MIMO BC with Imperfect and Delayed Channel State Information at the Transmitter and Receivers

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***Abstract*—In the setting of the two-user (M,N) multiple-input multiple-output (MIMO) broadcast channel (BC), recent work by Maddah-Ali and Tse, and Vaze and Varanasi have revealed the usefulness of delayed channel state information at the transmitter (perfect delayed CSIT). Our work studies the general case of communicating with imperfect delayed CSIT, and proceeds to present novel precoding schemes and degrees-of-freedom (DoF) bounds that are often tight, and to constructively reveal that even substantially imperfect delayed-CSIT, is in fact sufficient to achieve the optimal DoF performance previously associated to perfect delayed CSIT. Going one step further, we also constructively show that, this same optimal performance can in fact be achieved in the presence of additional imperfection of the global CSIR - i.e., even with imperfect receiver estimates of the channel of the other receiver.**

Specifically, for feedback-quality exponent β describing the high-SNR asymptotic rate-of-decay of the mean square error of the delayed CSIT estimate, the derived DoF $d(\beta)$ for a given exponent $\beta \in [0,1]$, reveals that the optimal two-user MIMO-BC DoF region previously associated to perfect delayed CSIT, can in fact be achieved for any imperfect $\beta \geq \frac{N}{\min(M,2N)+N}$. Interestingly, for all the cases studied here, the derived quality threshold $\beta^* \triangleq \arg\min_\beta\{d(\beta) = d'\}$ for any given symmetric DoF d', accepts the simple form of $\beta^* = (d'-d(0))/(d(1)-d(0))$, describing the fraction of the DoF gap - between the no-CSIT and the full delayed CSIT case - that is covered to reach the target d'. The potential of an up to $\frac{1}{\beta^*}$-fold reduction in feedback bits, can be advantageous in the presence of feedback links with limited reliability and limited capacity.

I. Introduction

We here consider the two-user (M,N) multiple-input multiple-output (MIMO) broadcast channel (BC), where a M-antenna transmitter communicates information to two N-antenna receivers. In this setting, the channel model takes the form

$$\boldsymbol{y}^{(1)} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z}^{(1)} \quad (1a)$$

$$\boldsymbol{y}^{(2)} = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{z}^{(2)} \quad (1b)$$

where vectors $\boldsymbol{H}, \boldsymbol{G} \in \mathbb{C}^{N\times M}$ represent the transmitter-to-user 1 and transmitter-to-user 2 channels respectively, where $\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}$ represent unit power AWGN noise at the two receivers, where $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ is the input signal with power constraint $\mathbb{E}[||\boldsymbol{x}||^2] \leq P$, and where in this case, P also takes the role of the signal-to-noise ratio (SNR).

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With channel state information at the transmitter (CSIT) being a crucial ingredient that facilitates improved performance, and with CSIT often being limited, imperfect and delayed, we here explore the effects of the *quality of delayed CSIT*, corresponding to how well the transmitter knows the same $\boldsymbol{H}, \boldsymbol{G}$ after this channel state has fully changed. Naturally, reduced CSIT quality relates to limitations in the capacity and reliability of the feedback links. Similar issues, which additionally motivate this work, and which are addressed here, pertain to the quality of delayed global CSIR, i.e., to the quality of the estimates, at a given receiver, of the channel of the other receiver (see for example the work of [1], [2] on the challenge of obtaining such global CSIR).

A. Related Work

It is well known that in the two-user (M,N) MIMO BC setting of interest, the presence of perfect CSIT allows for the optimal sum degrees-of-freedom (DoF) $\min\{M, 2N\}$ (this is with perfect global CSIR, cf. [3]), whereas the complete absence of CSIT causes a substantial degradation to just $\min\{M,N\}$ (cf. [4]) [1].

An interesting scheme that mitigates this degradation by utilizing partial CSIT knowledge, was recently presented in [5] by Maddah-Ali and Tse, which showed that in the absence of current CSIT, delayed CSIT knowledge can still be useful in improving the DoF region of the multiple-input single-output (MISO) broadcast channel ($N = 1$). This result was later generalized by Vaze and Varanasi in [6] to the MIMO case (again, this is with perfect global CSIR). [2]

Our work extends the work in [6], and studies the general case of communicating with imperfect delayed CSIT. Specifically this work reveals that even substantially imperfect delayed-CSIT, is in fact sufficient to achieve the optimal DoF performance previously associated to perfect delayed CSIT. Going one step further, we also constructively show that, this same optimal performance can in fact be achieved in the presence of additional imperfection of the global CSIR - i.e., even with imperfect receiver estimates of the channel of the other receiver.

[1] We remind the reader that for an achievable rate pair (R_1, R_2), the corresponding DoF pair (d_1, d_2) is given by $d_i = \lim_{P\to\infty} \frac{R_i}{\log P}$, $i = 1, 2$. The corresponding DoF region is then the set of all achievable DoF pairs.

[2] Other interesting works in the context of utilizing delayed and current CSIT, can be found in [7]–[9] which explored the setting of combining perfect delayed CSIT with immediately available imperfect CSIT, the work in [10] which additionally considered the effects of the quality of delayed CSIT for the MISO BC, the work in [11] which considered delayed and progressively evolving (progressively improving) current CSIT, and the works in [12]–[14] and many other publications.

