Contention Resolution and Channel Estimation in Satellite Random Access Channels

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***Abstract*—We consider the random access channel (RACH) of a multi-beam satellite system with universal frequency reuse. We benefit from the spatial diversity and the strong directivity of multiple antennas at the satellite to resolve contentions and perform channel estimation. We propose two approaches to detect active satellite terminals (ST) and estimate the channel coefficients of some or all the colliding STs: Grid Reduction (GR) and Successive Channel Cancelation (SCC) algorithm. The performance of the proposed approaches is assessed by numerical results and compared with the conventional satellite RACH system. Both approaches outperform the conventional RACH with Slotted-ALOHA and SCC algorithm shows better performance than GR approach.**

I. Introduction

The enhancement of the satellite RACH is going to play a crucial role in the development and success of modern Mobile Satellite Systems (SS). Already in current standards for interactive broadband networks, the RACH finds a wider utilization thanks to mechanisms that enable the transmission of short bursts and support capacity reservation for transmission of longer packets in the RACH. Additionally, in modern SS, with interactive consumer-type STs, traffic aggregation at STs will be greatly reduced and the utilization of RACH will become even more frequent [1].

Many recent studies focus on the RACH and several schemes have been proposed. Slotted Aloha (S-Aloha) [2], [3] was proposed 40 years ago, and its variant, Diversity Slotted Aloha (DS-Aloha) studied in [4] is currently widely used in SS. In [5], [6], Selective Reject Aloha (SR-Aloha) is proposed as an alternative to S-Aloha and DS-Aloha to avoid network synchronization. Thanks to a fragmentation of the packets and a selective Automatic Repeat Request (ARQ) mechanism working on fragments, it enables the retransmission only of colliding fragments instead of the full packet retransmission. In [1],[7], an enhanced RACH scheme for satellite access packet networks dubbed Contention Resolution Diversity Slotted ALOHA (CRDS-Aloha) is proposed. As the DS-Aloha, it requires packet retransmission but adopts iterative interference cancelation to solve contentions. The above mentioned techniques improve the performance of the RACH by modifying the protocol at the multiple access channel (MAC) layer. In this contribution, we present techniques applied at the physical layer to improve the RACH throughput. Thus, they are complementary to the mentioned techniques at MAC layer and can coexist.

In conventional multi-beam satellite RACH, a fixed beamforming network (BFN) is usually utilized and fixed beams are generated. They point to different adjacent positions to serve the full coverage area of the SS. In order to keep the interference from adjacent beams limited, frequency reuse is adopted. In S-Aloha, a beam can support a single ST per slot. Otherwise, a collision occurs and none of these STs can be detected by the system. The signal from each beam is independently processed and spatial diversity is not exploited.

We assume universal frequency reuse and utilize the spatial diversity and strong directivity of multiple antennas at the satellite to resolve contentions. In other words, diversity and directivity of the antennas provide an unique "signature" to each ST that is exploited for multiuser detection, i.e, contention resolution. It is obvious that a fundamental step of this technique requires the estimation of the unique signature for each ST. We propose two algorithms for the detection of transmitting STs, possible resolution of collisions by estimation of the unique signature and channel estimation, namely, Grid Reduction (GR) approach and Successive Channel Cancelation (SCC) approach. By utilizing strong directionality of the satellite antennas (SA), the knowledge of the radiation diagram, we can estimate the directivity vectors of the active STs. Then, based on this intermediate estimation and diversity in space and time, we estimate the instantaneous channel state information. In the Grid Reduction approach, we estimate the STs sequentially. At each iteration, the estimation of the directivity vector for the ST of interest, enables to narrow the searching area of the remaining undetected active STs. In the SCC approach, we estimate the channel realizations iteratively and remove the interference caused by the ST estimated in the current iteration. We evaluate the performance of the two approaches through numerical simulations. Compared to the conventional RACH system, both approaches provide significant improvements. Furthermore, SCC approach always outperforms GR approach. The performance of the RACH are also compared to the one of connection oriented channel studied in [8].

Notation remarks. Vectors and matrices are written in boldface lower case and capital letters, respectively. Superscript ^{H} and $\mathfrak{Re}(\cdot)$ denote conjugate transposition of a matrix and the real part operator, respectively. $\|\cdot\|_l$ denotes the norm l vector, $\|\cdot\|$ denotes the Euclidean norm. $\boldsymbol{A}^{(\backsim i)}$ represents the submatrices of the matrix $\boldsymbol{A}$ obtained by removing the i-th column and the i-th row.

II. System Model

We consider a satellite system consisting of a gateway, a bent-pipe satellite equipped with N SAs, and STs endowed with R antennas. All the antennas transmit in left and right polarizations. We focus on a RACH. Synchronized transmissions are assumed. The STs that want to transmit, initiate the transmission at the beginning of a certain slot and the signals are received synchronously at the gateway. During each slot, each ST transmits Q frames. In general, the duration of a slot transmission is longer than the coherence time of the channel while the frame duration is shorter. The gateway is oblivious of the number K of STs actually transmitting. Furthermore, the gateway is oblivious of the area where the transmitting STs are located. The STs share a training sequence set $\mathbb{X}$. The set $\mathbb{X}$ is partitioned into U groups. Each group consists of $2R$ different training sequences and the partition is known to the gateway. ST k selects randomly one group and transmit $2R$ training sequences, one for each antenna and polarization. The gateway is also oblivious of the specific training sequences chosen by the STs.

In this work, we adopt the same channel model and notation adopted in [8]. For the sake of completeness, we define it again here. Further discussions on the underlying assumptions for the system model can be found in [8].

