On the Fundamental Feedback-vs-Performance Tradeoff over the MISO-BC with Imperfect and Delayed CSIT

Jinyuan Chen
Mobile Communications Dep.
EURECOM
Sophia Antipolis, France
Email: jinyuan.chen@eurecom.fr

Sheng Yang
Telecommunications Dep.
SUPELEC
Gif sur Yvette, France
Email: sheng.yang@supelec.fr

Petros Elia
Mobile Communications Dep.
EURECOM
Sophia Antipolis, France
Email: petros.elia@eurecom.fr

***Abstract*—This work considers the multiuser multiple-input single-output (MISO) broadcast channel (BC), where a transmitter with M antennas transmits information to K single-antenna users, and where - as expected - the quality and timeliness of channel state information at the transmitter (CSIT) is imperfect. Motivated by the fundamental question of how much feedback is necessary to achieve a certain performance, this work seeks to establish bounds on the tradeoff between degrees-of-freedom (DoF) performance and CSIT feedback quality. Specifically, this work provides a novel DoF region outer bound for the general K-user $M\times 1$ MISO BC with partial current CSIT, which naturally bridges the gap between the case of having no current CSIT (only delayed CSIT, or no CSIT) and the case with full CSIT. The work then characterizes the minimum CSIT feedback that is necessary for any point of the sum DoF, which is optimal for the case with $M \geq K$, and the case with $M = 2,\ K = 3$.**

I. Introduction

We consider the multiuser multiple-input single-output (MISO) broadcast channel (BC), where a transmitter with M antennas, transmits information to K single-antenna users. In this setting, the received signal at time t, is of the form

$$y_{k,t} = \boldsymbol{h}_{k,t}^{\mathsf{T}} \boldsymbol{x}_t + z_{k,t}, \quad k = 1, \cdots, K \tag{1}$$

where $\boldsymbol{h}_{k,t}$ denotes the $M \times 1$ channel vector for user k, $z_{k,t}$ denotes the unit power AWGN noise, and where $\boldsymbol{x}_t$ denotes the transmitted signal vector adhering to a power constraint $\mathbb{E}[||\boldsymbol{x}_t||^2] \leq P$, for P taking the role of the signal-to-noise ratio (SNR). We here consider that the fading coefficients $\boldsymbol{h}_{k,t},\ k = 1, \cdots, K$, are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, and are i.i.d. over time.

It is well known that the performance of the BC is greatly affected by the timeliness and quality of feedback; having full CSIT allows for the optimal $\min\{M, K\}$ sum degrees-of-freedom (DoF) (cf. [1])[1], while the absence of any CSIT reduces this to just 1 sum DoF (cf. [2], [3]). This gap has spurred a plethora of works that seek to analyze and optimize BC communications in the presence of delayed and imperfect feedback. One of the works that stands out is the work by Maddah-Ali and Tse [4] which recently revealed the benefits of employing delayed CSIT over the BC, even if this CSIT is completely obsolete. Several interesting generalizations followed, including the work in [5] which showed that in the BC setting with $K = M+1$, combining delayed CSIT with perfect (current) CSIT (over the last $\frac{K-1}{K}$ fraction of communication period) allows for the optimal sum DoF M corresponding to full CSIT. A similar approach was exploited in [6] which revealed that, to achieve the maximum sum DoF $\min\{M, K\}$, each user has to symmetrically feed back perfect CSIT over a $\frac{\min\{M,K\}}{K}$ fraction of the communication time, and that this fraction is optimal. Other interesting works in the context of utilizing delayed and current CSIT, can be found in [7]–[10] which explored the setting of combining perfect delayed CSIT with immediately available imperfect CSIT, the work in [11] which additionally considered the effects of the quality of delayed CSIT, the work in [12] which considered alternating CSIT feedback, the work in [13] which considered delayed and progressively evolving (progressively improving) current CSIT, and the works in [14]–[16] and many other publications.

Our work here generalizes many of the above settings, and seeks to establish fundamental tradeoff between DoF performance and CSIT feedback quality, over the general K-

[1]We remind the reader that for an achievable rate tuple $(R_1, R_2, \cdots, R_K)$, where R_i is for user i, the corresponding DoF tuple $(d_1, d_2, \cdots, d_K)$ is given by $d_i = \lim_{P\to\infty} \frac{R_i}{\log P},\ i = 1, 2, \cdots, K$. The corresponding DoF region $\mathcal{D}$ is then the set of all achievable DoF tuples $(d_1, d_2, \cdots, d_K)$.

