# Sum Rate Maximization in the Noisy MIMO Interfering Broadcast Channel with Partial CSIT via the Expected Weighted MSE

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***Abstract*—We consider the maximization of Weighted Sum Rate (WSR) for a Noisy Multiple-Input-Multiple-Output (MIMO) K cell Interfering Broadcast Channel (IBC) when only partial channel state information is available at the transmit side (CSIT) while assuming perfect CSI at the receiver (CSIR). In this work, CSIT is modeled by means of a Gaussian prior representing the mean and the covariance of the channel. The expected WSR is then maximized exploiting the relationship between WSR and the Weighted Mean Square Error (WMSE). This leads to an approximate solution but allows to capitalize on a convenient iterative algorithm for the optimization of the transmit beamforming (BF) matrices. The algorithm uses alternating minimization between BF, receiver (RX) filters and WMSE weights by exploiting convexity properties of each sub problem.**

***Index Terms*—MIMO, MMSE, weighted sum rate, interference channel, interference alignment, deterministic annealing, Partial CSIT**

## I. Introduction

In modern wireless systems, multiple transmitter and receiver pairs attempt to communicate using the same communication resources. These communicating pairs interfere with each other resulting in an interference network also called in information theory, an Interference Channel (IFC). While systematic study of the $K$-user IFC has been the focus of intense research in recent times, its capacity in general remains unknown. The common approach to handling interference in the weak interference regime is to treat it as additional noise. In the strong interference regime, on the other hand, previous work recommends decoding interference and then subtracting it from the received signal [1]. More difficult to handle is the case when the useful signal and interference levels compare. This framework is typically studied as a problem where interference is recognized as an impairment but no attempt is made to explicitly decode it. We call this setting the Noisy Interference Channel.

Recent developments in the infinite Signal to Noise ratio (SNR) regime [2] suggests designing interferer signals such that their contributions align in reduced dimensions at the unintended receivers. This approach is termed Interference Alignment (IA). In this way an interference free subspace is more easily freed up for the desired signal to reside in.

To make IA practical requires perfect channel state information at each transmitter (CSIT). Furthermore global CSIT knowledge is in general required. This assumption is of course difficult to justify in practical time varying channels. Several recent works have therefore focused on making IA practical. In [3] and [4], the authors consider SISO and MISO IFC in a frequency selective environment. Using quantized channel fed back they show that full multiplexing gain can be achieved if the feedback bit-rate scales sufficiently fast with the SNR. Authors of [5] propose using analog feedback for the acquisition of full CSIT. Here channel coefficients are directly fed back to the base station (BS) without any quantization, offering the advantage that the underlying complexity does not increase with the SNR as opposed to quantized digital feedback. Furthermore, it is shown that IA when acquiring full CSIT by means of analog feedback incurs no loss in multiplexing gain if feedback power scales with SNR.

Even though IA promises maximum degrees of freedom (DoF) in a MIMO IFC, it remains suboptimal at finite SNRs. A more appropriate approach in finite SNR regimes is to maximize the weighted sum rate (WSR). The problem of sum rate maximization for a general multi-stream MIMO IFC has been addressed in [6]. The algorithm proposed is based on a previous work on broadcast channel (BC) [7] and extended to a MIMO IFC. It was extended to the interfering broadcast channel (IBC) in [8], where the MIMO IFC is a special case. Nevertheless, the solutions derived in [6] and [8] also require perfect CSIT. In practical systems only imperfect (estimated and often fed back) CSI is available at each transmitter implying that more robust schemes need to be considered for beamforming design. In [9] the authors propose a robust beamforming approach. Their iterative algorithm alternately computes transmit and receive filters with the objective of maximizing the worst-case signal-to-interference-plus-noise ratio (SINR) using semidefinite relaxation in presence of CSI uncertainties.

The beamformer design problem for a multiuser MIMO MAC channel, in presence of statistical CSIT, has been studied in [10]. The authors found that the optimal transmit directions, when the minimization of the average sum mean squared error (MSE) is studied, were given by the eigenvectors of the channel mean or correlation matrix. In this paper we introduce an iterative algorithm to optimize the BF matrices, to maximize the sum rate, for the noisy MIMO IBC when only stochastic CSIT is available. The solution proposed for robust BF design is based on the relationship between WSR and Weighted MSE (WMSE) [7], [6]. In [11] the problem of robust beamforming design for single user MIMO with different types of CSI has been studied where the designing criterion was the minimization of the mean squared error. The main difference with the approach taken here resides in the multi user approach that makes impossible to use the results derived in [11]. Here the optimal expressions of the BF filters are obtained from the expectation of the WMSE w.r.t. the channel realizations. In a recent paper [12] a similar approach is considered where the objective is the sum MSE minimization and not the sum rate maximization. In addition we show that minimizing the expected value of the WMSE corresponds to the maximization of a lower-bound of the WSR. We compare the proposed solution with an IA solution obtained from partial and perfect CSIT.

