MISO Broadcast Channel with Delayed and Evolving CSIT

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***Abstract*—The work considers the two-user MISO broadcast channel with a gradual and delayed accumulation of channel state information at the transmitter (CSIT), and addresses the question of how much feedback is necessary, and when, in order to achieve a certain degrees-of-freedom (DoF) performance. Motivated by limited-capacity feedback links with delays, that may not immediately convey perfect CSIT, and focusing on the block fading scenario, we consider a gradual accumulation of feedback bits that results in a progressively increasing CSIT quality as time progresses across the coherence period (T channel uses - current CSIT), or at any time after (delayed CSIT).**

Specifically, for any set $\{\alpha_t\}_{t=1}^T$ of feedback quality exponents describing the high-SNR rates-of-decay of the mean square error of the current CSIT estimates at time $t \leq T$ ($0 \leq \alpha_1 \leq \cdots \leq \alpha_T \leq 1$), given an average $\bar{\alpha} = \sum_{t=1}^T \alpha_t / T$, and given perfect delayed CSIT (received at any time $t > T$), the work here derives the optimal DoF region to be the polygon with corner points $\{(0,0),(0,1),(\bar{\alpha},1),(\frac{2+\bar{\alpha}}{3},\frac{2+\bar{\alpha}}{3}),(1,\bar{\alpha}),(1,0)\}$. Aiming to now reduce the overall number of feedback bits, we also prove that the above optimal region holds even with imperfect delayed CSIT for any (delayed-CSIT) quality exponent $\beta \geq \frac{1+2\bar{\alpha}}{3}$.

The results are supported by novel multi-phase precoding schemes that utilize gradually improving CSIT. The approach here incorporates different settings such as the delayed CSIT setting of Maddah-Ali and Tse ($\beta = 1, \alpha_t = 0, \ \forall t \leq T$), the imperfect current CSIT setting of Yang et al. and of Gou and Jafar ($\beta = 1, \alpha_1 = \cdots = \alpha_T > 0$), and the not-so-delayed CSIT setting of Lee and Heath ($\beta = 1, \alpha_1 = \cdots = \alpha_\tau = 0$ for some $\tau < T$).

I. Introduction

A. Channel model

We consider the multiple-input single-output broadcast channel (MISO BC) with an M-transmit antenna ($M \geq 2$) transmitter communicating to two receiving users with a single receive antenna each. Within the block fading setting, we consider a coherence period of T channel uses, during which the channel remains the same. For $\boldsymbol{h}_\ell$ and $\boldsymbol{g}_\ell$ denoting this channel during the ℓth coherence block for the first and second user respectively, and for $\boldsymbol{x}_{\ell,t}$ denoting the transmitted vector during timeslot t of this ℓth block, the corresponding received signals at the first and second user take the form

$$y_{\ell,t}^{(1)} = \boldsymbol{h}_\ell^\intercal \boldsymbol{x}_{\ell,t} + z_{\ell,t}^{(1)} \quad (1)$$

$$y_{\ell,t}^{(2)} = \boldsymbol{g}_\ell^\intercal \boldsymbol{x}_{\ell,t} + z_{\ell,t}^{(2)} \quad (2)$$

($t = 1, 2, \cdots, T$), where $z_{\ell,t}^{(1)}, z_{\ell,t}^{(2)}$ denote the unit power AWGN noise at the receivers. The above transmit vectors accept a power constraint $\mathbb{E}[||\boldsymbol{x}_{\ell,t}||^2] \leq P$, for some power P which also here takes the role of the signal-to-noise ratio (SNR). The fading coefficients are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, and are assumed to remain fixed during a coherence block, and to change independently from block to block.

B. Delay-and-quality effects of feedback

As in many multiuser wireless communications scenarios, the performance of the broadcast channel depends on the timeliness and quality of channel state information at the transmitter (CSIT). This timeliness and quality though may be reduced by limited-capacity feedback links, which may offer consistently low feedback quality, or may offer good quality feedback which though comes late in the communication process and can thus be used for only a fraction of the communication duration. The corresponding performance degradation, as compared to the case of having perfect feedback without delay, forces the delay-and-quality question of how much feedback is necessary, and when, in order to achieve a certain performance.

