. A lot of work on estimating FRoI signals has focussed on non-causal approaches [13]. However, what is needed for the application of FRoI to channel feedback is a design with prediction in mind.

For a number of applications (handling of multiple users with different T_{fb} or different T_c, see further also), the use of FRoI models with multiple basis functions might be desirable. In this case the FRoI model becomes

$$h_k = \sum_{m=1}^{M} g_k^{(m)} \ast a_k^{\uparrow(m)} \tag{6}$$

where the $a_k^{\uparrow(m)}$ are M sequences of parameters that are now MT_c times upsampled, to preserve a RoI of F_c. As the $g_k^{(m)}$ represent M different basis functions that are essentially bandlimited and also time limited, there might be some connection with prolate spheroidal wave functions [4], [9]. However, to limit FB delay, the first M coefficients of these basis functions play a particularly important role.

V. DoF obtained with FRoI Channel Models (CMs)

As mentioned above, DoF obtained with block fading CMs can immediately be extended to FRoI CMs. Hence the DoF of the MAT-ZF scheme of [8], obtained in [8] for block fading, also apply for FRoI. This allows to reproduce the DoF of [2] for the 2-user MISO BC, and furthermore extend these DoF results to any MIMO single-hop multi-user network (Interfering Broadcast Channel, MAC, etc.) by simply combining the DoF of MAT and ZF for such networks (when known):

$$\mathrm{DoF}_{MAT-ZF} = \frac{T_{fb}}{T_c}\,\mathrm{DoF}_{MAT} + (1 - \frac{T_{fb}}{T_c})\,\mathrm{DoF}_{ZF}\ . \tag{7}$$

These DoF can furthermore be improved by switching to FRoI models with $M > 1$ basis functions. As the RoI in these models is unchanged, the (average) feedback rate is unchanged. However, with $M > 1$, feedback needs to occur only once every MT_c, and hence FB delay is suffered only once every MT_c. Hence the weight of the MAT portion in the DoF_{MAT-ZF} is reduced to $\frac{T_{fb}}{MT_c}$, bringing the DoF_{MAT-ZF} closer to DoF_{ZF}. In theory M could be made arbitrarily large, but not in practice.

The main characteristic of FRoI CMs though is that they closely approximate stationary (BL) signals. *This means that if a FRoI CM is a good model, so is an arbitrary time shift of the FRoI model.* This can be exploited to overcome the FB delay as explained in Fig. 2. Consider FRoI CM with $M = 1$ basis function. While the current coherence period is running, as the Channel FB (CFB) is going to take a delay of T_{fb}, instead of waiting for the end of the current T_c, we start the next coherence period T_{fb} samples early. This means jumping from the subsampling grid of the FRoI model to the shifted subsampling grid of another instance of the same FRoI model. This involves recalculating the (finite number of past) FRoI parameters $a_k^{\uparrow}$ for the new grid from the past channel evolution on the old grid, plus a new channel estimate at the start of the T_c on the new grid. In this way the FB (sampling) "rate" increase from $\frac{1}{T_c}$ to $\frac{1}{T_c - T_{fb}}$. But the CSIT is available at the Tx all the time, with an SNR proportional to the general SNR.

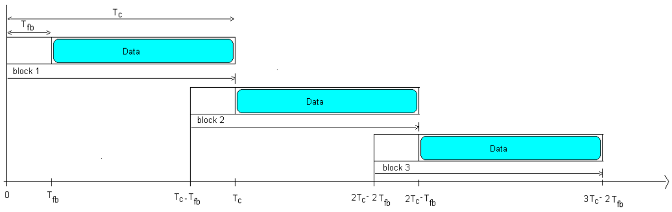

Fig. 2. Foresighted Channel Feedback (FCFB).

By increasing M, the number of basis functions, this approach continues to work for any $T_{fb} < M\,T_c$, and hence for any T_{fb}.

VI. Attainable (Sum) NetDoF of MISO BC with FRoI CMs

In order to evaluate the performances that can be expected in actual systems we now account for training overhead as well as the DoF consumption due to the feedback on the reverse link. For the K Rxs to estimate their channel, a common training of length $T_{ct} \geq N_t$ is needed as explained in [14]. To maximize the DoF we take $T_{ct} = N_t$. According to [15], an additional dedicated training of 1 pilot is required when coherent reception is needed resulting in N_t+1 symbol periods per block devoted to training in order to perform ZF.

