MIMO Interference Alignment Algorithms With Hierarchical CSIT

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***Abstract*—This work[1] deals with interference alignment (IA) in a K-users MIMO interference channel with only incomplete Channel State Information at the Transmitters (CSIT). Incompleteness of CSIT is defined by the perfect knowledge of only a submatrix of the global channel matrix. Additionally, each Transmitter (TX) may have different incomplete CSIT. Most IA techniques are developed under a full (complete) CSIT assumption -either explicitly or implicitly when the CSI is progressively acquired in the form of RX-to-TX feedback iterations. In contrast, we are interested here in the feasibility of IA based only on incomplete CSIT. We show that even in antenna settings where no extra-antenna is available in terms of feasibility of IA, perfect IA can be achieved when some TXs do not have the complete CSIT. Especially, for each antenna setting, we provide a sufficient incomplete CSIT sharing and we adapt IA algorithms from the literature to achieve perfect IA under this condition of incomplete CSIT. We confirm by simulations that the proposed IA algorithm based on incomplete CSIT achieves no significant losses compared to the algorithm based on perfect CSIT sharing.**

I. Introduction

One of the critical issues with multi-transmitter coordinated transmission in general, and with IA in particular, is the fact that coordination benefits go at the expense of acquiring CSIT and sharing it across all TXs [1]. In the case of multi-antenna based IA, CSIT acquisition and sharing is exploited to compute the precoders at each one of the TXs and can result in a significant overhead in practice.

The study of how CSIT requirements in IA methods can somehow be alleviated has become an active research topic in its own right [2]–[5]. Several approaches have been proposed in this direction. One strategy consists in developing iterative methods that can exploit local measurements made by the TXs on the reverse link or progressive feedback mechanisms [2]. Such methods rely on the fact that, through iterations, enough CSIT is acquired to allow convergence in a distributed manner toward a global IA solution. Another line of work consists in studying the minimal CSI quantization bits (scaling with SNR) that should be conveyed to the TXs to achieve the maximal Degree-of-Freedom (DoF) obtained with IA [4], [6]. In such works, it should be noted however that each one of the TXs is assumed to be provided with the *same* quantized CSIT, resulting in a perfect CSIT sharing. Alternatively, authors have investigated the scenario of TXs having access to outdated CSIT [7], [8]. But there again, imperfect CSIT is assumed to be perfectly shared across all TXs.

In this work we introduce another CSIT framework whereby the TXs have limited CSIT sharing capability. Given an MIMO IC, the term *incomplete* CSIT is coined, which refers to a situation in which each of the TXs acquires, through an arbitrary feedback and exchange mechanism left to be specified, a subset of the multi-user channel coefficients in an unquantized form. Hence, the fading coefficients which are available at a given TX are known perfectly, while the remaining coefficients are completely unknown at that TX. In general, different TXs will be provided with different subsets of the global CSIT.

Our main contributions read as follows. Firstly, we show that IA can be achieved without full CSIT at all TXs, i.e., with a *strictly incomplete* CSIT allocation, and we provide a sufficient criterion for testing the feasibility of incomplete CSIT. Secondly, we develop an algorithm returning an incomplete CSIT allocation preserving IA feasibility and we adapt an IA algorithm from the literature to that incomplete CSIT setting. Only the sketchs of the proofs are given and more details are available in the extended version [9].

II. System Model

A. Transmission Model

We study the transmission in a K-user MIMO IC where all the RXs and the TXs are linked by a wireless channel. We consider a conventional model for the MIMO static IC [10] with the particularity of our model lying in the structure of the CSIT since we consider that each TX has its *own* CSIT in the form of a submatrix of the multi-user channel matrix. This specific information structure is referred in this paper as *incomplete CSIT* and will be detailed in Subsection II-B. TX j is equipped with M_j antennas, RX i has N_i antennas, and TX j transmits d_j streams to RX j. This IC is then denoted as $\prod_{k=1}^{K}(M_k, N_k, d_k)$. As a first step, we focus exclusively in this work on the single stream transmission for all users so that $\forall k \in \{1,\dots,K\}, d_k = 1$. Consequently, we use the short notation $\prod_{k=1}^{K}(M_k, N_k)$.

In this work, we consider that there are no single-antenna RX and no single-antenna TX. This hypothesis is done to make the exposition more clear but does not represent any

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real limitation. This restriction is removed in the extended version [9].

