Slot Timing Maximum Likelihood Estimation with Bursty Pilot Signals for DS-CDMA in Multipath Fading Channels

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March 6, 2000

Contact author: Giuseppe Caire.
Designated Symposium: Communication Theory.
General Conference Topics: Communication Theory, Radio Communications.

ABSTRACT

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Abstract

This paper focuses on the initial slot timing acquisition in DS-CDMA with bursty pilot signals, when the propagation channel is affected by multipath and by fading. Subject to certain simplifying working assumptions, we obtain a nice form for the likelihood function and we derive the maximum likelihood estimator by solving a constrained maximization problem via the Lagrange-Kuhn-Tucker method. Our maximum likelihood slot timing estimator has linear complexity in the observation window length (i.e., constant complexity per observation sample). The relation to other estimation methods is addressed, and performance comparisons are provided by simulation. When our assumptions are not satisfied, the performance of the proposed estimator may deteriorate. Then, we provide an extremely simple method to cope with the model mismatch. The resulting estimator offers good results over all fading channels with delay spread not larger than a given maximum spread. Finally, the computation of the Cramer-Rao bound for slot timing estimation based on bursty pilot signals is addressed.

Keywords: Synchronization, CDMA systems, Fading channels.

1 Introduction and Motivation

In wireless mobile communication systems a mobile terminal (MT) must acquire the time reference of the base station (BS) before starting communication. When data transmission occurs with a slot structure the basic time reference is slot timing. In third generation wireless communication systems initial synchronization is facilitated by a pilot signal transmitted by the BS. In particular, all BSs transmit a common synchronization signal (with different time offsets). When the MT is switched on, it detects the presence and the timing of this signal. If the slot time reference of at least one BS is successfully acquired, the MT searches for some secondary synchronization signals carrying additional information (frame synchronization, BS identification, etc ...). Once the BS is identified, the MT can send a call request on the BS random access channel.

In both frequency-division duplex and time-division duplex modes of UMTS [1, 2], the synchronization signal is bursty, i.e., it is non-zero only for a small fraction of time. In particular, the primary synchronization signal consists of the same *bursty pilot* signal repeated indefinitely.

Motivated by this scheme, we consider the general problem of initial slot timing acquisition in a DS-CDMA system with a bursty pilot signal. We restrict our treatment to the case where there is exactly one transmitting BS. [1] Many algorithms for timing estimation have been derived in case of flat fading channels (e.g., see [3, 4, 5] and references therein). A simple approach to the case of multipath channels is to apply the same algorithms developed for flat fading, hoping that the estimator will lock to the timing of at least one of

[1] The more general problem where the number of transmitting BSs can be zero or larger than one will be addressed in [17].

the propagation paths. Other works derive timing estimators in the presence of multipath fading by making quite restrictive assumptions on the *a priori* knowledge of the channel statistics. For example, in [6] an ML timing estimator is derived by assuming that the channel multipath intensity profile (MIP) is known at the receiver, and in [7, 8, 5] the ML criterion is applied by modeling the channel as unknown but constant. We observe that in practice the multipath intensity profile is not known *before* initial acquisition, and that, since the initial synchronization phase may last several slots, the algorithms based on the constant channel assumption might perform poorly in the presence of time-varying fading.

In this paper, we obtain a low-complexity slot timing estimator requiring a minimum amount of prior knowledge and yielding good performances. The channel is modeled as a set of random Rayleigh fading discrete multipath components and the noise plus interference is modeled as a zero-mean white Gaussian process. Since both the channel multipath intensity profile and the noise plus interference power spectral density are not known at the receiver, we formulate a joint ML problem where all these parameters are estimated. Then, we derive the maximum likelihood estimator by solving a constrained maximization problem via Lagrange-Kuhn-Tucker (LKT) conditions. In order to obtain a tractable solution we make several simplifying working assumptions. When these are not satisfied, our estimator is not exactly ML and may suffer from mismatch. We address this problem and provide a heuristic modified estimator that copes with the model mismatch, and provides good results over all fading channels with delay spread not larger than a given maximum spread. The proposed estimator is compared to the overly optimistic algorithm of [6] and to the simple algorithm that selects the maximum of the squared magnitude of the matched filter output. The calculation of the Cramer-Rao bound (CRB) for the case of a bursty pilot signal is also addressed in detail.

2 Signal Model

The continuous-time baseband received signal in given by

$$y(t) = x(t,\, \theta) + v(t) \tag{1}$$

where $v(t)$ represents noise plus interference, modeled as a zero-mean complex circularly-symmetric Gaussian process with power spectral density I_0, and $x(t,\, \theta)$ is the received synchronization signal component, given by

$$x(t,\, \theta) = \sum_{m=0}^{M-1} \int h(t,\, t-\tau) s(\tau - \theta - mT) d\tau \tag{2}$$

where θ is the slot timing to be estimated, $s(t)$ is the bursty pilot waveform of duration T_s, $T \gg T_s$ is the period of repetition of $s(t)$, $h(t,\, \tau)$ is the time-varying multipath channel impulse response and M is the number of transmitted pilot bursts.

The channel is wide-sense stationary uncorrelated scattering (WSS-US [9]) Rayleigh fading with discrete multipath impulse response

$$h(t,\,\tau) = \sum_{p=0}^{P-1} c_p(t)\delta(\tau - \tau_p) \tag{3}$$

Since T_s is much shorter than T, we assume that the channel coherence time [9] satisfies $T_s < T_{\rm coh} \leq T$. This implies that the channel is almost constant during each m-th burst, but changes independently from burst to burst. [2] Therefore, $x(t,\theta)$ in (2) can be conveniently rewritten as

$$x(t,\theta) = \sum_{m=0}^{M-1}\sum_{p=0}^{P-1} c_{mp}s(t-\theta-\tau_p-mT) \tag{4}$$

where c_{mp} are complex zero-mean circularly-symmetric Gaussian mutually uncorrelated random variables such that $E\{c_{mp}c_{nq}^*\} = \sigma_p^2\delta_{m,n}\delta_{p,q}$. The channel MIP is defined by the delays $\boldsymbol{\tau} = (\tau_0,\ldots,\tau_{P-1})$ and by the variances $\boldsymbol{\sigma} = (\sigma_0^2,\ldots,\sigma_{P-1}^2)$. Notice that θ must be estimated modulo T, and that without loss of generality we can consider $\tau_0 = 0$.

The pilot waveform is given by

$$s(t) = \sum_{n=0}^{N-1} s_n\psi(t-nT_c) \tag{5}$$

where T_c is the chip period, s_n is a sequence of N chips known at the receiver and $\psi(t)$ is a root-raised-cosine [9] chip-shaping pulse with roll-off $\alpha \in [0,\,1]$ (e.g., $\alpha = 0.22$ in UMTS). In a digital receiver implementation the signal is low-pass filtered and sampled at a convenient rate $W > (1+\alpha)/(2T_c)$. Hence, baseband processing is performed in discrete time. We assume $W = n_c/T_c$, where $n_c > 1$ is the number of samples per chip.

Let $Q = WT$ denote the number of samples per pilot repetition period, and define the discrete-time observed signal

$$\boldsymbol{y} = (y[0],\ldots,y[MQ-1])^T$$

After a straightforward but tedious derivation, it is possible to write $\boldsymbol{y}$ in the compact form

$$\boldsymbol{y} = \boldsymbol{S}(\theta,\,\boldsymbol{\tau})\boldsymbol{c} + \boldsymbol{v} \tag{6}$$

where

1. $\boldsymbol{v} = (v[0]\,\ldots\,v[MQ-1])^T$ is the interference plus noise sampled vector.

[2] We assume that the delays τ_p are constant over the whole observation window of duration MT. This is satisfied in practice since the multipath delays vary at a rate much slower than the slot rate.

