ON THE NOISY MIMO INTERFERENCE CHANNEL WITH CSI THROUGH ANALOG FEEDBACK

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ABSTRACT

In this paper we propose a transmission protocol through which base stations (BSs) and user equipments (UEs) acquire the necessary channel state information (CSI) for Interference Alignment (IA) transmit (Tx) and receive (Rx) filter design. We focus our attention on the frequency-fla noisy MIMO interference channel (IFC) without symbol extension and with initial assumption of no CSI neither at the BS nor at the UE. Each device acquires the necessary CSI through channel training and analog feedback. We consider optimizing the sum rate by focusing in particular on the resulting degrees of freedom (DoF). This approach allows us to easily optimize any set of parameters to unveil the trade-off between the cost and the gains associated to CSI acquisition overhead.

1. INTRODUCTION

The interference channel, including related interference mitigation techniques, has recently attracted intense research interest. This is because cellular communication systems, which are severely affected by intercell interference, can be modeled using the K-user Interference Channel (IFC). In spite of the efforts of several research groups over the past few decades, the capacity of a general K-user IFC remains an open problem and is not well understood even for simple cases [1]. In the seminal work [2] a new approach to handle interference with linear transmit and receive filter has been introduced. The authors have shown that the conventional approach of orthogonalizing the resource blocks can be overcome by the use of a new signaling technique called Interference Alignment (IA). This approach is based on designing (transmit) interferer signal subspaces such that their received contributions align in reduced dimension subspaces at the unintended receivers. They have proven that for (time or frequency) varying SISO channels a total of $\frac{K}{2}$ interference-free streams can be received with IA instead of the 1 obtained through orthogonalization. This significan increase in degrees of freedom (DoF) can be achieved by asymptotic signal-space expansion in time or frequency called *symbol extension*. The sum degrees of freedom for a general MIMO IFC is still an open problem, the only known result is given in [3] for a $K = 2$ user MIMO IFC. For the case $K > 2$ some bounds have been provided in [4]. IA requires perfect and global channel state information (CSI) at all Tx/Rx. This assumption does not come for free in practical time-varying channels. For this reason different studies have been conducted for more practical situations. In [5] the authors consider the SISO IFC with frequency selective channels. Using quantized channel feedback they show that the full multiplexing gain can be achieved if the feedback bitrate scales sufficientl fast with the SNR. This result is extended in [6] to the MISO and MIMO IFC. In [7] the author shows for different selected multiuser communication scenarios that it is possible to align the interference when the transmitters do not know the channel coefficient but they only have information about the channel autocorrelation structure of different users. In [7] a staggered block fading channel model is the only assumption required to achieve IA. The resulting multiplexing gain is much lower however than for the case of full CSI. These techniques are now known by the terms *delayed CSIT* or *retrospective IA*. The authors of [8] propose to use analog feedback for the acquisition of full CSIT. The channel coefficient are directly fed back to the base station (BS) without any quantization process. This has the advantage, in contrast to digital feedback, that the complexity does not increase with SNR. In [8] CSIT processing and transmitter computation is centralized, and CSIR issues are neglected. They show that using IA with the acquisition of CSIT using analog feedback incurs no loss of multiplexing gain if the feedback power scales with the SNR.

In this paper we introduce two transmission protocols for the distributed CSI acquisition at the BS and UE that are based on channel training and analog feedback (FB), for both TDD and FDD communication systems. The main difference between the two approaches is in the FB part: channel FB or output FB. In the channel FB solution, described also in [8] and [9], each UE feeds back to the BS the downlink channel estimates while in the output FB scheme, the UE feeds back directly the received samples of the DL training phase. In FDD communications uplink (UL) and downlink (DL) transmission can take place at the same time. Hence with output FB, it is possible to shrink the time overhead, reducing partially the silent periods.

