To Scan or Not To Scan: The Effect of Channel Heterogeneity on Optimal Scanning Policies

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***Abstract*—Cognitive Networks have been proposed to opportunistically discover and exploit (temporarily) unused licensed spectrum bands. For a number of applications, high throughput is the key figure of merit, while the application is still elastic enough to be supported at different rates. To this end, the cognitive node will try to discover and pool together a number of (at the time available) primary channels to provide a given target throughput. When a single radio is used for both transmission and channel scanning, an interesting tradeoff arises: when one or more channels of the currently available ones are lost (e.g. primary user returns), should the node start scanning immediately or continue transmitting over the remaining channels. Using renewal-reward theory, we show that if the goal is to maximize the average (long-term) throughput, the answer to this question depends on the statistics of the channel availability periods. Specifically, for relatively homogeneous channels, we show that it is optimal to start scanning immediately, while for heterogeneous channels, it is often better to defer scanning, even if multiple channels are lost. Simulations for a range of different channel characteristics validate our analytical findings and suggest that triggering the scanning function at the right times, can improve performance considerably.**

***Keywords*-Cognitive networks, Spectrum sensing, Channel scanning, Renewal theory.**

I. Introduction

In the last years there has been an increased spectrum demand from wireless applications and data. Due to the static spectrum allocation policy, spectrum scarcity is a major problem in today's wireless industry. At the same time, a large portion of the assigned spectrum is underutilized [1]. Dynamic spectrum access techniques have recently been proposed to overcome these problems, and Cognitive radio [2] is the key enabling technology. In a cognitive network, there exist licensed users which are provided the spectrum from the regulation authority, as well as users that utilize the spectrum opportunistically whenever they find it available. The former ones are known as primary users (PUs), while the later ones are known as cognitive (secondary) users (SUs). Various spectrum management functions [3] such as spectrum sensing, spectrum decision, spectrum sharing, and spectrum mobility, are then used to enable this seamless sharing.

A cognitive user can use one or more channels when no other primary users are active in that region. However, as soon as a primary user returns to the channel, the SU must interrupt its transmission at that moment[1]. Spectrum sensing [4] is a key feature of cognitive radios. It enables an SU to detect available frequency bands (not occupied by PUs), as well as the detection of the PU returning on that channel. Sensing techniques such as energy detection, matched filter approach, and waveform sampling [3] have been extensively studied.

Spectrum scanning, on the other hand, decides how to choose the channels to sense first, among the large number of possible frequencies a wideband cognitive radio could have access to. Intuitively, spending time to sense channels which have a high PU activity, and thus a low probability to be found available, can waste resources. As a result, there has been a considerable amount of work regarding the problem of channel scanning. Most of these works are concerned with finding the optimal sequence order to scan the channels, so that a certain parameter is optimized. In [5], authors try to minimize the scanning time, i.e. to get an available channel in the fastest way. In [6], the goal is to minimize the probability of link failure. Some other papers [7], [8] propose greedy algorithms for the sequence of channel scanning for both reactive and pro-active spectrum handoff.

Cognitive radios are also capable of utilizing multiple channels in parallel ("pooling"), in order to increase the aggregate capacity [9]. In this context, an interesting problem arises when a single radio is used for both sensing and transmission, due to the need for low cost, energy, and/or complexity for small cognitive devices [9]. This means that a node cannot simultaneously transmit and sense, i.e. when sensing, it must interrupt its transmission. Should the node then initiate the scanning function immediately after is loses one of its channels currently in use? Or should it continue with the channels left, until one (or more) channels are lost? The tradeoff is the following: if it chooses to scan, then the capacity on the remaining available channels is wasted, and nothing useful is sent; if on the other hand it defers scanning, then it will transmit for some time at a lower rate (than it potentially could) *and* it will then have to scan longer to find *more* available channels. This latter intuition suggests that perhaps it is best to trigger scanning immediately, and maximize the

This work is supported by the French National Research agency (ANR) under grant reference LICoRNe ANR-10-VERS-005.

[1]We will assume throughout that the "interweave" model of sharing is used [1].

amount of time one sends at the maximum rate.

Our goal in this paper is to analyze this tradeoff as a function of the characteristics of all the channels available to the SU. We will assume an application that requires a maximum amount of throughput, but is elastic (i.e. could also operate at lower rates - e.g. streaming, file downloading, etc.), and we will try to maximize the average (long-term) throughput it is offered by the SU. To achieve this, we apply renewal reward theory [10]. Our contributions can be summarized as follows: (i) In accordance with the above intuition, we prove that when channel availability/idle periods are exponentially distributed with similar mean duration (*homogeneous*), it is indeed optimal to scan immediately when a single channel is lost; (ii) Contrary to the above intuition, when there is an initial scanning cost, or channel availability periods are heterogeneous, we prove that it is not optimal to scan immediately, but an optimal threshold exists; (iii) We provide a method to predict this threshold based on the channel characteristics, and show that this depends on the coefficient of variation between the mean availability periods of different channels; (iv) We provide an online algorithm that can take advantage of knowledge of which channel was just lost, to further improve performance; (v) Finally, using simulations, we provide evidence that our conclusions and algorithms are valid even when the channel availability durations follow a general (non-memoryless) distribution.

The paper is organized as follows. In the next section, we discuss some related work. We present our problem setup and provide analytical results about the optimal scanning actions, for a specific class of channel availability distributions (exponential) in Section III. We then validate our theory against simulations in Section IV, and also explore the case of general availability periods. We conclude our work in Section V.