The discrete-time baseband received signal at the gateway at time t during the transmission of frame q is given by

$$\boldsymbol{y}[t] = \boldsymbol{D}\boldsymbol{P}(q)\boldsymbol{x}[t] + \boldsymbol{z}[t], \tag{1}$$

where $\boldsymbol{y}[t]$ is the column vector of received signals at the gateway, $\boldsymbol{D}$ is the $2N \times 2K$ directivity matrix accounting for the radiation diagram and assumed constant during a slot. $\boldsymbol{P}(q)$ is the propagation matrix constant in frame q, $\boldsymbol{x}[t]$ is the $2RK$ vector of transmitted signals, and $\boldsymbol{z}[t]$ is the zero mean additive white Gaussian noise vector with covariance matrix $\sigma_z^2\boldsymbol{I}$. Let $\boldsymbol{x}_k[t]$ be the $2R$-dimensional vector of symbols transmitted in left and right polarization by the R antennas of ST k. Then, the vector $\boldsymbol{x}[t]$ of transmitted signals is obtained by stacking together the K vectors $\boldsymbol{x}_k[t]$, i.e.,

$$\boldsymbol{x}[t] = \left(\boldsymbol{x}[t]_1^H, \boldsymbol{x}[t]_2^H, ..., \boldsymbol{x}[t]_K^H\right)^H. \tag{2}$$

The propagation matrix $\boldsymbol{P}(q)$ is a block diagonal matrix with K independent blocks $\boldsymbol{P}^k(q)$ of size $2 \times 2R$ and form

$$\boldsymbol{P}^k(q) = \begin{pmatrix} P_{k,r}^{(1)}(q) & 0 & \cdots & P_{k,r}^{(R)}(q) & 0 \\ 0 & P_{k,l}^{(1)}(q) & \cdots & 0 & P_{k,l}^{(R)}(q) \end{pmatrix},$$

where $P_{k,o}^{(\ell)}(q)$ denotes the fast fading coefficient affecting the link between the satellite and antenna ℓ at ST k in o-polarization[1].

The directivity matrix $\boldsymbol{D}$ can conveniently be structured in KN blocks of the form

$$\boldsymbol{D}_n^k = \begin{pmatrix} d_{n,rr}^k & d_{n,rl}^k \\ d_{n,lr}^k & d_{n,ll}^k \end{pmatrix} = \begin{pmatrix} \boldsymbol{d}_{n,r}^k \\ \boldsymbol{d}_{n,l}^k \end{pmatrix}, \tag{3}$$

where $d_{n,ov}^k$, with $\{o, v\} \in \{r, l\}$ represents the directivity coefficient of SA n in o polarization in direction of ST k in v polarization. It is common to assume $d_{n,rr}^k = d_{n,ll}^k$ and $d_{n,rl}^k = d_{n,lr}^k$. In (3), $\boldsymbol{d}_{n,o}^k = (d_{n,or}^k, d_{n,ol}^k)$ is the component in o-polarization at SA n. For further study, it is convenient to structure the matrix $\boldsymbol{D}$ in block columns of size $2N \times 2$, $\boldsymbol{D}^k = (\boldsymbol{D}_1^{kH}, \boldsymbol{D}_2^{kH}, \ldots \boldsymbol{D}_N^{kH})^H$, where $\boldsymbol{D}^k$ represents the directivity coefficients of ST k, and rows $\boldsymbol{d}_{n,o} = (\boldsymbol{d}_{n,o}^1, ..., \boldsymbol{d}_{n,o}^K)$ associated to SA n in o polarization.

[1]In this model we assume that the signal leakage from left to right polarization and vice versa is negligible at the STs.

The STs' detection and corresponding channel estimation are based on the synchronous transmissions of training sequences of length L by all active STs. The signal received at SA n in o-polarization, with $\{o\} \in \{l, r\}$, is given by

$$y_{n,o}[s_q + s] = \boldsymbol{d}_{n,o}\boldsymbol{P}(q)\boldsymbol{x}[s_q + s] + z_{n,o}[s_q + s], \tag{4}$$

where s_q is the time offset when the transmission of a training sequences for the q-th frame starts and $s = 0, \ldots, L-1$ is a time index. The received signal $\boldsymbol{\mathcal{Y}}_{n,o}(q) = (y_{n,o}[s_q], y_{n,o}[s_q + 1], \ldots, y_{n,o}[s_q + L - 1])$ corresponding to training sequences of frame q at SA n and o-polarization is given by

$$\boldsymbol{\mathcal{Y}}_{n,o}(q) = \boldsymbol{d}_{n,o}\boldsymbol{P}(q)\boldsymbol{X}_q + \boldsymbol{\mathcal{Z}}_{n,o}(q) \tag{5}$$

where $\boldsymbol{X}_q$ is the $2RK \times L$ matrix whose rows are the training sequences of the active STs and $\boldsymbol{\mathcal{Z}}_{n,o}(q)$ is the L-dimensional row vector of the noise $\boldsymbol{\mathcal{Z}}_{n,o}(q) = (z_{n,o}[s_q], z_{n,o}[s_q + 1], \ldots, z_{n,o}[s_q + L - 1])$.