2. $\boldsymbol{c} = (c_{00}, \ldots, c_{0(P-1)}, c_{10}, \ldots, c_{1(P-1)}, \ldots, c_{(M-1)0}, \ldots, c_{(M-1)(P-1)})^T$ is the vector containing all MP channel path coefficients over the M periods.

3. $\boldsymbol{S}(\theta, \boldsymbol{\tau})$ is the $MQ \times MP$ matrix whose mp-th column, for $m = 0, \ldots, M-1$ and $p = 0, \ldots, P-1$, is given by

$$\boldsymbol{s}_{mp}(\theta) = (\underbrace{0, \ldots, 0}_{mQ}, s_p[0], \ldots, s_p[Q-1], \underbrace{0, \ldots, 0}_{(M-m-1)Q})^T \tag{7}$$

where

$$s_p[i] = \frac{1}{\sqrt{W}} \sum_{n=0}^{N-1} s_n \psi(i/W - \theta - \tau_p - nT_c)$$

Since the columns of $\boldsymbol{S}(\theta, \boldsymbol{\tau})$ are obtained by translating the same waveform, they have all the same square magnitude, equal to the energy E_s of the pilot waveform. Then, without loss of generality we can include the term E_s into the channel path variances σ_p^2, and consider $|\boldsymbol{s}_{mp}(\theta)|^2 = 1$ for all m, p and θ. Under our assumptions, $\boldsymbol{y}$ is a zero-mean complex circularly-symmetric Gaussian random vector with covariance matrix

$$\boldsymbol{R}_{\boldsymbol{yy}} = \boldsymbol{S}(\theta,\, \boldsymbol{\tau})\boldsymbol{\Lambda}_{\boldsymbol{cc}}\boldsymbol{S}^H(\theta,\, \boldsymbol{\tau}) + I_0\mathbf{I} \tag{8}$$

where $\boldsymbol{\Lambda}_{\boldsymbol{cc}}$ is the diagonal matrix

$$\boldsymbol{\Lambda}_{\boldsymbol{cc}} = \mathbf{I}_M \otimes \boldsymbol{\Lambda}_{\boldsymbol{\sigma}^2}$$

with $\boldsymbol{\Lambda}_{\boldsymbol{\sigma}^2} = \mathrm{diag}(\sigma_0^2,\, \ldots,\, \sigma_{P-1}^2)$ ($\otimes$ denotes Kronecker product and $\mathbf{I}_M$ denotes the $M \times M$ identity matrix).

3 Maximum Likelihood Problem Formulation

The log-likelihood function for the parameters $(\theta,\, \boldsymbol{\tau},\, \boldsymbol{\sigma},\, I_0)$ is immediately obtained as

$$\mathcal{L}(\boldsymbol{y}|\theta,\, \boldsymbol{\tau},\, \boldsymbol{\sigma},\, I_0) = -\log\det(\boldsymbol{R}_{\boldsymbol{yy}}) - \boldsymbol{y}^H \boldsymbol{R}_{\boldsymbol{yy}}^{-1} \boldsymbol{y} \tag{9}$$

In order to proceed further in the derivation we need to find an analytical expression for both the determinant and the inverse covariance matrix in (9). By applying the matrix inversion lemma [10] to (8) we get

$$\boldsymbol{R}_{\boldsymbol{yy}}^{-1} = \frac{1}{I_0}\left[\mathbf{I} - \frac{1}{I_0}\boldsymbol{S}\boldsymbol{\Lambda}_{\boldsymbol{cc}}^{1/2}\left(\mathbf{I} + \frac{1}{I_0}\boldsymbol{\Lambda}_{\boldsymbol{cc}}^{1/2}\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{\Lambda}_{\boldsymbol{cc}}^{1/2}\right)^{-1}\boldsymbol{\Lambda}_{\boldsymbol{cc}}^{1/2}\boldsymbol{S}^H\right] \tag{10}$$

where we write $\boldsymbol{S}$ instead of $\boldsymbol{S}(\theta, \boldsymbol{\tau})$ for the sake of notation simplicity. Now, we make the key working assumption that the delays τ_p are sufficiently far apart (say, more than

T_c) and that the chip sequence s_n has good acyclic autocorrelation properties so that the columns of $\boldsymbol{S}$ can be considered mutually orthogonal. This assumption is motivated by the fact that in practice the chip sequence s_n is quite long ($N = 256$ in UMTS) and that paths spaced by less than one chip interval are substantially treated as a single path (they are not *resolvable* [9]). Thus, (10) reduces to

$$\boldsymbol{R}_{\boldsymbol{yy}}^{-1} = \frac{1}{I_0}\left(\mathbf{I} - \boldsymbol{S}\boldsymbol{\Lambda}^{-1}\boldsymbol{S}^H\right) \tag{11}$$

where

$$\boldsymbol{\Lambda}^{-1} = \mathbf{I}_M \otimes \mathrm{diag}\left(\frac{\sigma_0^2}{\sigma_0^2 + I_0}, \dots, \frac{\sigma_{P-1}^2}{\sigma_{P-1}^2 + I_0}\right)$$

Subject to the assumption of orthonormal columns of $\boldsymbol{S}$ the determinant of $\boldsymbol{R}_{\boldsymbol{yy}}$ is readily obtained as

$$\det(\boldsymbol{R}_{\boldsymbol{yy}}) = \left[\prod_{p=0}^{P-1}(\sigma_p^2 + I_0)\right]^M I_0^{M(Q-P)} \tag{12}$$

By substituting (10) and (12) into (9) and by defining the average signal-to-interference plus noise ratio per path $\xi_p = \sigma_p^2/I_0$ and the total received energy $E_y = |\boldsymbol{y}|^2$, we obtain the log-likelihood function in the form

$$\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\tau}, \boldsymbol{\xi}, I_0) = \frac{1}{I_0}\left[\sum_{p=0}^{P-1}\frac{\xi_p}{\xi_p+1}\sum_{m=0}^{M-1}|\boldsymbol{s}_{mp}^H(\theta)\boldsymbol{y}|^2 - E_y\right] - MQ\log I_0 - M\sum_{p=0}^{P-1}\log(\xi_p+1) \tag{13}$$

where $\boldsymbol{\xi} = (\xi_0, \dots, \xi_{P-1})$.

In order to find the ML estimate of the slot timing θ, we should jointly estimate also the other unknown parameters $\boldsymbol{\tau}, \boldsymbol{\xi}$ and I_0. The lack of knowledge of the delays τ_p's represents the major hurdle in the evaluation of the expression (13). In order to obtain a low-complexity solution, we make the working assumption that the delays are equally spaced by the chip interval T_c, i.e., that $\tau_p = pT_c$ for $p = 0, \dots, \lceil T_d/T_c \rceil - 1$, where T_d is the channel delay spread.

Remark 1. Regarding this assumption, some comments are in order. First, we notice that modeling multipath fading channels as chip-spaced FIR filters with uncorrelated taps is fundamentally wrong, since chip-spacing and uncorrelated taps are contrasting issues. In fact, by low-pass filtering, sampling and truncating a WSS-US channel with arbitrary delays τ_p we obtain a FIR random filter with correlated taps. We hasten to say that ours is just a working assumption, in order to simplify the likelihood function. In general, the channel does not satisfy this condition and our estimator is mismatched (this point is discussed further in the following). Second, we observe that, implicitly, the number of paths is considered as known and equal to $\lceil T_d/T_c \rceil$. The maximum channel delay spread T_d is an a priori known parameter, since the system is designed to work up to this delay spread. However, depending on the propagation environment, the actual delay spread might be considerably less than T_d. In order to stress the fact that $\lceil T_d/T_c \rceil$ is not the true number of channel paths, but just the maximum number of resolvable paths, we denote this by L instead of P. This is a second source of mismatch for the resulting timing estimator. A robust estimator should provide good performances over all channels with delay spread *not larger* than T_d. $\diamond$

In light of the above assumption, the likelihood function (13) takes the form

$$\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0) = \frac{1}{I_0}\left[\sum_{l=0}^{L-1} \frac{\xi_l}{\xi_l+1} X_l(\theta) - E_y\right] - MQ\log I_0 - M\sum_{l=0}^{L-1}\log(\xi_l+1) \tag{14}$$

where we let

$$X_l(\theta) = \sum_{m=0}^{M-1} |\boldsymbol{s}_{ml}^H(\theta)\boldsymbol{y}|^2 \tag{15}$$

and where $\mathbf{s}_{ml}(\theta)$ is defined as $\mathbf{s}_{mp}(\theta)$ in (7) with $\tau_p = lT_c$.