2. SIGNAL MODEL

Fig. 1 depicts a K-link MIMO interference channel with K transmitter receiver pairs. To differentiate the two transmitting and receiving devices we assume that each of the K pairs is composed of a Base station (BS) and a User equipment (UE). This is only for notational purposes. The k-th BS and its corresponding UE are equipped with M_k and N_k antennas respectively. The k-th transmitter generates in-

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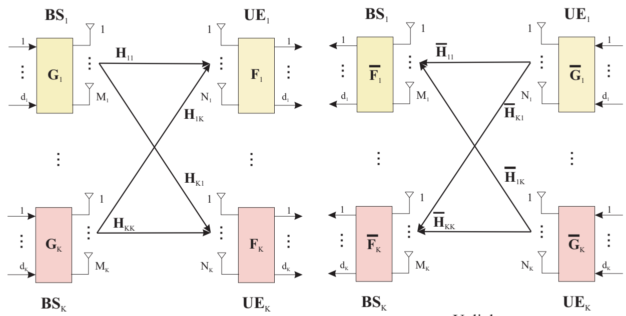

Downlink Uplink

(a) (b)

Fig. 1: MIMO DL/UL Interference Channel

terference at all $l \neq k$ receivers. The received signal in the Downlink (DL) phase $\mathbf{y}_k$ at the k-th UE, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1\\l\neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \tag{1}$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the l-th BS and k-th UE, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th BS and the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. The channel is assumed to follow a block-fading model having a coherence time of T symbol intervals without channel variation. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel matrices corresponding to its direct link and all the other cross-links in addition to the transmitter power constraints and the receiver noise covariances.

We denote by $\mathbf{G}_k$, the $\mathbb{C}^{M_k \times d_k}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where $\mathbf{s}_k$ is a $d_k \times 1$ vector representing the d_k independent symbol streams for the k-th user pair. We assume $\mathbf{s}_k$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_k \sim \mathcal{N}(0, \mathbf{I}_{d_k})$. The k-th receiver applies $\mathbf{F}_k \in \mathbb{C}^{d_k \times N_k}$ to suppress interference and retrieve its d_k desired streams. The output of such a receive filte is then given by

$$\mathbf{r}_k = \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{\substack{l=1\\l\neq k}}^{K} \mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k\mathbf{n}_k$$

In the reverse transmission link, Fig. 1(b) Uplink (UL) phase, the received signal at the k-th BS is given by:

$$\bar{\mathbf{r}}_k = \overline{\mathbf{F}}_k\overline{\mathbf{H}}_{kk}\overline{\mathbf{G}}_k\bar{\mathbf{s}}_k + \sum_{\substack{l=1\\l\neq k}}^{K} \overline{\mathbf{F}}_k\overline{\mathbf{H}}_{kl}\overline{\mathbf{G}}_l\bar{\mathbf{s}}_l + \overline{\mathbf{F}}_k\bar{\mathbf{n}}_k$$

where $\overline{\mathbf{F}}_k$ and $\overline{\mathbf{G}}_l$ denote respectively the $d_k \times M_k$ Rx filte at BS number k and the $N_l \times d_l$ BF matrix applied at Tx l. The UL channel form the l-th UE and the k-th BS is denoted as $\overline{\mathbf{H}}_{kl}$.

In this paper we design the transmit and receive filter according to IA. The objective then is to design spatial filter to be applied at the transmitters such that, the interference caused by all transmitters at each non-intended RX lies in a common *interference subspace* [2]. Since IA is a condition for joint transmit-receive linear ZF, the transmit and receive filter should satisfy the following conditions:

$$\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{2}$$

$$\text{rank}(\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{3}$$

3. TRANSMISSION PHASES

In this section we describe briefl the different phases of the transmission protocol for CSI acquisition proposed in [9]. This protocol can be used in both TDD and FDD communication systems but here we focus on FDD, refer to [9] for further details. We assume a block fading model, in which the channel is assumed to be constant over T channel uses. This time period T will need to be shared between the different training T_{ovrhd} and data transmission phases $T_{data} = T - T_{ovrhd}$ of the overall transmission scheme.

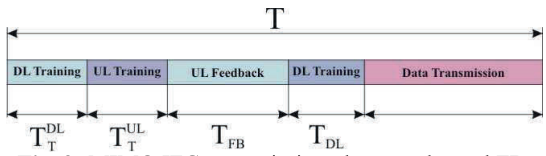

Fig. 2: MIMO IFC transmission phases - channel FB.

