Training Sequence Design for Adaptive Equalization of Multi-user Systems *

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Abstract

Preliminary work on training sequence design for block adaptive channel estimation methods for multi-user Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) systems is provided. The focus is on linear minimum-mean squared error estimation methods. First, a joint channel estimation scheme is considered for hybrid quasi-synchronous CDMA/Time-Division Multiple-Access systems. For the CDMA/TDMA hybrid optimal training sequences are designed. The second scenario considers a completely asynchronous classical DS/CDMA system. Bayesian channel estimation is considered for a single user of interest. Based on estimated interference statistics, an optimal training sequence is designed. Numerical results are provided which indicate that significant improvements in performance can be achieved when employing optimized training sequences.

1 Introduction

While there has been a significant amount of work on adaptive and block algorithms for Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) systems (see [8] and references therein), there has been limited effort devoted to optimizing training sequences for these adaptive algorithms.

Single-user approaches employing least-squares estimation [6] yield sequences that are "white" or self-orthogonal. Sequences with idealized auto and cross-correlation properties have been investigated in [7, 2, 5].

In the current work, two multi-user scenarios will be considered: joint channel estimation for a hybrid CDMA/Time-Division Multiple-Access scheme (C-TDMA) as considered in the European UMTS proposal [1] as well single user channel estimation for an asynchronous classical DS/CDMA system. There are a number of distinct differences between the two communication scenarios. In C-TDMA, the interference cannot be considered wide sense stationary, due to the bursty nature of transmission; furthermore there is quasi-synchronization even in the uplink. Thus, a slotted communication system is considered. The joint least squares channel estimation technique of [2] is improved to consider arbitrary synchronization between users within a slot. In addition, optimal training sequences are designed which obviate the need for an exhaustive search over all possible sequences. These optimal sequences are based on *periodic uncorrelated and complimentary* sequence sets [5].

For the more typical, asynchronous DS/CDMA scenario, single user channel estimation is performed. A Bayesian based, minimum-mean squared error channel estimator is devised. Optimal training sequences are designed through minimization of the resulting mean-squared error. While the true optimization requires an exhaustive search, a modified cost-metric is designed for which a closed-form solution exists. The resulting non-binary sequence is then mapped to a valid sequence. It will be observed that while sub-optimal, such a design offers solid improvement in performance for non-ideal covariance estimates.

This paper is organized as follows. Section 2 describes the model for the received signal. The joint channel estimation procedure is presented in Section 3. Section 4 describes the optimal sequences and their properties. The single user channel estimation scheme is derived in Section 5. Numerical results are provided in Section 6 and conclusions are drawn in Section 7.

2 Signal model

In this section we describe the signal model for the system under consideration. For simplicity, we consider a discrete-time model obtained by sampling the received continuous-time signal at the chip-rate. We presume that the system is arbitrarily synchronized. For simplicity, we further assume that *short* (*i.e.*, symbol-length) spreading sequences are employed.

The uncorrelated data symbols are denoted $b_k[n]$ and they are presumed zero mean and unit variance.

C-TDMA Case For the joint channel estimation scheme for the C-TDMA scenario, we presume band-limited chip-pulses, $\psi(t)$. Because of the bandlimited assumption, the effect of the multipath channel $c_k(t)$ concatenated with an ideal LPF is equivalent to the chip-spaced tapped delay-line channel. It is assumed that integers P and q_k exist for $k = 1, \ldots, K$ such that the samples of the low-pass filtered channel impulse response are non-negligible only for $t \in [q_k T_c, (q_k + P - 1)T_c]$. The delay of user k relative to the receiver zero timing reference is expressed by the integer q_k. After LPF filtering and sampling at iT_c, the k-th combined channel+ chip-pulse impulse response can be described as

$$g_k[i] = \sum_{p=0}^{P-1} c_{k,p} \psi[i - q_k - p] \qquad (1)$$

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and where $\psi[j] = \psi(jT_c)$. The function $\psi(t)$ is the chip-pulse shape which is common to all users.

The receiver processes a vector of input samples $\mathbf{y}[n]$ which is of length $M_1 + M_2 + 1$ to ensure sufficient contribution from the desired symbol $b_1[n]$. This length is a function of the delay spread, L_d, the spreading gain, $L = \frac{T}{T_c}$, and the channel length, $L_g = P + 2J$.