Remark 2. The term $X_l(\theta)$ is obtained by summing for $m = 0, \dots, M-1$ the squared magnitude of the output of a discrete time filter with impulse response matched to the delayed pilot waveform $s(t-\theta-mT-lT_c)$. Then, $X_l(\theta)$ is the output of a sort of *square-law diversity combiner* [9] collecting the signal energy over the M pilot repetition periods, for a given guess of the timing θ and multipath component with delay lT_c. $\diamond$

4 Maximum Likelihood Estimator

In order to maximize the likelihood function (14) we find the maximum with respect to $\boldsymbol{\xi}$ and I_0 for all possible θ, and then we select the overall maximum with respect to θ.

For fixed θ, the constrained maximization problem

$$\left\{\begin{array}{ll} \text{maximize} & \mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0) \\ \text{subject to} & \boldsymbol{\xi} \geq \mathbf{0}, \quad I_0 \geq 0 \end{array}\right. \tag{16}$$

can be solved by using the LKT conditions [11]. Necessary condition for $(\boldsymbol{\xi}, I_0)$ be a (local) constrained maximum point of $\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0)$ is that

$$\begin{aligned} \frac{\partial}{\partial \xi_i}\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0) &\leq 0 \qquad i = 0, \dots, L-1 \\ \frac{\partial}{\partial I_0}\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0) &\leq 0 \end{aligned} \tag{17}$$

where the inequality for ξ_i (resp. for I_0) holds with equality if $\xi_i > 0$ (resp. if $I_0 > 0$). The solution of the system of equations (17) yields

$$\xi_l = \left[\frac{1}{\beta}X_l - 1\right]_+, \qquad I_0 = \beta/M \tag{18}$$

where β is defined by

$$\beta = \frac{E_y - \sum_{l\in\mathcal{D}} X_l}{Q - |\mathcal{D}|} \tag{19}$$

and where we define the set of indexes $\mathcal{D} = \{l \in [0, L-1] : X_l > \beta\}$. In the above equations, $[\cdot]_+$ denotes positive part, $|\mathcal{D}|$ denotes the cardinality of the set $\mathcal{D}$ and we used X_l instead of $X_l(\theta)$ for the sake of notation simplicity.

Remark 3. The solution given by (18) and (19) has the following nice interpretation: β acts as an adaptive threshold level. As explained before, X_l represents the signal energy corresponding to the l-th delay, for a given tentative timing θ. If $X_l > \beta$ then the l-th delay is a good candidate for being a true channel path. $\Diamond$

Next, we have to prove that (18) is actually the global maximizer of $\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0)$. We proceed by first showing that (18) exists and its is the unique solution of (17). Then, by inspection of the likelihood function, we prove that it must be the unique (and therefore global) maximizer. Interestingly, the proof of existence provides also an efficient method for the maximum computation.

Proposition 1. The solution given by (18) and (19) to the system of inequalities (17) exists and is unique. $\Box$

Proof: existence. Consider the permutation π of the integers $\{0, \ldots, L-1\}$ sorting the X_l's in non-increasing order, so that

$$X_{\pi(0)} \geq X_{\pi(1)} \geq \cdots \geq X_{\pi(L-1)}$$

and define the sets

$$\mathcal{D}_d = \left\{i \in [0, L-1] : X_{\pi(i)} > \beta_d\right\}$$

where

$$\beta_d = \frac{E_y - \sum_{i=0}^{d-1} X_{\pi(i)}}{Q - d} \tag{20}$$

The solution (18) exists if the equation $|\mathcal{D}_d| = d$ holds for some $d \in [0, L]$.

If $X_{\pi(0)} \leq \beta_0$ then $|\mathcal{D}_0| = 0$ and the solution exists. Now, suppose that $X_{\pi(0)} > \beta_0$ and that there exists $1 \leq d \leq L-1$ such that $X_{\pi(d-1)} > \beta_{d-1}$ and $X_{\pi(d)} \leq \beta_d$. Then, we have

$$\beta_d \geq X_{\pi(d)} \geq X_{\pi(d+1)} \geq \cdots \geq X_{\pi(L-1)}$$

which yields $|\mathcal{D}_d| \leq d$. In order to establish equality, we need to show that $X_{\pi(i)} > \beta_d$ for all for all $0 \leq i \leq d-1$. This holds if $X_{\pi(d-1)} > \beta_d$. Hence, d is a solution if we can prove the implication

$$X_{\pi(d-1)} > \beta_{d-1} \Rightarrow X_{\pi(d-1)} > \beta_d \tag{21}$$

By using the definition (20) we can write

$$\begin{aligned} X_{\pi(d-1)} &> \beta_{d-1} \\ &= \frac{E_y - \sum_{i=0}^{d-2} X_{\pi(i)}}{Q-d+1} \\ &= \frac{Q-d}{Q-d}\left(\frac{E_y - \sum_{i=0}^{d-2} X_{\pi(i)}}{Q-d+1} + \frac{X_{\pi(d-1)}}{Q-d+1} - \frac{X_{\pi(d-1)}}{Q-d+1}\right) \\ &= \frac{Q-d}{Q-d+1}\beta_d + \frac{X_{\pi(d-1)}}{Q-d+1} \end{aligned} \tag{22}$$

so that

$$X_{\pi(d-1)}\left(1 - \frac{1}{Q-d+1}\right) > \frac{Q-d}{Q-d+1}\beta_d \tag{23}$$

which proves (21). We conclude that for any $1 \leq d \leq L-1$ such that $X_{\pi(d-1)} > \beta_{d-1}$ and $X_{\pi(d)} \leq \beta_d$, the equation $|\mathcal{D}_d| = d$ is verified. The last case to examine is when $X_{\pi(d)} > \beta_d$ for all $0 \leq d \leq L-1$. Then, by using again the implication (21), we have that $X_{\pi(L-1)} > \beta_L$ so that $|\mathcal{D}_L| = L$. Thus, we have shown the existence of a solution $|\mathcal{D}_d| = d$ for some $0 \leq d \leq L$. The corresponding solution (18) of (17) obviously exists and is obtained by letting $\mathcal{D} = \{\pi(i) : i = 0, \dots, d-1\}$ and $\beta = \beta_d$.