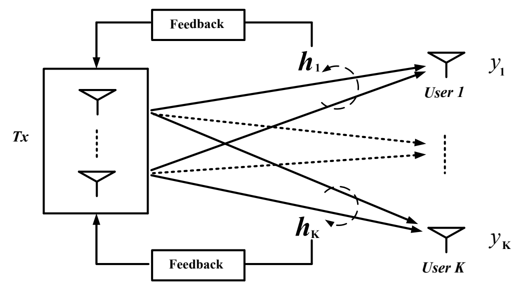

Fig. 1. System model of K-user MISO BC with CSIT feedback.

user $M \times 1$ MISO BC with general CSIT quality. It is noted that, in parallel to this work, [17] also considered a particular case of K-user MISO BC with the additional constraint that the CSIT quality is invariant over time and equal for all users, and provided an inner bound and an outer bound.

A. Structure of paper, notation and conventions

We proceed to first describe the quality and timeliness measure of CSIT feedback, and how this measure relates to existing work. After that, Section II provides the main results of this work, i.e., the novel outer bound on the DoF region and sum DoF, optimal DoF characterizations for many cases, and some inner bounds on the sum DoF. The sketches of the proofs are shown in the same section, as well as in Section III and Section IV, while most proof details are placed, due to the lack of space, in the journal version of this work [18].

Throughout this paper, we will consider communication over n coherence periods where, for clarity of notation, we will focus on the case where we employ a single channel use per such coherence period (unit coherence period). Furthermore, unless stated otherwise, we assume perfect delayed CSIT, as well as adhere to the common convention (see [4], [6], [8], [9], [12], [19]), and assume perfect and global knowledge of channel state information at the receivers. In terms of notation, $(\bullet)^{\mathsf{T}}$ will denote the transpose of a matrix or vector, while $||\bullet||$ will denote the Euclidean norm.

B. Quality and timeliness measure of CSIT feedback

We here use $\hat{\boldsymbol{h}}_{k,t}$ to denote the current channel estimate (for channel $\boldsymbol{h}_{k,t}$) at the transmitter at timeslot t, and use

$$\tilde{\boldsymbol{h}}_{k,t} = \boldsymbol{h}_{k,t} - \hat{\boldsymbol{h}}_{k,t}$$

to denote the estimate error assumed to be mutually independent of $\hat{\boldsymbol{h}}_{k,t}$ and assumed to have i.i.d. Gaussian entries with power

$$\mathbb{E}[\|\tilde{\boldsymbol{h}}_{k,t}\|^2] \doteq P^{-\alpha_{k,t}},$$

for some CSI quality exponent $\alpha_{k,t} \in [0,1]$ describing the quality of this estimate.

The approach extends over non-alternating CSIT settings in [4] and [7]–[10], as well as over an alternating CSIT setting (cf. [6], [12]) where CSIT knowledge alternates between perfect CSIT ($\alpha_{k,t} = 1$), and delayed or no CSIT ($\alpha_{k,t} = 0$).

In a setting where communication takes places over n such coherence periods ($t = 1, 2, \cdots, n$), this approach offers a natural measure of a per-user average feedback cost, in the form of

$$\bar{\alpha}_k \triangleq \frac{\sum_{t=1}^{n} \alpha_{k,t}}{n}, \quad k = 1, 2, \cdots, K,$$

as well as a measure of current CSIT feedback cost

$$\mathsf{C}_{\mathsf{C}} = \sum_{k=1}^{K} \bar{\alpha}_k,$$

accumulated over all users.

Furthermore, in a setting where delayed CSIT is always available, the above model captures the alternating CSIT setting where the exponents are binary ($\alpha_{k,t} = 0, 1$), in which case $\bar{\alpha}_k = \delta_{\mathrm{P},k}$ simply describes the fraction of time during which user k has perfect CSIT, with $\mathsf{C}_{\mathsf{C}} = \mathsf{C}_{\mathsf{P}} \triangleq \sum_{k=1}^{K} \delta_{\mathrm{P},k}$ describing the total perfect CSIT feedback cost.

II. Main results

A. Outer bounds

We first present the DoF region outer bound for the general K-user $M \times 1$ MISO BC.