These delay-and-quality effects of feedback, naturally fall between the two extreme cases of no CSIT and of full CSIT (immediately available and perfect CSIT), with full CSIT allowing for the optimal 1 degrees-of-freedom (DoF) per user (cf., [1])[1], while the absence of any CSIT reduces this to just $1/2$ DoF per user (cf., [2], [3]).

A valuable tool towards bridging this gap and further understanding the delay-and-quality effects of feedback, came with [4] showing that arbitrarily delayed feedback can still allow for performance improvement over the no-CSIT case. In a setting that differentiated between current and delayed CSIT - delayed CSIT being that which is available after the channel elapses, while current CSIT corresponded to feedback received during the channel's coherence period - the work in [4] showed that perfect delayed CSIT, even without any current CSIT, allows for an improved $2/3$ DoF per user.

Within the same context of delayed vs. current CSIT, the work in [5]–[8] introduced feedback quality considerations,

[1]We remind the reader that for an achievable rate pair (R_1, R_2), the corresponding DoF pair (d_1, d_2) is given by $d_i = \lim_{P\to\infty} \frac{R_i}{\log P}$, $i = 1, 2$. The corresponding DoF region is then the set of all achievable DoF pairs.

and managed to quantify the usefulness of combining perfect delayed CSIT with immediately available imperfect CSIT of a certain quality that remained unchanged throughout the entire coherence period. In this setting the above work showed a further bridging of the gap from $2/3$ to 1 DoF, as a function of this current CSIT quality.

Further progress came with the work in [9] which, in addition to exploring the effects of the quality of current CSIT, also considered the effects of the quality of delayed CSIT, thus allowing for consideration of the possibility that the overall number of feedback bits (corresponding to delayed plus current CSIT) may be reduced. Focusing again on the specific setting where the current CSIT quality remained unchanged for the entirety of the coherence period, this work revealed among other things that imperfect delayed CSIT can achieve the same optimality that was previously attributed to perfect delayed CSIT, thus equivalently showing how the amount of delayed feedback required, is proportional to the amount of current feedback.

A useful generalization of the delayed vs. current CSIT paradigm, came with the work in [10] which deviated from the assumption of having invariant CSIT quality throughout the coherence period, and allowed for the possibility that current CSIT may be available only after some delay, and specifically only after a certain fraction of the coherence period. Under these assumptions, in the presence of more than two users, and in the presence of perfect delayed CSIT, the above work showed that for up to a certain delay, one can achieve the optimal performance corresponding to full (and immediate) CSIT.

The above settings[2] addressed different instances of the more general problem of communicating in the presence of feedback with different delay-and-quality properties, with each of these settings being motivated by the fact that perfect CSIT may be generally hard and time-consuming to obtain, that CSIT precision may be improved over time[3], and that feedback delays and imperfections generally cost in terms of performance. The generalization here to the setting of time-evolving CSIT, incorporates the above considerations and motivations, and allows for insight on pertinent questions such as:

- How much CSIT feedback, and when, must one send to achieve a certain target DoF performance?
- How much current CSIT quality is necessary to achieve a certain performance?
- How much delayed CSIT quality is necessary to achieve the best possible performance?
- Can imperfect delayed CSIT achieve the same optimality that was previously attributed to perfect delayed CSIT?
- When is delayed feedback unnecessary?

C. Structure of paper, notation and conventions

Section I-D describes the quantification of evolving CSIT quality. Then Section II provides the main results, i.e., the optimal DoF regions for the different cases of evolving CSIT. In addition to the theorems, we also provide corollaries and examples that are meant to offer insight. The achievability and DoF outer bound proofs are shown in the journal version of this work [16], due to the lack of space here.

Throughout this paper, $(\bullet)^{\mathsf{T}}$ denotes the transpose, while $|| \bullet ||$ denotes the Euclidean norm. Finally we adhere to the common convention (see [4], [6], [7], [17]) of assuming perfect and global knowledge of channel state information at the receivers (perfect global CSIR), where the receivers know all channel states and all estimates.