In order to exploit the spatial resolution offered by multiple directional SAs, we assume that the radiation diagram on the coverage area is known or equivalently the directivity vectors of some reference STs in a grid are known at the gateway. We denote by $\boldsymbol{G}$ the matrix available at the gateway and containing all the directivity vectors of the points in the grid. The matrix $\boldsymbol{G}$ has a block structure similar to the one of $\boldsymbol{D}$ with blocks $\boldsymbol{G}_n^k$ of the form (3). Additionally, we assume that the directivity vector of a ST in an arbitrary position can be determined as a convex combination of the directivity vectors at some reference points. More specifically, let us consider ST k with coordinates $S_k \equiv (S_x, S_y)$, and let $G_{\tau(i)} \equiv (a_{\tau(i)}, b_{\tau(i)})$, with $i = 1, 2, 3$, be the three nearest reference points surrounding ST k. The point S_k can be expressed as convex combination of $G_{\tau(1)}, G_{\tau(2)}$, and $G_{\tau(3)}$

$$S_k = \alpha_1^k G_{\tau(1)} + \alpha_2^k G_{\tau(2)} + \alpha_3^k G_{\tau(3)}$$

with $0 \leq \alpha_i^k \leq 1$, for $i = \{1, 2, 3\}$, and $\sum_{i=1}^3 \alpha_i^k = 1$. If $\boldsymbol{G}^{\tau(i)}$ denotes the $\tau(i)$ block column of $\boldsymbol{G}$ corresponding to point $G_{\tau(i)}$, then, the directivity column block $\boldsymbol{D}^k$ of ST k is given by convex combination of the directivity column vectors with identical coefficients

$$\boldsymbol{D}^k = \alpha_1^k \boldsymbol{G}^{\tau(1)} + \alpha_2^k \boldsymbol{G}^{\tau(2)} + \alpha_3^k \boldsymbol{G}^{\tau(3)}. \tag{6}$$

III. Detection of active STs and multi-STs channel estimation

In this section, we describe our approaches to detect the active STs, identifying their directivity vector, and estimate their instantaneous CSI. They consist of three steps. In the first step, we detect the training sequence groups that have been utilized by the STs. In the second step, we estimate the sum of the channel coefficients of all STs utilizing the same training sequence group. The final step resolves contention among different STs by estimating the directivity of some or all colliding STs using the same training sequence and thus enabling multiuser detection.

A. Training Sequences Detection

The approach adopted to detect the training sequences is based on the correlation between the received signal at the gateway and the U groups of training sequences. The algorithm is summarized in Algorithm 1.

```
Set threshold ζ
for u = 1, ..., U do
    for q = 1, ..., Q do
        for n = 1, ..., N do
            Calculate c_{u,q,n} = || 𝓧_u 𝓨_n(q)^H ||^2
        end
        c_{u,q} = max_n c_{u,q,n}
    end
    if Σ_{q=1}^{Q} c_{u,q} > ζ then
        𝓧_u is utilized by at least one ST
    end
end
```

Algorithm 1: Algorithm 1: Estimation of utilized trainings

We denote the number of groups of training sequence transmitted at least by one ST as W. Furthermore, we denote the number of STs that utilize the w-th detected group as K_w and by $\mathcal{X}_w$ the $2R \times L$ matrix whose rows are the training sequences of group w. In the proposed algorithm, the group w is detected as transmitted if the correlation of $\mathcal{X}_w$ with the received signal is above a certain threshold. It is worth noticing that after the detection of training sequences, the gateway is still oblivious of K_w.

B. LSE Estimation of Transfer Matrix

Let us introduce $\boldsymbol{h}^k_{n,r}(q)$ and $\boldsymbol{h}^k_{n,l}(q)$. They are the transfer vectors in left and right polarizations from ST k to SA n. They are defined as

$$\begin{aligned}\boldsymbol{h}^k_{n,r}(q) &= \left(h^{k,(1)}_{n,rr}(q),\, h^{k,(1)}_{n,rl}(q), \cdots,\, h^{k,(R)}_{n,rr}(q),\, h^{k,(R)}_{n,rl}(q)\right),\\ \boldsymbol{h}^k_{n,l}(q) &= \left(h^{k,(1)}_{n,lr}(q),\, h^{k,(1)}_{n,ll}(q), \cdots,\, h^{k,(R)}_{n,lr}(q),\, h^{k,(R)}_{n,ll}(q)\right),\end{aligned} \tag{7}$$

with $h^{k,(\ell)}_{n,rr}(q) = d^k_{n,rr}P^{(\ell)}_{k,r}(q)$, $h^{k,(\ell)}_{n,rl}(q) = d^k_{n,rl}P^{(\ell)}_{k,l}(q)$, $h^{k,(\ell)}_{n,lr}(q) = d^k_{n,lr}P^{(\ell)}_{k,r}(q)$ and $h^{k,(\ell)}_{n,ll}(q) = d^k_{n,ll}P^{(\ell)}_{k,l}(q)$. Let

$$\mathfrak{h}^w_{n,o} = \sum_{k\in\Pi_w} \boldsymbol{h}^k_{n,o}(q) \qquad \{o\} \in \{r,l\},$$

where Π_w denotes the set of indices corresponding to STs transmitting the training sequence group w. Then, the received signal corresponding to the training sequence of the q-th frame can be written as

$$\boldsymbol{\mathcal{Y}}_{n,o}(q) = \sum_{w=1}^{W} \mathfrak{h}^w_{n,o}(q)\mathcal{X}_w + \boldsymbol{\mathcal{Z}}_{n,o}(q). \tag{8}$$

By applying standard results on linear LSE (see e.g. [9]), we obtain the LSE estimation of $\mathfrak{h}^w_{n,o}(q)$ given by

$$\hat{\mathfrak{h}}^w_{n,o}(q) = \boldsymbol{\mathcal{Y}}_{n,o}\boldsymbol{\mathcal{X}}^H_w(\boldsymbol{X}_q\boldsymbol{X}^H_q)^{-1}, \qquad \{o\} \in \{r,l\}. \tag{9}$$

The corresponding estimation error is $\varepsilon_{n,o}(q) = \hat{\mathfrak{h}}^w_{n,o}(q) - \mathfrak{h}^w_{n,o}(q)$, $\{o\} \in \{r,l\}$.