II. Related Work

There has been a large amount of research in the area of spectrum scanning in cognitive networks. Yet, most of these papers are concerned with determining the scanning sequence of the channels to be sensed, in order to optimize certain system parameters [11], [12], [13], [14]. In [7], the authors propose an algorithm based on dynamic programming for the optimal scanning sequence. However, this work refers to proactive spectrum mobility schemes, where the spectrum sensing process is initiated before losing the channel. In [8], a similar algorithm has been proposed for the reactive spectrum mobility schemes (like the ones we consider). However, the proposed algorithm is not optimal and intends to minimize only the time it takes for a packet to be successfully transmitted assuming that there will be spectrum handoffs.

In a more related work [5], the authors propose a scanning sequence in order to minimize the average time to get a new channel, after the current one in use is being lost. They prove that this can be accomplished if channels are sorted in descending order of their probabilities to be found available (related to the duty cycle of that channel). However, they do not consider how long the acquired channel will be available

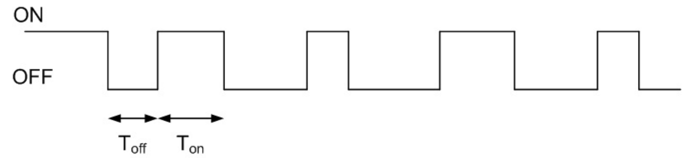

Fig. 1. The channel model

for (dependent on both the first and second moments of the availability period [15]). To this end, in [6], a channel scanning sequence is proposed to minimize the probability of link failure between two cognitive users, i.e. a transmitter and a receiver during a transmission session. This can be done by sorting channels in descending order according to their expected excess idle periods durations.

Furthermore, [5] does not consider when the scanning process should be triggered, but rather begins from the starting time of such a scanning with a goal to acquire an amount of "missing" bandwidth, and investigates the order of scanning (backup) channels so as to minimize the delay to acquire the required bandwidth (possibly pooling together multiple channels). While finding additional channels fast is an important problem, it is also orthogonal to our problem. Different scanning sequences (optimal or suboptimal) could be incorporated into our model, as long as we know the expected amount of time to acquire L channels using that sequence.

Summarizing, unlike many related works, we consider the scanning problem in scenarios when multiple channels are pooled together. More importantly, instead of optimizing the scanning sequence when scanning is triggered, we investigate the complementary optimization problem of *when* to trigger this scanning function in order to optimize the (long-term) throughput that can be maintained by the SU. To our best knowledge, this is the first work in this direction.

III. Analysis

A. Problem Setup

Consider a single channel used by one or more primary users. We assume that the state of this channel can be either active ('ON'), i.e. the primary user is active, or idle ('OFF'). A secondary user can only transmit during an OFF period (interweave model). The exact duration of the ON and OFF periods depends on the user behavior, the type of traffic, system details and protocol interaction. Such details cannot be known by the secondary user.

We will model the ON-OFF activity pattern of PUs as an alternating renewal process [10] $\left(T_{ON}^{(n)}, T_{OFF}^{(n)}\right), n \geq 1$, as shown in Figure 1. n denotes the number of ON-OFF cycles elapsed until time t. The duration of any ON (OFF) period $T_{ON}^{(n)}$ $\left(T_{OFF}^{(n)}\right)$, is a random variable distributed according to some probability distribution F_{ON} (F_{OFF}), and independently of other ON or OFF periods.

We assume that the spectrum, available to the SU, is divided into channels with identical bandwidths B. Furthermore, we consider applications that require a (maximum) bandwith of $C = NB$, but could also operate at lower rates. This

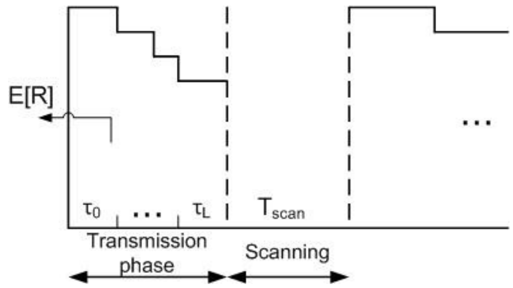

Fig. 2. The transmission and scanning phase

could be the case for example, for file downloading or P2P applications [16], where C (or N) is dictated by the cognitive radio technology (not more than N channels can be pooled together), or multimedia streaming applications with multiple rate codings [9]. In addition to these N channels used, we assume there are also other (backup) channels that can be scanned and used (if available), when one or more of the used channels are lost. After losing a certain channel, we add it to the list of backup channels.

In this context, we will consider the following problem: *A threshold value L is chosen, such that after* any *L channels (out of the N) are lost we must start scanning and regain L new channels. What is a good value for L?* Note that the choice of $L = 1$ corresponds to the usual case where scanning starts immediately after any of the available channels is lost.

We make here some final remarks about the practical implications of the above problem setting. We assume that a single radio and antenna is used. Although such a cognitive radio is usually wideband (e.g. some 10s of MHz), allowing for more than 1 channel to be grouped together, even if not adjacent (using a digital filter), it can only transmit or scan at any time, but not both (since scanning usually requires changing the center frequency as well, and introduces switching delays in the order of ms). In contrast, to detect that one or more channels out of the used ones is lost (i.e. a PU has started transmitting), we could just switch the radio periodically to receive mode (*on the same band*), take a short time sample (usually in the order of μs), and do an FFT to identify which band(s) out of the (up to) N used ones has high enough energy (implying activity). The energy threshold and the frequency of such sensing periods pose a tradeoff between the time lost not transmitting and false positive/negative for PU detection, and is beyond the scope of this paper. Yet, for the sake of the subsequent analysis, we can safely ignore these interleaved sensing periods as they are orders of magnitude shorter, and assume that PUs are detected correctly.

