Iterated Function Systems
for still image processing

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1 Introduction

Since the publication of Arnaud Jacquin's article [2] on still image coding using Iterated Function Systems (IFS) [1], more and more papers have appeared on IFS for image processing and coding [7]. Current work on this topic can be classified into four main categories, depending on the problem considered: statement, implementation, extension and functionality.

The category "statement" mainly covers work linked to contractivity constraint notions, general formulation and basic aspects of the algorithm. Although some theoretical problems in using IFS for image compression remain [3], the majority of studies currently available in the literature deal with implementation aspects such as segmentation, domain blocks classification, reduction of computation complexity, and combination of IFS with some other techniques such as DCT. Several relevant papers have proposed improvements on Jacquin's algorithm [3]. By extending the basic algorithm adapted for still grey-level images, some authors also work on a possible design of the method for video, color, and multispectral images. After a brief review of Jaquin's algorithm in section 2, we focus on the last category of studies. In particular, this paper deals with a possible implementation of some tools such as zoom [4], described in the section 3, and some security functionalities such as access control [6], described in section 4, as part of a coding scheme based on the IFS technique.

2 Recalls on image compression using IFS

Given an original image μ, the goal is to build a lossy representation τ of this image using a fractal technique. The image $\tilde{\mu}$ reconstructed from τ is obtained using an iterative process as follows [1]: $\mu_1 = \tau(\mu_0)$, $\mu_2 = \tau(\mu_1)$, ..., $\tilde{\mu} = \tau^\infty(\mu_0) = \tau(\tilde{\mu})$ for an arbitrary initial image μ_0. In order to obtain $\tilde{\mu} \approx \mu$, the coding stage is based on the minimization of the collage error $\varepsilon_c = d(\mu, \tau(\mu))$ (where d is the metric used), and then, according to the collage theorem, the reconstruction error $\varepsilon_r = d(\mu, \tilde{\mu})$ is upper bounded by the quantity:

$$\varepsilon_r \leq \frac{\varepsilon_c}{1 - s} \tag{1}$$

where s is the contractivity factor of the transform τ. In order to reduce the coding complexity, the image μ is divided into N non-overlapping blocks [2], called the range blocks. Each range block R_i, for $i \in \{1, ..., N\}$, is coded independently thanks to some matching τ_i with another bigger block D_i of the image μ, called a domain block. The global fractal code τ is then given by $\tau = \cup_{i=1}^{N} \tau_i$. Moreover, each local code τ_i is composed by a reduction, a discrete isometry and an affine transformation on the luminance. τ_i could be modelized by the following affine mapping:

$$
\tau_i \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} a_i & b_i \\ c_i & d_i \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \boxed{\begin{matrix} 0 & 0 \end{matrix}} & s_i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}
$$
$$
+ \begin{pmatrix} \boxed{\begin{matrix} t_{i,1} \\ t_{i,2} \end{matrix}} \\ o_i \end{pmatrix} \tag{2}
$$

where a_i, b_i, c_i, d_i, $t_{i,1}$, $t_{i,2}$ represent the geometric transforms and s_i, o_i the grey-levels transform ; x, y are the pixel coordinates and z the corresponding luminance value.

3 Zooming using IFS

Thanks to an iterative decoding process which uses a "fractal" transform τ, the corresponding attractor $\tilde{\mu}$ is a fractal object. The original image μ has a fixed size defined by its number of pixels, but the code τ, built by taking advantage of some image self-similarities, has no intrinsic size and is theoretically scale-independent. Then, by applying the transformation τ on an initial image μ_0, we could obtain a restitution $\tilde{\mu}$ of the original image with the same resolution as μ_0 (see Fig. 1). Hence, thanks to this coding, we eliminate the fixed resolution aspect of digitized images. The fractal zoom is mainly based on this remark. But the result of such a zoom is visually rather poor (see Fig. 2(c)), although there is no pixelization as when using the pixel duplication (see Fig. 2(a)). Indeed, the fractal zoom causes an important "blocking effect" due to independent and lossy coding of the range
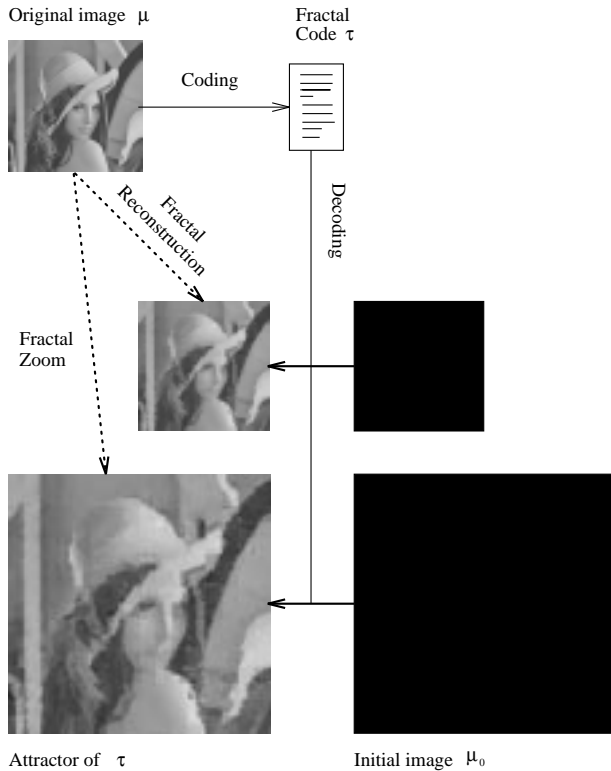
Figure 1: *Fractal zoom scheme.*

Figure 2: *Zoom by a factor 6 on Lenna's left eye using* (a) *nearest neighbor interpolation,* (b) *linear interpolation,* (c) *classical fractal technique, and* (d) *improved fractal technique without postprocessing.*

blocks. To obtain a good visual quality, some improvements have been made, such as the use of overlapping range blocks [4]. In this case, the coding becomes redundant in overlapping regions of the image μ, and then we average these parts in order to smooth the block effect and to reduce the collage error by chosing among several values for each considered pixel. This yields a zoomed image (see Fig. 2(d)) with a sharper quality than the classical linearly interpolated image (see Fig. 2(b)), obtained using a luminance continuity hypothesis. Unfortunately, this improvement is at the present time at the price of degrading the compression performance. However, this method allows the image to be displayed, from a unique code, at different levels of definition according to the service needs.