## II. Signal Model

We consider a $K$-cell MIMO interfering broadcast channel (IBC). For ease of exposition, we denote the transmitters as Base station (BS) and the receivers as Mobile user (MU). The $k$-th BS is equipped with $M_k$ transmitting antennas and wants to communicate with $L_k$ MUs in its own cell. We denote with $N_i^{(k)}$ the number of antennas at $i$-th MU in cell $k$. Because all Tx-Rx pairs share the same frequency bands each transmission generates interference at all non intended receivers. At Rx number $i$ in cell $k$ the received signal vector $\mathbf{y}_i^{(k)}$ can be written as

$$\mathbf{y}_i^{(k)} = \mathbf{H}_{ik}^{(k)}\mathbf{x}_i^{(k)} + \sum_{l\neq k}\mathbf{H}_{ik}^{(k)}\mathbf{x}_l^{(k)} + \sum_{j\neq k}\sum_{l=1}^{L_j}\mathbf{H}_{ij}^{(k)}\mathbf{x}_l^{(j)} + \mathbf{n}_i^{(k)} \quad (1)$$

where $\mathbf{H}_{ij}^{(k)} \in \mathbb{C}^{N_i^{(k)}\times M_j}$ represents the channel matrix between the $j$-th BS and $i$-th MU in cell number $k$, $\mathbf{x}_i^{(k)}$ is the $\mathbb{C}^{M_k\times 1}$ transmit signal vector of the $k$-th BS for its $i$-th MU and the $\mathbb{C}^{N_i^{(k)}\times 1}$ vector $\mathbf{n}_i^{(k)}$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. The channel is assumed to follow a block-fading model having a coherence time of $T$ symbol intervals without channel variation. Each entry of the channel matrix is an independent complex random variable drawn from a Gaussian distribution $\mathcal{CN}(0,1)$. We denote by $\mathbf{G}_i^{(k)}$, the $\mathbb{C}^{M_k\times d_i^{(k)}}$ precoding matrix of the $k$-th BS to the $i$-th receiver. Thus $\mathbf{x}_i^{(k)} = \mathbf{G}_i^{(k)}\mathbf{s}_i^{(k)}$, where $\mathbf{s}_i^{(k)}$ is a $d_i^{(k)}\times 1$ vector representing the $d_i^{(k)}$ independent symbol streams for the $(k,i)$-th user pair. We assume $\mathbf{s}_i^{(k)}$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_i^{(k)} \sim \mathcal{N}(0, \mathbf{I}_{d_i^{(k)}})$. The $i$-th receiver, to suppress interference and retrieve its $d_i^{(k)}$ desired streams, applies the filter matrix $\mathbf{F}_i^{(k)H} \in \mathbb{C}^{d_i^{(k)}\times N_i^{(k)}}$.

## III. WSR maximization for the MIMO IBC

In this paper we focus our attention on the maximization of the WSR of $K$-cell MIMO IBC when only partial CSIT is available. In the first part of this section we introduce some general results for a $K$-user MIMO IFC [6], extended to the MIMO IBC [8], that will be used for the case under investigation. We limit receiver complexity by modeling the interference as colored noise, whence the term Noisy IBC. As a result, linear receivers are sufficient.

Assuming Gaussian signaling, the WSR maximization problem can be mathematically expressed as follows:

$$\begin{aligned}\{\mathbf{G}_i^{\star(k)}\} &= \arg\min_{\{\mathbf{G}_i^{(k)}\}} \sum_{k=1}^{K}\sum_{i=1}^{L_k} -u_i^{(k)} \log|\mathbf{E}_i^{(k)-1}| \\ \text{s. t} &\sum_{i=1}^{L_k}\text{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) \leq P_k \;\forall k\end{aligned}$$