B. Quantification of CSI Quality

In this work we will consider the case without any current CSIT, but with imperfect delayed CSIT. In terms of delayed CSIT, we consider the case where the transmitter's delayed estimates $\check{\boldsymbol{H}}, \check{\boldsymbol{G}}$ come with estimation errors

$$\ddot{\boldsymbol{H}} = \boldsymbol{H} - \check{\boldsymbol{H}}, \quad \ddot{\boldsymbol{G}} = \boldsymbol{G} - \check{\boldsymbol{G}} \tag{2}$$

having independent and identically distributed (i.i.d.) Gaussian entries with power

$$\mathbb{E}[||\ddot{\boldsymbol{H}}||_F^2] \doteq \mathbb{E}[||\ddot{\boldsymbol{G}}||_F^2] \doteq P^{-\beta}$$

for some CSI quality exponent β describing the general quality of the delayed estimates.

In this setting, an increasing exponent β implies an improved delayed CSIT quality, with $\beta = 0$ implying very little delayed CSIT knowledge, and with $\beta = \infty$ implying perfect delayed CSIT.

In addition to the challenge of communicating CSIT over feedback channels with limited capacity and limited reliability, another known bottleneck is the non-negligible cost of distributing global CSIR across the different receiving nodes (see [1], [2]). For this reason, we explore the case where, in addition to limited and imperfect CSIT, we also have the additional imperfection of the global CSIR, which means that each user has imperfect estimates of the other user's channel, as well as, in this case, no access to the estimates of the transmitter. In the spirit of communicating global CSIR across feedback links ([1]) we also focus on the associated case of having imperfect delayed global CSIR corresponding to the same quality exponent β, and additionally having no receiver access to the CSIT estimates of the transmitter. With $\check{\check{\boldsymbol{H}}}$ denoting the delayed estimate of $\boldsymbol{H}$ at user 2, and $\check{\check{\boldsymbol{G}}}$ denoting the delayed estimate of $\boldsymbol{G}$ at user 1, we maintain as before that the estimation errors

$$\ddot{\ddot{\boldsymbol{H}}} = \boldsymbol{H} - \check{\check{\boldsymbol{H}}}, \quad \ddot{\ddot{\boldsymbol{G}}} = \boldsymbol{G} - \check{\check{\boldsymbol{G}}} \tag{3}$$

have i.i.d. Gaussian entries with power

$$\mathbb{E}[||\ddot{\ddot{\boldsymbol{H}}}||_F^2] \doteq \mathbb{E}[||\ddot{\ddot{\boldsymbol{G}}}||_F^2] \doteq P^{-\beta}$$

again for the same β as before, now also describing the quality of the global CSIR delayed estimates.

Remark 1: We here note that without loss of generality, we can restrict our attention to the range $0 \leq \beta \leq 1$ (cf. [15]), where again $\beta = 1$ corresponds the case of perfect delayed CSIT.

C. Notation, Conventions and Structure of Paper

In Section II, for the aforementioned two user MIMO BC, and for the general case of imperfect delayed CSIT and imperfect global-CSIR, we derive a DoF region inner bound, which turns out to be tight for any $\beta \geq \frac{N}{\min(M,2N)+N}$ previously associated to perfect delayed CSIT and perfect global-CSIR. Section III then presents the novel multi-phase precoding schemes associated to the aforementioned DoF regions.

In this work we assume that the elements of the channel vectors $\boldsymbol{H}$ and $\boldsymbol{G}$, are spatially and temporally i.i.d. Gaussian random variables, with zero mean and unit variance. Finally adhering to the common convention, we consider a unit coherence period, as well as assume that each receiver knows perfectly its own channel (perfect local CSIR).

In terms of the notation, throughout this paper, $(\bullet)^{\mathsf{T}}$, $(\bullet)^{\mathsf{H}}$ and $||\bullet||_F$ denote the transpose, conjugate transpose and Frobenius norm of a matrix respectively, while $||\bullet||$ denotes the Euclidean norm, and $|\bullet|$ denotes the magnitude of a scalar. $o(\bullet)$ comes from the standard Landau notation, where $f(x) = o(g(x))$ implies $\lim_{x\to\infty} f(x)/g(x) = 0$. We also use $\doteq$ to denote *exponential equality*, i.e., we write $f(P) \doteq P^B$ to denote $\lim\limits_{P\to\infty} \dfrac{\log f(P)}{\log P} = B$. Logarithms are of base 2.

II. DoF of the MIMO BC with Imperfect Delayed CSIT and Imperfect global-CSIR

It is noted that, for the case with $M \leq N$, the DoF region is characterized as $d_1 + d_2 \leq M$, which is achievable by TDMA scheme without any CSIT and without any global-CSIR. Thus in the following, we focus on the case with $M > N$.