It can then be shown that

$$\mathbf{y}[n] = \sum_{k=1}^{K} \mathbf{P}_k \mathbf{b}_k[n] + \boldsymbol{\nu}[n] \qquad (2)$$

where $E[\boldsymbol{\nu}[n]\boldsymbol{\nu}[n]^H] = \sigma^2 \mathbf{I}$ and where $\mathbf{P}_k$ is the $(M_1 + M_2 + 1) \times (B_1 + B_2 + 1)$ matrix

$$\mathbf{P}_k = \mathbf{S}_k^H \mathbf{G}_k \qquad (3)$$

The data vectors $\mathbf{b}_k[n]$ are of length $B_1 + B_2 + 1$ and are such that $\mathbf{b}_k[n]$ contains all symbols of user k contributing to the elements of $\mathbf{y}[n]$. The data bits are mutually uncorrelated and have unit energy. The k-th spreading matrix $\mathbf{S}_k$, of dimension $(M_1 + M_2 + L) \times (M_1 + M_2 + 1)$, is the matrix whose j-th column is the cyclic down-shift by $j - 1$ positions of the vector

$$(s_k[0], \ldots, s_k[L-1], \underbrace{0, \ldots, 0}_{M_1+M_2})^H$$

where $\mathbf{s}_k$ is the spreading vector for user k. Note that the spreading vectors are normalized: $\|\mathbf{s}_k\|^2 = 1$. The combined channel matrix $\mathbf{G}_k$, of dimension $(M_1+M_2+L)\times(B_1+B_2+1)$, depends only on the $M_1 + M_2 + (B_1 + B_2 + 1)L$ elements of the vector

$$\mathbf{g}_k = (g_k[M_2+B_1L], g_k[M_2-1+B_1L], \ldots, g_k[-M_1-L+1-B_2L])^T \qquad (4)$$

which are defined in (1). Moreover, only L_g elements of this vector are non-zero. Since normally $L_g < L$, it is clear that trying to estimate $\mathbf{g}_k$ might end up with a large overparameterization of the problem. On the other hand, if the individual user delays q_k and the maximum channel spread L_g were known, the dimensionality of the estimation problem can be greatly reduced.

Asynchronous CDMA For the single-user channel estimation problem, the assumptions on the delays are loosened: completely asynchronous communication is presumed. However, rectangular pulse shapes and chip-matched filtering are considered. The resulting discrete-time channel model for this scenario is then,

$$\begin{aligned} c_k(t) &= \sum_{p=0}^{P} c_k[p]\delta(t - \tau_k[p]) & (5) \\ \tau_k[p] &= q_k[p]T_c + \gamma_k[p] & (6) \end{aligned}$$

The integer $q_k[j]$ is the integer part (with respect to the chip duration T_c) of the delay $\tau_k[p]$ and $\gamma_k[j]$ is the corresponding fractional part. Without loss of generality, we assume that $q_k[0] \le q_k[1] \le \cdots \le q_k[P-1]$. Thus, the channel matrix can be further parameterized as follows:

$$\begin{aligned} \mathbf{P}_k &= \mathbf{S}_k^H \mathcal{C}_k & (7) \\ \text{where } \mathcal{C}_k &= \mathbf{c}_k \otimes \mathbf{I}_{B_1+B_2+1} & (8) \end{aligned}$$

$\mathbf{I}_{B_1+B_2+1}$ is the identity matrix of dimension $B_1 + B_2 + 1$ and $\otimes$ is the Kronecker product operator. The contribution due to the discrete time multipath channel can be described by the vector $\mathbf{c}_k$ which is of dimension $L \times 1$:

$$\mathbf{c}_k = [\,0 \;\; c_k[P-1]\gamma_k[P-1] \;\; c_k[P-1](T_c - \gamma_k[P-1]) \;\; 0 \;\; \cdots \;\; 0 \;\; \cdots \;\; c_k[0]\gamma_k[0] \;\; c_k[0](T_c - \gamma_k[0]) \;\; 0\,]^T$$

The location of the entry $c_k[j](T_c - \gamma_k[j])$ is the $(L - q_k[j])$'th component of the channel vector.

