A Team Decisional Beamforming Approach for Underlay Cognitive Radio Networks

Miltiades C. Filippou, George A. Ropokis and David Gesbert

Mobile Communications Department, EURECOM, Sophia Antipolis, France
e-mail:{filippou,ropokis,gesbert}@eurecom.fr

***Abstract*—In this paper, the problem of the coexistence of two multiple-antenna wireless links is addressed in a cognitive radio scenario. The novelty brought by our setup is three-fold: First we consider a more realistic rate target constraint at the primary receiver instead of the less meaningful maximum interference temperature, second we propose a *limited* channel state information (CSI) structure whereby transmitters only have access to partly instantaneous feedback (i.e., about the direct channels) and partly statistical feedback (i.e., about the interference channels). Third, we formulate a distributed decision making scenario, by which channel information is not shared among primary and secondary transmitters. Instead, a transmitter must make a precoding decision based on local CSI only. The problem is recast as a team decision theoretic problem and the optimal precoders are obtained by solving semidefinite programs (SDPs). A distributed algorithm is derived and compared with classical precoding solutions and gains are illustrated over a range of scenarios.**

***Index Terms*—transmit beamforming, cognitive radio, semidefinite programming, team decision theory, distributed CSIT.**

I. INTRODUCTION

The problem of optimizing the coexistence of spectrum sharing devices in radio networks is emerging as an important one in view of improving the efficiency of future mobile communications systems. In the classical underlay cognitive radio context, a primary service provider allows the reuse of its spectral resource by a newcoming secondary system under a maximum tolerated interference level generated by the secondary transmitter [1] [2]. Under this setup, several approaches have been considered to optimize a beamforming (BF) vector at the transmitter side so as to strike a balance between maximizing the SNR at the served users and reducing the generated interference from the secondary transmitter [3]–[7]. In this paper, we take on the problem of optimizing multiple antenna combining at the transmitters in the downlink of an underlay cognitive radio network (CRN), however we make substantial proposals for revisions on the classical underlay model so far adopted in much of the literature. First, we propose to use a quality of service (QoS) constraint at the primary terminal in the form of a minimum data rate target. This is an alternative to the traditional interference power constraint which has the drawback of neglecting the strength of the primary link, hence oversimplifying the actual impact that the secondary link has over the primary system's performance. Secondly, we place the emphasis on more realistic channel state information scenarios at the transmitter (CSIT), whereby only a hybrid form of CSIT is available at the primary and secondary transmitters. More precisely, one assumes that direct channels between a serving transmitter to a served terminal are known in instantaneous form, while other channels are only known through second order statistics (covariance) information. Under this scenario, we consider the problem of BF design at both the primary and secondary transmitters so as to maximize secondary rate performance under a QoS target on the primary terminal. Thirdly, we are emphasizing *distributed* techniques where each transmitter makes a BF decision under its local available CSIT together with channel covariance information. We highlight the connection with team decision theory, i.e., distributed multi-agent decision making [8]. More concretely, our contributions are the following:

- We derive a closed-form expression for the users' expected rates, conditioned on the knowledge of the instantaneous direct channels as well as a simple approximation of this expression by focusing on interference-limited systems, i.e., systems in which interference is the dominant factor of signal degradation, compared to noise.
- We formulate the problem of optimal distributed transmit BF, with respect to a multiple-input single-output (MISO) CRN with distributed CSIT. We show an algorithm inspired from team decision methods. The BF solutions are reached by solving SDPs. We numerically evaluate its performance by making a comparison with known precoding solutions.

Throughout the paper, the following notations are adopted: all boldface letters indicate vectors (lower case) or matrices (upper case). Superscript $(\cdot)^H$ stands for Hermitian transpose and $\mathbb{E}\{\cdot\}$ stands for the expectation operator. For a random variable X, $X \sim \mathcal{CN}(\mu, \sigma^2)$ denotes that X follows the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. The identity matrix of dimension $n \times n$ is denoted by $\mathbf{I}_n$. Operator $\mathrm{tr}(\cdot)$ stands for the trace of a matrix and $\mathbf{A} \succeq 0$ means $\mathbf{A}$ is a positive semidefinite matrix. Moreover, $\|\cdot\|$ is the Euclidean norm, and $E_1(\cdot)$ stands for the exponential integral function, defined as in [9, 5.1.1]. Finally, $\mathbf{x} \perp \mathbf{u_A}(1)$ denotes a unit norm vector $\mathbf{x}$ perpendicular to the eigenvector corresponding to the largest eigenvalue of matrix $\mathbf{A}$.