Uniqueness. Suppose that there exist d and d', with $d' > d$, such that $|\mathcal{D}_d| = d$ and $|\mathcal{D}_{d'}| = d'$, i.e.,

$$\left\{ \begin{array}{ll} X_{\pi(i)} > \beta_d & \text{for } 0 \leq i \leq d-1 \\ X_{\pi(i)} \leq \beta_d & \text{for } d \leq i \leq L-1 \end{array} \right. \tag{24}$$

and

$$\left\{ \begin{array}{ll} X_{\pi(i)} > \beta_{d'} & \text{for } 0 \leq i \leq d'-1 \\ X_{\pi(i)} \leq \beta_{d'} & \text{for } d' \leq i \leq L-1 \end{array} \right. \tag{25}$$

By following the same steps of (22-23) we can show the converse implication of (21), namely

$$\beta_d > X_{\pi(d)} \Rightarrow \beta_{d+1} > X_{\pi(d)} \tag{26}$$

Then, starting from (24) we can write the chain of inequalities $\beta_d \geq X_{\pi(d)} \Rightarrow \beta_{d+1} \geq X_{\pi(d)} \geq X_{\pi(d+1)} \Rightarrow \beta_{d+2} \geq X_{\pi(d+1)} \geq X_{\pi(d+2)} \Rightarrow \cdots \Rightarrow \beta_{d'} \geq X_{\pi(d'-1)}$, which contradicts (25). We conclude that the solution must be unique. $\square$

Proposition 2. The solution given by (18) and (19) to the system of inequalities (17) is the global maximizer of $\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0)$ with respect to $\boldsymbol{\xi}$ and I_0 for θ given. □

Proof. Let $\mathbf{z} = (\boldsymbol{\xi}, I_0)$ and define the function $f(\mathbf{z}) = \mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0)$ (seen as a function of $\boldsymbol{\xi}$ and I_0 only). The function $f(\mathbf{z})$ is not concave. However, it is continuous in the constrain set $\mathbf{z} \geq \mathbf{0}$ and $f(\mathbf{z}) \to -\infty$ if any of its variables grows without bound. By continuity, the global maximum of $f(\mathbf{z})$ is finite and the maximizer of $f(\mathbf{z})$ must have all finite components, therefore it must satisfy the necessary LKT conditions. Finally, the existence and uniqueness of a solution for the LKT conditions (17) proved in Proposition 1 implies that this is the global maximizer. □

4.1 ML timing estimator implementation

The proof of Proposition 1 provides also a method to compute

$$\bar{\mathcal{L}}(\mathbf{y}|\theta) = \max_{\boldsymbol{\xi}, I_0} \mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0) \tag{27}$$

for any given θ. Namely, this is given by the following algorithm:

1. Sort $X_1, \dots, X_{L-1}$ in non-increasing order (let π denote the sorting permutation).
2. If $\beta_L < X_{\pi(L)}$ then let $d = L$, otherwise let $d = \min\{0 \leq i \leq L-1 : \beta_i \geq X_{\pi(i)}\}$.
3. Let $I_0 = \beta_d / M$, and $\xi_{\pi(i)} = X_{\pi(i)}/\beta_d - 1$, for $i = 0, \dots, d-1$ and $\xi_{\pi(i)} = 0$ for $i = d, \dots, L-1$.
4. Substitute $\boldsymbol{\xi}$ and I_0 found in $\mathcal{L}(\boldsymbol{y}|\theta, \boldsymbol{\xi}, I_0)$ given in (14).

If only the slot timing estimate is needed, explicit computation of the ML estimates of $\boldsymbol{\xi}$ and I_0 can be avoided. By substituting directly the solution for $\boldsymbol{\xi}$ and I_0 into (14) and by dropping the terms which do not depend on θ, the ML estimate of θ is obtained by

$$\widehat{\theta} = \arg \max_{\theta \in [0,T]} \widetilde{\mathcal{L}}(\mathbf{y}|\theta) \tag{28}$$

where

$$\widetilde{\mathcal{L}}(\mathbf{y}|\theta) = -(Q-d)\log \beta_d - \sum_{i=0}^{d-1} \log X_{\pi(i)} \tag{29}$$

(the dependence of the RHS of (29) on θ is contained in the permutation π and in d, β_d and $X_{\pi(i)}$).

As far as implementation is concerned, some consideration are in order.

- Up to now, we have considered the parameter θ as continuous. However, in a low-complexity digital implementation, the search for the maximum of $\widetilde{\mathcal{L}}(\mathbf{y}|\theta)$ is done on a discrete set of values. The most computationally demanding operation is the computation of the matched filter output $z_{ml}(\theta) = \boldsymbol{s}_{ml}^H(\theta)\boldsymbol{y}$ involved in the calculation of X_l (see (15)). Ideally, the estimator must be equipped with a bank of matched filters, one for each value of θ, whose outputs are sampled with delay $mT + lT_c$, for $m = 0, \dots, M-1$ and $l = 0, \dots, L-1$. Since the signal is sampled at rate $W = n_c/T_c$, by discretizing θ with step $\Delta = 1/(n_s W)$ we can implement efficiently the bank of matched filters as a polyphase filter with n_s phases. However, in most practical applications it is sufficient to acquire the slot timing with an error of less than one chip. Therefore, a very fine discretization of θ is not needed. In our numerical examples, we discretize θ with step $1/W$, so that we need just a single matched filter operating at the signal sampling rate W, whose output is given by

$$z[i] = \sum_j y[j]s((j-i)/W)^*$$

By definition, we have $z_{ml}(k/W) = z[mQ + ln_c + k]$. The receiver accumulates the squared matched filter outputs in a vector buffer $\mathbf{b} = (b_0, \dots, b_{Q-1})$ such that

$$b_i = \sum_{m=0}^{M-1} |z[mQ + i]|^2$$

Finally, the search for the maximum in (28) is performed over the discrete values $\theta_k = k/W$, for $k = 0, \dots, Q-1$ by processing the data buffer $\mathbf{b}$ (notice that, by definition, $X_l(\theta_k) = b_{ln_c+k}$).

- Up to now, we have implicitly assumed that the pilot bursts fall approximately in the middle of the observation intervals $[mT, (m+1)T]$. However, the initial timing reference of the MT is arbitrary, and it may happen that the pilot bursts fall across the boundaries of the observation intervals. Since the slot timing is defined modulo T, in order to solve this problem it is sufficient to apply the ML algorithm (28) by treating $\mathbf{b}$ as a *circular buffer*.

- The complexity of the proposed algorithm is linear in the observation size QM, as opposed to other timing algorithms based on least-squares or subspace decomposition, which require matrix-vector multiplication or matrix eigen-analysis.

5 MSE estimator properties

The CRB is a lower bound to the variance of any unbiased estimator [12, 13]. Since we have no guarantee that our estimator is unbiased (certainly it is not in its quantized version), comparing the mean-square error (MSE) $\epsilon^2 = E[|\widehat{\theta} - \theta|^2]$ with the CRB for θ is questionable. However, since the estimator works on a sequence of i.i.d. observations (namely, the subvectors of $\mathbf{y}$ received during the interval $[mT, (m+1)T]$, for $m = 0, \ldots, M-1$), from the general properties of ML estimation we have that the unquantized version of $\widehat{\theta}$ is asymptotically unbiased and approaches the CRB as $M \to \infty$. The variance of the quantized estimator is lower bounded by the quantization error variance, given by $\epsilon_q^2 = \frac{1}{12}W^{-2}$, where we assume a uniform distribution for θ over $[-1/(2W), 1/(2W)]$.

We model the effect of quantization as an independent additive error, uniformly distributed over $[-1/(2W), 1/(2W)]$, added to the unquantized ML estimator. Subject to this assumption, we can lower bound the quantized estimator MSE for large M by

$$\epsilon^2 \geq \mathrm{CRB} + \epsilon_q^2 \tag{30}$$

where CRB denotes the Cramer-Rao bound calculated for $\theta = \theta_k$ (for an arbitrary $k \in [0, Q-1]$). In Appendix A, we give the details of the computation of the CRB in the case of bursty pilot. Obviously, the CRB decreases as the inverse of M (this is also implied by the fact of having M i.i.d. observations). Therefore, we expect that the performance of our estimator for sufficiently large M is dominated by the quantization error. As we shall show in our simulation, this is not the case for other slot synchronizers. For example, a slot synchronizer that selects the maximum of the accumulated data buffer $\mathbf{b}$ incurs in very large synchronization errors for channels with some strong and well separated paths.