An alternative parameterization of the channel and training bits is found by noting that that,

$$\mathbf{S}_1^H \mathcal{C}_1 \mathbf{b}_1 = \mathbf{S}_1^H \mathcal{B}_1 \mathbf{c}_1, \qquad (9)$$

where $\mathcal{B}_1$ is given by

$$\begin{aligned} \mathcal{B}_1 &= \left[\; \Gamma_0^H \;\; \Gamma_1^H \;\; \cdots \;\; \Gamma_{B-1} \;\right]^H, & (10) \\ \text{where } \Gamma_i &= \operatorname{diag} \underbrace{[b_k(i), \cdots, b_k(i)]}_{\text{dimension } 1\times L} & (11) \end{aligned}$$

3 Joint Channel Estimation

Channel estimation is based on the training sequences inserted in the middle of each user burst, as currently done in the GSM standard [3]. Training sequences are defined at the chip-rate, thus there is no notion of "symbols" and the sequence length need not be a multiple of the spreading gain, L

Training sequence structure. Let Q and T be integers such that $Q \ge L_g$. For each user k, consider an ordered set of T sequences $\mathcal{A}_k = \{\mathbf{a}_k^{(t)} : t \in \mathbb{Z}_T\}$ of length Q over the complex numbers[1]. The k-th user training sequence $\mathbf{a}_k$ is formed from the set $\mathcal{A}_k$ as

$$\begin{aligned} \mathbf{a}_k = &\; (\underbrace{(\mathbf{a}_k^{(0)})^T, \ldots, (\mathbf{a}_k^{(0)})^T}_{D}, \underbrace{(\mathbf{a}_k^{(1)})^T, \ldots, (\mathbf{a}_k^{(1)})^T}_{D}, \ldots, \\ &\; \underbrace{(\mathbf{a}_k^{(T-1)})^T, \ldots, (\mathbf{a}_k^{(T-1)})^T}_{D})^T \end{aligned} \qquad (12)$$

where $D = 2 + \lceil L_d/Q \rceil$ guarantees that any pair of such sequences (say, $\mathbf{a}_{k_1}$ and $\mathbf{a}_{k_2}$, for all possible $1 \le k_1, k_2 \le K$) with a maximum time offset of L_d chips, have at least T blocks of length $2Q$ chips whose elements belongs to pairs of sequences with the same t index, i.e., to pairs $(\mathbf{a}_1^{(t)}, \mathbf{a}_2^{(t)})$, for $t \in \mathbb{Z}_T$.

[1] In the following, $\mathbb{Z}_m$ denotes the set of integers modulo m, i.e., $\{0, 1, \ldots, m-1\}$.

Received signal structure for joint channel estimation. The receiver collects T blocks of Q consecutive samples, for $t \in \mathbb{Z}_T$, beginning at the nominal epochs $\tau_t = (tD + D - 1)Q + t_0$, where t_0 is the nominal starting time of the training sequence relative to the beginning of the current slot. Due to the repetition structure of the training sequences, the t-th signal block $\mathbf{y}^{(t)} = (y[\tau_t + Q - 1], y[\tau_t + Q - 2], \ldots, y[\tau_t])^T$ can be written as

$$\mathbf{y}^{(t)} = \mathbf{A}^{(t)}\widetilde{\mathbf{g}} + \boldsymbol{\nu}^{(t)} \tag{13}$$

where $\boldsymbol{\nu}^{(t)}$ is a vector of i.i.d. noise samples, where $\mathbf{A}^{(t)} = [\mathbf{A}_1^{(t)}, \ldots, \mathbf{A}_K^{(t)}]$ is a $Q \times QK$ block matrix whose k-th $Q \times Q$ block is the circulant matrix

$$\mathbf{A}_k^{(t)} = \begin{bmatrix} a_k^{(t)}[0] & a_k^{(t)}[1] & \cdots & a_k^{(t)}[Q-1] \\ a_k^{(t)}[Q-1] & a_k^{(t)}[0] & & a_k^{(t)}[Q-2] \\ \vdots & & \ddots & \vdots \\ a_k^{(t)}[1] & a_k^{(t)}[2] & \cdots & a_k^{(t)}[0] \end{bmatrix} \tag{14}$$

and where $\widetilde{\mathbf{g}} = (\widetilde{\mathbf{g}}_1^T, \ldots, \widetilde{\mathbf{g}}_K^T)^T$ is the block vector of length QK whose k-th subvector

$$\widetilde{\mathbf{g}}_k = (\widetilde{g}_k[0], \ldots, \widetilde{g}_k[Q-1])^T$$

has i-th element defined by

$$\widetilde{g}_k[i] = \sum_{\ell=-\infty}^{\infty} g_k[L_g - 1 - i + \ell Q] \quad i \in \mathbb{Z}_Q \tag{15}$$