The channel from TX j to RX i is represented by the channel matrix $\mathbf{H}_{ij} \in \mathbb{C}^{N_i \times M_j}$ with its elements distributed according to a generic distribution [11]. The global multi-user channel matrix is denoted by $\mathbf{H} \in \mathbb{C}^{N_{\text{tot}} \times M_{\text{tot}}}$ where $N_{\text{tot}} \triangleq \sum_{k=1}^{K} N_k$ and $M_{\text{tot}} \triangleq \sum_{k=1}^{K} M_k$:

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \dots & \mathbf{H}_{1K} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \dots & \mathbf{H}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{K1} & \mathbf{H}_{K2} & \dots & \mathbf{H}_{KK} \end{bmatrix}. \tag{1}$$

TX i uses the beamformer $\boldsymbol{t}_i \in \mathbb{C}^{M_i \times 1}$ to transmit the data symbol s_i (i.i.d. $\mathcal{CN}(0,1)$) to RX i. We consider the per-TX power constraint $\forall i \in \{1,\dots,K\}, \|\boldsymbol{t}\|_i^2 = P$ which corresponds to the TXs being non-colocated. The received signal $\boldsymbol{y}_i \in \mathbb{C}^{N_i \times 1}$ at the i-th RX reads then as

$$\boldsymbol{y}_i = \mathbf{H}_{ii}\boldsymbol{t}_i s_i + \sum_{j=1, j\neq i}^{K} \mathbf{H}_{ij}\boldsymbol{t}_j s_j + \boldsymbol{\eta}_i \tag{2}$$

and $\boldsymbol{\eta}_i \in \mathbb{C}^{N_i \times 1}$ is the normalized Gaussian noise at RX i and is i.i.d. $\mathcal{CN}(0,1)$. The received signal $\boldsymbol{y}_i$ is then processed by a Rx beamformer $\boldsymbol{g}_i^{\text{H}} \in \mathbb{C}^{1 \times N_i}$.

Our analysis deals with the achieveability of IA which means that at each RX the desired signal should be decoded free of interference. Equivalently, this means that the RX beamformer $\boldsymbol{g}_i^{\text{H}}$ should be able to zero force (ZF) the interference from all TXs. This is expressed at RX i by fulfilling for all interfering streams $\boldsymbol{t}_j$ with $j \neq i$ the *IA constraints*:

$$\boldsymbol{g}_i^{\text{H}} \mathbf{H}_{ij} \boldsymbol{t}_j = 0. \tag{3}$$

Thus, IA is feasible if the constraints (3) can be achieved at all the RXs for all the interfering streams. Note that this is equivalent to having the dimension of the interference subspace at each RX i lower or equal than $N_i - 1$.

B. Incomplete CSIT Model

We focus in this work on the feasibility of IA with incomplete CSIT allocation at the TXs. Each TX is assumed to receive using an unspecified feedback mechanism a fraction of the full multi-user channel matrix without any error. We do not consider here any practical quantization scheme but future works could exploit the large literature on CSI quantization on top of the approach developed here.

Under this model, a given channel element is either perfectly known by a TX or not at all. More specifically, we restrict ourselves to a CSIT sharing scheme where each TX receives the CSI relative to the fading coefficients *between a subset of users*. This restriction on the structure of the CSIT simplifies greatly the exposition and we will show later on that it is well adapted to the structure of the IA algorithms.

We denote by $\mathbf{H}^{(j)}$ the channel estimate at TX j defined such that $\{\mathbf{H}^{(j)}\}_{ij} = \{\mathbf{H}\}_{ij}$ if the channel element is known at TX j and otherwise $\{\mathbf{H}^{(j)}\}_{ij} = 0$. More specifically, let us assume that TX j receives the CSI relative to the subset of users $\mathcal{I}_j$, we then have $\mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{I}_j}$ with $\mathbf{H}_{\mathcal{I}_j}$ having its only nonzero elements set to verify

$$\forall i \neq j, \left(\mathbf{E}_{\mathcal{I}_j}^{\text{RX}}\right)^{\text{T}} \mathbf{H}_{\mathcal{I}_j} \left(\mathbf{E}_{\mathcal{I}_j}^{\text{TX}}\right) = \left(\mathbf{E}_{\mathcal{I}_j}^{\text{RX}}\right)^{\text{T}} \mathbf{H} \left(\mathbf{E}_{\mathcal{I}_j}^{\text{TX}}\right) \tag{4}$$