C. Contention resolution and multiuser channel estimation

Our multi-ST channel estimation is based on the following system of equation obtained from (7), the definition of the estimation error, and by utilizing the assumptions $d^k_{n,ll} = d^k_{n,rr}$ and $d^k_{n,lr} = d^k_{n,rl}$

$$\begin{cases} \sum_{k\in\Pi_w} d^k_{n,rr}P^{(1)}_{k,r}(q) &= \hat{\mathfrak{h}}^{w,(1)}_{n,rr}(q) + \varepsilon^{w,(1)}_{n,rr}(q)\\ \sum_{k\in\Pi_w} d^k_{n,lr}P^{(1)}_{k,r}(q) &= \hat{\mathfrak{h}}^{w,(1)}_{n,lr}(q) + \varepsilon^{w,(1)}_{n,lr}(q)\\ \sum_{k\in\Pi_w} d^k_{n,rr}P^{(1)}_{k,l}(q) &= \hat{\mathfrak{h}}^{w,(1)}_{n,ll}(q) + \varepsilon^{w,(1)}_{n,ll}(q)\\ \sum_{k\in\Pi_w} d^k_{n,rl}P^{(1)}_{k,l}(q) &= \hat{\mathfrak{h}}^{w,(1)}_{n,rl}(q) + \varepsilon^{w,(1)}_{n,rl}(q)\\ &\vdots\\ \sum_{k\in\Pi_w} d^k_{n,rr}P^{(R)}_{k,l}(q) &= \hat{\mathfrak{h}}^{w,(R)}_{n,ll}(q) + \varepsilon^{w,(R)}_{n,ll}(q)\\ \sum_{k\in\Pi_w} d^k_{n,rl}P^{(R)}_{k,l}(q) &= \hat{\mathfrak{h}}^{w,(R)}_{n,rl}(q) + \varepsilon^{w,(R)}_{n,rl}(q) \end{cases} \tag{10}$$

where the indices of the components of the estimation $\hat{\mathfrak{h}}^w_{n,o}(q)$ and the estimation error vector $\boldsymbol{\varepsilon}^w_{n,o}(q)$ are defined consistently with the ones of vector $\boldsymbol{h}^k_{n,o}(q)$ in (7). If we had known the directivity vectors of the active STs, the channel estimation would have reduced to a standard linear multiuser channel estimation. However, the gateway does not know neither the directivity vectors nor the number of colliding STs. Let us denote by $\mathcal{T}$ the set of all adjacent triplets τ on the reference grid and let τ_i denotes the index of the directivity block column of the i-th element of τ in the matrix $\boldsymbol{G}$. If the active ST k^* is located in the triangle identified by the triplet of points τ^*, we can obtain an estimation of $\boldsymbol{D}^{k^*}$ and $\boldsymbol{P}^{k^*}$ by minimum norm-two fitting. This problem has been investigated in the case when the set Π_w is singleton in [8] and its application to this problem is straightforward if we consider the remaining ST channels as noise. For the sake of completeness, we report the algorithm proposed in [8] in Algorithm 2. In the case of singleton Π_w, [8] performs channel estimation by exhaustive search over $\mathcal{T}$ and selects the triplet for which $f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}^w_r(0), ..., \hat{\mathfrak{h}}^w_l(Q-1))$, defined in Algorithm 2, is maximum. In the case of Π_w multiple elements, we are interested in determining all the local maxima of the piecewise function $f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}^w_r(0), ..., \hat{\mathfrak{h}}^w_l(Q-1))$ obtained by considering all possible triplets in $\mathcal{T}$. In order to solve this problem, we benefit from the additive nature of the transmitted signals and the strong directionality of the SAs to propose two heuristic approaches.

Successive Channel Cancelation (SCC) Approach : The rational behind SCC approach is to iterate over the channel estimation and the subsequent cancelation of the corresponding signal contribution from the received signal.

Let $\boldsymbol{\mathcal{Y}}^{(0)}_{n,o}(q) = \boldsymbol{\mathcal{Y}}_{n,o}(q)$. At iteration j, we estimate the sum of the channels of the colliding STs that have not been detected yet by applying

$$\hat{\mathfrak{h}}^w_{n,o}(q;j) = \boldsymbol{\mathcal{Y}}^{(j-1)}_{n,o}\mathcal{X}^H_w(\boldsymbol{X}_q\boldsymbol{X}^H_q)^{(-1)}, \quad \{o\} \in \{r,l\}. \tag{17}$$

Then, we determine the triplet $\tau \in \mathcal{T}$ and the vector $\boldsymbol{\alpha}$ solving the optimization problem $\mathbf{P_2}$:

$$\begin{aligned}\textbf{maximize}\quad & f^{(j)}(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}^w_r(0;j), \hat{\mathfrak{h}}^w_l(0;j), ..., \hat{\mathfrak{h}}^w_l(Q-1;j))\\ &= \frac{\boldsymbol{\alpha}^H\mathfrak{Re}(\boldsymbol{\Theta}(\tau, \hat{\mathfrak{h}}^w_r(0;j), ..., \hat{\mathfrak{h}}^w_l(Q-1;j)))\boldsymbol{\alpha}}{\boldsymbol{\alpha}^H\mathfrak{Re}(\boldsymbol{\Gamma}(\tau))\boldsymbol{\alpha}}\\ \textbf{subject to}\quad & \textstyle\sum_{i=1}^{3}\alpha_i = 1 \quad 0 \le \alpha_i \le 1, \qquad i = 1,2,3\end{aligned}$$