6 Results

In our examples, the pilot sequence has length $N = 256$ and is defined by the UMTS norm [14], and the chip-shaping pulse is root-raised cosine with roll-off $\alpha = 0.22$. The receiver sampling rate is $W = 4/T_c$ and the pilot repetition interval has length $Q \approx 2500$ samples (in reality Q may be much larger but we were limited by the simulation time). As performance measure of timing estimators we use the root-MSE (RMSE). This is normalized with respect to the chip interval (i.e., it is expressed in fraction of T_c).

We considered the three channels CH1 with MIP given by $\boldsymbol{\tau}_1 = (0, 0.96, 1.92, 2.88)$, $\boldsymbol{\sigma}_1 = (0, -2.3, -6.5, -9.6)$, CH2 with MIP given by $\boldsymbol{\tau}_2 = (0, 0.9, 2.8, 4.7)$, $\boldsymbol{\sigma}_2 = (0, -2, -7, -8.5)$ and CH3 with MIP given by $\boldsymbol{\tau}_3 = (0, 4, 20)$, $\boldsymbol{\sigma}_3 = (0, 0, 0)$. The channels CH1 and CH2 are given in [15] while CH3 is given in [16], the delays are normalized with respect to T_c and variances are expressed in dB.

Figs. 1 and 2 show the timing RMSE vs. the pilot energy to interference plus noise ratio E_s/I_0, for $M = 5$ and $M = 20$ observation intervals. The CRB plus quantization limit given in (30) is shown for reference. The proposed estimator is denoted by "JML" (joint-ML). For the sake of comparison, we considered also the estimator of [6] which assumes perfect knowledge of the channel MIP (denoted by "ML with known MIP") and the simple peak detector (denoted by "Max"), consisting of selecting the maximum of the accumulated data buffer $\mathbf{b}$, i.e.,

$$\widehat{\theta} = \frac{1}{W} \arg \max_{0 \leq i \leq Q-1} b_i$$

Interestingly, the unquantized version of this estimator is the ML estimator in the case of flat Rayleigh fading independent from burst to burst. Fig. 3 shows the RMSE vs. M for $E_s/I_0 = 12$ dB, for CH1.

Figs. 4, 5 and 6 show analogous results for CH2, and Fig. 7 shows the RMSE vs. M for $E_s/I_0 = 12$ dB, for CH3.

CH3 is a fairly long channel with equally strong paths. In this case, the Max detector performs poorly since it selects any path with nearly uniform probability. CH1 and CH2 have delays non-integer multiples of T_c. Then, the JML estimator is mismatched. In this case, the Max estimator may outperform the JML estimator (as shown by the results for CH2).

In order to cope with mismatch, we propose a simple modification of our estimator as follows. Let $\mathbf{a}$ be the vector of Q values of the likelihood function $\widetilde{\mathcal{L}}(\mathbf{y}|\theta)$ defined in (29), evaluated for $\theta = \theta_k = k/W$, for $k = 0, \ldots, Q-1$. Then, we can combine the Max estimator and the JML by forming a linear combination of the vectors $\mathbf{a}$ and $\mathbf{b}$ as

$$\mathbf{c} = \kappa_1 \mathbf{a} + \kappa_2 \mathbf{b}$$

where $\kappa_1 = 1/(\max \mathbf{a} - \min \mathbf{a})$ and $\kappa_2 = 1/(\max \mathbf{b} - \min \mathbf{b})$. Finally, the estimated slot timing is given by

$$\widehat{\theta} = \frac{1}{W} \arg \max_{0 \leq i \leq Q-1} c_i$$

The rationale behind this choice is provided by Fig. 8, where snapshots of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are shown (on a normalized scale) for the case of a channel with two separated paths with equal average strenght with delay spread less than the maximum T_d (used by the JML algorithm). The upper figure shows $\mathbf{a}$ vs. the number of samples. A delay spread shorter than T_d causes the likelihood function to have a flat top, whose width is approximately given by the difference between T_d and the actual channel delay spread. Because of the noise, the maximum of $\mathbf{a}$ can be located anywhere on the flat top, with roughly uniform probability. This is essentially why the mismatch deteriorates the performance of JML. The middle figure shows $\mathbf{b}$ vs. the number of samples. Here, as explained before, the two large peaks are selected by the Max estimator with roughly uniform probability, yielding a large timing RMSE. The bottom figure shows $\mathbf{c}$ vs. the number of samples. We notice that the effect of the linear combination of $\mathbf{a}$ and $\mathbf{b}$ is to create a large peak on the flat top of $\mathbf{a}$, and at the same time introduce more margin between the two peaks. In this way, the estimator based on the maximization of $\mathbf{c}$ provides good results for all channels with delay spread up to T_d, and it is robust with respect to the model mismatch.

In Figures from 1 to 6, the RMSE achieved by the modified estimator are denoted by "JML+Max". We observe that this estimator performs well in all cases, and it is often close to the ML with known MIP.

7 Conclusions

Motivated by the initial BS acquisition procedure of UMTS, we considered the problem of slot timing estimation based on bursty pilot signals. Subject to some simplifying working assumptions, we derived a low-complexity algorithm based on joint ML estimation of the slot timing, of the multipath intensity profile and of the interference plus noise power spectral density and we solved the likelihood function maximization by using the Lagrange-Kuhn-Tucker conditions. We derived also the CRB for the problem at hand, and we solved some problems related to its numerical computation with very large data size (typical of wideband CDMA, with high sampling rate and long observation intervals). Fortunately, the pilot signal burstyness provides the solution. We tested the proposed algorithm in a UMTS environment, by using propagation channels and the synchronization chip sequence defined in the UMTS norm. Comparison with other algorithms such as the simple peak detection and an ideal algorithm that exploits perfect knowledge of the channel multipath intersity profile are provided. The assumptions of perfect knowledge of the channel delay spread and of equally chip-spaced multipath components, that are essential in the derivation of our estimator, are not in general satisfied in reality. Then, the proposed estimator suffers from mismatch, which can degrades its performance. For this reason, we proposed a simple low-complexity heuristic modification that actually makes the estimator robust to the model mismatch. In conclusions, the proposed modified estimator is an attractive solution for the initial synchronization of UMTS-like system, because of its low complexity, no need for a priori information, and robustness.

APPENDIX

A On the computation of the Cramer-Rao Bound

In this appendix we derive the CRB for the joint ML estimation of the parameters $\boldsymbol{\alpha} = (\theta, \boldsymbol{\sigma}, \boldsymbol{\tau}, I_0)$ defining the signal model (6).

We shall limit the evaluation of the CRB to the case of $M = 1$. Since the subvectors of $\mathbf{y}$ received during the interval $[mT, (m+1)T]$, for $m = 0, \ldots, M-1$, are i.i.d., the CRB for larger M is just $1/M$ times the CRB computed for $M = 1$. From the Gaussianity of the observation, the (i, j) elements of the Fisher information matrix (FIM) $\mathbf{J}$ are given by [13]

$$[\mathbf{J}]_{i,j} = \operatorname{tr}\left(\boldsymbol{R}_{\boldsymbol{yy}}^{-1}\left(\frac{\partial \boldsymbol{R}_{\boldsymbol{yy}}}{\partial \alpha_i}\right)\boldsymbol{R}_{\boldsymbol{yy}}^{-1}\left(\frac{\partial \boldsymbol{R}_{\boldsymbol{yy}}}{\partial \alpha_j}\right)\right) \tag{31}$$

where α_i denotes the ith element of the parameter vector $\boldsymbol{\alpha}$. For the case of $M = 1$ we define $\boldsymbol{s}_p(\theta) = \boldsymbol{s}_{0p}(\theta)$ and we get

$$\boldsymbol{R}_{\boldsymbol{yy}} = \sum_{p=0}^{P-1} \sigma_p^2 \boldsymbol{s}_p(\theta)\boldsymbol{s}_p^H(\theta) + I_0\mathbf{I}$$