with

$$\mathbf{E}_i^{\text{TX}} \triangleq \begin{bmatrix} \mathbf{0}_{\sum_{k=1}^{i-1} M_k \times M_i} \\ \mathbf{I}_{M_i} \\ \mathbf{0}_{\sum_{k=i+1}^{K} M_k \times M_i} \end{bmatrix} \tag{5}$$

and the matrix $\mathbf{E}_i^{\text{RX}}$ defined similarly, solely with N_i replacing M_i. Upon defining $\mathcal{K} \triangleq \{1,\dots,K\}$, TX j receives the complete CSI if $\mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{K}}$ and no CSI if $\mathbf{H}^{(j)} = \mathbf{H}_{\emptyset}$.

C. CSIT-Sets

The CSIT allocations will be shown later in this work to be increasing by inclusion. Consequently, we define a new representation of the CSIT allocation to better represent the structure in the CSIT sharing. Thus, we define the *CSIT-set* as a subset of users correspondings to the CSIT allocation at some TXs. Thus, the CSIT-sets $\mathcal{S}_j^{\text{CSI}}$ are defined so that

$$\forall k \in \{1,\dots,K\}, \exists j, \mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{S}_k^{\text{CSI}}} \tag{6}$$

and defining n_{CSI} as the number of CSIT-sets, the CSIT-sets are indexed to verify

$$\forall k \in \{1,\dots,n_{\text{CSI}}-1\}, \mathcal{S}_k^{\text{CSI}} \subset \mathcal{S}_{k+1}^{\text{CSI}}. \tag{7}$$

From this definition, if TX j receives $\mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{I}}$, then it belongs to all the CSIT-sets $\mathcal{S}_k$ which verify $\mathcal{I} \subseteq \mathcal{S}_k^{\text{CSI}}$.

If the CSIT-sets are given, the CSIT allocation $\mathbf{H}^{(j)}$ can be obtained by setting $\mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{S}_k^{\text{CSI}}}$ where k is the smallest index for which j belongs to $\mathcal{S}_k^{\text{CSI}}$.

Therefore, determining the CSIT allocations $\{\mathbf{H}^{(j)}\}_j$ is equivalent to determining the CSIT-sets $\{\mathcal{S}_k^{\text{CSI}}\}_k$. The CSIT allocation algorithm derived in Section IV will use the CSIT-sets.

D. Feasibility Results

We start by recalling some results from the literature on the feasibility of IA in conventional IC with full CSIT sharing for single stream transmissions.

Theorem 1. *[12] IA is feasible in the IC* $\prod_{k=1}^{K}(M_k, N_k)$ *if and only if*

$$\sum_{k:(k,j)\in\mathcal{I}} (M_k - 1) + \sum_{j:(k,j)\in\mathcal{I}} (N_j - 1) \geq |\mathcal{I}|, \forall \mathcal{I} \subseteq \mathcal{J} \tag{8}$$

with $\mathcal{J} \triangleq \{(i,j) | 1 \leq i,j \leq K, i \neq j\}$. *In the homogeneous IC* $(M,N)^K$, *this condition reads simply as* $M + N \geq K + 1$.

This theorem can be understood intuitively as the condition that each subset of IA equations needs to contain at least as many variables as constraints but the rigorous proof in [12] is based on algebraic geometry arguments.

Note that our interest lies on the incomplete CSIT allocation, not on the feasibility problem in itself. Consequently, we assume in the following that the IC considered are *always* feasible with full CSIT sharing. Hence, the conditions (8) are always verified.

E. Interference Alignment Algorithm

We will use for the simulations a simple generalization of the original max-SINR algorithm [2] where the noise and the strength of the direct signal are taken into account in the optimization to improve the performance at finite SNR. Yet, this corresponds only to one possible choice and any other IA algorithm from the vast literature on IA algorithms could be used instead without any change for our approach. This precoding scheme is based on the maximization of the per-stream SINR, iteratively between the TX side and the RX side. We recall briefly the main steps for the sake of completeness and we refer to [2] for more details. The algorithm is based on the introduction of a reciprocal network where the roles of the TXs and the RXs are exchanged. In that reciprocal network, the RX beamformer becomes the TX beamformer and the TX beamformer is used as RX beamformer while the power constraint of the TX is transferred to the RX.