Theorem 1: For the $(M > N, N)$ MIMO BC with imperfect delayed CSIT and imperfect global-CSIR, the optimal DoF region takes the form

$$\frac{d_1}{\min\{M,N\}} + \frac{d_2}{\min\{M,2N\}} \leq 1$$
$$\frac{d_2}{\min\{M,N\}} + \frac{d_1}{\min\{M,2N\}} \leq 1$$

when $\beta \geq \frac{N}{\min(M,2N)+N}$, while when $\beta < \frac{N}{\min(M,2N)+N}$ this region is inner bounded by the achievable region

$$\frac{d_1}{\min\{M,N\}} + \frac{d_2}{\min\{M,2N\}} \leq 1$$
$$\frac{d_2}{\min\{M,N\}} + \frac{d_1}{\min\{M,2N\}} \leq 1$$
$$d_1 + d_2 \leq \min\{M,N\} + \beta(\min\{M,2N\} - \min\{M,N\})$$

which, for $\beta' \triangleq \min\{\beta, \frac{N}{\min(M,2N)+N}\}$, takes the form of a polygon with corner points $\{(0,0), (0,N), (\min\{M,2N\}\beta', N(1-\beta')), (N(1-\beta'), \min\{M,2N\}\beta'), (N,0)\}$.

At this point we can draw an interesting conclusion on the amount of delayed CSIT needed to achieve a certain symmetric DoF performance d'. For all the cases considered here, the derived threshold value $\beta^* \triangleq \arg\min_\beta\{d(\beta) = d'\}$, accepts the simple form of

$$\beta^* = (d' - d(0))/(d(1) - d(0)) \tag{4}$$

describing the fraction of the DoF gap - between the no-CSIT and the perfect delayed CSIT case - that is covered to reach d'.

As an example of this derived threshold quality, we see that for the MIMO case with $N < M < 2N$, the target optimal $d' = d_1^* = MN/(M+N)$ (cf. [6]) corresponds to the aforementioned

$$\beta^* = \frac{d' - d(0)}{d(1) - d(0)} = \frac{MN/(M+N) - N/2}{M/2 - N/2} = \frac{N}{M+N}.$$

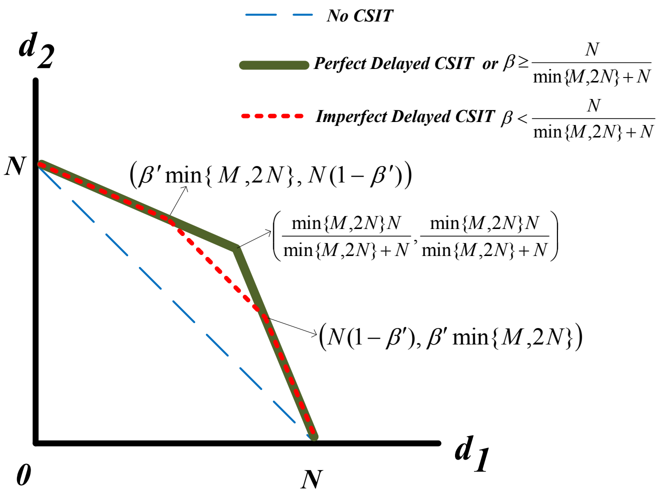

Fig. 1. DoF region of MIMO BC with imperfect delayed CSIT and imperfect global-CSIR ($M > N$).

III. Precoding Schemes for the MIMO BC

We proceed to describe precoding schemes that achieve the corresponding DoF corner points, by properly utilizing different combinations of superposition coding, successive cancellation, power allocation, and phase durations, and do so with imperfect global CSIR. Specifically, for $\beta' \triangleq \min\{\beta, \frac{N}{\min(M,2N)+N}\}$, scheme $\mathcal{X}_1$ will achieve DoF corner point $(2N\beta', N(1-\beta'))$ and point $(N(1-\beta'), 2N\beta')$ for the case with $M \geq 2N$, while scheme $\mathcal{X}_2$ will achieve DoF corner point $(M\beta', N(1-\beta'))$ and point $(N(1-\beta'), M\beta')$ for the case with $N < M < 2N$. It is noted that, DoF corner points $(\min\{M,N\}, 0)$ and $(0, \min\{M,N\})$ are easily achievable by single-user transmission scheme without any CSIT and without any global CSIR.

The schemes are designed to have S phases, where the sth phase ($s = 1, 2, \cdots, S$) consists of T_s channel uses, which will be designed later from scheme to scheme. At this point, and to more clearly reflect the division of time into phases, we will adopt a double time index where, for example, $\boldsymbol{H}_{s,t}$ and $\boldsymbol{G}_{s,t}$ will denote the channel vectors during timeslot t of phase s. Similarly, in terms of delayed CSIT (cf. (2)), $\check{\boldsymbol{H}}_{s,t}$ and $\check{\boldsymbol{G}}_{s,t}$ will be the delayed estimates of $\boldsymbol{H}_{s,t}$ and $\boldsymbol{G}_{s,t}$, where these estimates become known to the transmitter with unit delay (at time $t+1$ of the same phase - recall that we follow the unit-coherence period convention), and are stored and recalled thereafter. Finally $\ddot{\boldsymbol{H}}_{s,t} = \boldsymbol{H}_{s,t} - \check{\boldsymbol{H}}_{s,t}$, $\ddot{\boldsymbol{G}}_{s,t} = \boldsymbol{G}_{s,t} - \check{\boldsymbol{G}}_{s,t}$ will denote the estimation errors corresponding to delayed CSIT.