D. Quantification of evolving CSIT quality

In terms of current CSIT, i.e., in terms of CSIT corresponding to feedback received during the coherence period of the channel in question, we consider the case where at time t of the ℓth coherence block, the transmitter has estimates $\hat{\boldsymbol{h}}_{\ell,t}, \hat{\boldsymbol{g}}_{\ell,t}$ of $\boldsymbol{h}_\ell$ and $\boldsymbol{g}_\ell$ respectively, with estimation errors

$$\tilde{\boldsymbol{h}}_{\ell,t} = \boldsymbol{h}_\ell - \hat{\boldsymbol{h}}_{\ell,t}, \quad \tilde{\boldsymbol{g}}_{\ell,t} = \boldsymbol{g}_\ell - \hat{\boldsymbol{g}}_{\ell,t} \tag{3}$$

respectively having i.i.d. circularly symmetric complex Gaussian entries with zero mean and power

$$\frac{1}{M}\mathbb{E}[\|\tilde{\boldsymbol{h}}_{\ell,t}\|^2] = \frac{1}{M}\mathbb{E}[\|\tilde{\boldsymbol{g}}_{\ell,t}\|^2] = P^{-\alpha_t} \tag{4}$$

for some non-negative parameter α_t describing the quality of the estimates at any given time $t = 1, 2, \cdots, T$ during the channel's coherence period[4]. In this setting, a possibly increasing α_t implies an improving CSIT quality, with $\alpha_t = 0$ implying very little current CSIT knowledge up to time t, and with $\alpha_t = \infty$ - and for all DoF-related purposes, $\alpha_t = 1$ ([18]) - implying that starting at a given time t, the transmitter has access to perfect CSIT.

In terms of delayed CSIT, and again focusing on the aforementioned channels $\boldsymbol{h}_\ell, \boldsymbol{g}_\ell$ appearing during the ℓth coherence block, we consider the case where at any time after the end of the ℓth block, the transmitter has delayed estimates $\check{\boldsymbol{h}}_\ell, \check{\boldsymbol{g}}_\ell$ with estimation errors

$$\ddot{\boldsymbol{h}}_\ell = \boldsymbol{h}_\ell - \check{\boldsymbol{h}}_\ell, \quad \ddot{\boldsymbol{g}}_\ell = \boldsymbol{g}_\ell - \check{\boldsymbol{g}}_\ell \tag{5}$$

again having i.i.d. Gaussian entries, but this time with power

$$\frac{1}{M}\mathbb{E}[\|\ddot{\boldsymbol{h}}_\ell\|^2] = \frac{1}{M}\mathbb{E}[\|\ddot{\boldsymbol{g}}_\ell\|^2] = P^{-\beta}$$

for some non-negative parameter β.

We here adhere to the common convention (see [5]–[7]) of assuming that all the estimates up to time t, are independent of the current estimate errors $\tilde{\boldsymbol{h}}_t$ and $\tilde{\boldsymbol{g}}_t$ at time t [5].

[2]In describing existing work, we focused only on immediately related work, thus neglecting other results in the context of delayed CSIT, such as those in ([11]–[15]) and in many other publications.

[3]Such gradual improvement could be sought in FDD settings with limited-capacity feedback links that can be used more than once during the coherence period to progressively refine CSIT, as well as in TDD settings that use reciprocity-based prediction that improves over time.

[4]We clarify that the power of the error is averaged over channel realizations and noise, and is naturally a function of t but not of ℓ.

[5]It is noted that this assumption is valid in the block fading case where the estimates of the channel occur in a progressively improving manner.

Remark 1: We here note that the choice of invariant (non evolving) delayed CSIT, is meant to reflect the fact that - unlike the case of evolving current CSIT - delayed CSIT can, without loss of generality, be assumed to be received with any delay, after which any further improvement of feedback-quality may be unrealistic.

Remark 2: We also note that without loss of generality, in the DoF setting of interest, we can restrict our attention to the range $0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_T \leq 1$ and $0 \leq \beta \leq 1$, as well as to the case where $\alpha_T \leq \beta$ since delayed CSIT with $\beta < \alpha_T$ can be readily improved to delayed CSIT with $\beta = \alpha_T$, simply by recalling current CSIT estimates at a later time. As a result, we will consider the general setting where

$$0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_T \leq \beta \leq 1,$$

where $\beta = 1$ corresponds to having perfect delayed CSIT, and where $\alpha_1 = 1$ corresponds to the optimal case of perfect and immediately available CSIT.