B. Analytical Model

Figure 2 illustrates our model, assuming that at each instant at most one channel can be lost. A cycle consists of the transmission phase and the scanning phase. τ_i represents the time during which $N - i$ channels are being used for transmission ($i < L$). The scanning phase represents the time needed to regain the L missing channels. The area under the stair-case curve represents the total amount of data transmitted during a cycle. Our goal is to maximize the long-term throughput, namely the size of this area up to a time t, when t is large.

To this end, we will use renewal-reward theory [10]. We model the process shown in Figure 2 as a renewal process, where a renewal occurs each time the neccessary N channels are gathered and we can resume our transmission. A renewal cycle consists of two phases: transmission and scanning phase[2]. We can define a reward for each cycle as the amount of data transmitted during the transmission phase of the cycle.

If we denote by $R(t)$ the reward earned by time t, the average (long-term) throughput is the mean reward rate $\frac{R(t)}{t}$, which from renewal-reward theory we know it to be

$$lim_{t\to\infty}\frac{E\left[R\left(t\right)\right]}{t}=\frac{E\left[R\right]}{E\left[T\right]}. \tag{1}$$

In Eq.(1), $E\left[R\right]$ is the average reward per cycle, while $E\left[T\right]$ is the average cycle duration. Let's denote the reward rate by X. So, the average reward rate is given by

$$E[X]=\frac{E[R]}{E[T]}. \tag{2}$$

Our objective is to maximize the average reward rate, that is the average data rate per cycle, by choosing the right threshold L, as a function of the OFF (idle) period characteristics of the channels available to an SU node.

To calculate the values of $E[R]$ and $E[T]$, we need to understand the random variables τ_i. At the beginning of a transmission phase, N channels are available that were either already being used, or were found to be available during the scanning phase. As a result, the remaining availability time for each of these channels, say channel j, is an *excess* random variable (excess OFF period) denoted as $T^{(e)}_{OFF,j}$. Then, τ_0 corresponds to the time until *any* of the N channels is lost,

$$\tau_0=\min\left(T^{(e)}_{OFF,1},T^{(e)}_{OFF,2},\dots T^{(e)}_{OFF,N}\right). \tag{3}$$

Similarly, τ_1 denotes the amount of time exactly $N-1$ channels are in use (time between the 1$^{\text{st}}$ and the 2$^{\text{nd}}$ channels are lost),

$$\tau_1=\min\left(T^{(e)}_{OFF,1},T^{(e)}_{OFF,2},\dots T^{(e)}_{OFF,(N-1)}\right). \tag{4}$$

Similarly for the rest of τ_i.

We can thus express the average area below the curve (mean reward) as

$$E[R]=NB\sum_{i=0}^{L}E[\tau_i]-B\sum_{i=0}^{L}iE[\tau_i]. \tag{5}$$

The average cycle duration is

$$E[T]=\sum_{i=0}^{L}E[\tau_i]+E[T_{scan}(L)], \tag{6}$$

where $T_{scan}(L)$ is the scanning time needed to acquire the missing channels.

[2]We stress here that we do not claim this process to be a renewal process, but rather use the renewal-reward theory as a tool to derive analytical insight regarding the throughput achievable by different policies. This insight will be validated against simulations.

We can now say that our goal is to find the threshold value L that will maximize

$$E[X] = \frac{C\sum_{i=0}^{L} E[\tau_i] - B\sum_{i=0}^{L} iE[\tau_i]}{\sum_{i=0}^{L} E[\tau_i] + E[T_{scan}(L)]}. \quad (7)$$

This is quite involved in the general case. In the remainder, we will consider analytically the cases of arbitrary ON periods and exponential OFF periods (with the same or different mean durations). The assumption for exponentially distributed OFF periods is made for analytical tractability. Nevertheless, this assumption is often not far from reality, as measurements from [17], [18] suggest that the OFF periods can be approximatted quite well with exponential distributions. In Section IV, we will further consider generic OFF periods as well (with increasing and decreasing failure rates).

We make here a final remark about scanning. We will assume that the scanning periods for each channel are considered to be identical. Additional features could be included in our theory. The duration of a channel scanning period is T_I, and we also assume that during the scanning period the probability that other channels are lost is low; this is reasonable (and also supported by our simulations) since the scanning period is usually lower than the durations of the OFF periods for each channel. For example, assume that the scanning period is $T_I = 1$ ms. For a duty cycle (the ratio of time the primary user is active on that channel) of 0.5, on average we need to scan two channels in order to get one free channel. So, the average scanning time is $T_s = 2$ ms. If the OFF periods are exponentially distributed with mean 1 s, then the probability that an available channel will be lost while scanning is $P[T_{OFF} < T_s] = 1 - e^{-T_s} = 0.002$. While this value can be larger for some channels with shorter OFF periods, this is still small enough to ignore it in our analysis and only reintroduce this in simulations.

Finally, the duration of the scanning phase will also be a function of the number of channels that must be acquired:

$$E[T_{scan}] = l(L)T_I, \quad (8)$$

where $l(L)$ is the average number of channels that need to be scanned, in order to regain the L missing ones. It is easy to see that $l(L) \geq L$, since one might need to sense more than one channel to find one that is available. The exact function depends on the sequencing algorithm, and can be simply plugged into the above equations. Unless otherwise stated, we will assume w.l.o.g. throughout that $l(L)$ is linear, that is, if T_s is the total time to acquire one channel, then kT_s is the total time to acquire k channels.