II. System Model - Derivation of the Conditional Expected Rates

The system under investigation is shown in Fig.1. It consists of a primary base station (BS), BS_1, that communicates with a primary user (PU), U_1, in the presence of a secondary BS, BS_2 that communicates with a secondary user (SU), U_2. Assuming an underlay scenario, both BS_1 and BS_2 are sharing the same frequency resources while they are both equipped with M antennas, whereas the two users use single antenna terminals. For such a system, a distributed CSIT architecture is examined with emphasis on downlink communication. The signal received at user U_i can be expressed as

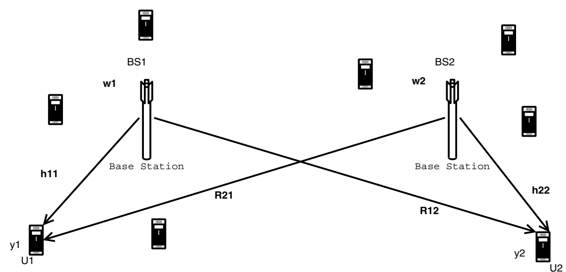

Fig. 1. System model.

$$y_i = \mathbf{h}_{ii}^H \mathbf{w}_i s_i + \mathbf{h}_{ji}^H \mathbf{w}_j s_j + n_i,\ i,j = \{1,2\},\ j \neq i \quad (1)$$

where $\mathbf{h}_{ii},\ i = \{1,2\}$ is the direct $M \times 1$ MISO Rayleigh fading channel between BS_i and U_i, $\mathbf{h}_{ji},\ j \neq i$ is the $M \times 1$ interfering Rayleigh fading channel between BS_j and U_i, whereas $\mathbf{w}_1$ and $\mathbf{w}_2$ are the $M \times 1$ complex transmit BF vectors at BS_1 and BS_2, respectively, with $\|\mathbf{w}_1\|^2 = P_{max,1}$ and $\|\mathbf{w}_2\|^2 \leq P_{max,2}$, $P_{max,1}$ and $P_{max,2}$ being the maximum available power levels at BS_1 and BS_2. Additionally, $n_i \sim \mathcal{CN}(0, N_0)$ is the additive Gaussian noise at the receiver side[1] and $s_i,\ s_j$ are the information symbols, for the transmission of which, Gaussian codebooks are used, i.e., $s_i \sim \mathcal{CN}(0,1),\ i = \{1,2\}$.

Correlated Rayleigh fading is assumed for both direct and interfering channels. As a result, the channel vector $\mathbf{h}_{ji}, i,j \in \{1,2\}$ can be expressed as

$$\mathbf{h}_{ji} = \mathbf{R}_{ji}^{1/2} \mathbf{h}_{ji}^{(w)} \quad (2)$$

where $\mathbf{R}_{ji}^{1/2}$ is the symmetric square root of the covariance matrix $\mathbf{R}_{ji}$ of vector $\mathbf{h}_{ji}$, and $\mathbf{h}_{ji}^{(w)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. It should be noted that the covariance matrix includes the parameters determining the SNR, such as the average received power. By analyzing (1), one can easily show that the achievable instantaneous rate for communication between BS_i and U_i is defined as[2]

$$R_i = \log\left(1 + \frac{|\eta_i|^2}{1 + |\zeta_i|^2}\right) \quad (3)$$

where $\eta_i = \mathbf{h}_{ii}^H \mathbf{w}_i$ and $\zeta_i = \mathbf{h}_{ji}^H \mathbf{w}_j,\ j \neq i$. A realistic assumption that can be made is that BS_i collects instantaneous CSI for its direct channel $\mathbf{h}_{ii},\ i = 1,2$, as well as statistical CSI for all links i.e., the covariance matrices $\mathbf{R}_{ji},\ i,j = \{1,2\}$[3]. Assuming that BF vectors $\mathbf{w}_1, \mathbf{w}_2$ are functions of this information, given any pair of such BF vectors, one can calculate the expected rates conditioned on the instantaneous direct channels, i.e., expectation is taken over the interference terms, in closed form for the two cells. Specifically, by following the steps of [10, Theorem 1], it is easy to prove that the expected rate of U_i, hereby denoted as $\mathbb{E}_{|\mathbf{h}_{ii}}\{R_i\},\ i = \{1,2\}$, conditioned on the instantaneous direct channel is expressed as

$$\mathbb{E}_{|\mathbf{h}_{ii}}\{R_i\} = \log(\alpha_i) + e^{\frac{\alpha_i}{\sigma^2_{\zeta_i}}} E_1\left(\frac{\alpha_i}{\sigma^2_{\zeta_i}}\right) - e^{\frac{1}{\sigma^2_{\zeta_i}}} E_1\left(\frac{1}{\sigma^2_{\zeta_i}}\right) \quad (4)$$