We can now see how the evolving CSIT generalization naturally incorporates different settings such as the perfect-delayed CSIT setting in [4] ($\beta = 1, \alpha_t = 0, \ \forall t \leq T$), the perfect-delayed and imperfect current CSIT setting in [5]–[7] ($\beta = 1, \alpha_1 = \cdots = \alpha_T < 1$), the bounded-overall-feedback setting with imperfect current and imperfect delayed CSIT [9] ($\beta < 1, \alpha_1 = \cdots = \alpha_T < 1$), as well as the 'not-so-delayed' CSIT setting in [10] corresponding to having $\beta = 1, \alpha_1 = \cdots = \alpha_\tau = 0, \alpha_{\tau+1} = \cdots = \alpha_T = 1$ for some integer $\tau < T$.

II. Main results

We proceed with the main results. As stated, the corresponding schemes and corresponding outer bound proof can be found in [16].

A. Evolving current CSIT and perfect delayed CSIT

We here consider the case of evolving current CSIT with perfect delayed CSIT. For notational convenience, we define

$$\bar{\alpha} \triangleq \frac{1}{T} \sum_{t=1}^{T} \alpha_t \tag{6}$$

to be the average (current) CSIT quality exponent.

Theorem 1: The optimal DoF region for the two-user MISO BC with symmetrically evolving current CSIT and perfect delayed CSIT, takes the form

$$d_1 \leq 1, \quad d_2 \leq 1 \tag{7}$$

$$2d_1 + d_2 \leq 2 + \bar{\alpha} \tag{8}$$

$$2d_2 + d_1 \leq 2 + \bar{\alpha} \tag{9}$$

and corresponds to the polygon with corner points

$$\{(0,0), (0,1), (\bar{\alpha}, 1), (\frac{2+\bar{\alpha}}{3}, \frac{2+\bar{\alpha}}{3}), (1, \bar{\alpha}), (1,0)\}.$$

This is depicted in Fig. 1.

Drawing from the above, the following corollary is partially motivated by the possibility of having imperfect feedback

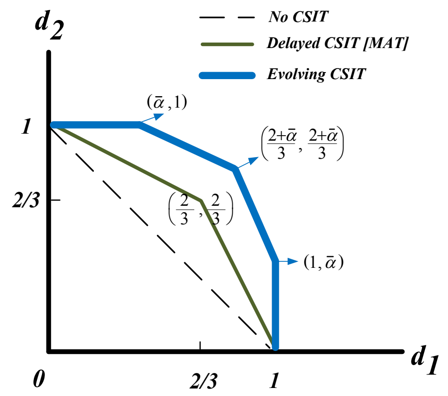

Fig. 1. Optimal DoF region of two-user MISO BC with evolving current CSIT and perfect delayed CSIT.

and/or having feedback with delays. The use of the term *symmetric DoF* is meant to correspond to the case where the two users have equal DoF.

Corollary 1a: In the setting of the two-user MISO BC, the optimal symmetric DoF $d' = 1$ (DoF pair $(d', d') = (1, 1)$) requires $\bar{\alpha} = 1$, i.e., requires perfect and immediately available CSIT.

The above applies to settings such as that in [10] which considers delays in receiving current CSIT, thus corresponding to having $\alpha_1 = \cdots \alpha_\tau = 0$ for some $\tau > 0$, and thus having $\bar{\alpha} < 1$. The corollary shows that, unlike in the $(M+1)$-user case in [10] where the optimal sum DoF is achieved even in the presence of the aforementioned (current feedback) delays, in the two-user case here, any delay or imperfection in the current CSIT, will result in suboptimal DoF performance.

The following examples provides insight.