In terms of general notation, the transmitted vector generally takes the form

$$\boldsymbol{x}_{s,t} = \boldsymbol{c}_{s,t} + \boldsymbol{a}_{s,t} + \boldsymbol{b}_{s,t} \tag{5}$$

where $\boldsymbol{a}_{s,t} \in \mathbb{C}^{M\times 1}$ will denote the independent information symbol vector for user 1, while symbol vector $\boldsymbol{b}_{s,t} \in \mathbb{C}^{M\times 1}$ is meant for user 2, and where $\boldsymbol{c}_{s,t} \in \mathbb{C}^{M\times 1}$ denotes the common information symbol vector. For any symbol $\bullet_{s,t}$, we will use $P_s^{(\bullet)} \triangleq \mathbb{E}|| \bullet_{s,t} ||^2$ to denote the power, and we will use $r_s^{(\bullet)}$ to denote the prelog factor of the number of bits $r_s^{(\bullet)} \log P - o(\log P)$ carried by $\bullet_{s,t}$, for the phase s.

In addition, we will use

$$\iota_{s,t}^{(1)} \triangleq \boldsymbol{H}_{s,t}\boldsymbol{b}_{s,t}, \ \iota_{s,t}^{(2)} \triangleq \boldsymbol{G}_{s,t}\boldsymbol{a}_{s,t}, \tag{6}$$

to denote the interference experienced by user 1 and user 2 respectively, during timeslot t of phase s, and we will use

$$\check{\iota}_{s,t}^{(1)} \triangleq \check{\boldsymbol{H}}_{s,t}\boldsymbol{b}_{s,t} \triangleq \bar{\iota}_{s,t}^{(1)} + \tilde{\iota}_{s,t}^{(1)}, \ \check{\iota}_{s,t}^{(2)} \triangleq \check{\boldsymbol{G}}_{s,t}\boldsymbol{a}_{s,t} \triangleq \bar{\iota}_{s,t}^{(2)} + \tilde{\iota}_{s,t}^{(2)}, \tag{7}$$

to denote transmitter's (delayed) estimates of $\iota_{s,t}^{(1)}, \iota_{s,t}^{(2)}$, and will use $\bar{\iota}_{s,t}^{(1)}, \bar{\iota}_{s,t}^{(2)}$ to denote a quantized version of these estimates, with $\tilde{\iota}_{s,t}^{(1)}, \tilde{\iota}_{s,t}^{(2)}$ denoting the corresponding quantization errors.

Furthermore, we will use $\boldsymbol{e}(m:n)$ to denote the $(n-m+1)$-length subvector that consists of the mth-to-nth elements of a vector $\boldsymbol{e}$, and we will similarly use $\boldsymbol{E}(m:n)$ to denote the submatrix that consists of the mth-to-nth rows of a matrix $\boldsymbol{E}$.

A. Scheme $\mathcal{X}_1$ for the MIMO BC with $M \geq 2N$

For scheme $\mathcal{X}_1$, the phase durations $T_1, T_2, \cdots, T_S$ are chosen to be integers from a geometric progression [3]

$$\begin{aligned} T_s &= T_{s-1}\xi = T_1\xi^{s-1}, \ s = 2, 3, \cdots, S-1, \\ T_S &= T_{S-1}\zeta = T_1\xi^{S-2}\zeta \end{aligned} \tag{8}$$

for $\xi = \frac{2\beta}{1-\beta}$ and $\zeta = 2\beta$.

We proceed to describe scheme $\mathcal{X}_1$ from phase to phase.

1) Phase 1: During phase 1 (T_1 channel uses), the transmitter sends

$$\boldsymbol{x}_{1,t} = \boldsymbol{c}_{1,t} + \boldsymbol{a}_{1,t} + \boldsymbol{b}_{1,t}, \tag{9}$$

with power and rate set as

$$\begin{aligned} P_1^{(\boldsymbol{c})} &\doteq P, & r_1^{(\boldsymbol{c})} &= N(1-\beta) \\ P_1^{(\boldsymbol{a})} &\doteq P_1^{(\boldsymbol{b})} \doteq P^{\beta}, & r_1^{(\boldsymbol{a})} &= r_1^{(\boldsymbol{b})} = 2N\beta. \end{aligned} \tag{10}$$

Then the received signal vectors take the form

$$\begin{aligned} \boldsymbol{y}_{1,t}^{(1)} &= \underbrace{\boldsymbol{H}_{1,t}\boldsymbol{c}_{1,t}}_{P} + \underbrace{\boldsymbol{H}_{1,t}\boldsymbol{a}_{1,t}}_{P^{\beta}} + \underbrace{\overbrace{\check{\boldsymbol{H}}_{1,t}\boldsymbol{b}_{1,t}}^{\check{\iota}_{1,t}^{(1)}}}_{P^{\beta}} + \underbrace{\overbrace{\ddot{\boldsymbol{H}}_{1,t}\boldsymbol{b}_{1,t}}^{\iota_{1,t}^{(1)} - \check{\iota}_{1,t}^{(1)}}}_{P^{0}} + \underbrace{\boldsymbol{z}_{1,t}^{(1)}}_{P^0}, \\ \boldsymbol{y}_{1,t}^{(2)} &= \underbrace{\boldsymbol{G}_{1,t}\boldsymbol{c}_{1,t}}_{P} + \underbrace{\boldsymbol{G}_{1,t}\boldsymbol{b}_{1,t}}_{P^{\beta}} + \underbrace{\overbrace{\check{\boldsymbol{G}}_{1,t}\boldsymbol{a}_{1,t}}^{\check{\iota}_{1,t}^{(2)}}}_{P^{\beta}} + \underbrace{\overbrace{\ddot{\boldsymbol{G}}_{1,t}\boldsymbol{a}_{1,t}}^{\iota_{1,t}^{(2)} - \check{\iota}_{1,t}^{(2)}}}_{P^{0}} + \underbrace{\boldsymbol{z}_{1,t}^{(2)}}_{P^0}, \end{aligned} \tag{11}$$