C. Exponential IID OFF Periods

We will consider first the simpler case of PU activities having independent indentically distributed (IID) OFF periods that are exponentially distributed, with mean $E[T_{OFF}] = \frac{1}{\lambda_{off}}$. The average durations of ON and OFF periods can be inferred in different ways [18]. Our first result is the following:

Result 1: *For channels with homogeneous (i.i.d.) exponentially distributed OFF periods it is always optimal to scan immediately after one channel is lost.*

To derive this, we start by the fact that for N independent exponentially distributed random variables $X_1, X_2, \ldots, X_N$ with parameters $\lambda_1, \lambda_2, \ldots, \lambda_N$, the expectation of the minimum of these random variables is

$$E[\min(X_1, X_2, \ldots, X_N)] = \frac{1}{\lambda_1 + \lambda_2 + \ldots + \lambda_N}. \quad (9)$$

Since $\lambda_i = \lambda_{off}, \forall i$, it holds that

$$E[\tau_0] = \frac{1}{N\lambda_{off}},$$
$$E[\tau_1] = \frac{1}{(N-1)\lambda_{off}},$$

and similarly for $E[\tau_i]$.

The mean reward for the duration τ_0 is

$$\frac{NB}{N\lambda_{off}} = \frac{B}{\lambda_{off}}. \quad (10)$$

Similarly, the mean reward for the duration τ_1 (i.e. if the node continues transmitting using the remaining $N-1$ channels) is

$$\frac{(N-1)B}{(N-1)\lambda_{off}} = \frac{B}{\lambda_{off}}. \quad (11)$$

It is easy to see then that, based on Eq.(7), the following inequality must hold so that postponing the scanning process after a channel is lost gives a higher expected rate per cycle, compared to immediately scanning

$$\frac{\frac{B}{\lambda_{off}}}{\frac{1}{N\lambda_{off}} + T_s} < \frac{\frac{B}{\lambda_{off}} + \frac{B}{\lambda_{off}}}{\frac{1}{N\lambda_{off}} + \frac{1}{(N-1)\lambda_{off}} + 2T_s}. \quad (12)$$

After rearranging, this gives

$$N\lambda_{off} < (N-1)\lambda_{off}, \quad (13)$$

which can not be satisfied. This proves that we cannot increase our average throughput by stopping after 2 channels are lost (instead of 1). Since the above argument holds for any N, it is easy to see that we can only lose more by increasing L further, which proves our claim.

So far, we have considered that there is no initial cost for the scanning process. However, it is expected that switching to scanning mode (adjust the devices, determine the first channel to sense, etc.) will incur some initial setup cost, before the actual sensing process can commence for the first channel. In this case, we show that, depending on this initial cost, the optimal value for L (the scanning threshold) might be larger than 1.

Result 2. *For i.i.d. exponentially distributed OFF periods, where an initial scanning cost T_0 exists, scanning immediately ($L = 1$) is not optimal when*

$$T_0 \geq \frac{1}{N(N-1)\lambda_{off}}. \quad (14)$$

The analysis is exactly the same as before, except the factor

T_0 added to the scanning time for both policies. Hence the inequality that needs to be satisfied is

$$\frac{\frac{B}{\lambda_{off}}}{\frac{1}{N\lambda_{off}}+T_0+T_s} < \frac{\frac{B}{\lambda_{off}}+\frac{B}{\lambda_{off}}}{\frac{1}{N\lambda_{off}}+\frac{1}{(N-1)\lambda_{off}}+T_0+2T_s}. \quad (15)$$

Rearranging again gives us the above value for T_0. When the setup (initial) scanning cost is higher than this value, then it is better to not scan immediately, so as to amortize this cost.

As explained earlier, we have assumed that the expected cost (time) to acquire k channels is linear in k. This could be the case for example, if the channels to be sensed during the scanning phase are picked randomly from the list of backup channels. If a better or optimal sequence is provided (e.g. as in [5]) then the time to get a second channel would be higher on average than the time to get the first channel (since, channels with high availability probability are scanned first). This could be easily included in our model by adding an extra cost Δ to the time to get the second channel in the above inequalities. Obviously, for the case of no initial scanning cost, this would not change Result 1. For the case of an initial scanning cost, it is easy to see that the required condition changes to

$$T_0 \geq \frac{1}{N(N-1)\lambda_{off}}+\Delta. \quad (16)$$

1) Finding an optimal threshold: We have so far proven conditions for which scanning immediately is optimal or not. In the case of IID exponential OFF periods, we can also find the optimal threshold explicitly. If we assume a threshold L, the average transmission time during a cycle is

$$\sum_{i=0}^{L} E[\tau_i] = \frac{1}{N\lambda_{off}}+\frac{1}{(N-1)\lambda_{off}}+\ldots+\frac{1}{(N-i)\lambda_{off}}. \quad (17)$$

This gives

$$\sum_{i=0}^{L} E[\tau_i] = \frac{1}{\lambda_{off}}\left(\frac{1}{N}+\frac{1}{N-1}+\ldots+\frac{1}{N-L}\right) = \frac{1}{\lambda_{off}}\sum_{i=0}^{L}\frac{1}{N-i}. \quad (18)$$

The last equation can be rewritten as

$$\sum_{i=0}^{L} E[\tau_i] = \frac{1}{\lambda_{off}}\left(\sum_{i=1}^{N}\frac{1}{i}-\sum_{i=1}^{N-L-1}\frac{1}{i}\right). \quad (19)$$

Using the Euler's approximation [19] $H_n=\sum_{i=1}^{n}\frac{1}{i}=\ln n+\frac{1}{2n}+0.57721$ we obtain

$$\sum_{i=0}^{L} E[\tau_i] = \frac{1}{\lambda_{off}}\left(H_N - H_{N-L-1}\right). \quad (20)$$

After simple calculus operation we can obtain

$$\sum_{i=0}^{L} E[\tau_i] = \frac{1}{\lambda_{off}}\left(\ln\frac{N}{N-L-1}-\frac{1}{2}\frac{L+1}{N(N-L-1)}\right). \quad (21)$$

The expected amount of transmitted data before L channels are lost is

$$\frac{1}{N\lambda_{off}}C+\frac{1}{(N-1)\lambda_{off}}(C-B)+\ldots+\frac{1}{(N-L)\lambda_{off}}(C-LB). \quad (22)$$

Since $C=NB$, after rearraning the last equation we obtain

$$E[R]=\frac{(L+1)B}{\lambda_{off}}. \quad (23)$$

Replacing (21), (23) and (8) into (7) we have

$$E[X]=\frac{\frac{(L+1)B}{\lambda_{off}}}{\frac{1}{\lambda_{off}}\left(\ln\frac{N}{N-L-1}-\frac{1}{2}\frac{L+1}{N(N-L-1)}\right)+l(L)T_I}. \quad (24)$$

We could thus differentiate $E[X]$ above, with respect to L (and round to the closest integer), in order to find the optimal value, when one exists.