where $\mathbf{E}_i^{(k)} = (\mathbf{I} + \mathbf{G}_i^{(k)H}\mathbf{H}_{ik}^{(k)H}\mathbf{R}_{\bar{i}}^{(k)-1}\mathbf{H}_{ik}^{(k)}\mathbf{G}_i^{(k)})^{-1}$, $u_i^{(k)} \geq 0$ denotes the weight assigned to the $(k,i)$-th user's rate and $P_k$ the corresponding transmit power constraint. We use $|\,.\,|$ to denote the determinant of a matrix. The interference plus noise covariance matrix $\mathbf{R}_{\bar{i}}^{(k)}$ is:

$$\mathbf{R}_{\bar{i}}^{(k)} = \mathbf{R}_{n_k n_k} + \sum_{(l,j)\neq(k,i)} \mathbf{H}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\mathbf{H}_{il}^{(k)H}.$$

WSR maximization is known to be a non-convex problem, hence finding a solution is a complex task. In [7] the authors show that for a MIMO Broadcast Channel the maximization of the WSR corresponds to the minimization of the WMSE. The principle has been extended for a MIMO IFC in [6] and to IBC in [8]. From there our approach is based on introducing an augmented cost function in which two additional optimization variables appear, receive filters and weighting matrices $\mathbf{W}_i^{(k)}$. The optimization problem that we consider is

$$\begin{aligned}\arg\max_{\{\mathbf{G}_i^{(k)},\,\mathbf{F}_i^{(k)},\,\mathbf{W}_i^{(k)}\}} &\sum_{(k,i)} -u_i^{(k)}(\text{Tr}(\mathbf{W}_i^{(k)}\boldsymbol{\mathcal{E}}_i^{(k)}) - \log|\mathbf{W}_i^{(k)}|) \\ \text{s. t} &\sum_{i=1}^{L_k}\text{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) \leq P_k.\end{aligned} \quad (2)$$

Assuming $\mathbb{E}\{\mathbf{s}_i^{(k)}\mathbf{s}_i^{(k)H}\} = \mathbf{I}_{d_i^{(k)}}$, the MSE covariance matrix for general Tx and Rx filters is

$$\begin{aligned}\boldsymbol{\mathcal{E}}_i^{(k)} =& \mathbb{E}[(\mathbf{s}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{y}_i^{(k)})(\mathbf{s}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{y}_i^{(k)H}]\} \\ =& \mathbf{I} - \mathbf{G}_i^{(k)H}\mathbf{H}_{ik}^{(k)H}\mathbf{F}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{H}_{ik}^{(k)}\mathbf{G}_i^{(k)} \\ &+ \sum_{(l,j)}\mathbf{F}_i^{(k)H}\mathbf{H}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\mathbf{H}_{il}^{(k)H}\mathbf{F}_i^{(k)} + \mathbf{F}_i^{(k)H}\mathbf{R}_{n_k n_k}\mathbf{F}_i^{(k)}\end{aligned} \quad (3)$$

Cost function (2) is concave or even quadratic in one set of variables, keeping the other two fixed. Hence we shall optimize it using alternating maximization. Here we consider a channel knowledge at the transmitter side that can be modeled in terms of channel mean and variance, that can represent the channel estimate and estimation error:

$$\mathbf{H}_{ij}^{(k)} = \widehat{\mathbf{H}}_{ij}^{(k)} + (\mathbf{R}_i^{(k)r})^{\frac{1}{2}}\widetilde{\mathbf{H}}_{ij}^{(k)}(\mathbf{R}_j^t)^{\frac{H}{2}} \quad (4)$$

where $\widehat{\mathbf{H}}_{ij}^{(k)}$ can model the channel estimate for the channel between Tx $j$ and Rx $i$ in cell $k$. $\mathbf{R}_j^t$ is the Tx side covariance matrix while $\mathbf{R}_i^{(k)r}$ represents the covariance matrix at the Rx side. $\widetilde{\mathbf{H}}_{ij}^{(k)}$ is a matrix with iid Gaussian, zero mean and unit variance, entries. We should underline here that this restrictive Kronecker covariance model is not required for the technique described in this paper to be applicable. We only assume this model to simplify some of the expressions.