Thus, in a first step, the TX beamformers are considered as being fixed and the RX beamformers are updated to their optimal value maximizing the per-stream SINR:

$$\forall i, \boldsymbol{g}_i = \frac{\left(\mathbf{I}_{N_i} + \sum_{j=1,j\neq i}^{K} \mathbf{H}_{ij}\boldsymbol{t}_j\boldsymbol{t}_j^{\mathrm{H}}\mathbf{H}_{ij}^{\mathrm{H}}\right)^{-1}\mathbf{H}_{ii}\boldsymbol{t}_i}{\|\left(\mathbf{I}_{N_i} + \sum_{j=1,j\neq i}^{K} \mathbf{H}_{ij}\boldsymbol{t}_j\boldsymbol{t}_j^{\mathrm{H}}\mathbf{H}_{ij}^{\mathrm{H}}\right)^{-1}\mathbf{H}_{ii}\boldsymbol{t}_i\|}\sqrt{P}. \quad (9)$$

In a second step, the RX beamformers are fixed and the transmission is considered in the reciprocal network so that it corresponds to fixed TX beamformers and we can apply the same approach as for the first step and obtain:

$$\forall i, \boldsymbol{t}_i = \frac{\left(\mathbf{I}_{M_i} + \sum_{j=1,j\neq i}^{K} \mathbf{H}_{ji}^{\mathrm{H}}\boldsymbol{g}_j\boldsymbol{g}_j^{\mathrm{H}}\mathbf{H}_{ji}\right)^{-1}\mathbf{H}_{ii}^{\mathrm{H}}\boldsymbol{g}_i}{\|\left(\mathbf{I}_{M_i} + \sum_{j=1,j\neq i}^{K} \mathbf{H}_{ji}^{\mathrm{H}}\boldsymbol{g}_j\boldsymbol{g}_j^{\mathrm{H}}\mathbf{H}_{ji}\right)^{-1}\mathbf{H}_{ii}^{\mathrm{H}}\boldsymbol{g}_i\|}\sqrt{P}. \quad (10)$$

This process is repeated until convergence or a maximal number of steps is reached.

III. Interference Alignment With Incomplete CSIT

A. Tightly-feasible Settings

In this work, we consider only settings where the number of antennas available at the TXs and the RXs is the minimal one which allows for IA to be feasible.

Definition 1. *An IC* $\prod_{k=1}^{K}(N_k, M_k)$ *is called* tightly-feasible *if removing one antenna at any TX or RX makes IA unfeasible. An IC is tightly-feasible if and only if* $\sum_{k=1}^{K} N_k + M_k = K(K+1)$.

The characterization follows directly from the feasibility conditions (8). In terms of feasibility of IA under incomplete CSIT, the tightly-feasible setting corresponds to the worst case since additional antenna cannot reduce the feasibility and can even potentially be used to reduce the CSIT requirements. As an example, no CSIT is required if every RX has K antennas.

B. Feasibility of IA with Incomplete CSIT

We will now state one of the main results on the feasibility of IA under incomplete CSIT allocation.

Theorem 2. *In the IC* $\prod_{k=1}^{K}(N_k, M_k)$*, there exist a strictly incomplete CSIT allocation preserving IA feasibility if there exist a strictly included subset of users* $\mathcal{I}_{\mathrm{IC}}$ *such that*

$$\sum_{i\in\mathcal{I}_{\mathrm{IC}}} N_i + M_i = |\mathcal{I}_{\mathrm{IC}}|\left(|\mathcal{I}_{\mathrm{IC}}| + 1\right). \quad (11)$$

Proof: If equation (11) is verified, the subset of users $\mathcal{I}_{\mathrm{IC}}$ forms itself a tightly-feasible IC included in the original one. Indeed, equation (11) can be seen to corresponds to the particular choice of sets $\mathcal{I} = (\mathcal{I}_{\mathrm{IC}}, \mathcal{I}_{\mathrm{IC}})$ in (8). The relation (11) is fulfilled with equality meaning that the number of equations is the same as the number of variables. Hence, the optimization of the TX beamformers and the RX beamformers can be made without taking into account the users outside the set without reducing the feasibility of IA in the total IC. ■

Interpretation: In a tightly-feasible IC, IA is possible with incomplete CSI if and only if all the TX beamformers are not inter-dependent. Otherwise, each TX requires clearly to know the CSIT at all the other TXs which implies having the complete CSIT. This occurs without reducing IA unfeasible only if each TX and each RX can exploit all its ZF capabilities. This means letting TX j align its interference at $M_j - 1$ RXs and letting RX i ZF an interference subspace of dimension $N_i 1 - 1$. This is achieved if the subset of chosen forming the smaller IC inside which the beamformers are optimized forms itself a tightly-feasible IC.