3.1. Downlink Training Phase

During this phase each BS_k sends orthogonal pilot sequences that can be received by all the UE for a total duration of T_T^{DL}. In this way UE_i can easily estimate the DL channels $\mathbf{H}_i = [\mathbf{H}_{i1}, \ldots, \mathbf{H}_{iK}]$ directly connected to it. Because the compound channel matrix $\mathbf{H}_i$ has dimensions $N_i \times \sum_k M_k$ the minimum total duration of this training phase is

$$T_T^{DL} \geq \sum_{k=1}^{K} M_k.$$

Each BS independently transmits an orthogonal matrix $\mathbf{\Psi}_k$ of dimension $M_k \times T_T^{DL}$ with power P_T^{DL} hence the total received $N_i \times T_T^{DL}$ matrix at Rx i is:

$$\mathbf{Y}_i = \sum_{k=1} \sqrt{P_T^{DL}}\mathbf{H}_{ik}\mathbf{\Psi}_k + \mathbf{V} \tag{4}$$

where $\mathbf{V}$ represents the zero mean additive white Gaussian noise with variance σ_v^2. The DL Tx power can be related to the time duration of the corresponding Tx phase as

$$P_T^{DL} = \frac{T_T^{DL}}{\sum M_k}\overline{P}_T^{DL}. \tag{5}$$

where $\overline{P}_T^{DL}$ represents the DL power constraint. From the Rx signal (4) UE_i performs an MMSE estimate of the DL channels.

3.2. Uplink Training Phase

This phase can be seen as the dual of the DL training where now all UE send orthogonal pilots to each BS for the estimation of the UL channel matrices. The time duration of this phase is:

$$T_T^{UL} \geq \sum_{k=1}^{K} N_k.$$

Then BS_k can estimate the compound channel matrix $\overline{\mathbf{H}}_i = [\overline{\mathbf{H}}_{i1}, \ldots, \overline{\mathbf{H}}_{iK}]$ using an MMSE estimator as described for the DL training phase. We are describing all the transmission phases for the FDD transmission scheme, hence different frequency bands are used for UL and DL communications. This separation implies that transmission and reception can take place at the same time. If we take advantage of this possibility the two training phases, UL and DL, can collapse in only one training slot that have duration $T_T = \max\{T_T^{DL}, T_T^{UL}\}$. Accounting for this new training phase implies a reduction of the total overhead T_{ovrhd}.

3.3. Uplink Feedback Phase

Once the UL and DL training phases are completed each terminal knows the channel directly connected to it in the UL and DL respectively. In order to compute the IA BF matrices full DL CSI is required. In FDD case, the one under investigation, each UE has to FB the DL channel estimate (CFB) $\widehat{\mathbf{H}}_i$ to all BS, this task can be done using Analog FB. This particular transmission phase should be designed according to the particular type of processing used for the computation of the BF matrices. We can describe two approaches: centralized and distributed. In the former a central controller acquires the necessary CSI, computes the BFs and then it disseminates this information among the K BSs. In the latter approach each BS should have full CSI to compute the IA BF, using e.g. the approach of [10]. This solution can be also called *Duplicated* because each BS essentially solves the same problem and find the complete solution, all the IA BFs, and then it will use only its own transmit filte .

Centralized Processing
The Rx signal vector at each BS is sent to the centralized controller that retrieves the useful channel information and computes the BF matrices. If we stack all the received vector from the K BSs in $\overline{\mathbf{Y}}$ we get:

$$\overline{\mathbf{Y}}=P_{FB}^{\frac{1}{2}}\underbrace{\begin{bmatrix}\overline{\mathbf{H}}_{11} & \dots & \overline{\mathbf{H}}_{1K}\\ \vdots & \ddots & \vdots\\ \overline{\mathbf{H}}_{K1} & \dots & \overline{\mathbf{H}}_{KK}\end{bmatrix}}_{M\times N}\underbrace{\begin{bmatrix}\widehat{\mathbf{H}}_1 & \mathbf{0} & \dots & \mathbf{0}\\ \mathbf{0} & \widehat{\mathbf{H}}_2 & \dots & \mathbf{0}\\ \vdots & & \ddots & \mathbf{0}\\ \mathbf{0} & \dots & \dots & \widehat{\mathbf{H}}_K\end{bmatrix}}_{N\times KM}\underbrace{\begin{bmatrix}\mathbf{\Phi}_1\\ \vdots\\ \mathbf{\Phi}_K\end{bmatrix}}_{KM\times T_{FB}}+\underbrace{\begin{bmatrix}\mathbf{V}_1\\ \vdots\\ \mathbf{V}_K\end{bmatrix}}_{\overline{\mathbf{V}}}$$