With the given parametrization of the channel we can obtain the expected value of the MSE:

$$\begin{aligned}\overline{\boldsymbol{\mathcal{E}}}_i^{(k)}=&\mathbb{E}_{\mathbf{H}|\widehat{\mathbf{H}}}\{\boldsymbol{\mathcal{E}}_i^{(k)}\} = \mathbf{I} - \mathbf{G}_i^{(k)H}\widehat{\mathbf{H}}_{ik}^{(k)H}\mathbf{F}_i^{(k)} - \mathbf{F}_i^{(k)H}\widehat{\mathbf{H}}_{ik}^{(k)}\mathbf{G}_i^{(k)}\\ &+\sum_{(l,j)}\mathbf{F}_i^{(k)H}\widehat{\mathbf{H}}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\widehat{\mathbf{H}}_{il}^{(k)H}\mathbf{F}_i^{(k)}\\ &+\mathbf{F}_i^{(k)H}\left[\sum_{(l,j)}Tr\{\mathbf{R}_l^t\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\}\mathbf{R}_i^{(k)r} + \mathbf{R}_{n_k n_k}\right]\mathbf{F}_i^{(k)}\end{aligned}$$

where the expectation above is taken w.r.t. the CSI uncertainty. We should remark that assuming a Tx side covariance matrix of the form $\mathbf{R}_l^t = \mathbf{I}$, it is possible to interpret the expected MSE above as the original MSE in (3) with $\mathbf{H}_{ij}^{(k)} = \widehat{\mathbf{H}}_{ij}^{(k)}$ and an augmented noise covariance matrix contribution of the form $\mathbf{R}'_{n_k n_k} = \sum_l P_l \mathbf{R}_i^{(k)r} + \mathbf{R}_{n_k n_k}$. Hence the proposed partial CSI algorithm solves exactly the perfect CSI WSR for a system with modified channel and noise covariance matrices! The optimization problem (2) now becomes the following:

$$\begin{aligned}&\arg\max_{\{\mathbf{G}_i^{(k)},\,\mathbf{F}_i^{(k)},\,\mathbf{W}_i^{(k)}\}}\sum_{(k,i)} -u_i^{(k)}(\mathrm{Tr}(\mathbf{W}_i^{(k)}\overline{\boldsymbol{\mathcal{E}}}_i^{(k)}) - \log|\mathbf{W}_i^{(k)}|)\\ &\qquad \mathrm{s.\,t}\sum_{i=1}^{L_k}\mathrm{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) \le P_k.\end{aligned} \tag{5}$$

The corresponding Lagrangian can be written as:

$$\begin{aligned}&J(\{\mathbf{G}_i^{(k)},\mathbf{F}_i^{(k)},\mathbf{W}_i^{(k)},\lambda_k\}) = -\sum_k\lambda_k(\sum_{i=1}^{L_k}\mathrm{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) - P_k)\\ &-\sum_{(k,i)}u_i^{(k)}(\mathrm{Tr}(\mathbf{W}_i^{(k)}\overline{\boldsymbol{\mathcal{E}}}_i^{(k)}) - \log|\mathbf{W}_i^{(k)}|)\end{aligned} \tag{6}$$

This new cost function will be optimized w.r.t. one set of variables, keeping the other two fixed. The first step is the calculation of the optimal Rx filters assuming fixed the matrices $\mathbf{G}_i^{(k)}$ and $\mathbf{W}_i^{(k)}$. From the derivative of $J$ w.r.t. $\mathbf{F}_i^{(k)}$ the optimal receiver results to be an MMSE filter of the form:

$$\begin{aligned}\mathbf{F}_i^{(k)}=&(\sum_{(l,j)}[\widehat{\mathbf{H}}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\widehat{\mathbf{H}}_{il}^{(k)H}+\mathrm{Tr}\{\mathbf{R}_l^t\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\}\mathbf{R}_i^{(k)r}]\\ &+\mathbf{R}_{n_k n_k})^{-1}\widehat{\mathbf{H}}_{ik}^{(k)}\mathbf{G}_i^{(k)}\end{aligned} \tag{7}$$

The following step is the determination of the optimal expression for the matrix $\mathbf{W}_i^{(k)}$ while keeping the other two variable sets fixed. Then, equating the derivative of the Lagrangian (6) w.r.t. $\mathbf{W}_i^{(k)}$ to zero, we get :

$$\mathbf{W}_i^{(k)} = \overline{\boldsymbol{\mathcal{E}}}_i^{(k)-1} \tag{8}$$

The final step is the maximization of the given cost function w.r.t. the BF matrix. To accomplish this task we derive the Lagrangian w.r.t. the matrix $\mathbf{G}_k$ and equate it to zero:

$$\begin{aligned}&\frac{\partial \mathrm{J}(\{\mathbf{G}_i^{(k)},\lambda_k\})}{\partial\mathbf{G}_i^{(k)*}} = -\sum_{(l,j)}u_j^{(l)}\mathrm{Tr}\{\mathbf{W}_j^{(l)}\mathbf{F}_j^{(l)H}\mathbf{R}_j^{(l)r}\mathbf{F}_j^{(l)}\}\mathbf{R}_k^t\mathbf{G}_i^{(k)} - \lambda_k\mathbf{G}_i^{(k)}\\ &u_i^{(k)}\widehat{\mathbf{H}}_{ik}^{(k)H}\mathbf{F}_i^{(k)}\mathbf{W}_i^{(k)} - \sum_{(l,j)}u_j^{(l)}\widehat{\mathbf{H}}_{jk}^{(l)H}\mathbf{F}_j^{(l)}\mathbf{W}_j^{(l)}\mathbf{F}_j^{(l)H}\widehat{\mathbf{H}}_{jk}^{(l)}\mathbf{G}_i^{(k)} = \mathbf{0}.\end{aligned}$$

This leads to the following expression for the optimal BF:

$$\begin{aligned}\mathbf{G}_i^{(k)}=&\left(\sum_{(l,j)}u_j^{(l)}[\widehat{\mathbf{H}}_{jk}^{(l)H}\mathbf{Q}_j^{(l)}\widehat{\mathbf{H}}_{jk}^{(l)}+\mathrm{Tr}\{\mathbf{Q}_j^{(l)}\mathbf{R}_j^{(l)r}\}\mathbf{R}_k^t]+\lambda_k\mathbf{I}\right)^{-1}\\ &\times\widehat{\mathbf{H}}_{ik}^{(k)H}\mathbf{F}_i^{(k)}\mathbf{W}_i^{(k)}u_i^{(k)}\end{aligned} \tag{9}$$

where $\mathbf{Q}_j^{(l)} = \mathbf{F}_j^{(l)}\mathbf{W}_j^{(l)}\mathbf{F}_j^{(l)H}$. The only variable that still needs to be optimized is the Lagrange multiplier $\lambda_k$. First check if the power constraint is satisfied for $\lambda_k = 0$. If yes, then $\lambda_k = 0$. If not, the Tx power equality constraint is active. To derive the optimal value of $\lambda_k$ we can use the results derived in [6] for the IFC. If we define the following compound quantities $\mathbf{F}_k = diag\{\mathbf{F}_1^{(k)},\ldots,\mathbf{F}_{L_k}^{(k)}\}$, $\mathbf{G}_k = [\mathbf{G}_1^{(k)},\ldots,\mathbf{G}_{L_k}^{(k)}]$, $\mathbf{H}_{ij} = [\mathbf{H}_{1j}^{(i)T},\ldots,\mathbf{H}_{L_k j}^{(i)T}]^T$, $\mathbf{W}_k = diag\{u_1^{(k)}\mathbf{W}_1^{(k)},\ldots,u_{L_k}^{(k)}\mathbf{W}_{L_k}^{(k)}\}$ and $\mathbf{R}_k^r = diag\{\mathbf{R}_1^{(k)r},\ldots,\mathbf{R}_{L_k}^{(k)r}\}$ we can read the IBC studied above as a traditional IFC. Then to find the optimal value of $\lambda_k$ we pre-multiply the derivative of the Lagrangian $J$ w.r.t. the compound BF matrix by $\mathbf{G}_k^H$. Thanks to the first order optimality condition taking the trace of that product we get:

$$\mathrm{Tr}\left\{\mathbf{G}_k^H\frac{\partial \mathrm{J}(\{\mathbf{G}_k,\lambda_k\})}{\partial\mathbf{G}_k^*}\right\} = 0.$$

Imposing the power constraint to be satisfied with equality, hence the contribution $\lambda_k Tr\{\mathbf{G}_k^H\mathbf{G}_k\} = \lambda_k P_k$, we are able to derive the value of the optimal Lagrange multiplier [6]. To overcome the convergence difficulties in non-convex optimization problems, like WSR, several heuristic approaches have been proposed. Among them we have *Deterministic Annealing* (DA) [6]. This method takes its name from the physical annealing process in which a system is first "melted" and then slowly cooled down in order to allow the atoms in the system to find a state with lower energy until the system is "frozen" in a globally optimum state. In our problem, the role of temperature is played by the *noise power* $\sigma_k^2$, which starting from now we assume, without losing generality (w.l.g.), equal to $\sigma_k = \sigma^2\ \forall k$.

