# DEGREES OF FREEDOM IN THE MISO BC WITH DELAYED-CSIT AND FINITE COHERENCE TIME: A SIMPLE OPTIMAL SCHEME

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## ABSTRACT

We consider the multi-input single-output (MISO) Broadcast Channel (BC), the multi-user (MU) downlink in a cell with a multi-antenna base station and mobile terminals equipped with a single antenna. Most techniques designed for this channel require accurate current channel state information at the transmitter (CSIT). However, a radically different approach has been proposed by Maddah-Ali and Tse (MAT), in which significant gain can be obtained by solely relying on perfect but outdated CSIT. This approach is proven to yield the optimal multiplexing gain when the channel current state is completely independent of the fed back channel state. A recent work focused on an intermediate case, channels exhibiting some temporal correlation, proposed a complex scheme shown to be optimal in terms of multiplexing gain for the 2-user case. Using a different but equivalent channel model, we propose a simple transmission scheme for the general case $K \in \mathbb{N}$ that reaches the optimal number of degrees of freedom.

## 1. INTRODUCTION

Interference is a major limitation in wireless networks and the search for efficient ways to transmit in this context has been productive and diversified [1–3]. Numerous techniques allow to increase the multiplexing gain. For instance in a multi-user context, dirty paper coding allows the transmitter to send information to multiple users simultaneously with the interferences pre-canceled [4]. In the interference channel, channel state information at the transmitter (CSIT) can be used to align the interferences from multiple receivers thereby reducing or even eliminating their impact. However these techniques rely on perfect current CSIT which is not practical. CSIT is by nature delayed and imperfect. Though interesting results have been found concerning imperfect CSIT [5], feedback delay can still be an issue especially if it approaches the coherence time of the channel. However a recent work [6] completely changed the paradigm by proposing a scheme yielding degrees of freedom (DoF) greater than one by relying solely on perfect but outdated CSIT, thus allowing for some multiplexing gain even if the channel state changes independently over the feedback delay. Their technique is referred to hereafter as the Maddah-Ali-Tse (MAT) scheme. The range of coherence time in which the sole use of the MAT scheme yields an increased multiplexing gain is determined in [7, 8].

The assumption of totally independent channel variation is overly pessimistic for numerous practical scenarios. Therefore another scheme was proposed in [9] for the time correlated MISO broadcast channel with 2 users. This scheme optimally combines delayed CSIT and current CSIT (both imperfect) but has not been generalized for a larger number of users. The scheme we propose combines zero-forcing (ZF) beamforming (BF) and MAT schemes to reach the optimal multiplexing gain accounting for CSIT delay only; we shall denote our scheme for the MISO BC with $K$ users by MAT-ZF$_K$. Our scheme essentially performs ZF and superposes MAT only during the dead times of ZF. We will show that the MAT-ZF scheme recovers the results of optimality of [9] for $K = 2$ but MAT-ZF is valid and optimal in terms of DoF for any number of users. The MAT-ZF scheme is based on a block fading model but we will show that stationary fading can be modeled exactly as a special block fading model.

We will then compare the multiplexing gains that ZF, MAT and MAT-ZF can be expected to yield in actual systems, accounting for training overhead as well as the DoF loss due to the feedback on the reverse link. As opposed to [8], in the net DoF we also subtract the DoF consumed in the reverse link, as in [10]. In general, weighted net DoF could be considered as in [10] since forward and reverse link rates could have different weights. We consider here

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unweighted net DoF from which weighted net DoF can easily be extrapolated. Note that the ZF scheme considered in [8] is different and does not have any dead time. However, ZF BF in [8] is based on predicted CSIT only, leading to some DoF loss. Also note that the channel model in [8] is somewhat approximate as it considers the channel variation as piecewise constant in transmit blocks, and a stationary variation between the blocks.