where $N=\sum_i N_i$ and $M=\sum_i M_i$ and

$$P_{FB}=\overline{P}_{FB}\frac{T_{FB}}{N_i M} \tag{6}$$

with $\overline{P}_{FB}$ is the FB power constraint. Using a centralized controller to gather all Rx data the entire system can be interpreted as a unique single user MIMO link with a BS that is equipped with M total antennas and a UE with N antennas. With this interpretation we can calculate the total amount of time necessary to satisfy the identifia bility conditions. In particular we get:

$$T_{FB}\geq\frac{N\times M}{\min\{N,M\}}=\max\{N,M\}\propto\ K. \tag{7}$$

Distributed Processing
In this case the CFB transmission is organized in such a way that each BS can gather full channel knowledge from all UE. The Rx matrix at BS_k can be written as:

$$\overline{\mathbf{Y}}_k=\sqrt{P_{FB}}\underbrace{\begin{bmatrix}\overline{\mathbf{H}}_{k1} & \dots & \overline{\mathbf{H}}_{kK}\end{bmatrix}}_{M_k\times N}\underbrace{\begin{bmatrix}\widehat{\mathbf{H}}_1 & \mathbf{0} & \dots & \mathbf{0}\\ \mathbf{0} & \widehat{\mathbf{H}}_2 & \dots & \mathbf{0}\\ \vdots & & \ddots & \mathbf{0}\\ \mathbf{0} & \dots & \dots & \widehat{\mathbf{H}}_K\end{bmatrix}}_{N\times KM}\underbrace{\begin{bmatrix}\mathbf{\Phi}_1\\ \vdots\\ \mathbf{\Phi}_K\end{bmatrix}}_{KM\times T_{FB}}+\mathbf{V}_k$$

where

$$P_{FB}=\overline{P}_{FB}\frac{T_{FB}}{N_i M} \tag{8}$$

with $\overline{P}_{FB}$ is the FB power constraint. In the distributed approach to satisfy the identifiabilit conditions the CFB length should be:

$$T_{FB}\geq\frac{N\times M}{\min_i\{M_i,N_i\}}\propto K^2 \tag{9}$$

Another possible strategy to receive the channel FB is to use linear MMSE estimate instead of the least square approach described in this section. The two solutions will be identical at high SNR but in different SNR regimes LMMSE should give better performances.

The analog FB transmission described here is based on the assumption that the number of Tx and Rx antennas satisfy the relation that $\min\{M_i\}\geq N_j,\forall j$. If this condition is not satisfie then a different transmission scheme should be applied. In particular each UE should apply a precoding matrix such that the identifiabilit conditions should be satisfie at all BS, this requires a more careful precoding design. A possible design criterion could be to optimize the performance of the worst FB link. This solution can be also used to introduced more redundancy in the transmission that can increase the performances of the FB reception. A simple approach could be to use a Kronecker model precoder at each UE of the form:

$$\mathbf{T}_k=\mathbf{S}_k^{T_{FB}\times M\frac{N_k}{s_k}}\otimes\mathbf{B}_k^{N_k\times s_k}$$

where $\mathbf{S}_k$ and $\mathbf{B}_k$ are optimize according to the channel conditions and s_k represents the number of transmitted streams such that the identifiabilit conditions are satisfie at all BS. With this model the compound channel matrix from UE_k to BS_i can be written as

$$\mathbf{G}_{ik}^{T_{FB}M_i\times MN_k}=(\mathbf{I}_{T_{FB}}\otimes\overline{\mathbf{H}}_{ik})\mathbf{T}_k=\mathbf{S}_k\otimes\overline{\mathbf{H}}_{ik}\mathbf{B}_k$$

then the equivalent channel matrix is designed for the transmission of the total number of FB $\mathbf{h}_k^{MN_k\times 1}=\mathbf{vec}\{\widehat{\mathbf{H}}_k\}$.