1 Determine $\boldsymbol{\alpha}^{k^*} \equiv (\alpha_1^{k^*}, \alpha_2^{k^*}, \alpha_3^{k^*})$ has the maximizer of the eigenvalue complementarity problem [11]:

$$\begin{aligned}
\textbf{maximize} \quad & f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}_r^w(0), \hat{\mathfrak{h}}_l^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1)) \\
& = \frac{\boldsymbol{\alpha}^H \mathfrak{Re}(\boldsymbol{\Theta}(\tau, \hat{\mathfrak{h}}_r^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1)))\boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \mathfrak{Re}(\boldsymbol{\Gamma}(\tau))\boldsymbol{\alpha}} \\
\textbf{subject to} \quad & \textstyle\sum_{i=1}^3 \alpha_i = 1 \quad 0 \le \alpha_i \le 1, \qquad i = 1,2,3 \quad \textbf{Problem P}_1
\end{aligned}$$

being $\boldsymbol{\Theta}(\tau, \hat{\mathfrak{h}}_r^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1))$ and $\boldsymbol{\Gamma}(\tau)$ the 3×3 matrices defined as:

$$\boldsymbol{\Theta}(\tau, \hat{\mathfrak{h}}_l^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1)) = \tag{11}$$

$$\widetilde{\boldsymbol{G}}^{\tau,H} \left(\sum_{q=0}^{Q-1} \sum_{\ell=1}^{R} \left(\hat{\mathfrak{h}}_r^{w,(\ell)}(q) \hat{\mathfrak{h}}_r^{w,(\ell)H}(q) + \hat{\mathfrak{h}}_l^{w,(\ell)}(q) \hat{\mathfrak{h}}_l^{w,(\ell)H}(q) \right) \right) \widetilde{\boldsymbol{G}}^{\tau}, \tag{12}$$

$$\boldsymbol{\Gamma}(\tau) = \widetilde{\boldsymbol{G}}^{\tau,H} \widetilde{\boldsymbol{G}}^{\tau} \tag{13}$$

with $\hat{\mathfrak{h}}_o^{w,(\ell)}(q) = \left(\hat{\mathfrak{h}}_{1,o}^{w,(\ell)H}(q), ..., \hat{\mathfrak{h}}_{N,o}^{w,(\ell)H}(q) \right)^H$,
$\widetilde{\boldsymbol{G}}^{\tau} = \left(\widetilde{\boldsymbol{G}}_1^{\tau,H}, ..., \widetilde{\boldsymbol{G}}_N^{\tau,H} \right)^H$ and $\widetilde{\boldsymbol{G}}_n^{\tau,H} = (\boldsymbol{g}_{n,r}^{\tau_1}, \boldsymbol{g}_{n,r}^{\tau_2}, \boldsymbol{g}_{n,r}^{\tau_3})$. Here $\boldsymbol{g}_{n,r}^{\tau_i}$ is the first row of the block $\boldsymbol{G}_n^{\tau_i}$ of matrix $\boldsymbol{G}$,
$\hat{\mathfrak{h}}_{n,r}^{w,(\ell)}(q) = (\hat{\mathfrak{h}}_{n,rr}^{w,(\ell)}, \hat{\mathfrak{h}}_{n,lr}^{w,(\ell)})^H$ and $\hat{\mathfrak{h}}_{n,l}^{w,(\ell)}(q) = (\hat{\mathfrak{h}}_{n,rl}^{w,(\ell)}, \hat{\mathfrak{h}}_{n,ll}^{w,(\ell)})^H$.

2 Determine

$$\hat{P}_{k^*,o}^{(\ell)}(q) = \frac{\boldsymbol{\alpha}^{k^*T} \widetilde{\boldsymbol{G}}^{\tau,H} \hat{\mathfrak{h}}_o^{w,(\ell)}(q)}{\boldsymbol{\alpha}^{k^*T} \widetilde{\boldsymbol{G}}^{\tau,H} \widetilde{\boldsymbol{G}}^{\tau} \boldsymbol{\alpha}^{k^*}}, \quad \{o\} \in \{r, l\} \quad \text{and} \quad \forall q. \tag{14}$$

3 Determine

$$\hat{\boldsymbol{D}}^{k^*} = \sum_{i=1}^{3} \alpha_i^{k^*} \boldsymbol{G}^{\tau_i}. \tag{15}$$

4 Determine the channel $\hat{\boldsymbol{H}}^{w,k^*}$ of ST k^*

$$\hat{\boldsymbol{H}}^{k^*} = \hat{\boldsymbol{D}}^{k^*} \hat{\boldsymbol{P}}^{k^*}(q), \tag{16}$$

with

$$\hat{\boldsymbol{P}}^{k^*}(q) = \begin{pmatrix} \hat{P}_{k^*,r}^{(1)}(q) & 0 & \cdots & \hat{P}_{k^*,r}^{(R)}(q) & 0 \\ 0 & \hat{P}_{k^*,l}^{(1)}(q) & \cdots & 0 & \hat{P}_{k^*,l}^{(R)}(q) \end{pmatrix}.$$