D. Heterogeneous exponentially distributed channels

We have shown that, unless a large enough initial setup cost for the scanning function exists, when channel OFF periods are exponential and of similar duration it is always optimal to scan immediately when any channel is lost. Here, we will consider the more realistic case of heterogeneous availability periods. We will still consider exponential durations to maintain tractability, and assume that the rate λ_i[3] for the OFF duration of channel i is drawn from some distribution ($G(\lambda)$). The main result of this section is the following:

Result 3: *For heterogeneous exponentially distributed OFF periods, it is not always optimal to scan immediately. Instead, the transmission should proceed with the remaining channels, if they satisfy the following relation:*

$$c_\lambda^2 \geq 1, \quad (25)$$

where c_λ is the (sample) coefficient of variation of the OFF periods for the channels remaining available.

There are two interesting things to notice about the above result. First, unlike the homogeneous channel case, scanning immediately is suboptimal and a better threshold than $L=1$ can be found. Second, this threshold increases when the variability of the pool of channels available to the SU increases. We will now go ahead and derive this result.

Since the OFF periods are exponential, we can derive the average throughput for $L=1$ (scan immediately) as

$$E[X_1]=\frac{\frac{BN}{\sum_i\lambda_i}}{\frac{1}{\sum_i\lambda_i}+T_s}. \quad (26)$$

Assume now that a channel with rate λ_{lost} is lost first. This is a random variable with probability $\frac{\lambda_{lost}}{\sum_i\lambda_i}$. If the node continues transmitting until another channel is lost, then the average throughput achieved is given by

$$E[X_2|\lambda_{lost}]=\frac{\frac{BN}{\sum_i\lambda_i}+\frac{B(N-1)}{\sum_i\lambda_i-\lambda_{lost}}}{\frac{1}{\sum_i\lambda_i}+\frac{1}{\sum_i\lambda_i-\lambda_{lost}}+2T_s}. \quad (27)$$

[3]From now on, we will denote $\lambda_{off,i}$ simply as λ_i.

We would like to uncondition and get the mean value of $E[X_2]$. If we denote $E[X_2|\lambda_{lost}]$ as a function $f(\lambda_{lost})$, then we would like to know $E[f(\lambda_{lost})]$. Rearranging Eq.(27), we obtain

$$E[f(\lambda_{lost})] = E\left[\frac{a - b\lambda_{lost}}{c - d\lambda_{lost}}\right], \tag{28}$$

where

$$a = B(2N-1)\sum_i \lambda_i, \quad b = BN,$$

$$c = 2\sum_i \lambda_i \left(1 + T_s \sum_i \lambda_i\right), \ \ d = 1 + 2T_s \sum_i \lambda_i.$$

The above expectation depends on the distribution of λ_i values and is not easy to calculate in the general case. We can use however Jensen's inequality to convert this to a function of $E[\lambda_{lost}]$ itself. We thus check for the convexity of the function $f(\lambda_{lost})$. It's second derivative is

$$f''(\lambda_{lost}) = \frac{2d(ad - bc)}{(c - d\lambda_{lost})^3}. \tag{29}$$

We can easily prove that the term $c - d\lambda_{lost}$ is always larger than 0. For convexity, the term in the numerator must satisfy

$$ad - bc = B\sum_i \lambda_i \left[2(N-1)T_s \sum_i \lambda_i - 1\right] > 0.$$

This implies that the function $f(\lambda_{lost})$ is always convex under the condition $T_s \sum_i \lambda_i \geq \frac{1}{2(N-1)}$. This condition could be satisfied when the number of channels N used is large and/or when there are enough "bad" channels in the pool of available ones, so that the product $T_s \sum_i \lambda_i$ exceeds 1 ("bad" channels would correspond to channels with quick fluctations between ON and OFF states, unlike e.g. the case of TV white spaces).

Using Jensen's inequality [10], and the convexity of $f(\lambda_{lost})$ ($E[f(x)] \geq f(E[x])$), we can now replace the *necessary* condition for the $L = 1$ to not be optimal

$$E[X_1] \leq E[f(\lambda_{lost})], \tag{30}$$

with the *sufficient* condition

$$E[X_1] \leq f(E[\lambda_{lost}]). \tag{31}$$

This yields

$$\frac{a - bE[\lambda_{lost}]}{c - dE[\lambda_{lost}]} \geq \frac{BN}{1 + T_s \sum_i \lambda_i}. \tag{32}$$

After solving the above inequation we obtain

$$E[\lambda_{lost}] \geq \frac{2bc - a(d+1)}{b(d-1)}, \tag{33}$$

and replacing the expressions for a, b, c, and d we get

$$E[\lambda_{lost}] \geq \frac{1 + T_s \sum_i \lambda_i}{NT_s}. \tag{34}$$

This already provides a condition related to the statistics of the existing and the lost channels. If it holds that $T_s \sum_i \lambda_i >> 1$, from Eq.(34) we get

$$E[\lambda_{lost}] \geq \frac{\sum_i \lambda_i}{N}. \tag{35}$$

However, since the probability of losing a channel is proportional to its OFF period rate λ, $E[\lambda_{lost}]$ is always greater than the sample average of the current N channels $\frac{\sum_i \lambda_i}{N}$, and it is always better (on average) to continue transmitting.