As is apparent from the above equation, $\widetilde{\mathbf{g}}_k$ is obtained by wrapping the combined channel impulse response vector $\mathbf{g}_k$ modulo Q. Since $\mathbf{g}_k$ is non-zero for at most $L_g \le Q$ consecutive samples, it can be uniquely determined up to a time shift of an integer multiple of Q chips from the wrapped channel response $\widetilde{\mathbf{g}}_k$ (we refer to this inverse operation as *channel unwrapping*).

Finally, we define the block vector $\mathbf{y}$ of length QT obtained by stacking the vectors $\mathbf{y}^{(t)}$ for $t \in \mathbb{Z}_T$. This can be written as

$$\mathbf{y} = \mathbf{A}\widetilde{\mathbf{g}} + \boldsymbol{\nu} \tag{16}$$

where $\mathbf{A}$ is the $QT \times QK$ block matrix obtained by stacking the matrices $\mathbf{A}^{(t)}$ for $t \in \mathbb{Z}_T$ correspondingly.

Joint wrapped channel estimation. Assuming that $T \ge K$ and that $\mathbf{A}$ has full column-rank, the least-squares estimate of the composite wrapped channel vector is given by

$$\widehat{\widetilde{\mathbf{g}}} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{y} \tag{17}$$

Notice that the matrix $(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ depends only on the known training sequences, thus it can be pre-computed. It is noted that the unwrapping operation can be done in several ways. Herein, channel unwrapping is performed by minimizing the mean-square delay spread.

4 Optimal training sequence sets

We focus on the LS estimation of the wrapped channel response (17) and we exploit the block-circulant structure of the matrix $\mathbf{A}$ in order to find optimal sets of training sequences.

For $\mathbf{M} = \mathbf{A}^H\mathbf{A}$ invertible, the normalized estimation MSE is given by

$$\epsilon^2 = \frac{\sigma^2}{QK}\text{tr}\left(\mathbf{M}^{-1}\right) \tag{18}$$

Since both $\mathbf{M}$ and $\mathbf{M}^{-1}$ are positive definite and Hermitian, we have $\text{tr}(\mathbf{M}) = \sum_{i=1}^{QK}\lambda_i = \sum_{i=1}^{=} QKm_{i,i}$ and $\text{tr}(\mathbf{M}^{-1}) = \sum_{i=1}^{QK} 1/\lambda_i$, where λ_i and $m_{i,i}$ are the eigenvalues and the diagonal elements of $\mathbf{M}$, respectively. Because of the block-circulant structure of $\mathbf{A}$, the $\{m_{i,i}\}$ satisfy

$$m_{(k-1)Q+\ell,(k-1)Q+\ell} = \frac{1}{D}|\mathbf{a}_k|^2 \quad \text{for } k = 1, \ldots, K,\ \ell = 1, \ldots, Q \tag{19}$$

We impose the constraint that the total normalized training sequence energy is a constant Γ, i.e.,

$$\frac{1}{K}\sum_{k=1}^{K}|\mathbf{a}_k|^2 = \frac{D}{KQ}\sum_{i=1}^{QK} m_{i,i} = \Gamma$$

Then, the training sequence optimization reduces to the constrained minimization problem:

$$\begin{cases} \text{minimze} & \text{tr}(\mathbf{M}^{-1}) = \sum_{i=1}^{QK}\frac{1}{\lambda_i} \\ \text{subject to} & \text{tr}(\mathbf{M}) = \sum_{i=1}^{QK}\lambda_i = \text{constant} \end{cases} \tag{20}$$

and subject to the positivity constraints $\lambda_i > 0$. This can be solved by the method of Lagrange's multipliers. The solution is of the form: $\lambda_i = $ constant. The only positive definite matrix with constant eigenvalues is a diagonal matrix with constant diagonal elements. We conclude that an optimal set of training sequences must satisfy $\mathbf{A}^H\mathbf{A} = \mathcal{E}\mathbf{I}$, where $\mathcal{E}$ is proportional to the common energy of the training sequences. Then, without loss of generality, we look for block-circulant matrices $\mathbf{A}$ whose columns are orthonormal (up to a normalization factor).