4   Hierarchical access control using IFS

The advent of multimedia applications has brought new requirements, especially in the security field [5]. Here we propose a hierarchical access control, a system that allows different levels of qualities according to the access fee paid. All the receivers of a broadcast channel can display an image, but only at low quality with no commercial value. This message received through the public channel is not totally hidden in order to attract potential customers who would apply for the commercial service to get the highest quality image. The IFS method is suitable to do this because it offers the possibility to control the reconstruction process. Let s_i be the scale factor of the grey level affine transform associated to the range block R_i (see Eq. 2). The quality o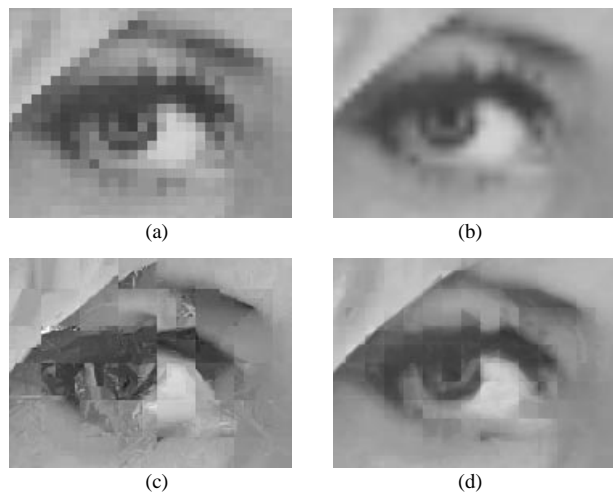f the image reconstruction is strongly related to this factor. Thus, our system is based on the degree of ciphering of scale factors. In this way, we introduce the quality level of visualization according to the number of bits of s_i that are readable (see Fig. 3). For instance, if s_i is quantized with 8 bits, the different levels of qualities are obtained through the masking from 5 up to 8 bits of s_i. Hence, we propose a hierarchical access control scheme which provides both compression and security functions within a single algorithm. Note that the multi-resolution access can be achievied without any degradation of compression performance. The security evaluation of this scheme is performed in [6]. Nevertheless, we note that, since the protection mechanism is integrated with coding, specific waveforms of images are taken into account in the design of the access control mechanism as opposed to a general encryption algorithm. Thus, instead of encrypting the whole encoded image, only key parameters of the fractal code τ are kept secret in order to get a sufficient level of security.

5   Concluding remarks

Although IFS is not a fully understood technique, fractal image coding has been used successfully to encode still grey level images. Until now, efforts have mainly focused on the compression aspect of IFS.
Nethertheless, with the growth of multimedia applications and communications, some future coding schemes, such as MPEG-4, have emerged. These schemes consider some functionalities and tools in addition to compression. It is very recognized that IFS is a new and interesting technique for image coding. In addition, in this paper, we have tried to demonstrate that IFS may also be a very useful technique for simultaneously performing image coding and processing. Two methods have been described: the first one provides a way to zoom a picture and the second one to control its access. These methods exploit some particular properties inherent in fractal signal processing
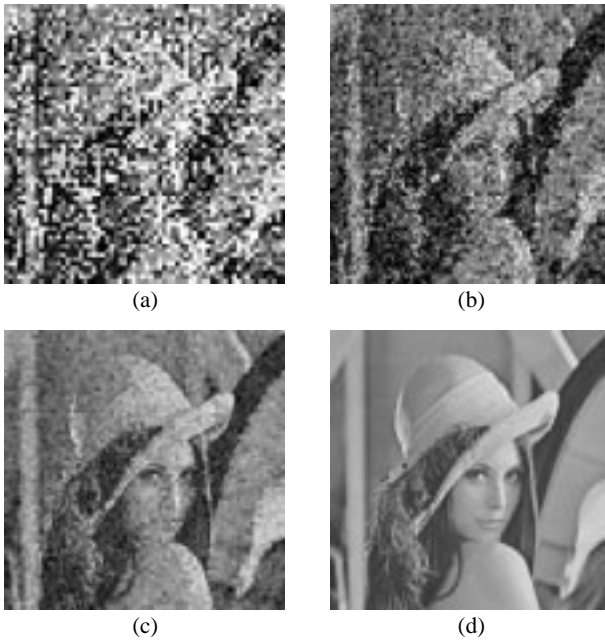
(a)   (b)

(c)   (d)

Figure 3: *Fractal compression of Lenna face with access control using* (a) *8/8 protected bits,* (b) *6/8 protected bits,* (c) *5/8 protected bits, and with* (d) *no access control (0/8 protected bits).*

(scale-independent) and more specifically to the IFS technique. In practice, both zoom and security could be combined for a single way of controlling the image definition.

One future direction for this work could consist of extending the use of these tools and functionalities from still images to video. Moreover, this study may also indirectly contribute to improvements in the general understanding for the use of IFS in the field of image coding.

6 Acknowlegments

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