3.4. Downlink Training Phase

Once the beamformers have been computed, using a centralized or distributed approach, they can be used for the DL communications. According to IA each UE should apply a ZF receiver, in order to compute the Rx filter each UE requires some additional information on the DL communication. On this purpose two approaches are possible: DL training or analog transmission of the entire Rx filters In the former case BS_k sends a set of beamformed pilots that allow UE_i to estimate the cascade $\mathbf{H}_{ik}\mathbf{G}_k$. This phase lasts

$$T_{DL}\geq\sum_k d_k.$$

Then each UE can estimate the interference subspace and the signal subspace for the Rx filte design. The other possibility consists in the transmission to the i-th UE of the entire Rx filte matrix $\mathbf{F}_i$ using analog transmission. This solution requires a duration

$$T_{DL}\geq\sum_k\frac{N_k d_k}{\min\{N_k,M_k\}}$$

The two solutions proposed here are not equivalent. Training is shorter but the estimation error will have a bigger impact in the calculation of the Rx filte compare to the one in the analog transmission. Which solution should be preferred depends also on the operating SNR point. For example in high SNR, where we are interested more in maximizing the total degrees of freedom the duration of this phase has a bigger impact compare to the estimation error then DL training is the preferable solution.

4. OUTPUT FEEDBACK

In the previous sections we have described the scheme proposed in [9] to acquire the necessary channel state information at each BS using analog FB based on the DL channel estimates obtained at each UE. A different strategy consists to FB directly to BSs the noiseless version of the received signal at each UE during the DL training phase instead of the DL channel estimates. This technique is called output FB (OFB). Then, once each BS accumulates enough FB samples, it estimates directly the required DL channels. The advantage of this strategy, compare to the traditional channel FB, is that the FB phase can start one time instant after the reception of the fir DL training samples. In FDD transmission schemes UL and DL communications can take place at the same time. Assuming the DL frame aligned with the end of the UL training phase, the difference between the two schemes can be pictorially represented as in Fig. 3. At time t the received signal at UE$_k$ during the DL training phase is

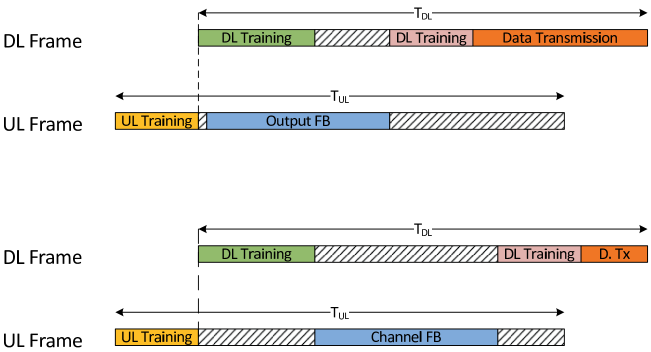

Fig. 3: Output Feedback and Channel Feedback

$$\mathbf{y}_k[t] = \sum_{i=1}^{K} \mathbf{H}_{ki}\boldsymbol{\psi}_i[t] + \mathbf{n}_k[t]. \tag{10}$$

In the next time instant [t+1] UE$_k$ transmits back to all BSs the noiseless version of the RX signal at time instant [t]. So BS number l receives:

$$\begin{aligned} \overline{\mathbf{y}}_l[t+1] &= \sum_{j=1}^{K} \overline{\mathbf{H}}_{lj}\overline{\mathbf{x}}_j[t+1] + \overline{\mathbf{n}}_l[t+1] \\ &= \sum_{j=1}^{K} \overline{\mathbf{H}}_{lj}\alpha_j \sum_{i=1}^{K} \mathbf{H}_{ji}\boldsymbol{\psi}_i[t] + \overline{\mathbf{n}}_l[t+1] \end{aligned}$$

where α_j denotes a scaling factor that takes into account the TX power constraint at j-th UE. In order to being able to separate the different contributions coming from different UEs we assume to use time multiplexing. Each BS has to estimate all the matrices $\mathbf{H}_i = [\mathbf{H}_{i1}, \ldots, \mathbf{H}_{iK}]^{N_i \times M}$. To estimate this many coefficient the required total length of the output FB phase is:

$$T^o_{FB} \geq \frac{N \times M}{\min_i\{N_i, M_i\}} \tag{11}$$

Comparing equation (11) with (8) we can see that there is no reduction in the length of the FB phase using OFB comparing to traditional channel FB. The reduction of the overhead comes from partial elimination of silent periods, as shown in Fig.3. The DL overhead time due to CSI acquisition for the case of OFB can be quantifie as:

$$T^{DL}_{ovrhd} = (T^o_{FB} + 1) + T_{DL}$$

while for CFB we have:

$$T^{DL}_{ovrhd} = T^{DL}_T + T_{FB} + T_{DL}.$$

From the equations above we see that using OFB we save $T^{DL}_T - 1$ time instants.