## 2. SYSTEM MODEL

We consider a MISO BC with a base station (BS) with $M$ transmit antennas and $K$ single antenna receivers. Below we shall typically assume $K = M$ (often leading to an interchangeable use of $K$ and $M$) since this leads to maximum DoF, unless the relative CSI overhead becomes too important in which case the optimal number of users $K$ (and corresponding active BS antennas) decreases. The channel is modeled by

$$y_k[t] = \mathbf{h}_k^*[t]\mathbf{x}[t] + z_k[t]$$

where $y_k[t]$ is the received signal of user $k$ at symbol time $t$, depends on $\mathbf{h}_k^*[t] \in \mathcal{C}^{1\times M}$ the channel state vector, $\mathbf{x}[t] \in \mathcal{C}^{M\times 1}$ the transmit signal and $z_k[t]$ the additive white Gaussian noise (AWGN) and $(\cdot)^*$ denote the Hermitian transpose. The channel matrix is defined as $\mathbf{H}[t] = [\mathbf{h}_1[t], \cdots, \mathbf{h}_K[t]]^* \sim \mathcal{CN}(0,1)^{K\times M}$ and remains constant over $T_c$ symbols i.e., the channel is assumed to be Rayleigh block-fading. We assume delayed (delay $T_{fd}$ symbol periods) but otherwise perfect CSIT and perfect instantaneous CSIR.

The performance metric is the number of degrees of freedom (DoF) (also called multiplexing gain), it is the prelog of the sum rate. Let $R(P)$ be the ergodic throughput of a MISO BC with $K$ receivers and transmit power $P$ then:

$$\text{DoF}(K) = \lim_{P\to\infty} \frac{R(P)}{\log_2(P)}, \quad \text{DoF}_{FB}(K) = \lim_{P\to\infty} \frac{F(P)}{\log_2(P)}$$

where in order to take into account the feedback cost we define the feedback overhead with $F(P)$ being the total feedback rate.

## 3. UNIFICATION STATIONARY & BLOCK FADING

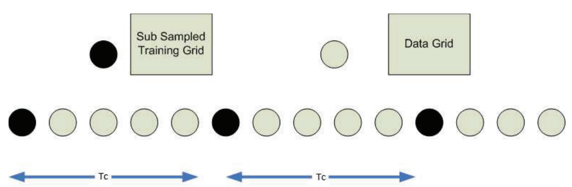


**Fig. 1**. Subsampling and polyphase representation of a bandlimited channel coefficient signal.

In what follows we focus on any scalar channel coefficient separately, which is sufficient for DoF considerations (to be optimal at finite SNR, all correlated channel coefficients should be treated jointly though). Assume the channel coefficient $h[t]$ has a Doppler spectrum strictly bandlimited to $1/T_c$. Assume for a moment $T_c$ to be an integer number of symbol periods. Then according to Nyquist's theorem, we can subsample it with a factor $T_c$ and the remaining $T_c$-1 polyphase components can be obtained from the first one by filtering (linear interpolation). So if $\mathbf{h}[iT_c] = [h[iT_c]\, h[iT_c+1] \cdots h[(i+1)T_c - 1]]^T$ is the vectorized process at subsampled time instants $i$, then $\mathbf{h}[iT_c] = \mathbf{g}(q)\, h[iT_c]$ for some SIMO filter $\mathbf{g}(z) = [1\; g_2(z) \cdots g_{T_c}(z)]^T$ where $g_j(z)$ represents the interpolation filter for obtaining the $j^{\text{th}}$ polyphase component from the first one. So, the matrix spectrum $S_{\mathbf{hh}}(z) = \mathbf{g}(z)\, S_{hh}(z)\, \mathbf{g}^\dagger(z)$ has rank one. As a result, the infinite order MIMO prediction error $\widetilde{\mathbf{h}}[iT_c]$ has a rank one covariance matrix $R_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}} = \widetilde{\mathbf{g}}\,\widetilde{\mathbf{g}}^*$ where all entries of the vector $\widetilde{\mathbf{g}}$ are non-zero. Hence, the channel evolution in the current coherence period can be represented as

$$\mathbf{h}[iT_c] = \widehat{\mathbf{h}}[iT_c] + \widetilde{\mathbf{h}}[iT_c] = \widehat{\mathbf{h}}[iT_c] + \widetilde{\mathbf{g}}\,\widetilde{h}[iT_c] \tag{1}$$

where $\widetilde{h}[iT_c]$ is a scalar white noise process. In order to learn $h[t]$ over the current coherence period $i$, it suffices to learn the first component $h[iT_c]$, which together with the prediction $\widehat{\mathbf{h}}[iT_c]$ (which at the scalar level represents multi-lag ahead predictions from the previous coherence periods) allows to learn the scalar $\widetilde{h}[iT_c]$ and hence allows to determine the rest of the $T_c$-1 channel coefficients over the current coherence period. So the entries of the eigenvector $\widetilde{\mathbf{g}}$ determine a basis vector, replacing the rectangular window basis function in the usual block fading model (in which $\widetilde{\mathbf{g}}$ would be a vector of ones, and $\widehat{\mathbf{h}} = 0$). In the case of rational $T_c = m/n$, a similar reasoning would lead to a prediction error rank $m$ over a block length $n$. In case of multiple users with different $T_c$, the block length could be taken as their least common multiple (lcm) and the prediction error ranks would be different for different users. Here we shall continue to consider an identical integer $T_c$ for the users.