where under each term we noted the order of the summand's average power.

At this point, based on the received signal vectors in (11), each user decodes $\boldsymbol{c}_{1,t}$ by treating the other signals as noise, with $r_1^{(\boldsymbol{c})} = N(1-\beta)$. After decoding $\boldsymbol{c}_{1,t}$, user 1 removes $\boldsymbol{H}_{1,t}\boldsymbol{c}_{1,t}$ from $\boldsymbol{y}_{1,t}^{(1)}$, while user 2 removes $\boldsymbol{G}_{1,t}\boldsymbol{c}_{1,t}$ from $\boldsymbol{y}_{1,t}^{(2)}$. Then, at the end of the first phase, the transmitter uses

[3]The scheme description considers $0 < \beta < 1$ and rational β, while the rest of the cases ($\beta = 1, 0$, or where β is irrational) can be readily handled with minor modifications. To accommodate the choice of phase durations, S may be chosen to be large.

its knowledge of delayed CSIT to reconstruct $\{\check{\iota}^{(1)}_{1,t},\check{\iota}^{(2)}_{1,t}\}_{t=1}^{T_1}$, and quantizes them to $\bar{\iota}^{(1)}_{1,t},\bar{\iota}^{(2)}_{1,t}$ (cf. (7)), with each vector corresponding to $N\beta\log P$ bits, thus allowing for $\mathbb{E}||\tilde{\iota}^{(1)}_{1,t}||^2 \doteq \mathbb{E}||\tilde{\iota}^{(1)}_{1,t}||^2 \doteq 1$. At this point, the $2NT_1\beta\log P$ quantization bits are distributed evenly across the set $\{\boldsymbol{c}_{2,t}\}_{t=1}^{T_2}$ of newly constructed symbols which will be sequentially transmitted during the next (second) phase.

2) Phase s, $2\leq s\leq S-1$: Phase s ($T_s=T_{s-1}\frac{2\beta}{1-\beta}$ channel uses) is similar to phase 1, with similar transmit signals, rates, power values and received signals ((5),(9),(10),(11)).

After decoding $\boldsymbol{c}_{s,t}$ and respectively removing $\boldsymbol{H}_{s,t}\boldsymbol{c}_{s,t}$ and $\boldsymbol{G}_{s,t}\boldsymbol{c}_{s,t}$, the users reconstruct $\{\bar{\iota}^{(2)}_{s-1,t},\bar{\iota}^{(1)}_{s-1,t},\}_{t=1}^{T_{s-1}}$, allowing user 1 to subtract $\bar{\iota}^{(1)}_{s-1,t}$ from $\boldsymbol{y}^{(1)}_{s-1,t}$ to remove, up to bounded noise, the interference corresponding to $\check{\iota}^{(1)}_{s-1,t}$. The same user also employs the estimate $\bar{\iota}^{(2)}_{s-1,t}$ as an extra observation which, together with the observation $\boldsymbol{y}^{(1)}_{s-1,t}-\boldsymbol{H}_{s-1,t}\boldsymbol{c}_{s-1,t}-\bar{\iota}^{(1)}_{s-1,t}$, allow for decoding of $\boldsymbol{a}_{s-1,t}$. Specifically user 1 is presented, at this instance, with a $2N\times M$ equivalent MIMO channel of the form

$$\begin{bmatrix}\boldsymbol{y}^{(1)}_{s-1,t}-\boldsymbol{H}_{s-1,t}\boldsymbol{c}_{s-1,t}-\bar{\iota}^{(1)}_{s-1,t}\\ \bar{\iota}^{(2)}_{s-1,t}\end{bmatrix}=\begin{bmatrix}\boldsymbol{H}_{s-1,t}\\ \check{\boldsymbol{G}}_{s-1,t}\end{bmatrix}\boldsymbol{a}_{s-1,t}+\begin{bmatrix}\tilde{\boldsymbol{z}}^{(1)}_{s-1,t}\\ \tilde{\boldsymbol{z}}^{(2)}_{s-1,t}\end{bmatrix}$$

where $\tilde{\boldsymbol{z}}^{(1)}_{s-1,t}=\ddot{\boldsymbol{H}}_{s-1,t}\boldsymbol{b}_{s-1,t}+\boldsymbol{z}^{(1)}_{s-1,t}+\tilde{\iota}^{(1)}_{s-1,t}$ and $\tilde{\boldsymbol{z}}^{(2)}_{s-1,t}=(\ddot{\boldsymbol{G}}^{\mathsf{T}}_{s-1,t}-\ddot{\boldsymbol{G}}^{\mathsf{T}}_{s-1,t})\boldsymbol{a}_{s-1,t}-\tilde{\iota}^{(2)}_{s-1,t}$ are the equivalent noise, the powers of which are properly bounded, thus allowing for decoding of $\boldsymbol{a}_{s-1,t}$ with $r_1^{(\boldsymbol{a})}=2N\beta$, and doing so with imperfect global CSIR. Similar actions are performed by user 2 which manages to decode $\boldsymbol{b}_{s-1,t}$.