where $\alpha_i = 1 + |\eta_i|^2$ and $\sigma^2_{\zeta_i} = \|\mathbf{w}_j^H \mathbf{R}_{ji}^{\frac{1}{2}}\|^2,\ \ j \neq i$. Moreover, for interference dominated systems in which $\sigma^2_{\zeta_i} \gg N_0 = 1$, following an approach similar to [10, Proposition 1], it can be shown that (4) can be effectively approximated as

$$\mathbb{E}_{|\mathbf{h}_{ii}}\{R_i\} = \log(\frac{\alpha_i}{\sigma^2_{\zeta_i}}) + e^{\frac{\alpha_i}{\sigma^2_{\zeta_i}}} E_1\left(\frac{\alpha_i}{\sigma^2_{\zeta_i}}\right) + \gamma \quad (5)$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant [9, 5.1.12]. In the following section, by taking into account expression (5), the problem of joint optimal design of transmit BF vectors with limited CSIT, for both primary and secondary communication is presented and a distributed method for solving this problem is developed. The proposed design is based on the maximization of SU's transmission rate $\mathbb{E}_{|\mathbf{h}_{22}}\{R_2\}$ while satisfying a QoS constraint for the PU rate $\mathbb{E}_{|\mathbf{h}_{11}}\{R_1\}$.

III. Optimal Transmit BF - Distributed CSIT Case

With the assumption that BSs are allowed to exchange available CSIT, the problem of maximizing SU's rate $\mathbb{E}_{|\mathbf{h}_{22}}\{R_2\}$ subject to a QoS constraint for the PU rate $\mathbb{E}_{|\mathbf{h}_{11}}\{R_1\}$ would be mathematically formulated as

$$\begin{aligned} \underset{\mathbf{w}_1, \mathbf{w}_2 \in \mathbb{C}^M}{\text{maximize}} \quad & \mathbb{E}_{|\mathbf{h}_{22}}\{R_2\} \\ \text{subject to} \quad & \mathbb{E}_{|\mathbf{h}_{11}}\{R_1\} \geq T \\ & \|\mathbf{w}_1\|^2 = P_{max,1}, \quad \|\mathbf{w}_2\|^2 \leq P_{max,2} \end{aligned} \quad (6)$$

where T stands for the QoS threshold at U_1 and the expected rates are obtained from (5). Examining an interference dominated scenario, i.e., employing expression (5) and the fact that the function $f(x) = \log(x) + e^x E_1(x) + \gamma$, is a monotonically increasing function of x, it is easy to show that (6) is equivalent

[1] For the rest of our analysis and without any loss of generality, we will assume that noise density, N_0, is equal to one.

[2] Throughout the paper, information rates will be measured in nats/sec/Hz.

[3] Note that this assumption slightly extends the hybrid CSI scenario presented in [10] by also considering knowledge of covariance information regarding direct channels.

to the following problem

$$\begin{aligned}
&\underset{\mathbf{w}_1,\mathbf{w}_2\in\mathbb{C}^M}{\text{maximize}} && \frac{1+\mathbf{w}_2^H\mathbf{h}_{22}\mathbf{h}_{22}^H\mathbf{w}_2}{\mathbf{w}_1^H\mathbf{R}_{12}\mathbf{w}_1} \\
&\text{subject to} && \frac{1+\mathbf{w}_1^H\mathbf{h}_{11}\mathbf{h}_{11}^H\mathbf{w}_1}{\mathbf{w}_2^H\mathbf{R}_{21}\mathbf{w}_2}\geq\tau \\
& && \|\mathbf{w}_1\|^2=P_{max,1},\quad \|\mathbf{w}_2\|^2\leq P_{max,2}
\end{aligned} \tag{7}$$

where τ is the solution to the equation $T=f(\tau)$. However, in our analysis we consider a distributed architecture where we assume that no CSI exchange is allowed between BSs. Thus, each BS tries to solve the BF problem by exploiting only its own CSIT by applying the approach presented in the following section.