5. DOF OPTIMIZATION AS FUNCTION OF COHERENCE TIME

In [9] we considered optimizing the length of different training and FB phases to maximize the sum rate. Here the goal is different, we want to optimize the number of transmitted streams as a function of coherence time. The rationale behind this optimization problem is the following. If the coherence time is not long enough to host the total overhead due to CSI acquisition then the transmission of $d_{tot} = \sum_k d_k$ is no longer possible. Then we should use blind IA or noncoherent transmission techniques. Another possibility is to reduce the total amount of transmitted streams. The reduction of d implies a reduction of the required number of transmit and receive antennas so that the amount of CSI exchange is optimized as a function of the coherence time. To solve this problem we should be able to defin a relationship between the number of transmitted streams and antennas. Unfortunately this relation only exists for symmetric systems of the form $(N, N, d)^K$ where each user is equipped with the same number of antennas N and transmits the same number of streams d. According to feasibility conditions in [11] and [12] we can write:

$$N \geq \frac{d}{2}(K+1). \tag{12}$$

Using (12) we can express the total time overhead as a function of the number of transmitted streams d. According to the previous sections we can write the total DL time overhead for CFB as:

$$T_{ovrhd} = T^{DL}_T + T_{FB} + T_{DL} = \begin{cases} dK(K+2) & \text{(Centr.)} \\ \frac{Kd}{2}((K+1)^2+2) & \text{(Distr.)} \end{cases} \tag{13}$$

In the equations above we derived the overhead length for the centralized and distributed FB case as in section 3. The optimization problem that we need to solve is the following:

$$\max_d J(d) = \max_d (1 - \frac{T_{ovrhd}}{T}) Kd \log SNR \tag{14}$$

The cost function that should be maximized is concave in the optimization variable then it admits a unique maximum. To fin the optimal solution we calculate the derivative w.r.t. the optimization variable d, imposing:

$$\frac{\partial J}{\partial d} = 0$$

we finall obtain the optimal solution:

$$d^* = \begin{cases} \frac{T}{2K(K+2)} & \text{(Centr.)} \\ \frac{T}{K[(K+1)^2+2]} & \text{(Distr.)} \end{cases} \tag{15}$$

Now we should determine when is convenient to reduce the number of transmitted streams to maximize the sum rate in high SNR. The number of transmitted streams per user should satisfy the following:

$$d \leq \min\left\{d^*, \frac{2N}{K+1}\right\}.$$

The relation above can be specified for the two cases studied in (15), as:

$$d = \begin{cases} \min\left\{\frac{T}{2K(K+2)}, \frac{2N}{K+1}\right\} & \text{(Centr.)} \\ \min\left\{\frac{T}{K[(K+1)^2+2]}, \frac{2N}{K+1}\right\} & \text{(Distr.)} \end{cases} \tag{16}$$

From the equation above we can see that, for example in the centralized case, if

$$T \geq \frac{4NK(K+2)}{K+1} = 2T_{ovrhd}$$

then to optimize the DoF the number of transmitted streams should be kept at its maximum $d = \frac{2N}{K+1}$. On the contrary, if the given condition is not satisfie then d^* streams per user should be transmitted. This implies that the number of antennas, used for transmission and reception, should be shrank to

$$n \geq \frac{d^*(K+1)}{2}$$

with a consequent reduction of the time overhead for CSI acquisition.

The same analysis can be done to study the case when output FB is used instead of channel FB. Calling D the number of time instants after which the FB transmission starts (in section 4 we used $D = 1$), the CSI acquisition phase lasts

$$T_{ovrhd} = T_{FB} + D + T_{DL} = \begin{cases} \frac{Kd}{2}(K+3) + D & \text{(Centr.)} \\ \frac{Kd}{2}(K(K+1)+2) + D & \text{(Distr.)} \end{cases} \tag{17}$$

Now solving the optimization problem (5) using the time overhead length derived above we obtain:

$$d^* = \begin{cases} \frac{T-D}{K(K+3)} & \text{(Centr.)} \\ \frac{T-D}{K[K(K+1)+2]} & \text{(Distr.)} \end{cases} \tag{18}$$