**Algorithm 1** DA-MWSR Algorithm for MIMO IFC

set $t = 0$
Fix the initial set of precoding matrices $\mathbf{G}_i^{(k)}$
**repeat**
  increment SNR: $t^{(i+1)} = t^{(i)} + \delta t$
  **repeat**
    Given $\mathbf{G}_i^{(k)}$ compute $\mathbf{F}_i^{(k)}$ and $\mathbf{W}_i^{(k)}$, $\forall(k,i)$, as in (7)-(8)
    Given $\mathbf{F}_i^{(k)}$, $\mathbf{W}_i^{(k)}$, compute $\mathbf{G}_i^{(k)}$ $\forall(k,i)$, using (9)
  **until** convergence
**until** target SNR is reached

In the algorithm description, in Table **Algorithm 1**, $t^{(i)}$ represents the value of the SNR that is incremented at each step of the annealing procedure. For further details on DA applied to WSR maximization refer to [6] and reference therein. Here we should underline that the Rx filters calculated in the proposed algorithm are not the ones actually used at the Rx side. Those filters are based on perfect CSIR and hence they are different compare to the one in (7). The MMSE Rx filter used at receiver $i$ in cell $k$ will be of the form:

$$\mathbf{F}_i^{(k)} = (\sum_{(l,j)}[\mathbf{H}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\mathbf{H}_{il}^{(k)H} + \mathbf{R}_{n_k n_k})^{-1}\mathbf{H}_{ik}^{(k)}\mathbf{G}_i^{(k)} \quad (10)$$

## IV. WSR Lower Bound with Partial CSIT

In this section we first study how the approach presented in section III is related to the ergodic sum rate, then we introduce a new lower bound for the WSR when IA transmit and receive filter are computed using partial CSIT. To make the presentation more clear we specify the results of the previous section to a MIMO IFC. This does not reduce the validity of our results because the IBC can be interpreted as a traditional MIMO IFC using the compound quantities introduced in the previous section. To simplify the derivation of the bound we assume to work per stream instead of per user. As shown in [7] working per stream does not cause a reduction of performances.

As described in section III the rate for a $K$-user MIMO IFC with full CSIT can be written as:

$$\mathcal{R}_{FCSIT} = \sum_{(k,n)} \log \mathbf{E}_{kn}^{-1} \quad (11)$$

where $\mathbf{E}_{kn}$ indicates the MMSE of the $n$-th stream of user $k$. Then the minimum MSE for stream $n$ of user $k$ is $\mathbf{E}_{kn} = \min_{\mathbf{f}_{kn}} \boldsymbol{\mathcal{E}}_{kn} = (1 + \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk} \mathbf{g}_{kn})^{-1}$. Vector $\mathbf{g}_{kn}$ represents the $n$-th column of the BF matrix $\mathbf{G}_k$, matrix $\mathbf{R}_{\overline{kn}}$ is the interference plus noise covariance matrix for stream $(k,n)$. From Jensen's inequality we have:

$$\mathbb{E}_{\widehat{\mathbf{H}},\widetilde{\mathbf{H}}}\{\sum_{(k,n)} \log \mathbf{E}_{kn}^{-1}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \quad (12)$$

the expectation above is taken over all channel pdfs. On the other hand the rate that we obtain once we optimize the cost function in (5) is

$$\mathcal{R}_{PCSIT} = \sum_{(k,n)} u_k \log(\overline{\boldsymbol{\mathcal{E}}}_{kn}^{\star})^{-1} \quad (13)$$

where $\overline{\boldsymbol{\mathcal{E}}}_{kn}^{\star} = \min_{\mathbf{f}_{kn}} \overline{\boldsymbol{\mathcal{E}}}_{kn}$ and is equal to:

$$\mathbf{g}_{kn}^H \widehat{\mathbf{H}}_{kk}^H [\sum_{(l,m)} [\widehat{\mathbf{H}}_{kl}\mathbf{g}_{lm}\mathbf{g}_{lm}^H \widehat{\mathbf{H}}_{kl}^H + \mathrm{Tr}\{\mathbf{R}_{kl}^t \mathbf{g}_{lm}\mathbf{g}_{lm}^H\}\mathbf{R}_{kl}^r] + \mathbf{R}_{n_k n_k}]^{-1} \widehat{\mathbf{H}}_{kk}\mathbf{g}_{kn}$$

where $\mathbf{f}_{kn}$ is the $n$-th row of the Rx filter matrix $\mathbf{F}_k$. From the equation above we can see that $\mathbb{E}\{\mathbf{E}_{kn}\} \neq \overline{\boldsymbol{\mathcal{E}}}_{kn}^{\star}$. Calculating the expected value over the channel uncertainty we can show that:

$$\mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \leq \overline{\boldsymbol{\mathcal{E}}}_{kn}^{\star} \quad (14)$$