A. Distributed information structure and BF

Since the two BSs have different views (instantaneous or statistical) of the same global downlink channel, the optimal transmit BF problem can be examined within the framework of team decision theory [8], [11], [12]. Following a team decisional approach, we can then define the distributed BF design problem by means of the following components [8]: *a*) The **observations** (available CSIT) at BS_1 and BS_2, denoted as $\mathbf{z}_1=[\mathbf{h}_{11},\mathbf{R}_{11},\mathbf{R}_{12},\mathbf{R}_{21},\mathbf{R}_{22}]$ and $\mathbf{z}_2=[\mathbf{h}_{22},\mathbf{R}_{22},\mathbf{R}_{12},\mathbf{R}_{21},\mathbf{R}_{11}]$, respectively. *b*) A **transmission strategy** $g_i(\cdot), i=\{1,2\}$ available at BS_i that is used in order to calculate the BF $\mathbf{w}_i=g_i(\mathbf{z}_i)$. *c*) An estimated **model** $g_i^{(p)}(\cdot), i=\{1,2\}$ of strategy $g_i(\cdot), i=\{1,2\}$ available at $BS_j, j\neq i$. In our case this model is based on a weighted linear combination of Maximal Ratio Combining (MRC) and statistical zero-forcing (SZF) BF[4]. The reasoning for using such a model is based on the fact that in [13] it was shown that for the two-user case, any point of the Pareto boundary of the achievable rate region of the MISO interference channel corresponds to BF vectors that are linear combinations of the zero-forcing (ZF) and MRC BFs. Thus, since the interference channels are statistically known at each BS, one could argue that a linear combination of MRC and SZF is a meaningful approximation to strategies corresponding to the Pareto boundary. *d*) A **utility criterion** for the problem, which, in our case, is the expected rate of SU, $\mathbb{E}\{R_2\}$.

Given the above components, the target of the team decisional BF approach is to maximize the utility criterion subject to a QoS constraint for the PU as well as power constraints for the two BFs. To this end, our proposed team decisional approach is based on an iterative procedure applied at each BS, where at each iteration, BS_i redefines its strategy as well as the model for the strategy of $BS_j, j\neq i$ based solely on its own CSI. The key principle of the iterative procedure applied by $BS_i, i=1,2$ can be summarized as follows.

- Initialization: Set the iteration counter n to 1 and initialize model $g_j^{(p)}(\mathbf{z}_j)$ for the strategy followed by $BS_j, j\neq i$.
- Step 1: Given the model $\mathbf{w}_j^{(p)}(n-1)=g_j^{(p)}(\mathbf{z}_j)$ for the strategy followed by $BS_j, j\neq i$, estimate the optimum $\mathbf{w}_i(n)=g_i(\mathbf{z}_i)$ that maximizes an "analogous" to the utility criterion based on the available CSIT at BS_i.
- Step 2: Using the derived $\mathbf{w}_i(n)$, formulate a new model $g_j^{(p)}(\mathbf{z}_j)$ for the BF strategy applied by BS_j, based again on the utility maximization criterion and the available CSIT.
- Step 3: Increase the iteration counter n by one and if $n\leq N_{max}$, where N_{max} is a predefined maximum number of iterations, go back to Step 1, otherwise stop.

This generic procedure is applied as follows by the two BSs.

1) Transmit BF design at BS_1: Following the developed team decisional approach, BS_1 tries to find optimal values for $\mathbf{w}_1$ and for $\mathbf{w}_2^{(p)}=g_2^{(p)}(\mathbf{z}_2)$. To this end, in the n-th iteration, BS_1 uses the estimated model from step $n-1$ i.e., $\mathbf{w}_2^{(p)}(n-1)=\sqrt{P_{max,2}}\frac{\tilde{\mathbf{w}}_2^{(p)}(n-1)}{\|\tilde{\mathbf{w}}_2^{(p)}(n-1)\|}$ with $\tilde{\mathbf{w}}_2^{(p)}(n-1)=\alpha(n-1)\mathbf{v}+(1-\alpha(n-1))\tilde{\mathbf{h}}_{22}$[5], where $\mathbf{v}\perp\mathbf{u}_{\mathbf{R}_{21}}(1)$, $\tilde{\mathbf{h}}_{22}=\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$ and $\alpha(n)\in[0,1]$ in order to find an optimum $\mathbf{w}_1(n)$ that exploits the available CSI to solve the problem

$$\begin{aligned}
&\underset{\mathbf{w}_1\in\mathbb{C}^M}{\text{maximize}} && \frac{1+\mathbb{E}_{|\mathbf{z}_1}\{\mathbf{w}_2^{(p)H}(n-1)\mathbf{h}_{22}\mathbf{h}_{22}^H\mathbf{w}_2^{(p)}(n-1)\}}{\mathbf{w}_1^H\mathbf{R}_{12}\mathbf{w}_1} \\
&\text{subject to} && \frac{1+\mathbf{w}_1^H\mathbf{h}_{11}\mathbf{h}_{11}^H\mathbf{w}_1}{\mathbb{E}_{|\mathbf{z}_1}\{\mathbf{w}_2^{(p)H}(n-1)\mathbf{R}_{21}\mathbf{w}_2^{(p)}(n-1)\}}\geq\tau \\
& && \|\mathbf{w}_1\|^2=P_{max,1}.
\end{aligned} \tag{8}$$