IV. IA Algorithm with Incomplete CSIT Allocation

In this section, we build upon Theorem 2 to provide an IA algorithm requiring in some settings a strictly incomplete CSI allocation. We first provide a CSI allocation algorithm before showing how the max-SINR IA algorithm can be adapted to this incomplete CSIT allocation.

A. Incomplete CSIT Allocation Algorithm

This algorithm takes as input the antenna configuration and returns the n_{CSI} CSIT-sets $\mathcal{S}_k^{\mathrm{CSI}}$ (equivalent to the knowledge of the CSIT allocations $\{\mathbf{H}^{(j)}\}_j$).

Initialization: Firstly, the users are ordered by increasing number of antennas shared between the TX and the RX, i.e., with the permutation σ_{IC} verifying

$$\forall i = \{1,\ldots,K-1\}, N_{\sigma_{\mathrm{IC}}(i)} + M_{\sigma_{\mathrm{IC}}(i)} \leq N_{\sigma_{\mathrm{IC}}(i+1)} + M_{\sigma_{\mathrm{IC}}(i+1)}. \quad (12)$$

We define $k = 0$ and a subset of users $\mathcal{S}$ which we initialize with the 3 users having the smallest number of antennas, i.e.,

$$\mathcal{S} = \{\sigma_{\mathrm{IC}}(1), \sigma_{\mathrm{IC}}(2), \sigma_{\mathrm{IC}}(3)\}. \quad (13)$$

Update at step n: If equation (11) is verified with the set $\mathcal{S}$, then we update $k = k+1$ and we set

$$\mathcal{S}_k^{\text{CSI}} = \mathcal{S}. \tag{14}$$

If $|\mathcal{S}_k^{\text{CSI}}| = K$, the algorithm has reached its end and we set $n_{\text{CSI}} = k$. Otherwise, the set $\mathcal{S}$ is updated as

$$\mathcal{S} = \{\mathcal{S}, \sigma_{\text{IC}}(|\mathcal{S}|+1)\} \tag{15}$$

and the step $n+1$ starts.

B. Precoding with Incomplete CSIT

We have developed an algorithm to disseminate the CSIT and we will now provide a precoding algorithm exploiting this CSIT allocation to achieve IA. As a first step, we start by defining a modified IA max-SINR algorithm to allow for computation of only a subset of the beamformers.

1) IA Algorithm for Effective Channels: Hence, the modified IA function $f_{\text{Mod}}(\bullet)$ has for main difference with the conventional max-SINR algorithm that a subset of the TX beamformers and the RX beamformers are already fixed and taken as input parameter so that the resulting effective channel is considered.

Indeed, our modified IA function takes as input argument a channel matrix $\mathbf{H}_{\text{out}} \in \mathbb{C}^{K_{\text{out}} \times K_{\text{out}}}$ and a set of pairs of RX and TX beamformers $\mathcal{B}_{\text{in}} = (\boldsymbol{g}_i, \boldsymbol{t}_i)_{i=1}^{n_{\text{in}}}$ and provides as outputs the set of pairs of RX and TX beamformers $\mathcal{B}_{\text{out}} = (\boldsymbol{g}_i, \boldsymbol{t}_i)_{i=1}^{n_{\text{out}}}$. The set $\mathcal{B}_{\text{in}}$ contains the pairs of RX and TX beamformer which have already been fixed. This means that the effective channel created by these choices has to be considered. In contrast, $\mathcal{B}_{\text{out}}$ contains the K_{out} RX beamformers and K_{out} beamformers associated with $\mathbf{H}_{\text{out}}$. Consequently, it is clear that $\mathcal{B}_{\text{in}} \subset \mathcal{B}_{\text{out}}$.