Fortunately, there exists a fairly general algebraic construction [5] of set of root-of-unity sequences (i.e., sequences whose elements are of the form $\exp(j2\pi r)$ with r rational, which provide periodically uncorrelated and complimentary sequence sets (denoted as PUC sets). Optimal training sequence sets are derived as follows:

Corollary 1: construction of optimal training sequences. Let $\mathcal{A} = \{\mathbf{a}^{(t)} : t \in \mathbb{Z}_T\}$ be an ordered PUC set of root-of-unity sequences of cardinality $T \ge K$ and length Q. Then, optimal sets $\mathcal{A}_k$ defining the training sequences for all users $k = 1, \ldots, K$ are obtained from $\mathcal{A}$ by

$$\mathcal{A}_k = \left\{\mathbf{a}^{(t+k-1)_{\bmod T}} : t \in \mathbb{Z}_T\right\} \tag{21}$$

5 Single Channel Estimation

In the single channel estimation scenario, we consider channel estimation for the case where $K-1$ active users exist in the system and a new user wishes to initiate transmission. Thus, it is desired to estimate the channel of the new user. For this problem, a probabilistic model is assumed for the channel: each path is independent of the other paths; the delays are uniformly distributed on $[0,T)$ and the path coefficients are white complex Gaussian variables. The model on the delays and the paths further implies that $q_k[n] \in [0,1,\cdots,L-1]$ with probability $\frac{1}{L}$ and that $\gamma_k[n] \sim U[0,T_c)$. We consider linear estimation of the channel which minimizes the mean-squared error. Thus, we seek $\mathbf{\Lambda_1}$ such that

$$\hat{\mathbf{c}}_1 = \mathbf{\Lambda}_1 \mathbf{y}[n], \tag{22}$$

$$\text{where } \mathbf{\Lambda_1} = \arg\min \mathbf{E}\left[\text{trace}\left(\mathbf{\Lambda_1 r} - \mathbf{c}_1\right)\left(\mathbf{\Lambda_1 r} - \mathbf{c}_1\right)^H\right] = \mathbf{Q}\mathbf{R}^{-1} \tag{23}$$

$$\mathbf{R} = \mathbf{E}\left[\mathbf{y}\mathbf{y}^H\right] = \mathbf{S}_1 \mathcal{B}_1 \mathbf{E}\left[\mathbf{c}_1\mathbf{c}_1^H\right]\mathcal{B}_1^H \mathbf{S}_1^H + \sum_{k=2}^{K} \mathbf{S}_k \mathcal{C}_k \mathcal{C}_k^H \mathbf{S}_k^H + \sigma^2 \mathbf{I}_{(B_1+B_2-1)L} \tag{24}$$

$$\mathbf{Q} = \mathbf{E}\left[\mathbf{c}_1 \mathbf{r}^H\right] = \mathbf{E}\left[\mathbf{c}_1\mathbf{c}_1^H\right]\mathcal{B}_1^H \mathbf{S}_1^H \tag{25}$$

The matrices $\mathbf{R}$ and $\mathbf{Q}$ are determined under the assumption of uncorrelated data being transmitted by the existing users. Furthermore, it is assumed that the channels of the existing active users are known. The channel covariance matrix is given by

$$\mathbf{E}\left[\mathbf{c}_1\mathbf{c}_1^H\right] = \Delta, \tag{26}$$

where Δ is a tridiagonal matrix such that

$$\text{diag}(\Delta, 0) = \frac{1}{L-1}\left[\frac{T_c^2}{3}, \underbrace{\frac{T_c^2}{6}, \cdots, \frac{T_c^2}{6}}_{\text{dimension } P-2}, \frac{T_c^2}{3}\right] \tag{27}$$

$$\text{diag}(\Delta, 1) = \frac{1}{L-1}\left[\frac{T_c^2}{6}, \cdots, \frac{T_c^2}{6}\right] \tag{28}$$

and $\text{diag}(\Delta,-1) = \text{diag}(\Delta,1)$. The notation $\text{diag}(\Delta,k)$ refers to the k'th diagonal of the matrix Δ.