One can show that the following optimization problem is formed

$$\begin{aligned}
&\underset{\mathbf{w}_1\in\mathbb{C}^M}{\text{minimize}} && \mathbf{w}_1^H\mathbf{R}_{12}\mathbf{w}_1 \\
&\text{subject to} && \mathbf{w}_1^H\mathbf{H}_{11}\mathbf{w}_1\geq\tau K(n-1)-1 \\
& && \|\mathbf{w}_1\|^2=P_{max,1}
\end{aligned} \tag{9}$$

where $\mathbf{H}_{11}=\mathbf{h}_{11}\mathbf{h}_{11}^H$ is a rank-one positive semidefinite matrix and $K(n-1)=\mathbb{E}_{|\mathbf{z}_1}\{\mathbf{w}_2^{(p)H}(n-1)\mathbf{R}_{21}\mathbf{w}_2^{(p)}(n-1)\}$ can be numerically approximated via Monte Carlo (MC) iterations exploting knowledge of $\mathbf{R}_{21}$ and $\mathbf{R}_{22}$. Problem (9) is a non-convex quadratically constrained quadratic problem (QCQP), which by introducing $\mathbf{W}_1=\mathbf{w}_1\mathbf{w}_1^H$ can be expressed as follows

$$\begin{aligned}
&\underset{\mathbf{W}_1\in\mathbb{C}^{M\times M}}{\text{minimize}} && \mathrm{tr}(\mathbf{R}_{12}\mathbf{W}_1) \\
&\text{subject to} && \mathrm{tr}(\mathbf{H}_{11}\mathbf{W}_1)\geq\tau K(n-1)-1 \\
& && \mathrm{tr}(\mathbf{W}_1)=P_{max,1},\quad \mathbf{W}_1\succeq 0 \\
& && rank(\mathbf{W}_1)=1.
\end{aligned} \tag{10}$$

Solving (10) with the rank-one restriction, proves to be cumbersome. To overcome this problem, the rank-one constraint can be dropped by applying semidefinite relaxation (SDR) [14]. The resulting problem is

$$\begin{aligned}
&\underset{\mathbf{W}_1\in\mathbb{C}^{M\times M}}{\text{minimize}} && \mathrm{tr}(\mathbf{R}_{12}\mathbf{W}_1) \\
&\text{subject to} && \mathrm{tr}(\mathbf{H}_{11}\mathbf{W}_1)\geq\tau K(n-1)-1 \\
& && \mathrm{tr}(\mathbf{W}_1)=P_{max,1},\quad \mathbf{W}_1\succeq 0
\end{aligned} \tag{11}$$

[4]In the case of SZF, we assume that the selected BF vector is perpendicular to the dominant eigenvector of the interfering channel's covariance matrix.

[5]Recall that BS_i assumes that BS_j uses a BF that is a linear combination of MRC and SZF BF.

that can be solved by using well known optimization packages such as CVX [15]. The optimal BF vector $\mathbf{w}_1(n)$ can then be approximated by the eigenvector corresponding to the dominant eigenvalue of the solution obtained from (11).

Having calculated $\mathbf{w}_1(n)$, BS_1 then produces a new model $\mathbf{w}_2^{(p)}(n)$ that can be seen as the optimum "response" to the selection of $\mathbf{w}_1(n)$. This is achieved by setting $\alpha(n)$ to be the solution of the following problem[6]

$$\begin{aligned} \max_{\alpha\in\mathbb{R}} \quad & \mathbb{E}_{|\mathbf{z}_1}\left\{ \log\left(1+\frac{|\mathbf{w}_2^{(p)H}\mathbf{h}_{22}|^2}{1+\mathbf{w}_1^H(n)\mathbf{h}_{12}\mathbf{h}_{12}^H\mathbf{w}_1(n)}\right)\right\} \\ \text{subject to} \quad & \mathbb{E}_{|\mathbf{z}_1}\{\|\mathbf{w}_2^{(p)H}\mathbf{R}_{21}^{\frac{1}{2}}\|^2\} \leq \frac{1}{\tau}(\mathbf{w}_1^H(n)\mathbf{H}_{11}\mathbf{w}_1(n)+1) \\ & 0\leq\alpha\leq 1 \end{aligned} \tag{12}$$

where $\mathbf{w}_2^{(p)}$ is a function of α. It is easy to observe that the objective function in (12) is increasing as α decreases. Thus, one can solve problem (12) simply by finding the minimum possible value of α that satisfies the QoS-related inequality constraint in (12). The expectations involved in (12), can be computed by means of MC simulations. The value α that is calculated through this procedure at the n-th iteration is selected to be the value $\alpha(n)$ that updates the model $\mathbf{w}_2^{(p)}(n)$ that should be used in iteration $n+1$.