Since we are interested in the DoF consumed by the FB, which is the scaling of the FB rate with $\log_2(P)$ as $P \to \infty$, the noise in the fed back channel estimate can be ignored in the case of analog FB or of digital FB of equivalent rate. The FB can be considered accurate, suffering only from the delay T_{fb}. We consider analog output FB, the Rxs directly feed back the training signal they receive and the Tx performs the (downlink) channel estimation. The FB of N_t symbols per user consumes KN_t channel uses on the reverse link.

1) ZF_{FCFB}: With FB every $T_c - T_{fb}$, the netDoF by performing ZF precoding is then

$$\mathrm{netDoF}(\mathrm{ZF}_{N_t}) \quad = \quad K\left(1 - \frac{2N_t + 1}{T_c - T_{fb}}\right) \tag{8}$$

since with full CSIT, the full DoF can be achieved with ZF [16]. For sake of comparison we concisely review the netDoF yielded by other schemes in the MISO BC with delayed CSIT. When FB is done only every T_c, there are always two parts in each block, a first part with outdated CSIT a second part with current CSIT.

2) Classic ZF: Performing ZF only when CSIT is available, the netDoF is

$$\text{netDoF}(\text{ZF}_{N_t}) = N_t \left(1 - \frac{T_{fb}}{T_c} - \frac{2N_t + 1}{T_c}\right). \quad (9)$$

3) TDMA-ZF: TDMA-ZF is a direct extension of ZF. The only difference being that while the transmitter is waiting for the CSI, and not sending training symbols it performs TDMA transmission since this does not require any CSIT, thus yielding

$$\text{netDoF}(\text{TDMA-ZF}_{N_t}) = \text{netDoF}(\text{ZF}_{N_t}) + \frac{T_{fb}}{T_c} \quad (10)$$

4) MAT: The MAT scheme was proposed in [1]. The authors describe an original approach that yields a DoF $\frac{N_t D}{Q}$ with no *current* CSIT at all. Here $\{D, Q\} \in \mathbb{N}^2$ are such that $\frac{1}{1+\frac{1}{2}\cdots\frac{1}{N_t}} = \frac{D}{Q}$, where D is the least common multiple of $\{1, 2, \cdots, N_t\}$ and $Q = DH_{N_t}$ with $H_{N_t} = \sum_{m=1}^{N_t} \frac{1}{m}$. This scheme allows the transmission of D symbols in Q time slots for each user as noted in [17]. To perform this scheme the Rxs not only need to know their channel but also that of some other Rxs (a different subset in each block), resulting in the need for a CSIR distribution.

In [18], FB and training overheads as well as the cost of the CSIR distribution are determined. Assuming $K = N_t$, we get

$$\text{netDoF}(\text{MAT}_{N_t}) = \frac{N_t(T_c - N_t) - \sum_{j=1}^{K} \frac{1}{j}(K-j)(N_t - j + 1)}{H_K T_c + \left(\left(\frac{K-1}{K} \sum_{j=1}^{K} \frac{(K-j)(N_t-j+1)}{j}\right) + H_k - K\right)} \quad (11)$$

5) MAT-ZF: The idea behind the MAT-ZF scheme is essentially to perform ZF and superpose MAT only during the dead times of ZF. For that purpose we consider Q blocks of T_c symbol periods and split each block into two parts. The first part, the dead times of ZF, spans T_{fb} symbol periods and the second part, the $T_c - T_{fb}$ remaining symbols. We use the first part of each block to perform the MAT scheme T_{fb} times in parallel. During the second part of each block, ZF is performed.

The sum DoF for the MAT-ZF$_K$ scheme without accounting for the overhead is

$$\text{DoF}(\text{MAT-ZF}_{N_t}) = N_t \left(1 - \frac{(Q - D)T_{fb}}{QT_c}\right).$$

Indeed, per user, in QT_c channel uses, the ZF portion transmits $Q(T_c - T_{fb})$ symbols, whereas the MAT scheme transmits DT_{fb} symbols.