The derivatives of $\boldsymbol{R_{yy}}$ can be expressed as follows

$$\begin{aligned}
\frac{\partial \boldsymbol{R_{yy}}}{\partial \theta} &= \sum_{p=0}^{P-1} \sigma_p^2 \left[\boldsymbol{s}_p(\theta) \left(\frac{\partial \boldsymbol{s}_p(\theta)}{\partial \theta} \right)^H + \left(\frac{\partial \boldsymbol{s}_p(\theta)}{\partial \theta} \right) \boldsymbol{s}_p^H(\theta) \right] \\
\frac{\partial \boldsymbol{R_{yy}}}{\partial (\sigma_p^2)} &= \boldsymbol{s}_p(\theta) \boldsymbol{s}_p^H(\theta) \\
\frac{\partial \boldsymbol{R_{yy}}}{\partial \tau_p} &= \sigma_p^2 \left[\boldsymbol{s}_p(\theta) \left(\frac{\partial \boldsymbol{s}_p(\theta)}{\partial \tau_p} \right)^H + \left(\frac{\partial \boldsymbol{s}_p(\theta)}{\partial \tau_p} \right) \boldsymbol{s}_p^H(\theta) \right] \\
\frac{\partial \boldsymbol{R_{yy}}}{\partial I_0} &= \mathbf{I}
\end{aligned} \tag{32}$$

In order to compute $\boldsymbol{s}_p'(\theta) = \frac{\partial \boldsymbol{s}_p(\theta)}{\partial \theta}$ we write

$$\frac{\partial s(i/W - \theta - \tau_p)}{\partial \theta} = -\sum_{n=0}^{N-1} s_n \psi'(i/W - \theta - \tau_p)$$

where $\psi'(t) = \frac{d\psi(t)}{dt}$. The partial derivatives $\frac{\partial \boldsymbol{s}_p(\theta)}{\partial \tau_p}$ are also given by the above expression.

The CRB for θ and arbitrary M is obtained as

$$\text{CRB}_\theta = \frac{1}{M} \left[\mathbf{J}^{-1} \right]_{1,1} \tag{33}$$

If some elements of $\boldsymbol{\alpha}$ are known, the CRB is computed from the inverse of the reduced FIM obtained by removing the rows and the columns of $\mathbf{J}$ corresponding to the known parameters. For example, the CRB frothe setting of [6], which assumes $\boldsymbol{\sigma}, \boldsymbol{\tau}$ and I_0 known, is obtained simply by the inverse of the $(1,1)$-th element of $\mathbf{J}$.

A.1 Numerical evaluation

In order to compute the elements of the FIM according to (31) the inverse of $\boldsymbol{R_{yy}}$ is required. Unfortunately, even by limiting the observation to a single pilot repetition interval, the $\boldsymbol{R_{yy}}$ is $Q \times Q$ with Q very large. In practical UMTS (see e.g. [14]) Q is of the order of $2560 n_c$. With $n_c = 4$ samples per chip, this yields $Q = 10240$. The computational complexity of calculating $\boldsymbol{R_{yy}^{-1}}$ can easily become overwhelming and the result might be numerically ill-conditioned.

We can overcome the above problem by exploiting the special structure of $\mathbf{R}_{yy}$ due to the fact that the pilot is bursty. In the following, we derive closed form expressions for the elements of the FIM, which are much less computationally intensive and much better numerically conditioned.

The covariance matrix of $\mathbf{y}$ has the block diagonal structure $\mathbf{R}_{yy} = \text{diag}(\boldsymbol{\Lambda}_1, \mathbf{R}, \boldsymbol{\Lambda}_2)$, where $\boldsymbol{\Lambda}_1 = I_0 \mathbf{I}_{L_1}$ with $L_1 = \lfloor \theta / W \rfloor$, $\boldsymbol{R}$ is a full matrix with dimensions $L_R \times L_R$

and $\mathbf{\Lambda}_2 = I_0 \mathbf{I}_{L_1}$ with $L_2 = Q - L_1 - L_R$. The size of the middle block $\mathbf{R}$ is given by the length (in samples) of the convolution of the pilot waveform with the multipath channel, and since the pilot duration is much shorter than T, it satisfies $L_R \ll Q$. Since $\mathbf{R}_{yy}^{-1} = \text{diag}(\mathbf{\Lambda}_1^{-1}, \mathbf{R}^{-1}, \mathbf{\Lambda}_2^{-1})$, only the inverse of the smaller matrix $\mathbf{R}$ is needed.

Next, by using the trace relation $\text{tr}(\boldsymbol{AB}) = \text{tr}(\boldsymbol{BA})$, with $\boldsymbol{A}$ and $\boldsymbol{B}$ of proper dimensions, and the derivative expressions (32) we obtain explicit expressions for the elements of the FIM. Define $\tilde{\boldsymbol{s}}_p$ and $\tilde{\boldsymbol{s}}'_p$ as the non-identically zero subvectors of length L_R of $\mathbf{s}_p(\theta)$ and of $\frac{\partial \mathbf{s}_p(\theta)}{\partial \theta}$. Then the elements of the FIM are given by

$$\begin{aligned}
\mathfrak{J}_{\theta\theta} &= \sum_{p=0}^{P-1}\sum_{q=0}^{P-1} \sigma_p^2\sigma_q^2[(\tilde{\boldsymbol{s}}'_q)^H \boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q + \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q + \\
&\quad (\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q + \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q] \\
\mathfrak{J}_{\sigma_p^2\sigma_q^2} &= \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q \\
\mathfrak{J}_{\tau_p\tau_q} &= \sigma_p^2\sigma_q^2[(\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q + \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q + \\
&\quad (\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q + \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q] \\
\mathfrak{J}_{\theta\sigma_p^2} &= \sum_{q=0}^{P-1}\sigma_q^2[\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q(\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p + \tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q\tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p] \\
\mathfrak{J}_{\theta\tau_p} &= \sigma_p^2\sum_{q=0}^{P-1}\sigma_q^2[(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q(\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p + (\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q\tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p + \\
&\quad (\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q(\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p + \tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q\tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p] \\
\mathfrak{J}_{\theta I_0} &= 2\sum_{p=0}^{P-1}\sigma_p^2\mathbf{Re}\{(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p\} \\
\mathfrak{J}_{\sigma_p^2\tau_q} &= \sigma_q^2[(\tilde{\boldsymbol{s}}'_q)^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_q + \tilde{\boldsymbol{s}}_q^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p\tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_q] \\
\mathfrak{J}_{\sigma_p^2 I_0} &= \tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p \\
\mathfrak{J}_{\tau_p I_0} &= \sigma_p^2[(\tilde{\boldsymbol{s}}'_p)^H\boldsymbol{R}^{-1}\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_p + \tilde{\boldsymbol{s}}_p^H\boldsymbol{R}^{-1}\boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}'_p] \\
\mathfrak{J}_{I_0 I_0} &= \text{tr}(\boldsymbol{R}^{-1}\boldsymbol{R}^{-1}) + (Q - L_R)\frac{1}{I_0^2}
\end{aligned}$$

Despite of the apparent complications, the expressions above allow the computation of the FIM by mean of a few inner products and the inversion of a $L_R \times L_R$ matrix. Finally, note that the computational complexity does not increase with the slot length Q.