The second case of interest is when $T_s \sum_i \lambda_i \geq \frac{1}{2(N-1)}$, but it is not larger than 1. The average rate of the first lost channel is

$$E[\lambda_{lost}] = \sum_i \lambda_i \frac{\lambda_i}{\sum_i \lambda_i} = \frac{\sum_i \lambda_i^2}{\sum_i \lambda_i}. \tag{36}$$

Replacing Eq.(36) into Eq.(34) we have

$$\frac{\sum_i \lambda_i^2}{\sum_i \lambda_i} \geq \frac{1 + T_s \sum_i \lambda_i}{NT_s}. \tag{37}$$

After some simple calculus steps we get

$$\frac{\frac{1}{N}\sum_i \lambda_i^2}{\frac{1}{N^2}\left(\sum_i \lambda_i\right)^2} \geq \frac{1 + T_s \sum_i \lambda_i}{T_s \sum_i \lambda_i}. \tag{38}$$

On the left hand side of Eq.(38) we have the ratio of the second moment of the rates of the OFF periods for the channels in use and the square of their means. This can be written through the coefficient of variaton $c_v^2 = \frac{Var(X)}{(E[X])^2}$ as

$$c_\lambda^2 + 1 \geq \frac{1}{T_s \sum_i \lambda_i} + 1 \geq 2. \tag{39}$$

This means that the coefficient of variation must fullfill the condition

$$c_\lambda^2 \geq 1. \tag{40}$$

It is important to note that the above conditions we derived are sufficient, but not necessary. We may be allowing a lot of slack through the step where we use Jensen's inequality. However, our goal was to show that, contrary to the homogeneous case, scanning less frequently (but for more channels) can improve performance. The fact that we can find reasonable regimes for channel characteristics where this holds, despite the stricter condition, only strengthens our argument. Furthermore, while the above result proves conditions for the existence of a (non-trivial) scanning threshold, it could also be used in a recursive manner to derive the optimal threshold. We can already predict from the above theory that this threshold will increase when the variability of the channels accessible by the SU increases. In the remainder we will call this approach the *offline* algorithm. This algorithm maximizes the expected throughput, given that the threshold must be chosen only once and at the beginning.

E. The online (adaptive) algorithm

Based on the above insight, we can also take advantage of knowledge of which channel was in fact lost and propose a simple algorithm that we expect to further improve the average throughput. We will call this the *online* algorithm. The purpose of this algorithm is to decide about the threshold "on the fly", depending on which channel we lose. The condition regarding the lost channel can be derived departing from the comparison of $E[X_1] < X_2$ from Eq.(26) and Eq.(27). Solving this inequality, as before, we obtain

$$\lambda_{lost} > \frac{1 + T_s \sum_{i=1}^{N} \lambda_i}{N T_s}. \quad (41)$$

If the channel we lose has duration rates for the OFF periods that are larger than the right hand side of Eq.(41), then it is better to keep transmitting after losing that channel. Similarly, if for the second lost channel the above inequality still holds, transmission is not stopped. If after losing the ith channel, the above inequality does not hold any more, then we start to scan until we regain the missing i channels.

A special case would be if there exists the relation $T_s \sum_i \lambda_i >> 1$. Then, (41) reduces to

$$\lambda_{lost} > \frac{\sum_i \lambda_i}{N}. \quad (42)$$

Then, if the lost channel is worse (has higher λ) than the average of the channels in use, then it is better to resume transmission. This is rather intuitive, since by getting rid of channels with low availability we are left only with the ones with high availability, and if we decide to scan instead, the new channel is expected to only make the mean rate of the channel pool worse, *on average*. On the other hand, if we lose a good channel then we should interrupt, since the rest of the channels are probably worse than average and scanning could improve this situation.

There are a number of important design decisions involved in implementing this algorithm in practice, but this is beyond the scope of this paper.

IV. Simulation results

A. Homogeneous channels

We have already seen in Section III that for i.i.d. exponentially distributed OFF periods, it is better to start scanning immediately. So, there is no threshold value that provides higher data rate. In that case, it is better to scan as soon as we lose the first channel. Fig. 3 shows the average throughput for a radio that uses $N = 5$ channels to transmit its data. The ON periods for the primary users activities in all the channels are identical and chosen from a uniform distribution in the range between 10 and 20 s^{-1}. The OFF periods are also identical for all the channels with the average duration of $\frac{1}{\lambda_{off}} = 1\,s$. The number of backup channels is large enough. Unless otherwise stated, the channel bandwidth is $1\,MHz$, while the sensing period for each channel is $1\,ms$ for all the scenarios. We scan channels from the backup list sequentially one at a time. The x axis gives the threshold value used in each case. From Fig. 3 we can observe that the best result is achieved if we start scanning right away after losing the first channel, and by increasing the threshold value the average data rate decreases. Throughout this section we will user MATLAB as simulation tool.

Fig. 4 illustrates the dependence of the average data rate per cycle on the threshold value for i.i.d. exponentially distributed OFF periods, where $\lambda_{off} = 1\,s^{-1}$. We show the effect for a larger number of channels in use ($N = 10$). The other parameters are identical to the scenario of Fig. 3. Here also, the plot proves our claim that there is no threshold value that provides better results in terms of the data rate for exponentially i.i.d. channels.