Algorithm 2: PLSE algorithm proposed in [8]

where $\boldsymbol{\Theta}(\tau, \hat{\mathfrak{h}}_r^w(0;j), ..., \hat{\mathfrak{h}}_l^w(Q-1;j))$ and $\boldsymbol{\Gamma}(\tau)$ are 3×3 matrices defined as:

$$\begin{aligned}
&\boldsymbol{\Theta}(\tau^j, \hat{\mathfrak{h}}_r^w(0;j), ..., \hat{\mathfrak{h}}_l^w(Q-1;j)) \\
&= \widetilde{\boldsymbol{G}}^{\tau,H} (\sum_{q=0}^{Q-1} \sum_{\ell=1}^{R} (\hat{\mathfrak{h}}_r^{w,(\ell)}(q;j) \hat{\mathfrak{h}}_r^{w,(\ell)H}(q;j) \\
&+ \hat{\mathfrak{h}}_l^{w,(\ell)}(q;j) \hat{\mathfrak{h}}_l^{w,(\ell)H}(q;j))) \widetilde{\boldsymbol{G}}^{\tau}, && (18) \\
&\boldsymbol{\Gamma}(\tau) = \widetilde{\boldsymbol{G}}^{\tau,H} \widetilde{\boldsymbol{G}}^{\tau}, && (19)
\end{aligned}$$

$\widetilde{\boldsymbol{G}}^{\tau}$ is as in Algorithm 2. The optimum values of τ and $\boldsymbol{\alpha}$ enable the estimation of the channel along the same lines as in Algorithm 2, equations (14), (15) and (16). Let us denote by $\hat{\mathfrak{h}}_{n,o}^{w,(j)}$, $\{o\} \in \{r, l\}$, the estimation of the channel at step j. SCC approach removes from the signal $\boldsymbol{\mathcal{Y}}_{n,o}^{(j-1)}(q)$ the contribution from the detected ST to obtain

$$\boldsymbol{\mathcal{Y}}_{n,o}^{(j)}(q) = \boldsymbol{\mathcal{Y}}_{n,o}^{(j-1)}(q) - \hat{\mathfrak{h}}_{n,o}^{w,(j)} \mathcal{X}_w. \tag{20}$$

The algorithm terminates if the channel estimate $\hat{\mathfrak{h}}_{n,o}^{w,(j)}$ does not yield to a correct decoding of the transmitted information. SCC approach is detailed in Algorithm 3.

Set w.
Set threshold η.
Set $j = 0$.
for $q = 1, ..., Q$ **do**
    Calculate $\hat{\mathfrak{h}}_{n,o}^w(q)$, $\{o\} \in \{r, l\}$ according to (9).
    Set $\hat{\mathfrak{h}}_{n,o}^w(q;0) = \hat{\mathfrak{h}}_{n,o}^w(q)$.
end
Solve Problem P_2 and determine the maximizer $(\boldsymbol{\alpha}_0^*, \tau_0^*)$,
Determine directivity vector $\boldsymbol{D}^{k^*}$ of the given ST corresponding to the optimum triplet τ_0^* and optimum $\boldsymbol{\alpha}_0^*$ by applying (15);
Calculate $\hat{P}_{k,o}^{(\ell)}(q;0)$ according to (14);
Calculate $\hat{\boldsymbol{H}}^{k^*}$ according to (16);
Calculate $\boldsymbol{\mathcal{Y}}_{n,o}^{(j)}(q)$ according to (20);
Check if ST k^* can be successfully decoded;
$j = j + 1$.
while *ST is successfully decoded* **do**
    Solve Problem P_2 and determine the maximizer $(\boldsymbol{\alpha}_j^*, \tau_j^*)$,
    Determine directivity vector $\boldsymbol{D}^{k^*}$ of a ST corresponding to the optimum triplet τ_j^* and optimum $\boldsymbol{\alpha}_j^*$ by applying (15);
    Calculate $\hat{P}_{k,o}^{(\ell)}(q;j)$ according to (14);
    Calculate $\hat{\boldsymbol{H}}^{k^*}$ according to (16);
    Calculate $\boldsymbol{\mathcal{Y}}_{n,o}^{(j)}(q)$ according to (20);
    Check if ST k^* can be successfully decoded;
    $j = j + 1$.
end

Algorithm 3: SCC estimation for contention resolution and multiuser channel estimation

Grid Reduction (GR) Approach:As in [8], we compare the maximum of $f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}_r^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1))$ over all triplets τ in $\mathcal{T}$ and then select the triplet yielding the maximum $f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}_r^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1))$. The maximizer of $f(\boldsymbol{\alpha}, \tau; \hat{\mathfrak{h}}_r^w(0), ..., \hat{\mathfrak{h}}_l^w(Q-1))$ determines the estimation of the ST channel. If the channel estimation enables a successful decoding of the detected ST. We remove from $\mathcal{T}$ all the triplets containing reference points whose directivity vector have high correlation with the estimated directivity vector and we obtain a reduced set $\mathcal{T}^{(1)}$. In the following steps, we iterate along similar lines but adopting more and more reduced sets $\mathcal{T}^{(j)}$. The algorithm terminates when the obtained channel estimation does not allow a successful decoding. GR approach is detailed in Algorithm 4.

It is apparent that the algorithm resolves collisions by a successive interference decoding, which is enabled by the multiuser channel estimation.