Theorem 1 (DoF region outer bound): The DoF region of the K-user $M \times 1$ MISO BC, is outer bounded as

$$\sum_{k=1}^{K} \frac{d_{\pi(k)}}{\min\{k, M\}} \leq 1 + \sum_{k=1}^{K-1} \left(\frac{1}{\min\{k, M\}} - \frac{1}{\min\{K, M\}} \right) \bar{\alpha}_{\pi(k)} \quad (2)$$

$$d_k \leq 1, \quad k = 1, 2, \cdots, K \quad (3)$$

where π denotes a permutation of the ordered set $\{1, 2, \cdots, K\}$, and $\pi(k)$ denotes the kth element of set π.

Proof: A sketch of the proof is shown in Section III. ■

Remark 1: It is noted that the bound captures the results in [4] ($\alpha_{k,t} = 0, \ \forall t, k$), in [8], [9] ($K = 2, \ \alpha_{k,t} = \alpha, \ \forall t, k$), in [19] ($M = K = 2, \ \alpha_{1,t} = 1, \alpha_{2,t} = 0, \ \forall t$), in [10] ($K = 2, \ \alpha_{1,t} \neq \alpha_{2,t}, \ \forall t$), in [6], [12] ($\alpha_{k,t} \in \{0,1\}, \ \forall t, k$), as well as in [17] ($M \geq K, \ \alpha_{k,t} = \alpha, \ \forall t, k$).

Summing up from the above the K different bounds, where for bound $k (= 1, 2, \cdots, K)$ we have $\pi = \{\pi(i) = \mathrm{mod}(k + i - 2)_K + 1, i = 1, \cdots, K\}$ with $\mathrm{mod}(x)_K$ being the modulo operator, we directly have the following upper bound on the sum DoF $d_{\sum} \triangleq \sum_{k=1}^{K} d_k$, which is presented using the following notation

$$d_{\mathrm{MAT}} \triangleq \frac{K}{1 + \frac{1}{\min\{2,M\}} + \frac{1}{\min\{3,M\}} + \cdots + \frac{1}{\min\{K,M\}}}. \quad (4)$$

Corollary 1a (Sum DoF outer bound): For the K-user $M \times 1$ MISO BC, the sum DoF is outer bounded as

$$d_{\sum} \leq d_{\mathrm{MAT}} + \left(1 - \frac{d_{\mathrm{MAT}}}{\min\{K, M\}}\right) \sum_{k=1}^{K} \bar{\alpha}_k. \quad (5)$$

The above then readily translates onto a lower bound on the minimum possible total current CSIT feedback cost $\mathsf{C}_{\mathsf{C}} = \sum_{k=1}^{K} \bar{\alpha}_k$ needed to achieve the maximum sum DoF[2] $d_{\sum} = \min\{K, M\}$.

Corollary 1b (Bound on CSIT cost for maximum DoF): The minimum C_{C} required to achieve the maximum sum DoF $\min\{K, M\}$ of the K-user $M \times 1$ MISO BC, is lower bounded as

$$\mathsf{C}_{\mathsf{C}}^{\star} \geq \min\{K, M\}. \quad (6)$$

Transitioning to the alternating CSIT setting where $\alpha_{k,t} \in \{0, 1\}$, we have the following sum-DoF outer bound as a function of the perfect-CSIT duration $\bar{\alpha}_k = \delta_{\mathrm{P},k} = \delta_{\mathrm{P}}, \ \forall \ k$.

[2] Naturally the result is limited to the case where $\min\{K, M\} > 1$.

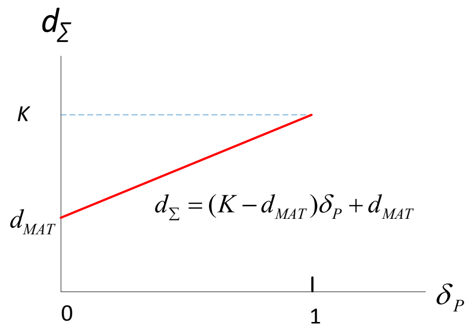

Fig. 2. Optimal sum DoF $d_{\sum}$ vs. δ_{P} for the MISO BC with $M \geq K$.