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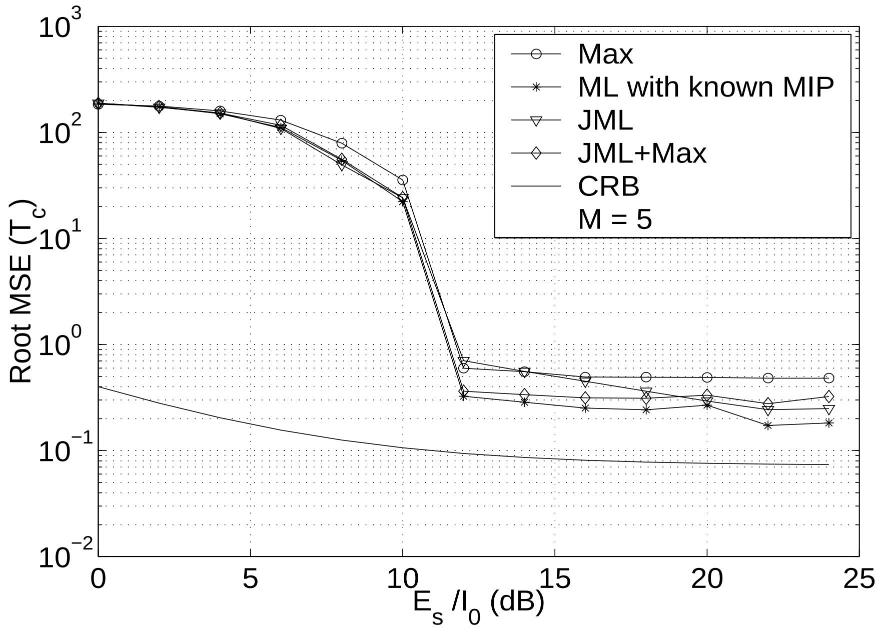

Figure 1: Normalized RMSE vs. E_s/I_0 for CH1, with $M = 5$ observation intervals.

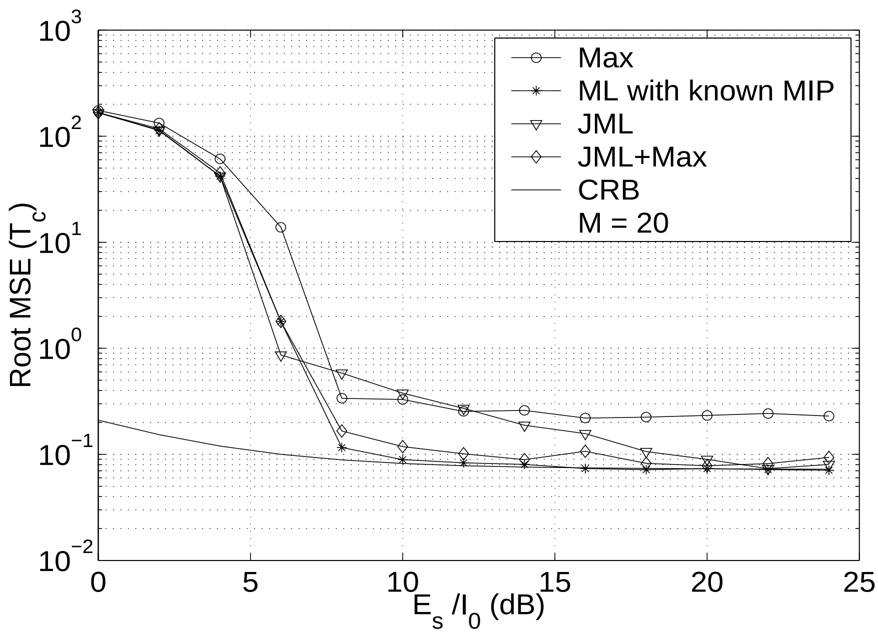

Figure 2: Normalized RMSE vs. E_s/I_0 for CH1, with $M = 20$ observation intervals.

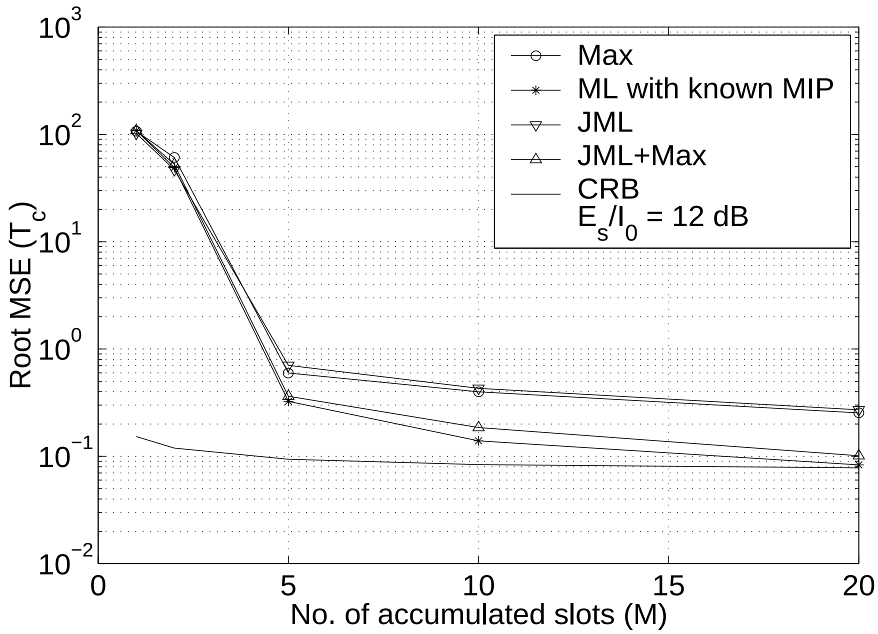

Figure 3: Normalized RMSE vs. number of observation intervals M for CH1, with E_s/I_0 = 12 dB

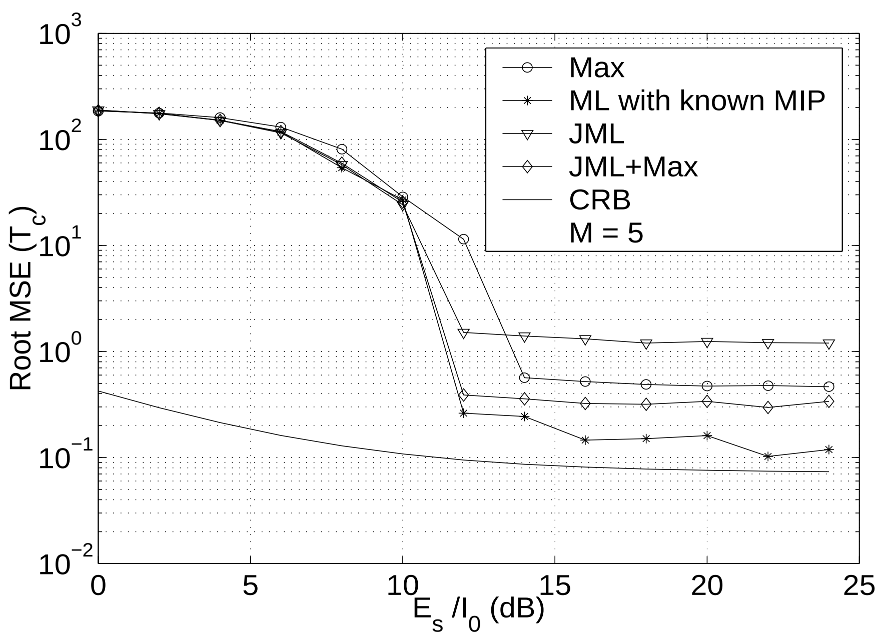

Figure 4: Normalized RMSE vs. E_s/I_0 for CH2, with $M = 5$ observation intervals.