As before, upon completion of phase s, the transmitter reconstructs and quantizes $\{\check{\iota}^{(2)}_{s,t},\check{\iota}^{(1)}_{s,t}\}_{t=1}^{T_s}$, to a total of $2NT_s\beta\log P$ quantization bits which are converted into $\{\boldsymbol{c}_{s+1,t}\}_{t=1}^{T_{s+1}}$ to be sequentially transmitted in the next phase (phase $s+1$).

3) Phase S ($T_S=T_{S-1}2\beta$): Here we have $\boldsymbol{x}_{S,t}=\boldsymbol{c}_{S,t}$ with power and rate set as $P_S^{(\boldsymbol{c})}\doteq P$ and $r_S^{(\boldsymbol{c})}=N$. As before, both receivers can decode $\boldsymbol{c}_{S,t}$ with the mentioned rate.

Then after reconstructing $\{\bar{\iota}^{(2)}_{S-1,t},\bar{\iota}^{(1)}_{S-1,t}\}_{t=1}^{T_{S-1}}$, with the knowledge of imperfect global CSIR, the first user decodes $\boldsymbol{a}_{S-1,t}$ and the second user decoders $\boldsymbol{b}_{S-1,t}$.

Finally, one can show that, for $\beta'\triangleq\min\{\beta,\frac{N}{\min(M,2N)+N}\}$, $\mathcal{X}_1$ achieves DoF point $(2N\beta',N(1-\beta'))$ by allocating the common information of the first phase $\{\boldsymbol{c}_{1,t}\}_{t=1}^{T_1}$ entirely for user 2. The same scheme achieves the point $(N(1-\beta'),2N\beta')$ by assigning all the common information of phase 1 to user 1, as well as achieves DoF point $(\frac{N(1+\beta')}{2},\frac{N(1+\beta')}{2})$ by evenly splitting this information between the two users. The three DoF points converge to the optimal DoF corner point $(\frac{2N}{3},\frac{2N}{3})$ for any $\beta\geq\frac{N}{\min(M,2N)+N}$.

B. Scheme $\mathcal{X}_2$ for the MIMO BC with $N<M<2N$

For scheme $\mathcal{X}_2$, given $\phi=\frac{M\beta}{N(1-\beta)}$, the phase durations are integers such that

$$T_s=T_{s-1}\phi=T_1\phi^{s-1}, s=2,\cdots,S-1,\ T_S=T_{S-1}\frac{M\beta}{N}.$$

We proceed to describe the phases.

1) Phase 1: The transmitter sends

$$\boldsymbol{x}_{1,t}=\boldsymbol{c}_{1,t}+\boldsymbol{a}_{1,t}+\boldsymbol{b}_{1,t} \tag{12}$$

with power and rates

$$\begin{aligned} P_1^{(\boldsymbol{c})}&\doteq P, & r_1^{(\boldsymbol{c})}&=N(1-\beta)\\ P_1^{(\boldsymbol{a})}&\doteq P_1^{(\boldsymbol{b})}\doteq P^{\beta}, & r_1^{(\boldsymbol{a})}&=r_1^{(\boldsymbol{b})}=M\beta\end{aligned} \tag{13}$$

resulting in an output signal

$$\begin{aligned}\boldsymbol{y}^{(1)}_{1,t}&=\underbrace{\boldsymbol{H}_{1,t}\boldsymbol{c}_{1,t}}_{P}+\underbrace{\boldsymbol{H}_{1,t}\boldsymbol{a}_{1,t}}_{P^{\beta}}+\underbrace{\overbrace{\check{\boldsymbol{H}}_{1,t}\boldsymbol{b}_{1,t}}^{\check{\iota}^{(1)}_{1,t}}}_{P^{\beta}}+\underbrace{\overbrace{\ddot{\boldsymbol{H}}_{1,t}\boldsymbol{b}_{1,t}}^{\iota^{(1)}_{1,t}-\check{\iota}^{(1)}_{1,t}}}_{P^0}+\underbrace{\boldsymbol{z}^{(1)}_{1,t}}_{P^0},\\ \boldsymbol{y}^{(2)}_{1,t}&=\underbrace{\boldsymbol{G}_{1,t}\boldsymbol{c}_{1,t}}_{P}+\underbrace{\boldsymbol{G}_{1,t}\boldsymbol{b}_{1,t}}_{P^{\beta}}+\underbrace{\overbrace{\check{\boldsymbol{G}}_{1,t}\boldsymbol{a}_{1,t}}^{\check{\iota}^{(2)}_{1,t}}}_{P^{\beta}}+\underbrace{\overbrace{\ddot{\boldsymbol{G}}_{1,t}\boldsymbol{a}_{1,t}}^{\iota^{(2)}_{1,t}-\check{\iota}^{(2)}_{1,t}}}_{P^0}+\underbrace{\boldsymbol{z}^{(2)}_{1,t}}_{P^0}.\end{aligned} \tag{14}$$