## 4. BACKGROUND

With the block fading model and feedback delay $T_{fd}$, each block can be split into two parts. The current channel state is unknown to the transmitter for $t < T_{fd}$ and then the transmitter has full CSI for $t \geq T_{fd}$. Our idea is merely to use two different techniques within each block, the MAT scheme when the current channel state is unknown and then ZF for $t \geq T_{fd}$. Both techniques have been proven to be optimal in terms of multiplexing gain in their respective settings. We first review the multiplexing gains achievable with these schemes.

### 4.1. ZF

When CSI is available at the transmitter full multiplexing gain can be achieved with ZF [11], in other words it is possible to transmit 1 symbol per channel use per user with this technique. It merely relies on the transmitter using a pseudo in-

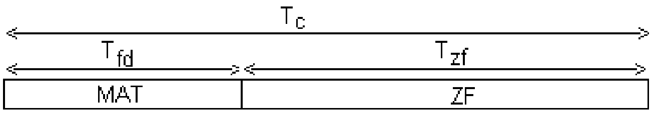


**Fig. 2**. The MAT-ZF scheme over one coherence period.

verse of the channel as precoder thereby zero-forcing all inter-user interferences. Doing only ZF would allow to transmit 1 symbol per channel use in the second part of each block and nothing in the first part, thus yielding

$$\text{DoF}(\text{ZF}_K) = K\text{DoF}(\text{ZF}_1) = K\left(1 - \frac{T_{fd}}{T_c}\right). \qquad (2)$$

### 4.2. MAT

The MAT scheme was proposed in [6]. The authors describe an innovative approach that allows to reach a multiplexing gain of

$$\frac{K}{1 + \frac{1}{2} \cdots \frac{1}{K}} = \frac{KD}{Q} \qquad (3)$$

with no *current* CSIT at all. Here $\{D, Q\} \in \mathbb{N}^2$ such that $\frac{1}{1+\frac{1}{2}\cdots\frac{1}{K}} = \frac{D}{Q}$, where $D$ is the least common multiple of $\{1, 2, \cdots, K\}$ and $Q = DH_K$ with $H_K = \sum_{k=1}^{K} \frac{1}{k}$. This scheme allows the transmission of $D$ symbols in Q channel uses for each user as noted in [7].

## 5. MAIN RESULTS

We first propose our scheme, MAT-ZF$_K$ then prove its optimality. In [9] and [12] the authors propose a scheme for the stationary fading model with 2 users. Using the equivalence of the two fading models we will see that our scheme achieves the same multiplexing gain.

### 5.1. General K user case

The idea is essentially to perform ZF and superpose MAT only during the dead times of ZF. For that purpose we consider $Q$ blocks of $T_c$ symbol periods and split each block into two parts as in Fig. 2. The first part, the dead times of ZF, spans $T_{fd}$ symbol periods and the second part, the $T_c - T_{fd}$ remaining symbols. We use the first part of each block to perform the MAT scheme $T_{fd}$ times in parallel. During the second part of each block, ZF is performed.

**Theorem 1** *The sum DoF for the MAT-ZF$_K$ scheme is*

$$DoF(MAT\text{-}ZF_K) = K\left(1 - \frac{(Q-D)T_{fd}}{QT_c}\right). \qquad (4)$$

**Proof** Per user, in $QT_c$ channel uses, the ZF portion transmits $Q(T_c - T_{fd})$ symbols, whereas the MAT scheme transmits $DT_{fd}$ symbols. ■