This statement can be proved easily. Assume $\boldsymbol{\mathcal{E}}_{kn} = \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$, and let $B(\mathbf{f}_{kn}) = \mathbb{E}_{\mathbf{H}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$. Now consider $B(\mathbf{f}_{kn}^o) = \min_{\mathbf{f}_{kn}} B(\mathbf{f}_{kn}) = \min_{\mathbf{f}_{kn}} \mathbb{E}_{\mathbf{H}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$. Then for any $\mathbf{H}_{ij}$,

$$\min_{\mathbf{f}_{kn}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij}) \leq \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}^o, \mathbf{H}_{ij})$$

hence

$$\mathbb{E}_{\mathbf{H}} \min_{\mathbf{f}_{kn}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij}) \leq \mathbb{E}_{\mathbf{H}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}^o, \mathbf{H}_{ij}) = \min_{\mathbf{f}_{kn}} \mathbb{E}_{\mathbf{H}} \boldsymbol{\mathcal{E}}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$$

From (14) we can conclude

$$\mathbb{E}_{\widehat{\mathbf{H}},\widetilde{\mathbf{H}}}\{\sum_{(k,n)} \log \mathbf{E}_{kn}^{-1}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \overline{\boldsymbol{\mathcal{E}}}_{kn}^{\star}$$

The overall inequality states that with our approach we are actually maximizing a sum rate lower-bound.

As shown in [13] for a BC system, also for a $K$-user MIMO IFC it is possible to derive a SR lower bound in case of IA transmissions. The rate for the $k$-th UE can be written as:

$$\mathcal{R}_k = \log(1 + \frac{1}{\sigma_k^2}|\mathbf{f}_k \mathbf{H}_{kk} \mathbf{g}_k|^2) \quad (15)$$

in the following we assume that $\sigma_k^2 = 1$. This is for the case where perfect CSIT are available at the BSs. In this paper, due to lack of space, we give only the main results for the case where each user sends only one stream $d_k = 1\ \forall k$.

To study the case with imperfect CSIT, we use the channel model (4) where now $\overline{\mathbf{H}}_{ij} = (\mathbf{R}_i^r)^{\frac{1}{2}} \widetilde{\mathbf{H}}_{ij} (\mathbf{R}_j^t)^{\frac{H}{2}}$. Due to IA design $\mathbf{f}_k \widehat{\mathbf{H}}_{ki} \mathbf{g}_i = 0$ , $\forall k \neq i$. Then we rewrite the Rx signal as:

$$\begin{aligned} r_k &= \mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \sum_{i \neq k} \mathbf{f}_k \overline{\mathbf{H}}_{ki} \mathbf{g}_i s_i + \mathbf{f}_k \mathbf{n}_k \\ &= \mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \mathbf{f}_k \mathbf{n}_k^{'} \end{aligned}$$

where the equivalent noise term $\mathbf{n}_k^{'}$ represents the residual interference plus noise contribution that can be model as a Gaussian noise zero mean and variance $\sigma_k^{'2} = 1 + \tilde{\sigma}^2$.

Now absorbing all the interference contributions in the noise (we are considering the noisy IFC) we get a rate lower bound (and close approximation) by ignoring the dependence of the term $\mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k$ on the signal $s_k$ and absorbing $\mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k$ into the noise also. Hence we get the rate lower bound

$$\mathcal{R}_k^{LB} = \log(1 + \frac{1}{\sigma_k^{'2}}|\mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k|^2) \quad (16)$$

In other words, this rate lower bound corresponds to the rate of an IFC with max WSR-IA design for the case in which the overall channel is $\widehat{\mathbf{H}}$ instead to $\mathbf{H}$ and the noise variances are increased by a term $\tilde{\sigma}^2$ in link $k$.