Example 1: Let us consider a setting where we seek to achieve a certain symmetric target DoF $d' = 7/9$. Noting directly from the theorem that this requires $\bar{\alpha} \geq 3d' - 2 = 1/3$, we identify possible sets of quality exponents to include:

- ($\alpha_t = 0$ for $t \leq 2T/3$, $\alpha_t = 1$ for $t > 2T/3$) which allows for maximal current-feedback delay that is equal to two thirds of the coherence block, and which asks for perfect feedback at the beginning of the last third of the block
- ($\alpha_t = 0$ for $t \leq T/3$, $\alpha_t = 4/9$ for $t \in (T/3, 2T/3]$, $\alpha_t = 5/9$ for $t > 2T/3$) which allows for some feedback delay and a gradual evolution of CSIT quality
- ($\alpha_t = 1/3$ for all $t \in (0, T]$) which asks for immediate feedback, but of lesser quality with fewer feedback bits.

Example 2: In the setting of the previous example and the aforementioned three options, let us assume for the sake of simplicity that channel quantization is simple scalar quantization, in which case a quantization rate of $\log P$ bits allows for (essentially) perfect feedback, and where $\alpha \log P$ bits allow for a quality exponent $\alpha \in [0, 1]$ ([19]). In this simplified quantization setting we observe the following.

- The first option is direct: send no feedback during the first two-thirds of the coherence block, and then send $\log P$ feedback bits right after that (no need for further delayed

TABLE I
SOME FEEDBACK OPTIONS ACHIEVING SYMMETRIC DOF $d' = \frac{7}{9}$.

α_1 to $\alpha_{\frac{T}{3}}$	$\alpha_{\frac{T}{3}+1}$ to $\alpha_{\frac{2T}{3}}$	$\alpha_{\frac{2T}{3}+1}$ to α_T	feedback delay	feedback bits in period $1 \to T$	extra bits after $t = T$
1/3	1/3	1/3	0	$1/3 \cdot \log P$	$2/3 \cdot \log P$
0	4/9	5/9	$T/3$	$5/9 \cdot \log P$	$4/9 \cdot \log P$
0	0	1	$2T/3$	$\log P$	0

feedback).
• To get the second option, we allow for feedback delay equal to a third of the coherence block, at the end of which we send $\frac{4}{9}\log P$ bits of feedback to get $\alpha_t = 4/9, t \in (T/3, 2T/3]$, and then at the beginning of the last third of the coherence block, send an additional $\frac{1}{9}\log P$ bits to increase the number of accumulated feedback bits to $\frac{5}{9}\log P$ bits and to get $\alpha_t = 5/9, t \in (2T/3, T]$. Sending, at any point after the end of the coherence block, an additional $\frac{4}{9}\log P$ bits of delayed feedback, would complement the existing $\frac{5}{9}\log P$ bits of feedback accumulated during the coherence block, would bring the total number of accumulated feedback bits to $\log P$ bits, and would allow for perfect delayed CSIT corresponding to $\beta = 1$.
• To get the third option, we immediately send $\frac{1}{3}\log P$ bits of feedback at the beginning of the coherence block in order to get $\alpha_t = 1/3, t \in [1, T]$. Sending an extra $\frac{2}{3}\log P$ bits of delayed feedback at any point $t > T$ after the end of the coherence block, would result in perfect delayed CSIT.

These are summarized in Table II where the second-to-last column describes the total number of feedback bits sent during the coherence block, and where the last column describes the number of extra (delayed) feedback bits required to refine the current CSIT estimates to the point of perfect delayed CSIT.

B. Evolving current CSIT with imperfect delayed CSIT

We now proceed to the more general case where, in addition to imperfections in the current CSIT, imperfections can be found in delayed CSIT estimates as well ($0 \leq \alpha_1 \leq \cdots \leq \alpha_T \leq \beta \leq 1$). Having $\beta \leq 1$ could reflect a limitation in the feedback link quality or a limitation in the total number of (current plus delayed) feedback bits.