The net DoFs yielded by this scheme is determined in [18],

$$\text{netDoF}(\text{MAT-ZF}_{N_t}) = \text{netDoF}(\text{ZF}_{N_t}) + \frac{T_{fb}}{T_c} \frac{N_t}{(H_{N_t} + \delta)} \quad (12)$$

where $\delta = \frac{\frac{K-1}{K} \sum_{j=1}^{K} \frac{D(K-j)(N_t-j+1)}{j})}{DT_{fb}}$ i.e., the netDoF of ZF plus an additional term, the DoF brought about by MAT but decreased by a factor due to the CSIR distribution.

6) ST-ZF: Lee and Heath [3] proposed a scheme to achieve N_t DoF in the MISO BC with $K = N_t + 1$ users when $\gamma = \frac{T_{fb}}{T_c} \leq \frac{1}{K}$. For $\gamma < \frac{1}{K}$ N_t DoF can also be reached by doing ZF the remaining time. We refer to this scheme as ST-ZF since it is a space-time (ST) precoding, which is combined with ZF for $\gamma < \frac{1}{K}$.

In [18] the net DoF yielded by the ST-ZF scheme is determined. Actually two values are proposed depending on how some data needed at the receiver is transmitted. The two variants, ST-ZF and ST-ZF2 being adapted for different values of the feedback delay. The net multiplexing gain of the first variant is

$$\text{netDoF}(\text{ST-ZF}_{N_t}) = N_t \left(1 - \frac{3(N_t + 1)}{T_c}\right) \quad (13)$$

as long as $\frac{T_{fb}+1}{T_c - 2(N_t+1)} \leq \frac{1}{K} \Leftrightarrow T_c \geq K(T_{fb} + 3)$ since ST-ZF needs a part with CSIT that is $K - 1 = N_t$ times longer than the no current CSIT part. With the second variant the netDoF is

$$\text{netDoF}(\text{ST-ZF2}_{N_t}) = \frac{N_t\left(1 - \frac{2(N_t+1)}{T_c}\right) + \text{netDoF}(\text{ZF}_{N_t})\frac{KN_t}{T_{fb}+1}}{1 + \frac{KN_t}{T_{fb}+1}} \quad (14)$$

as long as $\frac{T_{fb}+1}{T_c - (N_t+1)} \leq \frac{1}{K}$.

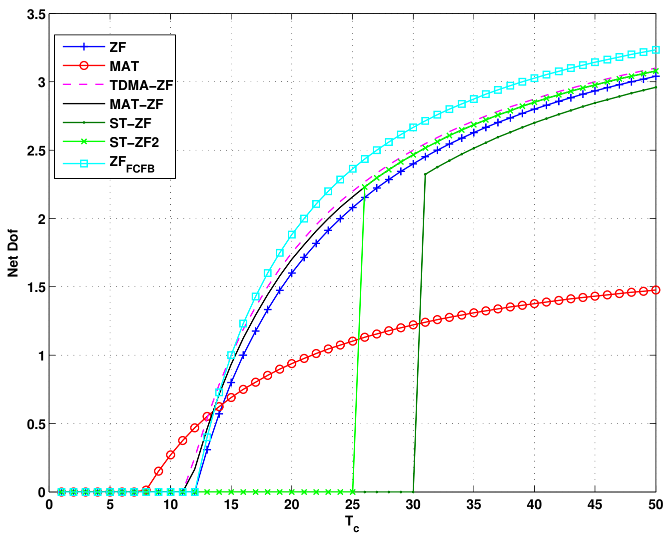

Fig. 3. NetDoF of ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA for $N_t = 4$, $T_{fb} = 3$ as a function of T_c.

VII. Numerical Results

In Fig. 3 we plot the netDoF provided by ZF$_{FCFB}$, ZF, MAT, TDMA-ZF, MAT-ZF, TDMA and ST-ZF for $N_t = 4$, $T_{fb} = 3$ as a function of T_c using (8) for ZF$_{FCFB}$, (9) for ZF, (11) for MAT, (10) for TDMA-ZF, (12) for MAT-ZF, (13) for ST-ZF and (14) for ST-ZF 2. A discussion regarding all the schemes except ZF$_{FCFB}$ is already conducted in [18], here we observe that as soon as $T_c > 15$ ZF$_{FCFB}$ outperforms all the other schemes. We also note that for $T_c < 15$ all schemes yield less than 1 net DoF meaning that simple TDMA transmission