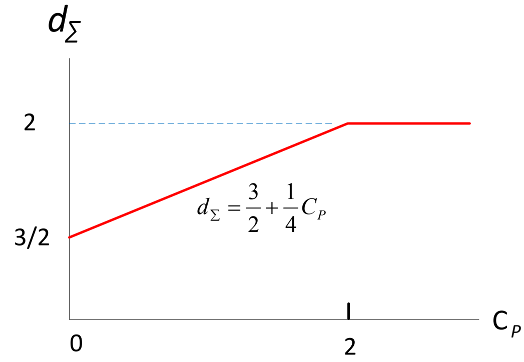

Fig. 3. Optimal sum DoF (d_{Σ}) vs. total perfect CSIT feedback cost (C_{P}) for three-user 2×1 MISO BC.

We note that the bound holds irrespective of whether, in the remaining fraction of the time $1-\delta_{\mathrm{P}}$, the CSIT is delayed or non existent.

Corollary 1c (Outer bound, alternating CSIT): For the K-user $M \times 1$ MISO BC, the sum DoF is outer bounded as

$$d_{\sum} \leq d_{\mathrm{MAT}} + \left(K - \frac{K d_{\mathrm{MAT}}}{\min\{K, M\}}\right) \min\left\{\delta_{\mathrm{P}}, \frac{\min\{K, M\}}{K}\right\}. \tag{7}$$

B. Optimal cases of DoF characterizations

We now provide the optimal cases of DoF characterizations. The case with $M \geq K$ is first considered in the following.

Theorem 2 (Optimal case, $M \geq K$): For the K-user $M \times 1$ MISO BC with $M \geq K$, the optimal sum DoF is characterized as

$$d_{\sum} = (K - d_{\mathrm{MAT}}) \min\{\delta_{\mathrm{P}}, 1\} + d_{\mathrm{MAT}}. \tag{8}$$

Proof: The converse and achievability proofs are derived from Corollary 1c and Proposition 2 (shown in the next subsection), respectively. ■

Remark 2: It is noted that, for the special case with $M = K = 2$, the above characterization captures the result in [12].

Moving to the case where $M < K$, we have the following optimal sum DoF characterizations for the case with $M = 2,\ K = 3$. The first interest is placed on the minimum $\mathsf{C}_{\mathrm{P}}^{\star}(d_{\sum})$ to achieve a sum DoF $d_{\sum}$, recalling that $\mathsf{C}_{\mathrm{P}} = \sum_{k=1}^{K} \delta_{\mathrm{P},k}$ describes the total perfect CSIT feedback cost.

Theorem 3 (Optimal case, $M = 2, K = 3$): For the three-user 2×1 MISO BC, the minimum total perfect CSIT feedback cost is given as

$$\mathsf{C}_{\mathrm{P}}^{\star}(d_{\sum}) = (4 d_{\sum} - 6)^{+}, \quad \forall\ d_{\sum} \in [0, 2] \tag{9}$$

where the total feedback cost $\mathsf{C}_{\mathrm{P}}^{\star}(d_{\sum})$ can be distributed among all the users with some combinations $\{\delta_{\mathrm{P},k}\}_k$ such that $\delta_{\mathrm{P},k} \leq \mathsf{C}_{\mathrm{P}}^{\star}(d_{\sum})/2$ for any k.

Proof: The converse proof is directly from Corollary 1a, while the achievability proof can be found in [18]. ■

Theorem 3 reveals the fundamental tradeoff between sum DoF and total perfect CSIT feedback cost (see Fig 3). The following examples are provided to offer some insights corresponding to Theorem 3.

Example 1: For the target sum DoF $d_{\Sigma} = 3/2,\ 7/4,\ 2$, the minimum total perfect CSIT feedback cost is $\mathsf{C}_{\mathrm{P}}^{\star} = 0,\ 1,\ 2$, respectively.

Example 2: The target $d_{\Sigma} = 7/4$ is achievable with asymmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/6 \quad 1/3 \quad 1/2]$, and symmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/3 \quad 1/3 \quad 1/3]$, and some other feedback such that $\mathsf{C}_{\mathrm{P}}^{\star}(7/4) = 1$.

Example 3: The target $d_{\Sigma} = 2$ is achievable with asymmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/3 \quad 2/3 \quad 1]$, and symmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [2/3 \quad 2/3 \quad 2/3]$, and some other feedback such that $\mathsf{C}_{\mathrm{P}}^{\star}(2) = 2$.