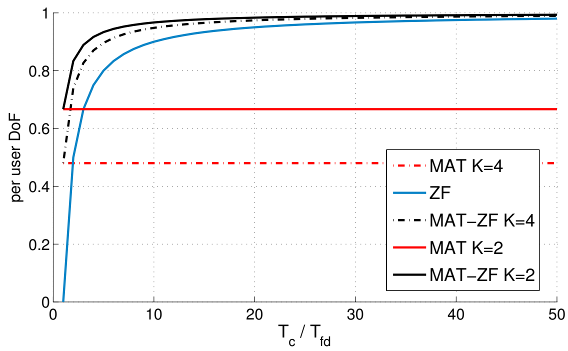


**Fig. 3**. Per user DoFs as a function of $T_c/T_{fd}$ for $K \in \{2, 4\}$.

**Theorem 2** *The MAT-ZF$_K$ scheme is optimal in terms of sum multiplexing gain i.e., for any transmission scheme $\psi_K$ for the MISO BC with $K$ users, DoF($\psi_K$) $\leq$ DoF(MAT-ZF$_K$).*

**Proof** The MAT-ZF$_K$ approach as in Fig. 2 decomposes the channel with feedback delay into two orthogonal parts: the ZF part in which CSIT is perfect, and the MAT part with delayed CSIT. In the ZF part, the relative portion of which is maximal, ZF allows to obtain the DoF of the full CSIT case. In the MAT part, the MAT scheme has been shown to maximize DoF for the case of delayed CSIT with block size equal to $T_{fd}$. ■

In Fig. 3 we plot the per user DoFs of the 3 schemes for $K \in \{2, 4\}$, using (2) for ZF, (3) for MAT and (4) for MAT-ZF. The DoFs are plotted as a function of the ratio between the coherence time of the channel and the feedback delay $\frac{T_c}{T_{fd}}$. For large values of the ratio, the MAT-ZF and ZF schemes are very close to the optimum, 1. For small values of the ratio we observe that by optimally combining ZF and MAT. the MAT-ZF scheme allows to reach a significantly larger DoF than ZF or MAT separately. Nervertheless, the DoF being an increasing function of the ratio, a longer coherence time or a smaller feedback delay is better. Since the coherence time is a fixed parameter of the channel we understand that the feedback delay should be reduced to its minimum in order to improve the multiplexing gain. We can already notice that for $K = 2$ the gap between MAT-ZF and pure ZF is larger than for $K = 4$ hinting that the gain due to the optimal combining of MAT and ZF could be decreasing with the number of users.

### 5.2. The two user case in detail

In the MISO BC with $K = 2$ the MAT scheme in [6] and variants in [9] have been proposed and allow to transmit 2 symbols to each receiver in 3 channel uses. In the original scheme during the first channel use a combination of the 2 symbols intended for the first receiver is sent and during the second a combination of the 2 symbols intended for the second receiver. The third symbol is sent once the transmitter has received the feedback from the two receivers allowing for the transmission of a combination of the interferences overheard by the two receivers which permits them to decode their two symbols. Then, during the second part of each MAT-ZF block, the channel is known by the receiver, therefore by performing ZF BF it is possible to transmit 1 symbol per channel use to each receiver without interference. According to Theorem 1, DoF(MAT-ZF$_2$) $= 2\left(1 - \frac{T_{fd}}{3T_c}\right)$

In [9] the authors proposed a scheme for the time correlated MISO BC with 2 users. The DoF reached by their scheme is expressed as a function of $\alpha$, a constant characterizing the predictability of the channel. This scheme is proved to be optimal and to yield $\frac{2+\alpha}{3}$ DoF per user. A practical example of $\alpha$ is given for a bandwidth-limited process with channel coefficients limited to $[-F, F]$, $\alpha = 1-2F$. We have the correspondence $\alpha = 1 - \frac{T_{fd}}{T_c}$ when a channel is limited by the coherence time. When doing ZF only one can achieve $\alpha = 1 - \frac{T_{fd}}{T_c}$ DoF . Using this equivalence in (4) leads to DoF(MAT-ZF$_2$) $= 2\left(\frac{2+\alpha}{3}\right)$, which is the same DoF as in [9].

### 5.3. DoF Region

Our scheme allows for a simple analysis of the DoF region. Let $d_k$ be a rate for user $k$ in MAT-ZF which uses $d_{MAT_k}$ in the MAT portion and $d_{ZF_k}$ in the ZF portion. Then the rate vector $\mathbf{d} = \{d_1, d_2, \cdots, d_K\}$ is attainable by MAT-ZF if and only if it can be written as the following weighted sum:

$$\mathbf{d} = \frac{T_{fd}}{T_c}\mathbf{d}_{MAT} + \mathbf{d}_{ZF} \tag{5}$$

where $\mathbf{d}_{MAT}$ is in the MAT$_K$ DoF region and $\mathbf{d}_{ZF}$ is in the ZF$_K$ DoF region. Note that $d_{ZF_k}$ here accounts for $T_{fd}$ as in (2). If on the other hand $d_{ZF_k}$ would refer to full CSI DoF, then we would get the convex combination $\mathbf{d} = \frac{T_{fd}}{T_c}\mathbf{d}_{MAT} + (1-\frac{T_{fd}}{T_c})\mathbf{d}_{ZF}$. In other words with decreasing feedback delay the DoF region smoothly goes from the MAT DoF region to the ZF DoF region.