The IA function f_{Mod} can be defined based on any IA algorithm f_{IA} from the literature and we use in the simulations the max-SINR algorithm recalled in Subsection II-E. The IA algorithm f_{IA} is simply applied on the effective channel obtained once the RX beamformers and the TX beamformers of the users inside $\mathcal{B}_{\text{in}}$ are fixed as given in input. Therefore, we can write

$$\mathcal{B}_{\text{out}} = f_{\text{Mod}}(\mathbf{H}_{\text{out}}, \mathcal{B}_{\text{in}}). \tag{16}$$

2) IA Algorithm with Incomplete CSIT: Let the CSIT-sets $\mathcal{S}_{k_j}^{\text{CSI}}$ be given and let us consider the precoding at TX j with the incomplete CSIT $\mathbf{H}^{(j)} = \mathbf{H}_{\mathcal{S}_{k_j}^{\text{CSI}}}$. If $k_j = 1$ (TX j belongs to the set of TXs which has the most incomplete CSI), TX j simply computes its precoder as

$$\mathcal{B}_1 = f_{\text{Mod}}(\mathbf{H}_{\mathcal{S}_1^{\text{CSI}}}, \mathcal{B}_0) \quad (= f_{\text{IA}}(\mathbf{H}_{\mathcal{S}_1^{\text{CSI}}})). \tag{17}$$

In fact, the TX computes at the same time the beamformers of all the TXs having the same incomplete CSIT, i.e., associated with the same CSIT-set.

If $k_j > 1$, TX j computes iteratively the beamformers and the RX beamformers associated with the CSIT-sets smaller by inclusion than its own CSIT-set.

$$\forall n = \{2, \dots, k_j\}, \mathcal{B}_n = f(\mathbf{H}_{\mathcal{S}_n^{\text{CSI}}}, \mathcal{B}_{n-1}). \tag{18}$$

From the set of TX and RX beamformers computed based on its own CSIT-set, TX j can extract its TX beamformer and implement it. It is then possible to show the following result

Proposition 1. *The IA algorithm with incomplete CSIT described in Subsection IV-B achieves IA.*

Proof: The proof is based on the fact that settings the beamformers inside a tightly-feasible set does not reduce the feasibility if all the beamformers are optimized at the same time. In our case, this is not the case since the precoding is distributed at the TXs but each TX starts by computing the beamformers of the TXs inside smaller tightly-feasible sets, thus ensuring the coherency of the precoding between the TXs. ■

V. Example and Symbolic Representation

In the following, we study an example to gain insight into how the algorithm works in practice. We take as example the IC $(2,2).(2,2).(2,2),(4,5).(4,5)$ which will be verified later on to be tightly-feasible. Applying our algorithm returns the two CSIT-sets

$$\mathcal{S}_1^{\text{CSI}} = \{1,2,3\}, \quad \mathcal{S}_2^{\text{CSI}} = \{1,2,3,4,5\}. \tag{19}$$

The first 3 users can be seen to form an homogeneous tightly-feasible setting with $M=2$, $N=2$, and $K=3$. These 3 TXs run the max-SINR IA algorithm without taking into account the users 4 and 5. Indeed, the ZF capabilities of these users are already completely consumed by the interference management inside this smaller IC.

The algorithm continues by using the effective channel created by the beamformers implemented at the 3 first users. In fact, after consideration of the IA constraints at RXs 1, 2, and 3, TXs 4 and 5 do have have any ZF possibilities left. Thus, TX 4, resp. TX 5, simply fills the last dimension left free for the interference at RX 5, resp. RX 4.

To verify intuitively how the precoding works, we introduce a symbolic representation of the IA scheme in Fig. 1. We represent the dimensions available at RX i by an array of N_i boxes. The first box on the right represents the dimension taken by the signal while the other boxes represent the dimensions left free for the interference, i.e., the ZF capabilities at the RX. For each RX, another box indicates if a TX precodes its signal to align with the interference subspace, thus creating no additional dimension of interference. If this is not the case, the transmission from this TX creates a dimension of interference at the RX considered. This is symbolized by filling one box at this RX meaning that this stream generates one dimension of interference at that RX.

In this symbolic representation, the precoding scheme achieves IA if the number of interfering dimensions at a RX can be contained in the boxes represented at the RX while fulfilling that each TX fulfills a number of IA constraint smaller or equal than its number of antennas. Note that this representation is symbolic and does not take into account the beamformer actually used. Yet, it allows to verify the feasibility of IA and to visualize the steps of the IA algorithm.