Each user decodes $\boldsymbol{c}_{1,t}$, allowing for user 1 to remove $\boldsymbol{H}_{1,t}\boldsymbol{c}_{1,t}$ and for user 2 to remove $\boldsymbol{G}_{1,t}\boldsymbol{c}_{1,t}$. Then, at the end of the first phase, the transmitter reconstructs $\{\check{\iota}^{(1)}_{1,t},\check{\iota}^{(2)}_{1,t}\}_{t=1}^{T_1}$ (cf. (7)) and then proceeds to only quantize

$$\check{\iota}^{(1)}_{1,t}(1:M{-}N),\ \check{\iota}^{(2)}_{1,t}(1:M{-}N),\ \left(\check{\iota}^{(1)}_{1,t}+\check{\iota}^{(2)}_{1,t}\right)(M{-}N{+}1:N)$$

with $(M-N)\beta\log P$ bits, $(M-N)\beta\log P$ bits, and $(2N-M)\beta\log P$ bits respectively. This variable rate quantization allows for bounded quantization noise. At this point, the $MT_1\beta\log P$ bits representing

$$\begin{Bmatrix}\bar{\iota}^{(1)}_{1,t}(1:M-N),\bar{\iota}^{(2)}_{1,t}(1:M-N)\\ \left(\bar{\iota}^{(1)}_{1,t}+\bar{\iota}^{(2)}_{1,t}\right)(M-N+1:N)\end{Bmatrix}_{t=1}^{T_1} \tag{15}$$

are distributed evenly across the set $\{\boldsymbol{c}_{2,t}\}_{t=1}^{T_2}$ to be sent in the next phase.

2) Phase s, $2\leq s\leq S-1$: The transmitted and received signals, as well as the rates and power values are as in phase 1 ((12),(13),(14)). As before each user decodes $\boldsymbol{c}_{s,t}$ and $\boldsymbol{H}_{s,t}\boldsymbol{c}_{s,t},\boldsymbol{G}_{s,t}\boldsymbol{c}_{s,t}$ are removed. After reconstructing the quantized delayed estimates accumulated during the previous phase $s-1$ (cf. (15)), user 1 subtracts $\bar{\iota}^{(1)}_{s-1,t}(1:M-N)$ from $\boldsymbol{y}^{(1)}_{s-1,t}(1:M-N)$, removes up to bounded noise the interference corresponding to $\check{\iota}^{(1)}_{s-1,t}(1:M{-}N)$, and subtracts $(\bar{\iota}^{(1)}_{s-1,t}{+}\bar{\iota}^{(2)}_{s-1,t})(M{-}N{+}1:N)$ from $\boldsymbol{y}^{(1)}_{s-1,t}(M{-}N{+}1:N)$ to remove interference corresponding to $\check{\iota}^{(1)}_{s-1,t}(M-N+1:N)$. The same user also employs the estimate $\bar{\iota}^{(2)}_{s-1,t}(1:M-N)$ as an extra observation which, together with the observations $(\boldsymbol{y}^{(1)}_{s-1,t}{-}\boldsymbol{H}_{s-1,t}\boldsymbol{c}_{s-1,t}{-}\bar{\iota}^{(1)}_{s-1,t})(1:M{-}N)$ and $(\boldsymbol{y}^{(1)}_{s-1,t}-\boldsymbol{H}_{s-1,t}\boldsymbol{c}_{s-1,t}-\bar{\iota}^{(1)}_{s-1,t}-\bar{\iota}^{(2)}_{s-1,t})(M-N+1:N)$, allow for decoding of $\boldsymbol{a}_{s-1,t}$. Specifically user 1 is presented, at this instance, with an $M\times M$ equivalent MIMO channel of the form in (16) (where we ignore the time index $(s-1,t)$ for simplicity), where the power of equivalent noise term is properly bounded, thus allowing for successful decoding $\boldsymbol{a}_{s-1,t}$, doing so with imperfect global CSIR. Similar actions apply for the second user, which can decode $\boldsymbol{b}_{s-1,t}$.