IV. Numerical Performance Assessment

In this section, we analyze the performance of the two proposed approaches through numerical simulations. The simulations are performed for STs equipped with two antennas, ($R = 2$). The satellite is endowed with $N = 163$ SAs. We utilize the actual directivity vectors of a geostationary system serving the European area. The propagation coefficients are generated according to the Surrey model in [10]. The power of the transmit signals is set to 0 dBW. We assume the thermal noise is absent and only co-channel interference is present. The number of transmitted frames Q is 50. The positions of the STs are generated randomly and uniformly in a rectangular region

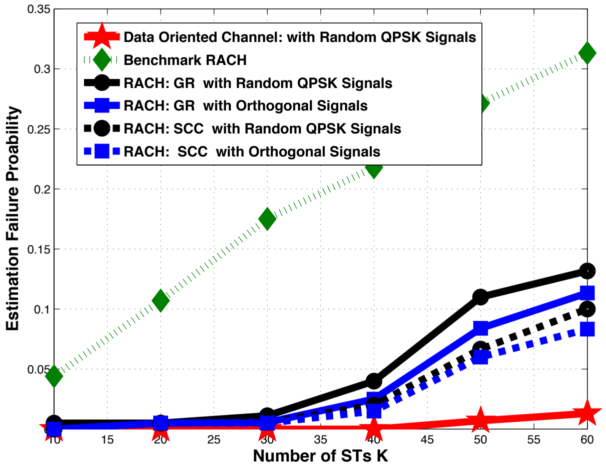

Figure 1: Estimation failure probability versus varying number of STs with different number of STs, $Q = 50$, thermal noise=$-\infty$dBW, traininglength= 200, $U = 50$

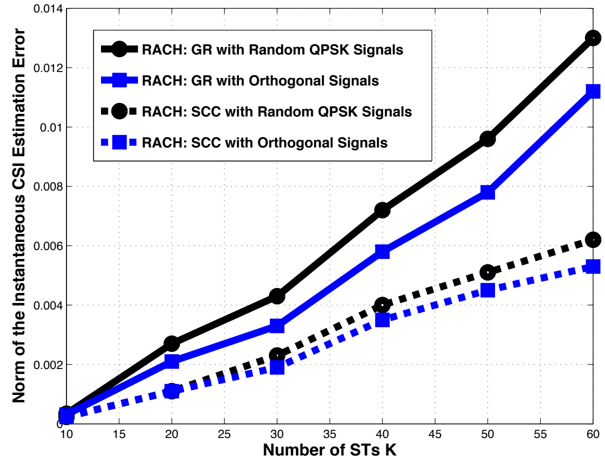

Figure 2: Estimation error norm of instantaneous CSI versus varying number of STs with different number of STs, Q=50, thermal noise=$-\infty$dBW, traininglength= 200, U=50

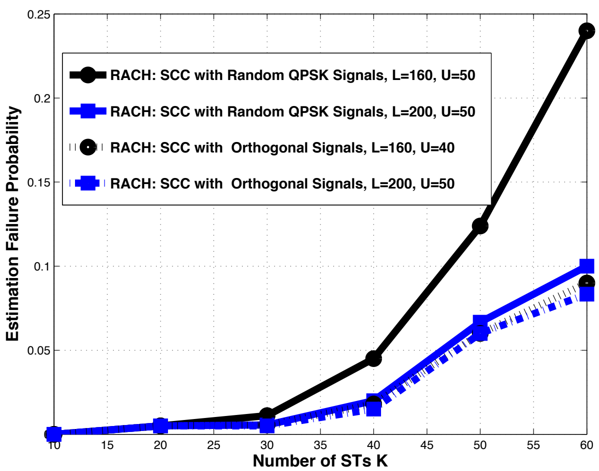

Figure 3: Estimation failure probability versus varying number of STs with different types of training sequences for SCC approach, $Q = 50$, thermal noise=$-\infty$dBW

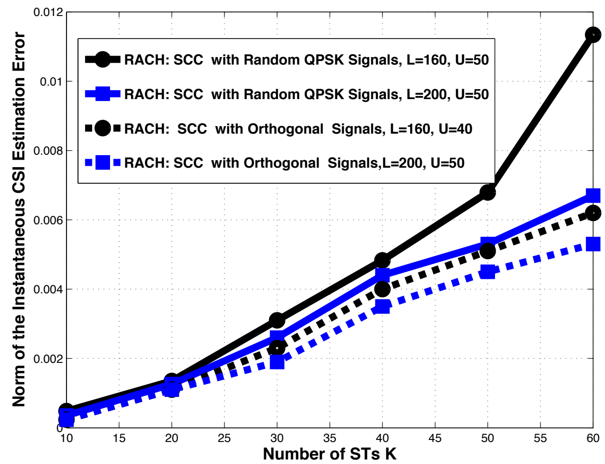

Figure 4: Estimation error of instantaneous CSI versus varying number of STs with different types of training sequences for SCC approach, $Q = 50$, thermal noise=$-\infty$dBW

covering the most of Europe. The results are obtained by averaging over 20 system realizations, i.e., 20 different groups of STs randomly generated. The pilot sequence length is either 160 or 200. We consider two types of training sequences in the simulation: 1) random QPSK training sequences. In this case, the training sequences set is always partitioned into 50 groups; 2) orthogonal training sequences. In this case, the training sequence set is partitioned into 40 and 50 groups for training lengths 160 and 200, respectively.

The perspective of this work is substantially different from other works [1]-[6] that improve the throughput of the RACH by proper design of the MAC layer. The proposed approaches modify the physical layer but are independent from the MAC layer. Then, to keep the analysis independent of a specific MAC protocol, the performance metrics that we use are necessarily different from the standard metrics utilized for the analysis of MAC protocols: throughput and offered channel traffic. Additionally, comparison of the proposed approaches with standard MAC protocols for MAC is further exacerbated by the fact that we consider here a network and exploit its spatial and user diversity while the methods in [2]-[7] consider a single receiver.