A case that requires special treatment appears when problem (11) is infeasible. Such an event can occur due to strict primary communication QoS constraints and/or deep fades for primary communication. In this case, in an attempt to protect the PU, a minimal information exchange is allowed between the BSs. Specifically, BS_1 reports the infeasibility to BS_2. BS_2 then decides upon using SZF, i.e., employs a BF vector orthogonal to the dominant eigenvector of $\mathbf{R}_{21}$ such as to minimize the interference caused to the PU, while at the same time BS_1 decides upon using MRC such as to maximize its SNR.

2) Transmit BF design at BS_2: In a similar fashion with BS_1, BS_2 starts its n-th iteration using an estimate $\mathbf{w}_1^{(p)}(n-1)=\sqrt{P_{max,1}}\frac{\tilde{\mathbf{w}}_1^{(p)}(n-1)}{\|\tilde{\mathbf{w}}_1^{(p)}(n-1)\|}$ with $\tilde{\mathbf{w}}_1^{(p)}(n-1)=\beta(n-1)\mathbf{u}+(1-\beta(n-1))\tilde{\mathbf{h}}_{11}$, where $\mathbf{u}\perp\mathbf{u}_{\mathbf{R}_{12}}(1), \tilde{\mathbf{h}}_{11}=\frac{\mathbf{h}_{11}}{\|\mathbf{h}_{11}\|}$ and $\beta(n)\in[0,1]$, and given its set of observations $\mathbf{z}_2$, it forms the following optimization problem

$$\begin{aligned} \underset{\mathbf{w}_2\in\mathbb{C}^M}{\text{maximize}} \quad & \frac{1+\mathbf{w}_2^H\mathbf{H}_{22}\mathbf{w}_2}{\mathbb{E}_{|\mathbf{z}_2}\{\mathbf{w}_1^{(p)H}(n-1)\mathbf{R}_{12}\mathbf{w}_1^{(p)}(n-1)\}} \\ \text{subject to} \quad & \frac{1+\mathbb{E}_{|\mathbf{z}_2}\{\mathbf{w}_1^{(p)H}(n-1)\mathbf{h}_{11}\mathbf{h}_{11}^H\mathbf{w}_1^{(p)}(n-1)\}}{\mathbf{w}_2^H\mathbf{R}_{21}\mathbf{w}_2}\geq\tau \\ & \|\mathbf{w}_2\|^2\leq P_{max,2} \end{aligned} \tag{13}$$

where $\mathbf{H}_{22}=\mathbf{h}_{22}\mathbf{h}_{22}^H, \;\; \mathbf{H}_{22}\succeq 0$. Problem (13) is equivalent to

$$\begin{aligned} \underset{\mathbf{w}_2\in\mathbb{C}^M}{\text{maximize}} \quad & \mathbf{w}_2^H\mathbf{H}_{22}\mathbf{w}_2 \\ \text{subject to} \quad & \mathbf{w}_2^H\mathbf{R}_{21}\mathbf{w}_2\leq\frac{1}{\tau}(L(n-1)+1) \\ & \|\mathbf{w}_2\|^2\leq P_{max,2} \end{aligned} \tag{14}$$

where $L(n-1)=\mathbb{E}_{|\mathbf{z}_2}\{\mathbf{w}_1^{(p)H}(n-1)\mathbf{h}_{11}\mathbf{h}_{11}^H\mathbf{w}_1^{(p)}(n-1)\}$ can be numerically evaluated, since BS_2 has knowledge of covariance matrices $\mathbf{R}_{12}$ and $\mathbf{R}_{11}$. Following the same steps as in the previous subsection, the optimization problem becomes

$$\begin{aligned} \underset{\mathbf{W}_2\in\mathbb{C}^{M\times M}}{\text{maximize}} \quad & \mathrm{tr}(\mathbf{H}_{22}\mathbf{W}_2) \\ \text{subject to} \quad & \mathrm{tr}(\mathbf{R}_{21}\mathbf{W}_2)\leq\frac{1}{\tau}(L(n-1)+1) \\ & \mathrm{tr}(\mathbf{W}_2)\leq P_{max,2}, \quad \mathbf{W}_2\succeq 0 \end{aligned} \tag{15}$$