$$
\begin{bmatrix} \left(\boldsymbol{y}^{(1)} - \boldsymbol{H}\boldsymbol{c} - \bar{\tilde{\iota}}^{(1)}\right)(1:M-N) \\ \left(\boldsymbol{y}^{(1)} - \boldsymbol{H}\boldsymbol{c} - \bar{\tilde{\iota}}^{(1)} - \bar{\tilde{\iota}}^{(2)}\right)(M-N+1:N) \\ \bar{\tilde{\iota}}^{(2)}(1:M-N) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}(1:M-N) \\ \left(\boldsymbol{H} - \check{\ddot{\boldsymbol{G}}}\right)(M-N+1:N) \\ \check{\ddot{\boldsymbol{G}}}(1:M-N) \end{bmatrix} \boldsymbol{a} + \begin{bmatrix} \tilde{\boldsymbol{z}}^{(1)}(1:M-N) \\ \tilde{\boldsymbol{z}}^{(1)}(M-N+1:N) \\ \left(-\tilde{\iota}^{(2)} + (\ddot{\ddot{\boldsymbol{G}}} - \ddot{\boldsymbol{G}})\boldsymbol{a}\right)(1:M-N) \end{bmatrix} \tag{16}
$$

$$
\text{where} \quad \tilde{\boldsymbol{z}}^{(1)}(1:M-N) = \left(\ddot{\boldsymbol{H}}\boldsymbol{b} + \boldsymbol{z}^{(1)} + \tilde{\iota}^{(1)}\right)(1:M-N) \tag{17}
$$

$$
\tilde{\boldsymbol{z}}^{(1)}(M-N+1:N) = \left(\ddot{\boldsymbol{H}}\boldsymbol{b} + \boldsymbol{z}^{(1)} + \tilde{\iota}^{(1)} + \tilde{\iota}^{(2)} + (\ddot{\ddot{\boldsymbol{G}}} - \ddot{\boldsymbol{G}})\boldsymbol{a}\right)(M-N+1:N). \tag{18}
$$

After being reconstructed at the transmitter, $\{\tilde{\iota}_{s,t}^{(2)}, \tilde{\iota}_{s,t}^{(1)}\}_{t=1}^{T_s}$ are quantized into

$$
\left\{ \begin{array}{c} \bar{\tilde{\iota}}_{s,t}^{(1)}(1:M-N), \bar{\tilde{\iota}}_{s,t}^{(2)}(1:M-N) \\ \left(\bar{\tilde{\iota}}_{s,t}^{(1)} + \bar{\tilde{\iota}}_{s,t}^{(2)}\right)(M-N+1:N) \end{array} \right\}_{t=1}^{T_s} \tag{19}
$$

corresponding to $MT_s\beta \log P$ quantization bits which are distributed evenly across the set $\{\boldsymbol{c}_{s+1,t}\}_{t=1}^{T_{s+1}}$, to be sent in the next phase.

3) Phase S: The transmitter sends $\boldsymbol{x}_{S,t} = \boldsymbol{c}_{S,t}$ with $P_S^{(\boldsymbol{c})} \doteq P$, $r_S^{(\boldsymbol{c})} = N$, resulting in received signals of the form

$$
\boldsymbol{y}_{S,t}^{(1)} = \boldsymbol{H}_{S,t}\boldsymbol{c}_{S,t} + \boldsymbol{z}_{S,t}^{(1)}, \; \boldsymbol{y}_{S,t}^{(2)} = \boldsymbol{G}_{S,t}\boldsymbol{c}_{S,t} + \boldsymbol{z}_{S,t}^{(2)}.
$$

As before, both receivers decode $\boldsymbol{c}_{S,t}$ and, with the knowledge of imperfect global CSIR, go back one phase to decode $\boldsymbol{a}_{S-1,t}$ at user 1, and $\boldsymbol{b}_{S-1,t}$ at user 2, all as described in the previous phase.

Finally, one can show that, for large S, and for $\beta' \triangleq \min\{\beta, \frac{N}{\min(M,2N)+N}\}$, $\mathcal{X}_2$ achieves DoF points $(M\beta', N - N\beta')$ by allocating the common information of the first phase $\{\boldsymbol{c}_{1,t}\}_{t=1}^{T_1}$ entirely for user 2. The same scheme achieves the point $(N - N\beta', M\beta')$ by assigning all the common information of phase 1 to user 1, as well as $(\frac{N+(M-N)\beta'}{2}, \frac{N+(M-N)\beta'}{2})$ by evenly splitting this information between the two users. The three DoF points converge to the optimal DoF corner point $(\frac{MN}{M+N}, \frac{MN}{M+N})$ for any $\beta \geq \frac{N}{\min(M,2N)+N}$, for the MIMO BC with $N < M < 2N$.

Remark 2: The specific quantization and transmission technique plays a key role in scheme $\mathcal{X}_2$. As in (15), instead of sending quantized $\bar{\tilde{\iota}}_{1,t}^{(1)}(M-N+1:N)$ and $\bar{\tilde{\iota}}_{1,t}^{(2)}(M-N+1:N)$ respectively, we choose to send quantized $\bar{\tilde{\iota}}_{1,t}^{(1)}(M-N+1:N) + \bar{\tilde{\iota}}_{1,t}^{(2)}(M-N+1:N)$, a choice which on the one hand reduces the number of quantization bits, and on the other hand allows the receivers to decode (cf. (16)).

IV. Conclusions

This work provided analysis and novel communication schemes for the setting of the two-user MIMO BC with imperfect delayed CSIT, as well as, in the presence of additional imperfections in the global CSIR. The derived DoF region is often optimal and, while corresponding to imperfect delayed CSIT and imperfect global CSIR, often matches the region previously associated to perfect delayed CSIT and perfect global CSIR. In addition to the theoretical limits and practical schemes, the work provided insight on how much delayed feedback is necessary to achieve a certain target performance, offering possible advantages in the presence of feedback links with limited capacity and limited reliability.

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