### 5.4. Asymptotic Analyses

The MAT-ZF DoF can be written as DoF(MAT-ZF$_K$) $= K\left(1 + \frac{T_{fd}}{H_K T_c} - \frac{T_{fd}}{T_c}\right)$. The difference in multiplexing gain between the optimal MAT-ZF and pure ZF schemes is:

$$\text{DoF(MAT-ZF}_K) - \text{DoF(ZF}_K) = \frac{K}{H_K}\frac{T_{fd}}{T_c} \approx \frac{K}{\ln K}\frac{T_{fd}}{T_c}$$

which approaches 0 when the ratio $\frac{T_c}{T_{fd}}$ becomes large, i.e., pure ZF gets close to optimal for small feedback delay. On the other hand, as for large K,$H_K \approx \ln K$, the per user DoF difference between ZF and MAT-ZF tends to 0 as the number of users $K$ increases.

## 6. NET DoF CHARACTERIZATION

In order to compare the multiplexing gains that MAT, ZF and MAT-ZF can be expected to obtain in actual systems, we derive their netDoFs, accounting for training overhead as well as the DoF loss due to the feedback on the reverse link.

### 6.1. General Feedback and Training Considerations

Since we are interested in the DoF$_{FB}$ which is the scaling of the feedback rate with $\log_2(P)$ as $P \to \infty$, the noise in the fed back channel estimate can be ignored in the case of analog feedback or of digital of equivalent rate. The feedback can be considered accurate, suffering only from the delay $T_{fd}$. We consider channel feedback and output feedback. Since for DoF only the channel vector direction is needed at the transmitter, by normalizing the channel vector only $M-1$ coefficients per channel vector need to be fed back. However, channel feedback requires determining a channel estimate at the end of a training period. A smaller feedback delay can be assured by using output feedback, in which the receivers directly feed back the training signal they receive and the transmitter performs the (downlink) channel estimation. This costs $M$ channel uses per user instead of $M-1$ but it reduces the feedback delay. For these reasons we will consider output feedback for ZF and MAT-ZF whereas MAT can benefit from the reduced feedback overhead of channel feedback since it is not sensitive to the feedback delay. For the three schemes in each block a common training of length $T_{ct} \geq M$ is needed to estimate the channel. To maximize DoF we take $T_{ct} = M = K$.

### 6.2. ZF

Once the common training and the output feedback have been done, the transmitter has the CSI and an additional dedicated training of only one channel use, $T_{dt} = 1$, is needed to insure coherent reception according to [13]. This results in $K + 1$ channel uses per block dedicated to the training yielding for $K$ users a training overhead of

$$\text{Tr}(ZF_K) = \frac{K(K+1)}{T_c}$$

and $KM$ channels uses for the output feedback yielding a feedback overhead of

$$\text{DoF}_{FB}(ZF_K) = \frac{KM}{T_c} = \frac{KK}{T_c}\,.$$

The net (sum) multiplexing gain is then:

$$\text{netDoF(ZF}_K) = K\left(1 - \frac{T_{fd}}{T_c} - \frac{2K+1}{T_c}\right). \tag{6}$$

### 6.3. MAT

In the MAT scheme the feedback is needed so that the transmitter can align the interferences for a user in subspace of dimension $Q - D$. Therefore each receiver needs to feed back