B. Initial cost

As we have shown in Section III when an initial (setup) scanning cost exists, the optimal threshold might move from $L = 1$ to higher values. Fig. 5 illustrates the case when the initial cost value is 0.06 s, that is slightly higher than the theoretical minimum value obtained from our theory (14). The other parameters are exactly the same as those from the scenario which corresponds to Fig. 3. From Fig. 5 we can see that the threshold value that provides the best result is $L = 2$.

By intuition, if the initial scanning cost is higher, then the ideal threshold value moves towards higher values. If the initial cost is 0.26 s, then the dependence of the average data rate per cycle on the threshold value is depicted in Fig. 6. All the simulation parameters are the same as in Fig. 5. From this plot we can observe that the best result is achieved for a threshold value of $L = 3$. As expected, by increasing the initial cost, the ideal threshold value has been increased, too.

C. Heterogeneous channels

Now, we will consider the case with channels where the OFF periods are not drawn from identically exponential distributions. Fig. 7 shows the average throughput for the scenario with 15 channels. The OFF periods of these channels are independent exponentially distributed with different λ's. The values of the channel parameters λ_i are drawn from a uniform distribution in the interval $[1, 200]\ s^{-1}$. There is no initial scanning cost, and the sensing period is $1\ ms$. From Fig. 7 we can observe that the ideal threshold value is $L = 7$. This was also expected, because from that point on the condition $T_s \sum_i \lambda_i \geq \frac{1}{2(N-1)}$, does not hold anymore.

Fig. 8 illustrates the average throughput for the same simulation scenario as in Fig. 7, with the exception that the upper bound of λ's is $50\,s^{-1}$. The ideal threshold value is now $L = 4$. From this plot we can observe that the throughput is higher compared to the case of Fig. 7, because we have channels with higher durations of their OFF periods. It is also interesting to observe that the threshold value that provides maximum data rates has decreased. This is because of the fact that the variability of channels has been decreased compared to the previous case. Or, in other words, there is less difference between the channels in terms of their utility.

To further enhance our previous claim, let us consider the case when we have only good channels, i.e. channels whose OFF periods are with relatively long durations. For that purpose we will again consider the scenario in which we use 15 channels, with heterogeneous exponentially distributed OFF periods. The inverses of the mean durations for the OFF periods of different channels (λ_i), are drawn from a Bounded Pareto distribution with shape parameter $\alpha = 1.2$, lower bound of $0.1\,s^{-1}$, and upper bound equal to $1\,s^{-1}$. We pick λ's from a different distribution to give another proof that our theory is correct. The average throughput for this case is shown in

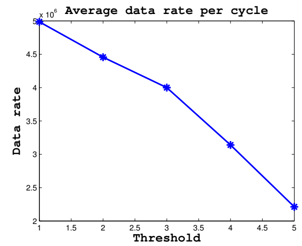

Fig. 3. Homogeneous exp. channels

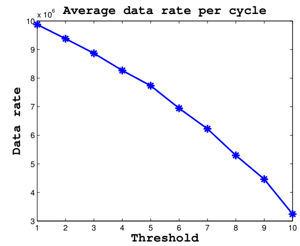

Fig. 4. Homogeneous exp. channels

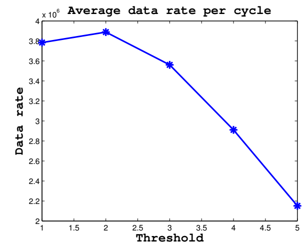

Fig. 5. Initial scanning cost

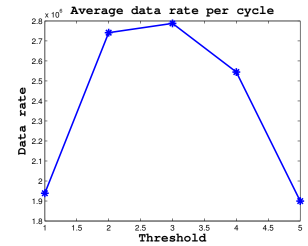

Fig. 6. Initial scanning cost

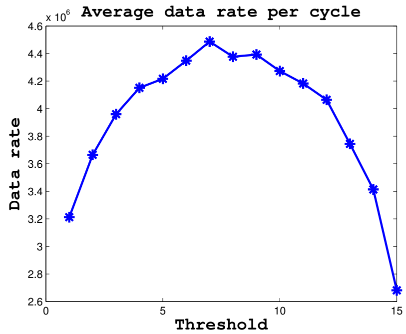

Fig. 7. Heterogeneous exp. channels

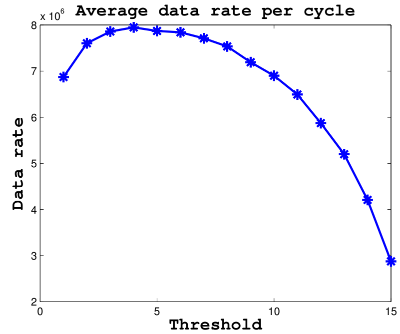

Fig. 8. Heterogeneous exp. channels

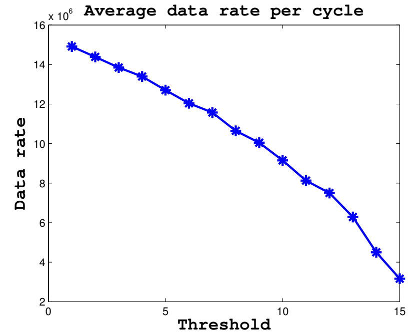

Fig. 9. Heterogenerous exp. channels with low var.