Transitioning to the symmetric setting where $\delta_{\mathrm{P},k} = \delta_{\mathrm{P}}\ \forall k$, from Theorem 3 we have the fundamental tradeoff between optimal sum DoF and CSIT feedback cost δ_{P}.

Corollary 3a (Optimal case, $M = 2, K = 3$, δ_P): For the three-user (2×1) MISO BC with symmetrically alternating CSIT feedback, the optimal sum DoF is given as

$$d_{\sum} = \min\left\{\frac{3(2+\delta_{\mathrm{P}})}{4}, 2\right\}. \tag{10}$$

Now we address the questions of what is the minimum $\mathsf{C}_{\mathrm{P}}^{\star}$ to achieve the maximum sum DoF $\min\{M, K\}$ for the general BC, and how to distribute $\mathsf{C}_{\mathrm{P}}^{\star}$ among all the users, recalling again that $\mathsf{C}_{\mathrm{P}}^{\star}$ is the total perfect CSIT feedback cost.

Theorem 4 (Minimum cost for maximum DoF): For the K-user $M \times 1$ MISO BC, the minimum total perfect CSIT feedback cost to achieve the maximum DoF is given by

$$\mathsf{C}_{\mathrm{P}}^{\star}(\min\{M,K\}) = \begin{cases} 0, & \text{if} \quad \min\{M,K\} = 1 \\ \min\{M,K\}, & \text{if} \quad \min\{M,K\} > 1 \end{cases}$$

where the total feedback cost $\mathsf{C}_{\mathrm{P}}^{\star}$ can be distributed among all the users with any combinations $\{\delta_{\mathrm{P},k}\}_k$.

Proof: For the case with $\min\{M, K\} = 1$, simple TDMA is optimal in terms of the DoF performance. For the case with $\min\{M, K\} > 1$, the converse proof is directly derived from Corollary 1b, while the achievability proof can be found in [18]. ■

It is noted that Theorem 4 is a generalization of the result in [6] where only symmetric feedback was considered. The following examples are provided to offer some insights corresponding to Theorem 4.

Example 4: For the case where $M = 2,\ K = 4$, the optimal 2 sum DoF performance is achievable, with asymmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/5 \quad 2/5 \quad 3/5 \quad 4/5]$, and symmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/2 \quad 1/2 \quad 1/2 \quad 1/2]$, and any other feedback such that $\mathsf{C}^{\star}_{\mathrm{P}} = 2$.

Example 5: For the case where $M = 3,\ K = 5$, the optimal 3 sum DoF performance is achievable, with asymmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [1/5 \quad 2/5 \quad 3/5 \quad 4/5 \quad 1]$, and symmetric feedback $\boldsymbol{\delta}_{\mathrm{P}} = [3/5 \quad 3/5 \quad 3/5 \quad 3/5 \quad 3/5]$, and any other feedback such that $\mathsf{C}^{\star}_{\mathrm{P}} = 3$.

The following corollary is derived from Theorem 4, where the case with $\min\{M, K\} > 1$ is considered.

Corollary 4a (Minimum cost for maximum DoF): For the K-user $M \times 1$ MISO BC, where J users instantaneously feed back perfect (current) CSIT, with the other users feeding back delayed CSIT, then the minimum number J is $\min\{M, K\}$, in order to achieve the maximum sum DoF $\min\{M, K\}$.

C. Inner bounds

In this subsection, we provide the following inner bounds on the sum DoF as a function of the CSIT cost, which are tight for many cases as stated.

Proposition 1 (Inner bound, $M = 2, K \geq 3$): For the $K(\geq 3)$-user 2×1 MISO BC, the sum DoF is bounded as

$$d_{\sum} \geq \frac{3}{2} + \frac{K}{4} \min\{\delta_{\mathrm{P}}, \frac{2}{K}\}. \tag{11}$$

Proof: The proof is shown in Section IV-A. ■

Proposition 2 (Inner bound, $M \geq K$ and $M < K$): For the K-user $M \times 1$ MISO BC, the sum DoF for the case with $M \geq K$ is bounded as

$$d_{\sum} \geq (K - d_{\mathrm{MAT}}) \min\{\delta_{\mathrm{P}}, 1\} + d_{\mathrm{MAT}}, \tag{12}$$

while for the case with $M < K$, the sum DoF is bounded as

$$d_{\sum} \geq (K - \frac{K\Gamma}{M}) \min\{\delta_{\mathrm{P}}, \frac{M}{K}\} + \Gamma \tag{13}$$

where

$$\Gamma \triangleq \frac{M}{\sum_{i=1}^{K-M} \frac{1}{i}(\frac{M-1}{M})^{i-1} + (\frac{M-1}{M})^{K-M}(\sum_{i=K-M+1}^{K} \frac{1}{i})}.$$