where $\mathbf{W}_2=\mathbf{w}_2\mathbf{w}_2^H$. This optimization problem can be also efficiently solved by using the CVX package. Having obtained an optimal $\mathbf{w}_2(n)$ for a given $\beta(n-1)$, BS_2 reestimates the strategy followed by BS_1, by exploiting its available observations $\mathbf{z}_2$ and finding an optimal $\beta(n)$ for the obtained $\mathbf{w}_2(n)$. Thus, the problem to be solved is the following

$$\begin{aligned} \max_{\beta\in\mathbb{R}} \quad & \mathbb{E}_{|\mathbf{z}_2}\left\{ \log\left(1+\frac{\mathbf{w}_2^H(n)\mathbf{H}_{22}\mathbf{w}_2(n)}{1+|\mathbf{w}_1^{(p)H}\mathbf{h}_{12}|^2}\right)\right\} \\ \text{subject to} \quad & \mathbb{E}_{|\mathbf{z}_2}\{|\mathbf{w}_1^{(p)H}\mathbf{h}_{11}|^2\}\geq\tau\mathbf{w}_2^H(n)\mathbf{R}_{21}\mathbf{w}_2(n)-1 \\ & 0\leq\beta\leq 1 \end{aligned} \tag{16}$$

where $\mathbf{w}_1^{(p)}$ is a function of β.[7] The resulting approximate solution of (16) that can be reached by discretizing the search space for β is the new estimate $\beta(n)$ that should be used to determine the model $\mathbf{w}_1^{(p)}(n)$ for iteration $n+1$, leading towards an iterative process of solving problems (15) and (16).

IV. Numerical Results

With the aim of evaluating the performance of the proposed BF scheme, extensive MC simulations have been performed for the studied system model. A CRN scenario is considered, in which the two BS coverage areas (CAs) overlap with each other by a factor of 50%. We assume that $P_{max,1}=P_{max,2}$ and that both CAs have the same radius. The covariance matrices are computed as a function of angle spread, antenna spacing and wavelength, according to [16]. In Table I, further simulation parameters are provided.

TABLE I
BASIC SIMULATION PARAMETERS

BS CA radius	1 km
Number of BS antennas	2
Path loss exponent	3
Carrier frequency	2 GHz
Antenna spacing	$\lambda/2$
AOA distribution	Gaussian
Multipath angle spread	20 degrees

In Fig. 2 the expected rate of the SU is depicted as a function of the prescribed average SNR at the CA edge for a QoS level, $T=1$ nat/sec/Hz, for the PU. We choose to compare our novel method ($N_{max}=2$) with other distributed BF methods exploiting the same hybrid CSIT available here. Specifically, we compare our proposed BF solution with BFs based on

[6]Given that different CSIT is availabe at BS_1 and BS_2, one can see this problem as the "analogous" to the problem that BS_2 is trying to solve.

[7]In problems (12) and (16), one could instead use the averaged version of the objective function of problem (7).

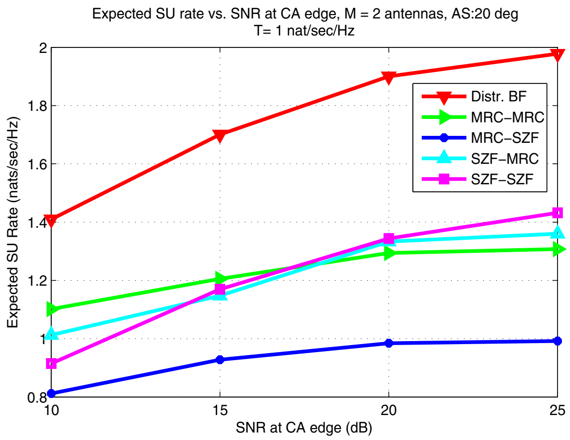

Fig. 2. Expected SU rate vs. average SNR at CA edge.

MRC and SZF BF. The four reference schemes are then given by MRC-MRC, MRC-SZF, SZF-MRC and SZF-SZF, where the first (resp. second) acronym in each pair denotes the BF solution implemented at BS_1 (resp. BS_2). The BF solutions obtained by applying our iterative method are such that a clear rate gain appears, compared with the classical BF solutions.