The relevant metrics in this work are the estimation failure probability and the normalized estimation error. The event that the gateway fails to detect an active ST is referred to as ‘estimation failure’. As metric to assess the performance of the instantaneous CSI estimation, we adopt the average ratio between the norm of the estimation error and the norm of the exact channel, i.e, $\xi = \mathbb{E}_k \left(\frac{\|\boldsymbol{\varepsilon}_{\boldsymbol{H}_k}\|^2}{\|\boldsymbol{H}_k\|^2} \right)$, where $\boldsymbol{\varepsilon}_{\boldsymbol{H}_k} = \boldsymbol{H}_k - \hat{\boldsymbol{H}}_k$.

As benchmark system, we consider a conventional RACH system with BFN as benchmark. In the conventional RACH system, we assume that four frequency bands are available. Each band support 40 fixed beams isolated by frequency reuse.

The impact of the number of active STs on the proposed estimation approaches is shown in terms of estimation failure probability in Figure 1 and in terms of estimation error

Set w.
Set threshold η.
Set threshold ϕ.
Set $j = 0$.
Set $\mathcal{T}^{(0)} = \mathcal{T}$.
for $q = 1, ..., Q$ **do**
  Calculate $\hat{\mathfrak{h}}^{w}_{n,o}(q)$, $\{o\} \in \{r, l\}$ according to (9).
end
Solve Problem P_2 over the set $\mathcal{T}^{(0)}$ and determine the maximizer $(\boldsymbol{\alpha}_0^*, \tau_0^*)$,
Determine the directivity vector $\boldsymbol{D}^{k^*}$ by applying (15).
Determine the channel estimation $\boldsymbol{H}^{k^*}$ by applying (16).
Check if ST k^* can be successfully decoded.
while *ST k^* is successfully decoded* **do**
  Remove from $\mathcal{T}^{j}$ all the triplets τ such that the directivity vector of a reference point τ_i has the correlation with the directivity vector $\boldsymbol{D}^{k^*}$ higher than ϕ to obtain $\mathcal{T}^{(j+1)}$;
  Solve Problem P_2 over the set $\mathcal{T}^{(j+1)}$ and determine the maximizer $(\boldsymbol{\alpha}_j^*, \tau_j^*)$,
  Determine the directivity vector $\boldsymbol{D}^{k^*}$ by applying (15).
  Determine the channel estimation $\boldsymbol{H}^{k^*}$ by applying (16).
  Check if ST k^* can be successfully decoded.
  $j = j + 1$.
end

Algorithm 4: GR Approach for contention resolution and multiuser channel estimation

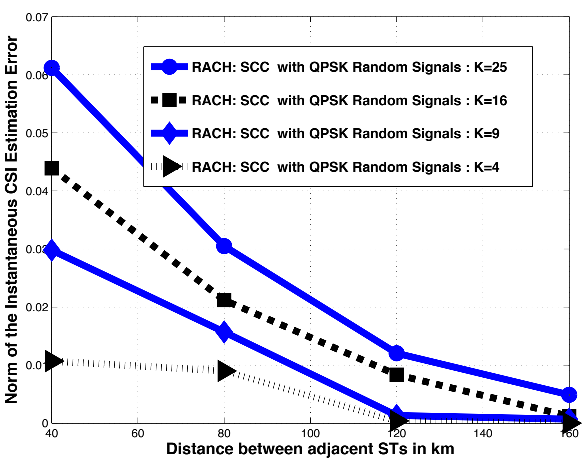

Figure 5: Estimation error norm of instantaneous CSI versus distance between adjacent STs in km for SCC approach, training length= 200, $U = 50$, thermal noise=$-\infty$dBW, $Q = 50$

in Figure 2. The estimation failure probability of the two proposed approaches is much lower than the conventional benchmark with obvious gain in serving a higher number of STs. Figure 1 also indicates that for each type of training sequences, SCC approach always outperforms GR approach. As expected, the use of orthogonal training sequences is beneficial when compared to the use of randomly generated training sequences[2]. Figure 2 shows a trend similar to the one in Figure 1. Furthermore, the performance of the RACH is compared to the one of the connection oriented channel studied in [8]. From Figure 1, the performance loss due to collisions in the RACH is apparent.

The impact of different training sequences and training length on SCC approach is analyzed in Figure 3 and 4. It is worth noticing that when K STs are transmitting, the channel consists of $2RK = 4K$ links and all of them have to be estimated. Figure 3 shows that if random QPSK training is adopted, the estimation failure probability grows rapidly as the number of STs increases and approaches the number of different training groups. On the contrary, if we adopt orthogonal training sequences, SCC approach still can provide good estimation performance in similar conditions. A similar trend is also shown in Figure 4.

Finally, we study the impact of the distance between adjacent STs on SCC approach. We generate the STs' positions on a square grid. Each ST has the same distance from its adjacent ST. Figure 5 shows the impact of the distance between adjacent STs in terms of the norm of instantaneous CSI estimation error. As expected, when the distance between adjacent STs increases, SCC approach achieves better performance and the norm of instantaneous CSI estimation error decreases. Note that Figure 5 cannot be compared with the simulation results obtained in Figure 2, since the positions of the STs are generated in a much smaller area and the interference among the active STs is larger.

[2]The use of orthogonal training implies an ideal system completely synchronized, since asynchronism destroy orthogonality. On the contrary, random generated training sequences do not suffer from this effect and offer a bound to the performance loss due to lack of synchronism/orthogonality. Techniques to limit the performance degradation due to asynchronism are known. However, the analysis of this aspect exceeds the scope of this work.

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