It can be shown that the minimum mean-squared error is given by,

$$\text{MMSE} = \text{trace}\left[\Delta - \mathbf{Q}\mathbf{R}^{-1}\mathbf{Q}^H\right] \tag{29}$$

$$= \text{trace}\left(\Delta^{-1} + \tilde{\mathbf{R}}\right)^{-1} \tag{30}$$

$$\text{where } \tilde{\mathbf{R}} = \mathcal{B}_1^H \mathbf{S}_1^H \left(\sum_{k=2}^{K} \mathbf{S}_k \mathcal{C}_k \mathcal{C}_k^H \mathbf{S}_k^H + \sigma^2 \mathbf{I}_{(B_1+B_2-1)L}\right)^{-1} \mathbf{S}_1 \mathcal{B}_1$$

The last statement follows from the application of the matrix inversion lemma. Thus to determine the optimal training sequence, we seek to minimize the MMSE over all possible training sequences. This activity requires knowledge of the statistics of the multiple access interference and that of the unknown channel.

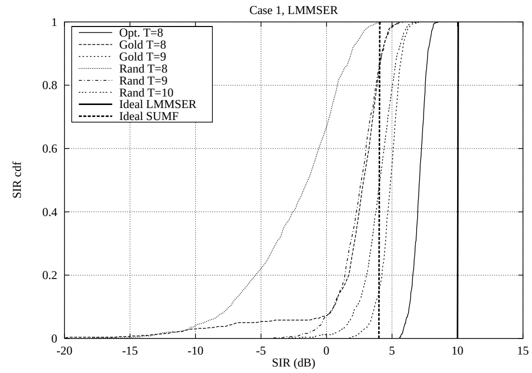

Figure 1: User 1 SIR cumulative distribution function for the MMSE receiver with joint channel estimation (SNR = 15dB for all users).

As the proposed training sequence design technique is based on an exhaustive search, we propose the following approximate method for determining the training sequence. The MMSE in (30) is convex in the training bits. For an unconstrained solution, it can be shown the B equations, $f(\underline{b}) = \mathbf{0}_{B\times 1}$, must be solved simultaneously to determine our desired training sequence, $\underline{b}$, where

$$f(\underline{b})_i = \text{trace}\left[\left(\sum_{j=1}^{B} b_j \left(\mathbf{W}(i,j) + \delta(i-j)\mathbf{W}(i,j)\right)\right) \times \left(\Delta^{-1} + \tilde{\mathbf{R}}\right)^{-2}\right]$$

The matrices $\mathbf{W}(i,j)$ are the i,jth $B \times B$ blocks of the matrix $\tilde{\mathbf{R}}$ which is $B^2 \times B^2$. Due to the lack of a closed form solution for the set of equations above we consider an approximate method. We consider the following property of the trace for Hermitian matrices A and B: $\text{trace}(AB) \leq \text{trace}(A)\text{trace}(B)$. Thus we shall seek to simultaneously minimize the trace $\left[\sum_{j=1}^{B} b_j \left(\mathbf{W}(i,j) + \delta(i-j)\mathbf{W}(i,j)\right)\right]$. This results in the following minimization,

$$\hat{\underline{b}} = \arg\min \underline{b}^T \mathbf{Z}\mathbf{Z}^H \underline{b}$$

The elements of the matrix $\mathbf{Z}$ are functions of the traces of the matrices $\mathbf{W}(i,j)$. Thus, the desired unconstrained solution is the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{Z}\mathbf{Z}^H$. The sequence employed is found by $\underline{b}^* = \text{sgn}(\hat{\underline{b}})$. In fact, the sum of the $\mathbf{W}(i,j)$ matrices are not explicitly Hermitian. However for long spreading sequences, the covariance matrix will be near-Toeplitz.