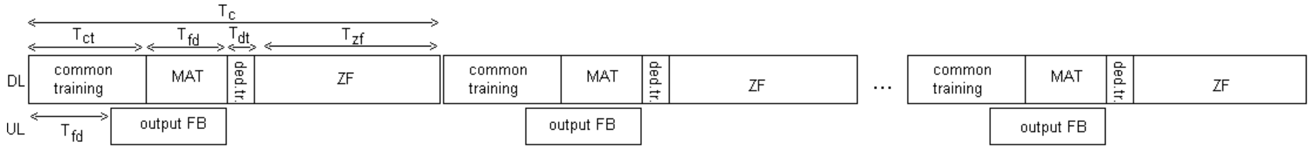


**Fig. 4**. The MAT-ZF scheme over a few coherence periods.

its channel vector $Q-D$ times. We assume channel feedback to reduce the overhead since this scheme is not sensitive to feedback delay. Then a total of $K(K-1)(Q-D)$ channel uses are needed to do the feedback over $Q$ blocks, yielding a feedback overhead of $K(K-1)(Q-D)/QT_c$ or

$$\mathrm{DoF}_{FB}(MAT_K) = \frac{K(K-1)(H_K-1)}{H_K T_c}$$

and the cost of the common training is the same as for ZF $\mathrm{Tr}(MAT_K) = \frac{K^2}{T_c}$.

To perform the MAT scheme each receiver needs to know the channels of all receivers resulting in the need for CSIR distribution. According to [8] the CSIR distribution for each block of phase $j$ can be limited to the transmission of $K-j$ channel states, since only $M-j+1$ antennas are active at phase $j$ and assuming channel feedback this results in a total CSIR distribution length of

$$\mathrm{L}_{CSIR}(MAT_K) = \sum_{j=1}^{K} \frac{D(K-j)^2}{j} = K^2(Q - \frac{3}{2}D) - \frac{DK}{2}.$$

The resulting net multiplexing gain is

$$\mathrm{netDoF}(\mathrm{MAT}_K) = K\frac{T_C - (K-1)(H_K-1) - KH_K}{H_K T_c + \sum_{j=1}^{K} \frac{(K-j)^2}{j}} \quad (7)$$

### 6.4. MAT-ZF

In the MAT-ZF scheme we perform ZF and MAT. Since the training for ZF comprises the training needed for MAT, the training cost for MAT-ZF is the same as for ZF, $\mathrm{Tr}(MAT-ZF_K) = \frac{K(K+1)}{T_c}$. In Fig. 4 we illustrate the composition of the blocks of this scheme with feedback and training. In order to perform MAT, the CSIR distribution is required. The scheme was initially meant to be done over $Q$ blocks to perform the MAT scheme but we add more blocks to do the CSIR distribution. We only use the MAT part of the additional coherence blocks to do the CSIR distribution while we still perform ZF when the transmitter has CSI in order to avoid any degradation of the ZF DoF. The MAT part then requires $\Delta = \frac{L_{CSIR}(MAT_K)}{T_{fd}}$ additional blocks. It should actually be the smallest integer not less than this fraction but by repeating the scheme more than once the number of blocks to add per scheme can be reduced to this exact value. Let $\delta = \frac{\Delta}{D}$, then the netDoF of this scheme is $\mathrm{netDoF}(\mathrm{ZF}_K) + \frac{KDT_{fd}}{T_c(Q+\Delta)}$ or

$$\mathrm{netDoF}(\text{MAT-ZF}_K) = \mathrm{netDoF}(\mathrm{ZF}_K) + \frac{T_{fd}}{T_c}\frac{K}{(H_K+\delta)} \quad (8)$$

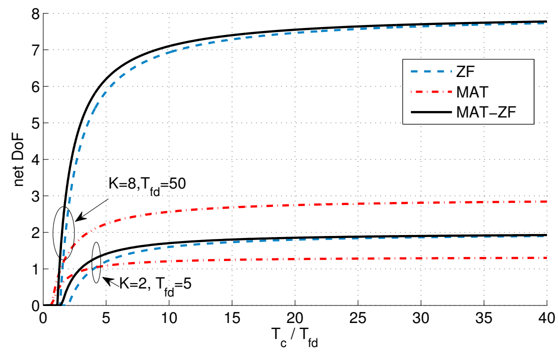


**Fig. 5**. netDoFs yielded by ZF, MAT and MAT-ZF for $K=8$, $T_{fd}=50$ and for $K=2$, $T_{fd}=5$ as a function of $\frac{T_c}{T_{fd}}$.

i.e., the netDoF of ZF plus an additional term, the DoF brought about by MAT but decreased by a factor due to the CSIR distribution. From (8) we can analyze the behavior of the expected gain of MAT-ZF over ZF, it increases with $T_{fd}$, decreases with $T_c$ and decreases with $K$ since $\delta = O(K^2 \ln K)$, .