Proof: The proof is shown in Section IV-B. ■

III. Converse Proof of Theorem 1

We first provide Proposition 3 to be used, the proof of which can be found in [18]. For simplicity we drop the time index.

Proposition 3: Let

$$y_k = \boldsymbol{h}_k^{\mathsf{T}} \boldsymbol{x} + z_k,$$
$$\boldsymbol{y}_k \triangleq [y_1 \ y_2 \ \cdots \ y_k]^{\mathsf{T}}, \ \boldsymbol{H}_k \triangleq [\boldsymbol{h}_1 \ \boldsymbol{h}_2 \ \cdots \ \boldsymbol{h}_k]^{\mathsf{T}}$$
$$\boldsymbol{H} \triangleq [\boldsymbol{h}_1 \ \boldsymbol{h}_2 \ \cdots \ \boldsymbol{h}_K]^{\mathsf{T}}, \ \boldsymbol{H} = \hat{\boldsymbol{H}} + \tilde{\boldsymbol{H}}$$

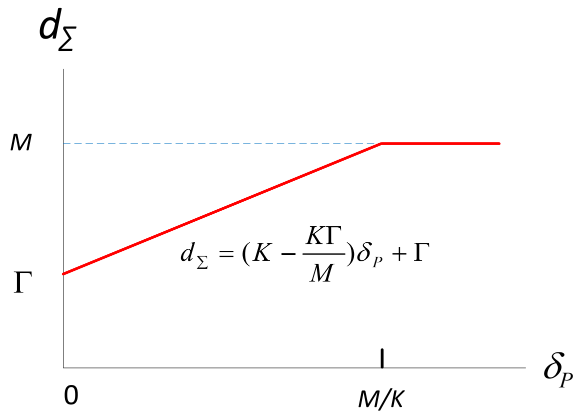

Fig. 4. Achievable sum DoF d_Σ vs. δ_{P} for the MISO BC with $M < K$.

where $\tilde{\boldsymbol{h}}_i \in \mathbb{C}^{M\times 1}$ has i.i.d. $\mathcal{N}_{\mathbb{C}}(0, \sigma_i^2)$ entries. Then, for any U such that $p_{X|U\hat{H}\tilde{H}} = p_{X|U\hat{H}}$ and $K \geq m \geq l$, we have

$$l' \, h(\boldsymbol{y}_m | U, \hat{H}, \tilde{H}) - m' \, h(\boldsymbol{y}_l | U, \hat{H}, \tilde{H})$$
$$\leq -(m' - l') \sum_{i=1}^{l} \log \sigma_i^2 + o(\log P) \tag{14}$$

where we define $l' \triangleq \min\{l, M\}$ and $m' \triangleq \min\{m, M\}$.

Giving the observations and messages of users $1, \ldots, k-1$ to user k, we establish the following genie-aided upper bounds on the achievable rates

$$nR_k \leq I(W_k; y_1^n, y_2^n, \ldots, y_k^n \,|\, W_1, \ldots, W_{k-1}, \Omega^n) + n\epsilon \tag{15}$$

where we apply Fano's inequality and some basic chain rules of mutual information using the fact that messages from different users are independent, and where we define

$$\Omega^n \triangleq \{\boldsymbol{S}_t, \hat{\boldsymbol{S}}_t\}_{t=1}^n, \ y_k^n \triangleq \{y_{k,t}\}_{t=1}^n$$
$$\boldsymbol{S}_t \triangleq \left[\boldsymbol{h}_{1,t} \ \cdots \ \boldsymbol{h}_{K,t}\right]^{\mathsf{T}}, \ \hat{\boldsymbol{S}}_t \triangleq \left[\hat{\boldsymbol{h}}_{1,t} \ \cdots \ \hat{\boldsymbol{h}}_{K,t}\right]^{\mathsf{T}}.$$