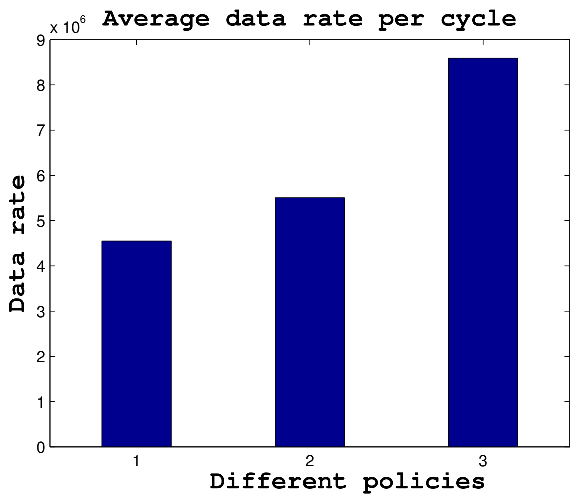

Fig. 10. Throughputs for different policies

Fig. 9. For these parameter values the data rate is decreased by increasing the threshold value. So, the best thing to do is to start scanning after the first lost channel. This is a consequence of the low coefficient of variation for the channels' OFF period durations.

D. The adaptive algorithm

Having validated our analytical conclusions that an optimal threshold exists when channels are heterogeneous, we now turn our attention to the proposed adaptive (online) algorithm to see if it can further improve performance. The number of channels in use is 15. The rates of the OFF periods are drawn from the uniform distribution in the interval 0.1 to 100 s^{-1}. Fig. 10 shows the data rates for three cases:

1) Threshold $L = 1$,
2) The ideal threshold for the offline algorithm, and
3) Online (adaptive) algorithm with the variable threshold.

The (offline) ideal threshold that provides maximum data rate (case 2) is $L = 5$. From Fig. 10 we can observe that our proposed online algorithm provides the highest average data rate per cycle. This is a consequence of the fact that we make the decision when to start scanning on the fly, depending on which channel we have lost. For the offline algorithm though, we must make the decision in advance, based on the average characteristics of the pool of channels and the expected quality of the lost ones. It can happen that a channel with average long duration is lost before a bad channel, which gives rise to this difference between the offline and online algorithms. We can also observe that the data rate is lowest if we trigger the scanning immediately.

E. Generic OFF periods

So far, we have only considered channels with exponential OFF periods in both analysis and simulations. While it is difficult to investigate analytically threshold policies for generic OFF periods, we can do so using simulations. Fig. 11 shows the average data rate for different threshold values for homogeneous OFF periods (uniformly distributed) with average duration of 1 s, and average ON duration of 0.1 s. There are 10 channels in use. We can observe that there does not exist a threshold value that provides higher average data rate compared to immediate scanning. This is consistent with the exponential homogeneous scenario, where it is also better to scan immediately. In Fig. 12 there are 8 channels in use, each with uniformly distributed OFF periods. Furthermore, this is a heterogeneous scenario where the mean OFF period for

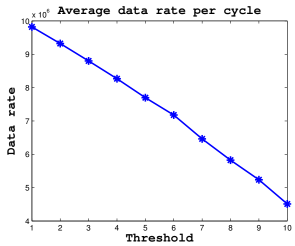

Fig. 11. Homogeneous uniform

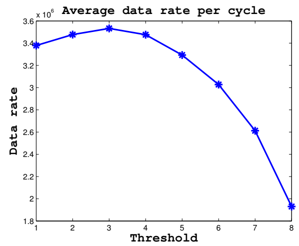

Fig. 12. Heterogeneous uniform

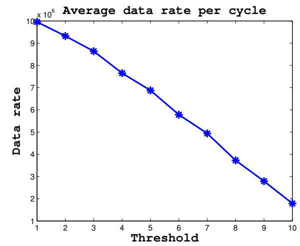

Fig. 13. Homogeneous Pareto

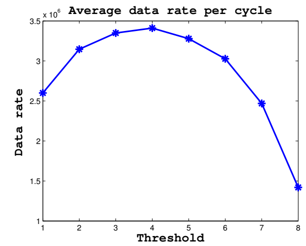

Fig. 14. Heterogeneous Pareto

each channel can be in the range 0.1 to 100 s^{-1}. For this case, as in the exponential heterogeneous case, an optimal threshold larger than one ($L = 3$) exists. The optimal value of the threshold depends on the failure rate of the distribution for the OFF periods. The uniform distribution has an increasing failure rate, so as time goes on, the probability that a channel in use will be available in the future is lower. Hence, we would expect that the threshold value is not too high ($L = 3$), since we lose quite quickly the good channels.

Fig. 13 shows the average throughput for homogeneous Pareto distributed OFF periods with shape parameter $\alpha = 1.2$, and with the rest of parameters identical to the scenario of Fig. 11. Here also, we can see that no ideal threshold value higher than 1 exists. This also means that it is not a sufficient condition for a threshold to exist, that the durations of homogeneous OFF periods to be drawn from a distribution with decreasing failure rate. Figure 14 shows the throughput for heterogeneous Pareto distributed OFF periods with the same average as in Fig. 12, and with identical number of channels ($N = 8$). The ideal threshold value in this case is $L = 4$, which is larger than that in Fig. 12. This can be explained as follows. Pareto distribution belongs to the class of distributions with decreasing failure rate, as opposed to the uniform distribution which has increasing arrival rate. This means that as time goes on, the chances to lose a good channel are lower and lower.

V. Conclusion

In this paper, we have analyzed the spectrum scanning process in cognitive radio networks and explored the ways to maximize the average throughput rate. We have introduced the notion of a threshold value as the number of channels we are allowed to lose, before the initiation of the scanning procedure. This threshold value provides the best results in terms of the average data rate. It is proven that no such threshold value exists for the case of homogeneous (i.i.d.) channels if there is no initial cost to be paid at the beginning of the scanning process. However, this value exists for heterogeneous independent channels and its value depends on the variability of the OFF periods of the channels in use. We have also proposed an adaptive algorithm that determines the moments when to stop transmission depending on which channel was lost, and have shown by simulations that this algorithm provides the highest throughput. In future work, we intend to extend our theoretical analysis to the generic OFF periods, as well as to consider joint scanning and sequencing optimization.

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