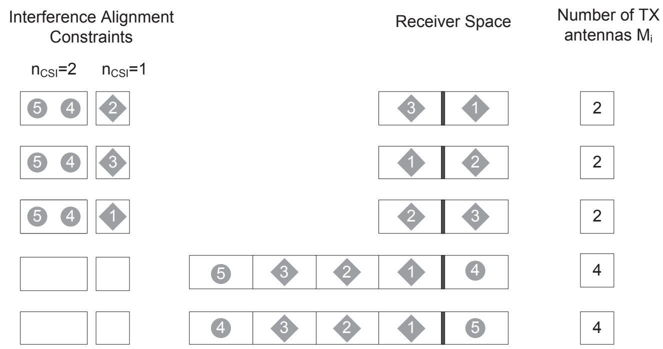

Fig. 1. Symbolic representation of the IA algorithm with incomplete CSIT for the tightly-feasible IC $(2,2).(2,2).(2,2).(4,5).(4,5)$.

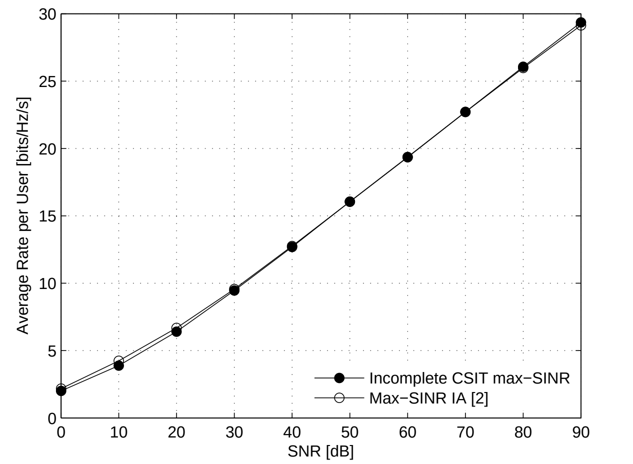

Fig. 2. Average rate per user in terms of the SNR for the tightly-feasible IC $(2,2).(2,2).(3,3).(3,3).(5,5).(6,6).(7,7)$.

We can see from Fig. 1 that each RX can receive its transmitted signal free of interference whilst TXs 1, 2, and 3 fulfill IA constraints solely inside the smaller IC formed by these 3 users and thus require only this incomplete CSIT. Furthermore, for all j, TX j align its interference at M_j-1 RXs, and for all i, the interference subspace at RX i spans N_i-1 dimensions. Thus, this symbolic representation confirms that our IA scheme achieves IA based on the strictly incomplete CSIT allocation provided.

VI. Simulation Results

To confirm the efficiency of our approach, we provide in Fig. 2 the simulations results for the IC setting $(2,2).(2,2).(3,3).(3,3).(6,6).(6,6).(6,6)$. The CSIT-sets for that example are $\mathcal{S}_1^{\text{CSI}} = \{1,2,3,4\}, \mathcal{S}_2^{\text{CSI}} = \{1,2,3,4,5,6,7\}$. Due to space constraints, we omit the symbolic diagram as in Fig. 1 and we let to the reader the verification that the provided CSIT-sets achieve IA in the tightly-feasible IC setting considered.

We plot the average rate per user achieved using the max-SINR IA algorithm described in Subsection II-E adapted as described in Subsection IV-B to the *incomplete CSIT* setting. Additionally, we show the performance of the same max-SINR algorithm but used directly in a conventional setting with full CSIT sharing. We can observe that the proposed algorithm using incomplete CSIT performs very close to the algorithm with full CSIT and achieves the same pre-log factor which means that it achieves IA. Furthermore, the small losses are balanced by an important property of our IA algorithm, which is a strong reduction of the complexity.

VII. Conclusion

We have discussed IA algorithms which do not require full CSIT at all TXs, thus alleviating strongly the impact on the architecture of the sharing of the CSIT. Furthermore, this reduction in CSIT comes at the cost of very reduced performance losses. Consequently, such algorithms have a strong potential for making IA more practical. Furthermore, in the extended version [9], the minimal CSIT allocation required to achieve IA is derived for tightly-feasible ICs. It is also shown there how it is possible in the settings with extra-antennas, called super-feasible, to exploit the additionnal antennas to reduce the size of the CSIT allocation. Finally, well known results on the analysis of the quantization scheme could be applied in the future to evaluate more accurately the amount of feedback required.

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