Theorem 2: The optimal DoF region takes the form

$$d_1 \leq 1, \quad d_2 \leq 1, \quad 2d_1 + d_2 \leq 2 + \bar{\alpha}, \quad 2d_2 + d_1 \leq 2 + \bar{\alpha}$$

when $\beta \geq \frac{1+2\bar{\alpha}}{3}$, while when $\beta < \frac{1+2\bar{\alpha}}{3}$ this region is inner bounded by the achievable region

$$d_1 \leq 1, \quad d_2 \leq 1 \tag{10}$$
$$2d_1 + d_2 \leq 2 + \bar{\alpha} \tag{11}$$
$$2d_2 + d_1 \leq 2 + \bar{\alpha} \tag{12}$$
$$d_2 + d_1 \leq 1 + \beta \tag{13}$$

which takes the form of a polygon with corner points $\{(0,0), (0,1), (\bar{\alpha}, 1), (2\beta - \bar{\alpha}, 1 + \bar{\alpha} - \beta), (1 + \bar{\alpha} - \beta, 2\beta - \bar{\alpha}), (1, \bar{\alpha}), (1, 0)\}$.

The following corollaries provide further insight and conclusions that hold in the same DoF context.

Corollary 2a: Having delayed-CSIT quality $\beta \geq \frac{1+2\bar{\alpha}}{3}$ is equivalent to having perfect delayed CSIT. Consequently whenever $\alpha_T \geq \frac{1+2\bar{\alpha}}{3}$, there is no need for any delayed CSIT, i.e., there is no utility in sending feedback after the end of the coherence block.

The above is direct from the theorem and simply considers that current CSIT estimates can be recalled at a later point in time. It applies towards answering the question of how many (delayed) feedback bits must be gathered after the channel changes in order to achieve the best possible performance.

Furthermore we have the following, which gives insight on how many feedback bits to send, and when, in order to achieve a certain performance d'. The proof is again direct.

Corollary 2b: To achieve a symmetric target DoF d', it is sufficient to have $\bar{\alpha} \geq 3d' - 2$ with $\beta \geq 2d' - 1$ or to have $\bar{\alpha} \geq 3d' - 2$ with $\alpha_T \geq 2d' - 1$ (and no extra delayed feedback).

In addition, the following corollary describes feedback delays that allow for a given target symmetric DoF d' in the presence of constraints on current and delayed CSIT qualities. We will be specifically interested in the allowable fractional delay of feedback

$$\gamma \triangleq \arg\max_{\gamma'} \{\alpha_{\gamma' T} = 0\} \tag{14}$$

i.e., the fraction $\gamma \leq 1$ for which $\alpha_1 = \cdots = \alpha_{\gamma T} = 0, \alpha_{\gamma T+1} > 0$. A constraint $\alpha_t \leq \alpha_{\max}$ on the current quality exponents, is meant to reflect a constraint on the total number of feedback bits sent during the coherence period, while bounding β corresponds to having a limited total number of (current plus delayed) feedback bits per coherence period[6].

Corollary 2c: Under a current CSIT quality constraint $\alpha_t \leq \alpha_{\max}$, a symmetric target DoF d' can be achieved with any fractional delay $\gamma \leq 1 - \frac{3d'-2}{\alpha_{\max}}$, by setting $\alpha_1 = \cdots = \alpha_{\gamma T} = 0, \alpha_{\gamma T+1} = \cdots = \alpha_T = \alpha_{\max} = 2d' - 1 = \beta$. Furthermore under a delayed CSIT quality constraint $\beta \leq \beta_{\max}$, a target DoF d' can be achieved with any $\gamma \leq 1 - \frac{3d'-2}{\beta_{\max}}$, by setting $\alpha_1 = \cdots = \alpha_{\gamma T} = 0, \alpha_{\gamma T+1} = \cdots = \alpha_T = \beta_{\max} = 2d' - 1$. Finally under no specific constraint on CSIT quality, the target DoF d' can be achieved with any $\gamma \leq 3(1 - d')$, using perfect (but delayed) feedback ($\alpha_1 = \cdots = \alpha_{\gamma T} = 0, \alpha_{\gamma T+1} = \cdots \alpha_T = \beta = 1$).

The following bounds the quality of current and of delayed CSIT needed to achieve a certain target symmetric DoF d'.

Corollary 2d: Having $\alpha_{\max} = 3d' - 2$ and $\beta = 2d' - 1$, is sufficient to achieve a symmetric DoF d'.