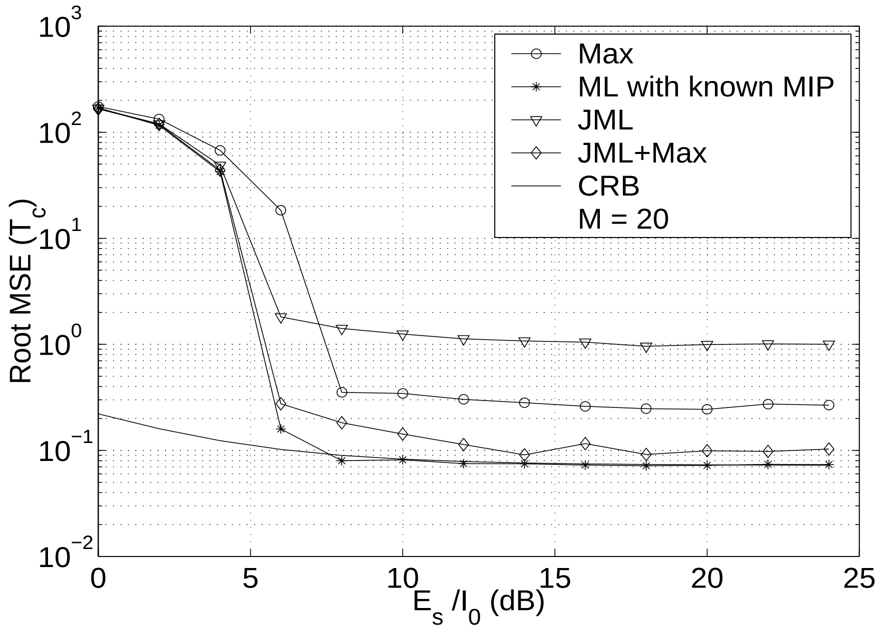

Figure 5: Normalized RMSE vs. E_s/I_0 for CH2, with $M = 20$ observation intervals.

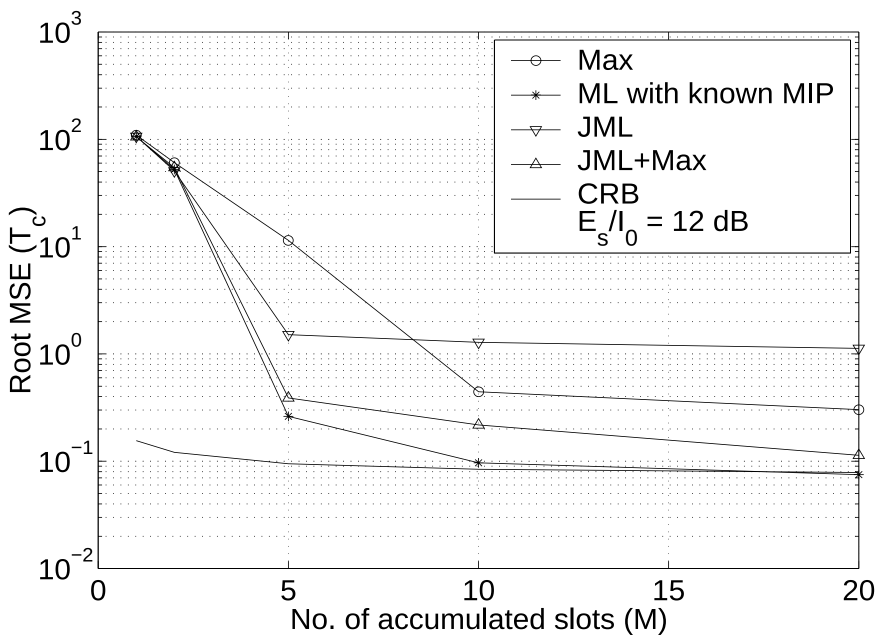

Figure 6: Normalized RMSE vs. number of observation intervals M for CH2, with E_s/I_0 = 12 dB

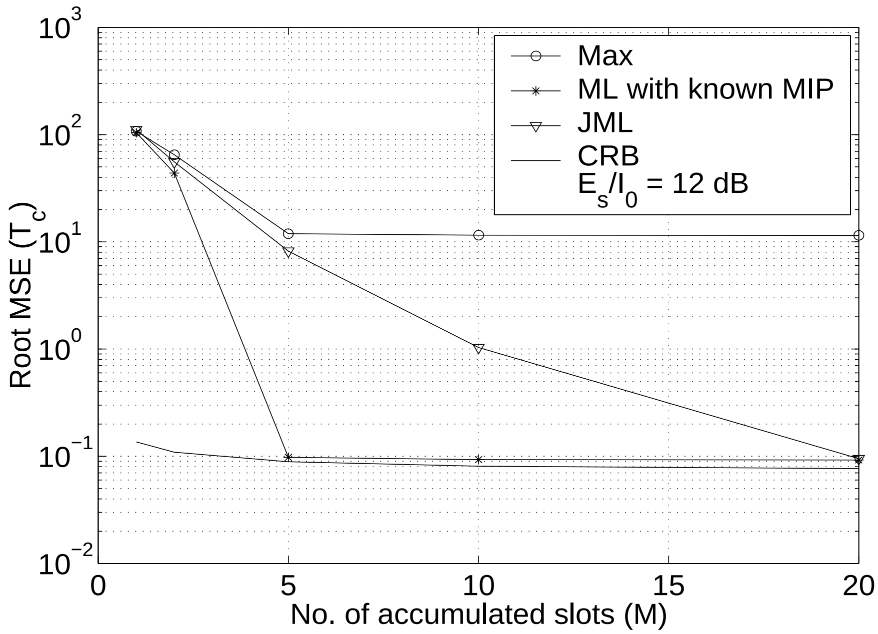

Figure 7: Normalized RMSE vs. number of observation intervals M for CH3, with E_s/I_0 = 12 dB

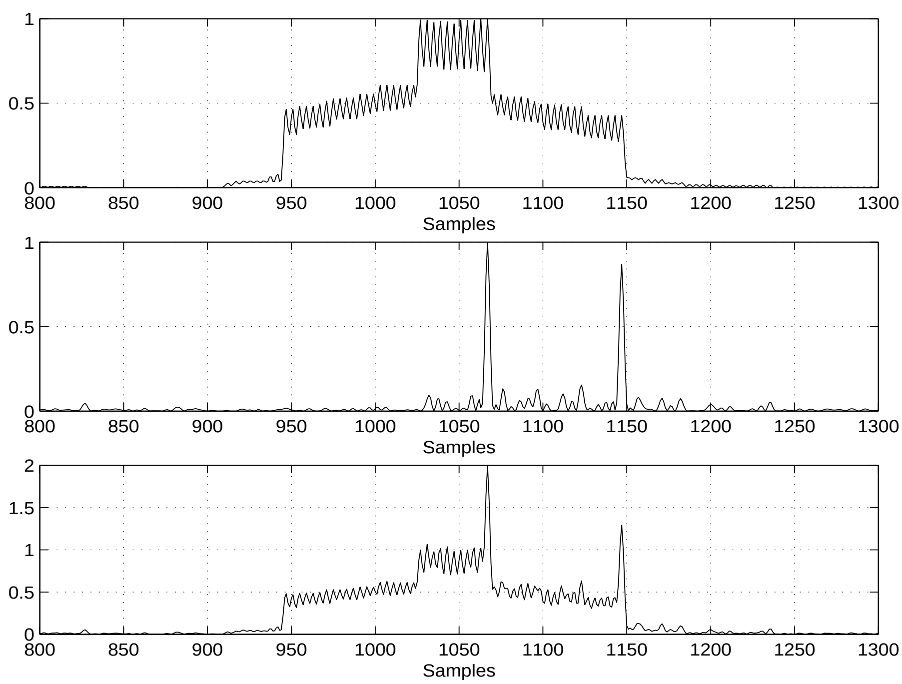

Figure 8: Data buffer, likelihood function and their weighted sum.