Alternatively, we have

$$nR_k \leq h(y_1^n, \ldots, y_k^n \,|\, W_1, \ldots, W_{k-1}, \Omega^n)$$
$$- h(y_1^n, \ldots, y_k^n \,|\, W_1, \ldots, W_k, \Omega^n) + n\epsilon. \tag{16}$$

Therefore, it follows that

$$\sum_{k=1}^{K} \frac{n}{k'}(R_k - \epsilon)$$
$$\leq \sum_{k=1}^{K-1}\sum_{t=1}^{n}\Bigg(\frac{h(y_{1,t}, \ldots, y_{k+1,t} \,|\, y_1^{t-1}, \ldots, y_k^{t-1}, W_1, \ldots, W_k, \Omega^n)}{(k+1)'}$$
$$- \frac{1}{k'} h(y_{1,t}, \ldots, y_{k,t} \,|\, y_1^{t-1}, \ldots, y_k^{t-1}, W_1, \ldots, W_k, \Omega^n)\Bigg)$$
$$+ n \log P + n\, o(\log P) \tag{17}$$
$$\leq \log P \sum_{k=1}^{K-1} \sum_{t=1}^{n} \frac{(k+1)' - k'}{k'(k+1)'} \sum_{i=1}^{k} \alpha_{i,t} + n \log P + n\, o(\log P) \tag{18}$$

$$= n \log P \sum_{i=1}^{K-1} \Big(\frac{1}{k'} - \frac{1}{K'}\Big)\bar{\alpha}_i + n\log P + n\, o(\log P) \quad (19)$$

where we define $k' \triangleq \min\{k, M\}$, the inequality (17) is due to 1) the chain rule of differential entropy, 2) the fact that removing condition does not decrease differential entropy, 3) $h(y_{1,t}\,|\,\Omega^n) \le \log P + o(\log P)$, i.e., Gaussian distribution maximizes differential entropy under covariance constraint, and 4) $h(y_1^n, \dots, y_K^n \,|\, W_1, \dots, W_K, \Omega^n) = h(z_1^n, \dots, z_K^n) > 0$; (18) is from Proposition 3 by setting $U = \{y_1^{t-1}, \dots, y_k^{t-1}, W_1, \dots, W_k, \Omega^n\} \setminus \{\boldsymbol{S}_t, \hat{\boldsymbol{S}}_t\}$, $H = \boldsymbol{S}_t$, and $\hat{H} = \hat{\boldsymbol{S}}_t$; the last equality is obtained after putting the summation over k inside the summation over i and some basic manipulations. Similarly, we can interchange the roles of the users and obtain the same genie-aided bounds. Finally, the single antenna constraint gives that $d_i \le 1,\ i = 1, \cdots, K$. With this, we complete the proof.

IV. Some achievability proofs

We here provide the sketches of some achievability proofs, leaving more details in [18] due to the lack of space.

A. Proof of Proposition 1

The achievability scheme is based on time sharing between two strategies of CSIT feedback, i.e., delayed CSIT feedback with $\delta_{\mathrm{P}} = 0$ and alternating CSIT feedback with $\delta_{\mathrm{P}} = \frac{2}{K}$, where the first strategy achieves $d_{\sum} = 3/2$ by applying Maddah-Ali and Tse (MAT) scheme (see in [4]), with the second strategy achieving $d_{\sum} = 2$ by using alternating CSIT feedback manner (see in [6]).

B. Proof of Proposition 2

For the case with $M \ge K$, the proposed scheme is based on time sharing between delayed CSIT feedback with $\delta_{\mathrm{P}} = 0$ and full CSIT feedback with $\delta_{\mathrm{P}} = 1$, where the first feedback strategy achieves $d_{\sum} = d_{\mathrm{MAT}}$ by applying MAT scheme, with the second one achieving $d_{\sum} = K$.

Similar approach is exploited for the case with $M < K$. In this case, we apply time sharing between delayed CSIT feedback with $\delta_{\mathrm{P}} = 0$ and alternating CSIT feedback with $\delta_{\mathrm{P}} = M/K$.

V. Conclusions

This work considered the general multiuser MISO BC, and established inner and outer bounds on the tradeoff between DoF performance and CSIT feedback quality, which are optimal for many cases. Those bounds, as well as some analysis, were provided with the aim of giving insights on how much CSIT feedback to achieve a certain DoF performance.

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