## 7. NUMERICAL EXAMPLES

In Fig. 5 we plot the netDoFs of the three schemes for $(K, T_{fd}) = (8, 50)$ and $(2, 5)$, as a function of $\frac{T_c}{T_{fd}}$, using (6) for ZF, (7) for MAT and (8) for MAT-ZF. We observe that $T_{fd} = 50$ is large enough for the MAT-ZF scheme to yield some gain with 8 users. Indeed the loss that results in doing ZF only is significant and the part of each block dedicated to the MAT in MAT-ZF is large enough to compensate for the CSIR distribution cost. However the DoFs are increasing with the ratio $\frac{T_c}{T_{fd}}$ so the feedback delay should actually be made as short as possible. For $K=2$ we observe some gains because less users means less CSIR to distribute.

In Fig. 6 we plot the netDoFs for $(K, T_{fd}) = (8, 5)$. We observe that ZF and MAT-ZF give almost the same netDoFs. Indeed, the CSIR distribution needed in MAT-ZF prolonged the scheme with a large number of blocks because $K$ is large and the $T_{fd}$ is too small for MAT to bring significant gains.

In Fig. 7 we plot the netDoFs for $T_c = 30$, $T_{fd} = 5$ as a function of $M = K$. We notice that all the 3 schemes reach a maximum and then decrease. For each scheme there is an optimum number of users, depending on the system parameters, beyond which the overhead becomes too large.

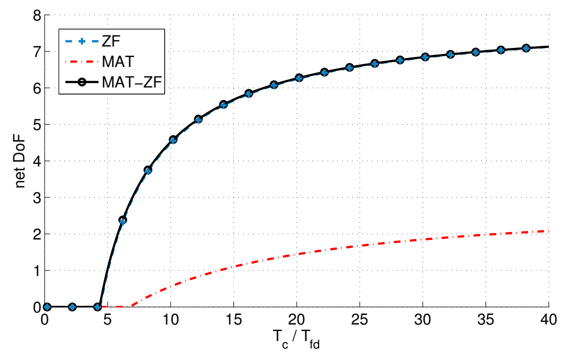


**Fig. 6**. netDoFs yielded by ZF, MAT and MAT-ZF for $K = 8$, $T_{fd} = 5$ as a function of $\frac{T_c}{T_{fd}}$.

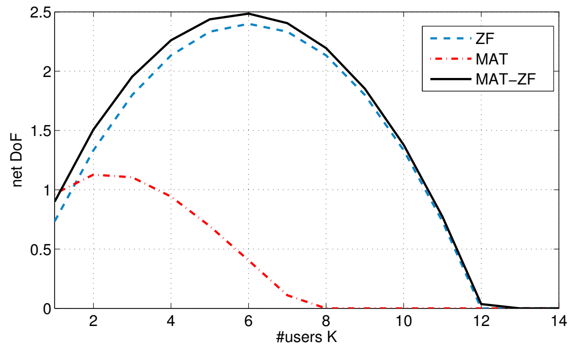


**Fig. 7**. netDoFs yielded by ZF, MAT and MAT-ZF for $T_c = 30$, $T_{fd} = 5$ as a function of $K$.

## 8. CONCLUDING REMARKS

We first showed that for the purpose of DoF analysis, working with a basic i.i.d. block fading model is equivalent to working with stationary fading (for a more precise analysis, the block fading equivalent of stationary fading exists but is more involved). We then proposed a scheme for the MISO BC with delayed CSIT based on the observation that we can divide each block into two parts and combine two techniques that are optimal in each part. Thereby we find a simple scheme for the general $K$ user case that is optimal in terms of DoF. The proposed MAT-ZF scheme with straightforward adaptations also yields the optimal multiplexing gain when not only accounting for feedback delay but also for feedback and training overhead. The relative benefit of adding the MAT component over just simply considering the ZF scheme depends on the system parameters. While leading to a theoretically optimal combination, the complication is often unjustified, given the time span over which the MAT scheme needs to be implemented, especially for a larger number of users $K$. And especially if the feedback delay is small. Which advocates for the use of analog output feedback, in which the feedback delay can be made as small as the roundtrip propagation delay.

When the coherence time gets really short, the MAT-ZF scheme needs to be further modified by optimizing the number of users $k$ and corresponding number of active antennas $m = k \in [1, M]$. This will lead for extremely short coherence times to TDMA with $k = 1$, which does not require any CSIT (apart from rate information).

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