The proof of this is straightforward; the corresponding quality exponents can be $\alpha_1 = \cdots = \alpha_T = 3d' - 2, \beta = 2d' - 1$. We proceed with some simple examples.

Example 3: Consider a symmetric target DoF $d' = \frac{7}{9}$. In the absence of any specific constraint on the quality of current

[6]Our ignoring integer rounding considerations is an abuse of notation that is only done for the sake of clarity, and it carries no real effect.

TABLE II
SOME FEEDBACK OPTIONS ACHIEVING SYMMETRIC DOF $d' = \frac{7}{9}$.

α_1 to $\alpha_{\frac{T}{3}}$	$\alpha_{\frac{T}{3}+1}$ to $\alpha_{\frac{2T}{3}}$	$\alpha_{\frac{2T}{3}+1}$ to α_T	β	feedback delay	extra bits after $t = T$
$1/3$	$1/3$	$1/3$	$5/9$	0	$2/9 \cdot \log P$
0	$4/9$	$5/9$	$5/9$	$T/3$	0
0	0	1	1	$2T/3$	0

and delayed CSIT, d' can be achieved with $\alpha_1 = \cdots \alpha_{2T/3} = 0,\ \alpha_t = \beta = 1, t \in (2T/3, T]$, corresponding to fractional feedback delay $\gamma = 3(1-d') = 2/3$ (Corollary 2c), and corresponding to sending perfect feedback at the beginning of the last third of the coherence period. If on the other hand, the feedback link only allows for $\alpha_t \leq \alpha_{\max} = 1/2$, then the desired $d' = 7/9$ can be achieved with feedback delay $\gamma = 1 - (3d'-2)/\alpha_{\max} = 1/3$, allowing for $\alpha_t = 0$ for $t \in [1, T/3]$ and then $\alpha_t = 1/2$ for $t > T/3$, and $\beta \geq \frac{1+2\bar{\alpha}}{3} = 2d' - 1 = 5/9$.

Example 4: If in the setting of the previous example, we loosened slightly the constraint, from $\alpha_t \leq 1/2$ to $\alpha_t \leq 5/9$, we could allow for an increase in the fractional delay, from $\gamma = 1/3$ to $\gamma = 1 - \frac{\bar{\alpha}}{\beta} = 1 - \frac{3d'-2}{2d'-1} = 1 - \frac{1/3}{5/9} = 2/5$ allowing for $\alpha_t = 0$ for $t \leq 2T/5$ and then $\alpha_t = 2d' - 1 = 5/9 = \beta$ for $t > 2T/5$.

Example 5: If feedback delay is not a priority, then we can substantially reduce the number of current feedback bits and achieve $d' = \frac{7}{9}$ with $\alpha_1 = \cdots = \alpha_T = \bar{\alpha} = 3d' - 2 = 1/3$ ($\beta = \frac{1+2\bar{\alpha}}{3} = 2d' - 1 = 5/9$).

Example 6: If feedback can only be sent every third of the coherence period, then possible feedback options for $d' = 7/9$ would include:

- ($\alpha_t = 0$ for $t \leq 2T/3,\ \alpha_t = 1 = \beta$ for $t > 2T/3$) which allows for increased feedback delay
- ($\alpha_t = 0$ for $t \leq T/3,\ \alpha_t = 4/9$ for $t \in (T/3, 2T/3],\ \alpha_t = 5/9 = \beta$ for $t > 2T/3$) which combines feedback delay and a reduced total amount of feedback bits
- ($\alpha_t = 1/3$ for all $t < T, \beta = 5/9$) which allows for reduced feedback within the duration of the coherence block.

These options are summarized in Table II, again corresponding to the simple aforementioned quantization setting. The last column describes the number of delayed feedback bits, sent at any point after the end of coherence block, to refine current CSIT estimates to the desired quality of delayed CSIT.

III. CONCLUSIONS

This work considered the two user MISO BC setting with gradually accumulated feedback that incrementally improves CSIT quality. This was done for the cases of perfect and imperfect delayed CSIT. The many corollaries and examples aimed to offer insight on many questions relating to the delay-and-quality effects of feedback.

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