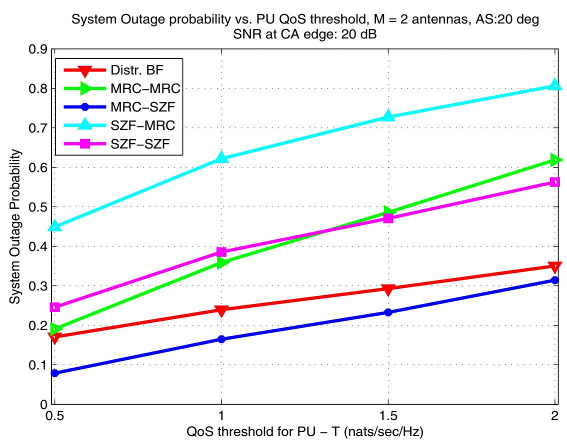

Fig. 3. System outage probability vs. QoS threshold, T for PU.

In Fig. 3, the system's outage probability is depicted for all the abovementioned BF design approaches, as a function of the QoS threshold, T, posed at the PU, for an SNR at the CA edge equal to 20dB. An outage event is declared when the initial QoS constraint in (6) is not satisfied for the PU. It is evident that as the value of T increases, the probability of the system being in outage will increase for all the examined BF approaches. Also, only the MRC-SZF BF method outperforms our method, since it is mostly focused on protecting the PU. It is worth mentioning that with a QoS value, $T = 1$ nat/sec/Hz for the PU, and for an SNR value of 20dB at the CA edge, the new BF scheme gives 41% rate increase over SZF-SZF BF and 38% outage probability decrease over the same BF method.

V. Conclusions

In this paper, an optimal MISO BF method is proposed, with respect to an underlay CRN setup, when the available CSIT is both instantaneous and statistical. First, expressions for the conditional expected rates of the users are derived, and then the problem of optimal MISO BF is described and solved in the presence of distributed CSIT at each BS, leading to the solution of a team decision problem. Substantial gains are depicted in comparison with other known distributed precoding solutions.

Acknowledgment

This work was supported by the French national ANR-VERSO funded project LICORNE.[8]

References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201–220, 2005.

[2] E. Biglieri, A. J. Goldsmith, L. J. Greenstein, N. Mandayam, and H. V. Poor, *Principles of Cognitive Radio*. Cambridge University Press, 2012.

[3] Y. Zhang and A.-C. So, "Optimal spectrum sharing in MIMO cognitive radio networks via semidefinite programming," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 2, pp. 362–373, 2011.

[4] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *Signal Processing, IEEE Transactions on*, vol. 58, no. 10, pp. 5450–5458, 2010.

[5] H. Du and T. Ratnarajah, "Transmit beamforming in MIMO cognitive radio network via semidefinite programming," in *Vehicular Technology Conference (VTC Spring), 2011 IEEE 73rd*, May 2011, pp. 1 –5.

[6] L. Zhang, Y.-C. Liang, Y. Xin, and H. Poor, "Robust cognitive beamforming with partial channel state information," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 8, pp. 4143–4153, 2009.

[7] K. Wang, N. Jacklin, Z. Ding, and C. Chi, "Robust MISO Transmit optimization under outage-based QoS constraints in two-tier heterogeneous networks," *Wireless Communications, IEEE Transactions on*, vol. PP, no. 99, pp. 1–15, 2013.

[8] Y.-C. Ho, "Team decision theory and information structures," *Proceedings of the IEEE*, vol. 68, no. 6, pp. 644–654, 1980.

[9] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions*. Dover publications, 1965.

[10] M. Filippou, D. Gesbert, and G. Ropokis, "Optimal combining of instantaneous and statistical CSI in the SIMO interference channel," in *Proc. of Vehicular Technology Conference (VTC-Spring), 2013 IEEE*, Jun. 2013.

[11] R. Zakhour and D. Gesbert, "Team decision for the cooperative MIMO channel with imperfect CSIT sharing," in *Information Theory and Applications Workshop (ITA), 2010*, 31 2010-Feb. 5, pp. 1–6.

[12] S. Han and C. Yang, "Downlink multicell cooperative transmission with imperfect CSI sharing," in *proc. of ICASSP 2011*, 2011, pp. 3024–3027.

[13] E. Jorswieck, E. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *Signal Processing, IEEE Transactions on*, vol. 56, no. 10, pp. 5292–5296, 2008.

[14] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *Signal Processing Magazine, IEEE*, vol. 27, no. 3, pp. 20 –34, May 2010.

[15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.0 beta," http://cvxr.com/cvx, Sep. 2012.

[16] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 2, pp. 264–273, 2013.

[8] Dr. Ropokis was funded by the MIMOCORD project. The project is implemented within the framework of the Action "Supporting Postdoctoral Researchers" of the Operational Program "Education and Lifelong Learning" (Action's Beneficiary: General Secretariat for Research and Technology), and is co-financed by the European Social Fund (ESF) and the Greek State.