6 Numerical Results

Joint Channel Estimation A C-TDMA system is examined with $K=8$ users and $L=16$ spreading gain. The channel coefficients are random and independent for all user, constant over the bursts. They are generated from a complex Gaussian distribution with zero mean and are normalized. The spreading sequences are orthogonal Walsh-Hadamard sequences that are further multiplied by a long pseudo-noise

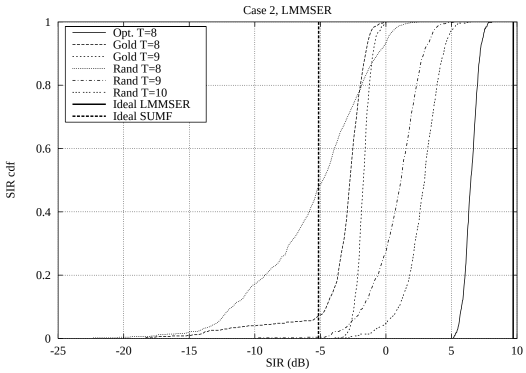

Figure 2: User 1 SIR cumulative distribution function for the MMSE receiver with joint channel estimation (SNR = 15dB for users 1 and 2, other users SNR = 25 dB).

sequence. For brevity, the LMMSER results are provided only for a perfect power control scenario (all users have a SNR of 15dB) and for a near far scenario where the desired user and one interferer have SNR of 15dB and the remaining interferers have an SNR of 25 dB. The receiver processing window was three symbols. The PUC set is constructed with $s = 2, T = 8, \gamma(t) = 0, \beta(t) = (3T)_{modQ}$ and $\alpha(t) = 1$. For comparison, random binary sequences are also generated for $T = 8, 9, 10$. Gold sequences are also considered. The LMMSER filter is constructed from the signal estimated covariance matrix and a vector based on the estimated channel [4]. It is clear from the numerical results in Figures 1 and 2 that the optimal sequences provide near-far resistant performance, while the sub-optimal sequences suffer performance degradation. The cumulative distribution function is estimated from 1000 realizations of the random channel.

Single-user Channel Estimation For the single-user estimation scenario, a $K = 7$ user system with $L = 15$ length spreading codes is considered. A perfect power control case (all users SNR = 20dB) is examined in Figure 3 and a near-far scenario is considered in Figure 4 (user 1 SNR = 20db, interfering users SNR = 30 dB). An exhaustive search over the cost function in (30) is considered for length 4 (Figure 3) and length 7 (Figure 4) training sequences. The minimum and maximum resulting MSE are provided; for exact channel statistics these two values are nearly equal. However, for the use of sample interference covariance, significant gain can be achieved through optimization. The approximate technique employing the minimizing eigenvector yields consistent results for the estimate interference covariance matrix case. The cumulative distribution function is estimated from 500 realizations of the random channel.

7 Conclusions

For two specific CDMA communication systems, it has been shown that optimization of training sequences can improve performance. The first scenario considered was a C-TDMA hybrid system where quasi-synchronism holds even in the uplink. For this system, optimal training sequences were designed for joint channel estimation. For the standard DS/CDMA scenario, cost functions were derived for optimization of single-user channel estimation for a new user entering communication. A key issue is that the optimal training sequence is a function of the statistics of the active users in the system. A heuristic method for designing sequences was also provided.

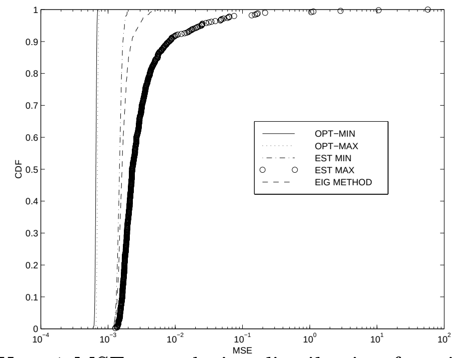

Figure 3: User 1 MSE cumulative distribution function for single user Bayesian channel estimation (SNR = 20dB for all users).

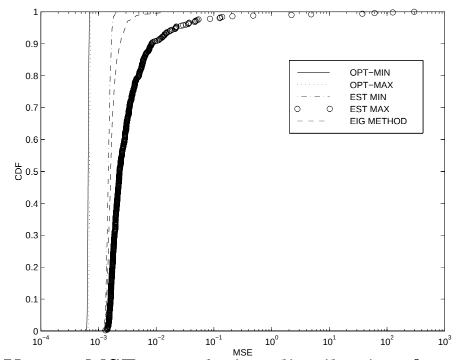

Figure 4: User 1 MSE cumulative distribution function for single user Bayesian channel estimation (SNR = 20dB for user 1, SNR=30dB for interferers.

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