## V. Simulation Results

We provide here some simulation results to compare the performances of the proposed algorithm for the maximization of the WSR using partial CSIT (MWSR$_{PCSI}$ in figure) for a 3-user MIMO IFC. Here DA is used to avoid to be trapped in local optimal solutions. To find the IA solution for the case with partial CSIT (IA$_{PCSI}$) we use the algorithm proposed in [14] where instead of the real channel matrix $\mathbf{H}_{ij}$ we use only the channel estimate $\widehat{\mathbf{H}}_{ij}$. The channels are generated according to model (4). The algorithm proposed in this paper can handle any general choice of the Tx and Rx covariance matrix but in the numerical examples proposed in this section we consider, as example, $\mathbf{R}_j^t = \mathbf{I}$ and $\mathbf{R}_i^r = \tilde{\sigma}^2\mathbf{I}$. $\tilde{\sigma}^2$ represents the estimation error variance and here is scaled inversely proportional with the SNR. In Fig. 1 the performances, in terms of sum rate, of the proposed max WSR algorithm and IA for the case of partial CSIT are depicted. Those curves are compared with sum rate obtained using the MWSR, from [6], and IA, from [14], with perfect CSIT (MWSR$_{FCSI}$

and IA$_{FCSI}$ respectively). It can be noted that the max WSR solution outperforms the IA solutions for both cases, perfect and partial CSIT. On the other hand the rate curves obtained with partial CSIT show a rate offset compare to the corresponding curves obtained with perfect CSIT. In Fig.2 we

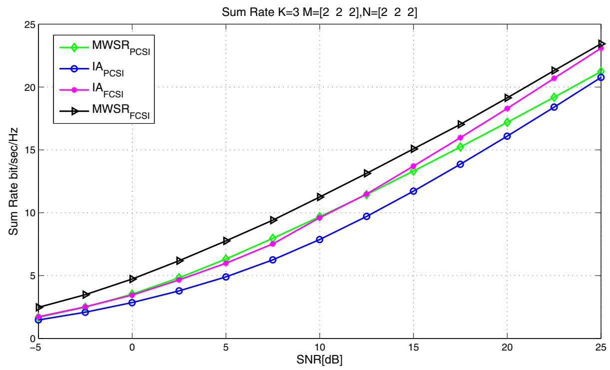


Fig. 1: Sum Rate comparisons for $K = 3$ $M_k = N_k = 2$, $\forall k$

report the sum rate curves for the same algorithms presented before. The main difference is that in this case the channel estimation error variance does not scale with the SNR but remains constant at $-6$dB. As we can see this implies that the performance obtained with algorithms based on partial CSIT are characterized by a saturation at high SNR. This is due to the fact that at high SNR to keep a finite gap with the full CSIT case the channel quality should scale with the SNR. In addition in Fig. 2 we report also the performance of the MWSR algorithm [6] when it is implemented using only the channel estimate $\widehat{\mathbf{H}}_{ij}$ to determine the optimal BF (MWSR w channel estimate). As we can see the proposed algorithm (MWSR$_{PCSI}$) outperforms the former solution manifesting a more robust behavior.

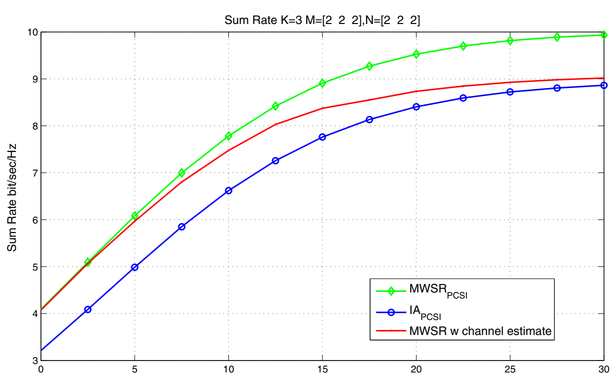


Fig. 2: Sum Rate comparisons for $K = 3$ $M_k = N_k = 2$, $\forall k$

## VI. Conclusions

In this paper we studied the maximization of the WSR in the case of partial CSIT for the Noisy MIMO IBC. The expectation of the WSR, w.r.t. the channel, is approximated with the expectation of the WMSE. This approximate solution gives an iterative algorithm based on alternating minimization between Tx, Rx filters and weighting matrices. The performances of the proposed algorithm, specified for a MIMO IFC, are compared with an IA solution calculated using the same partial CSIT. As we were expecting maximizing the WSR outperforms the IA solution also for the case of partial CSIT case. On the other hand using partial channel knowledge causes a loss in term of SNR offset but not in term of slope. So we can conclude that the proposed algorithm achieves, with partial CSIT, the same DoF of IA with perfect CSIT.


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