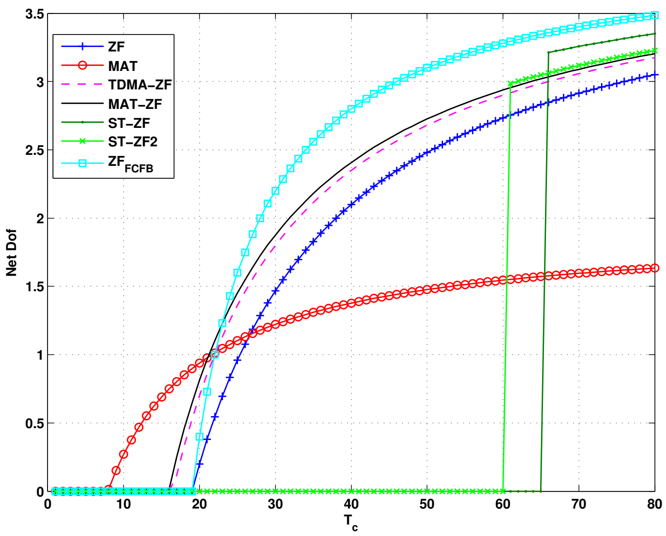

Fig. 4. NetDoF of ZF_{FCFB}, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA for $N_t = 4$, $T_{fb} = 10$ as a function of T_c.

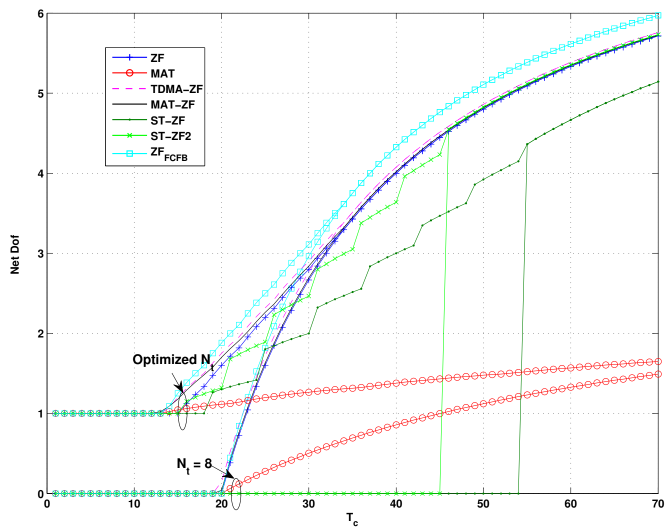

Fig. 5. NetDoF of ZF_{FCFB}, ZF, MAT, TDMA-ZF, MAT-ZF, ST-ZF and TDMA and their optimized variants for $N_t = 8$, $T_{fb} = 3$ as a fn of T_c.

would actually be better under these conditions. In Fig. 4 the same curves are plotted for $T_{fb} = 10$, a similar behavior is observed and the gap between ZF_{FCFB} and the other schemes is wider.

A. Optimization of K, the number of users

As it was noticed in [19] the number of users K (and hence active antennas N_t) needs to be optimized to find the right channel learning/using compromise because serving more users means a larger DoF but also larger overhead. All the net DoF of the schemes we reviewed reach a single maximum as a function of the number of antennas. For the scheme we proposed, ZF_{FCFB}, the net DoF are a simple quadratic function in N_t

$$f(N_t) \;=\; -\frac{2}{T_c - T_{fb}} N_t^2 + \left(\frac{T_c - T_{fb} - 1}{T_c - T_{fb}}\right) N_t$$

which is maximized for $N_t = \frac{T_c - T_{fb} - 1}{4}$. So in Fig. 3 for $T_c = 16$ better performances could be achieved with $N_t = \frac{16-3-1}{4} = 3$. To each scheme we associate its optimized version, in which the number of active antennas is optimized, either analytically or empirically to assure the maximum net DoF. In Fig. 5 we observe the net DoF of all considered schemes and of their optimized version for $K = 8$, $T_{fb} = 3$ as a function of T_c. We notice that if the optimization of the number antennas results in a gain for all schemes it also confirms that ZF_{FCFB} outperforms all the other schemes soon after having only one active antenna and one served user (simple TDMA) is not optimal anymore.

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