From the optimal number of transmitted streams above we can finall write :

$$d = \begin{cases} \min\left\{\frac{T-D}{K(K+3)}, \frac{2N}{K+1}\right\} & \text{(Centr.)} \\ \min\left\{\frac{T-D}{K[K(K+1)+2]}, \frac{2N}{K+1}\right\} & \text{(Distr.)} \end{cases} \tag{19}$$

The analysis develop above is based on the assumption of symmetric MIMO links, in the following we extend the results to the asymmetric MIMO case where each transmitter has M antennas, and each receiver is equipped with N antennas. In concise notatio we consider the $(M, N, d)^K$ interference channel. To study this more general case we assume that the relationship between number of antennas and transmitted streams

$$d \leq \frac{M+N}{K+1} \tag{20}$$

derived also in [11] for such a system configuration is still valid. This is true if the ratio $\frac{M}{N}$ is not too far from 1. This is due to the fact that, as recently shown in [13], for very rectangular MIMO links counting the total number of variables and constraints in the IA problem is not enough to determine the feasibility of the problem.

In the analysis below we optimize w.r.t. the number of active transmitting and receiving antennas, m and n respectively, instead of a direct optimization of the number of transmitted streams.

We firs study the case $M > N$. To simplify the analysis we consider that the overhead time for CSI acquisition is due to only DL training and CFB, then we obtain $T_{ovrhd} = 2Km$. Expressing d as a function of the number of antennas we can rewrite as

$$\max_{n,m} J(n,m) = \max_{n,m}(1 - \frac{2Km}{T})\frac{K(m+n)}{K+1}\log SNR \tag{21}$$

for $m \in [1, M]$ and $n \in [1, N]$ and $m \geq n$.
The optimization problem above is linear in the variable n, then to maximize the cost function the optimal value for the number of receiving antennas falls in the extremum of the optimization interval: $n^* = N$. To optimize w.r.t. m we need to equate the firs order derivative of J w.r.t. m to zero obtaining:

$$m^* = \frac{T}{4K} - \frac{N}{2}. \tag{22}$$

From the solution above, and the constraint $m = \min\{m^*, M\}$, then we can state that:

$$m = \begin{cases} M, & T \geq 4KM + 2KN \\ \frac{T}{4K} - \frac{N}{2}, & 6KN \leq T \leq 4KM + 2KN \end{cases} \tag{23}$$

To conclude the analysis we should study the other regime $M \leq N$. In this case the time overhead is $T_{ovrhd} = \frac{K(K+2)}{K+1}(m+n)$, where we also included the duration of the beamformed DL training phase. With this result the cost function that should be optimized is:

$$J(n,m) = (1 - \frac{1}{T}\frac{K(K+2)}{K+1}(m+n))\frac{K(m+n)}{K+1}\log SNR. \tag{24}$$

As we can see the cost function J depends only on the sum of the two optimization variables so we directly optimize w.r.t. $y = (m+n)$. Then from the firs order optimality condition we get:

$$(m+n)^* = \frac{T}{2}\frac{K+1}{K(K+2)}.$$

From the optimization problem (24) there is nothing that we can infer about the behavior of the single variables m and n and how $m+n$ is split over them as long as $1 \leq n \leq N$ and $1 \leq m \leq M \leq N$. On the other hand there is a slight preference to consider a square system since only for that case we are sure about the feasibility condition (12) [12]. Then :

$$m + n \leq M + N, \quad T \leq 2(M+N)\frac{K(K+2)}{K+1}$$

Form the optimal antennas distribution is possible to determine the corresponding stream allocation just using the feasibility condition (20).

In Fig. 4 we summarize all the results found in this section. It gives a qualitative description of the behavior of the antennas distribution $(m+n)$ as a function of the coherence time T.
In particular in Fig. 4(a) we describe the regime $M \geq N$. When T is long enough $m+n$ assumes its maximum value, then the number of Tx antenna starts to shrink up to the point where $m = n = N$. At this point the IFC becomes square and then the dimensions decrease as long as T decreases but the system remains square because the condition $m \geq n$ should be always satisfied
Fig. 4(b) depicts the situation where $M \leq N$. There we should underline that only the behavior of $m+n$ can be described but not how m and n behave separately.

Also for the rectangular MIMO interference channel the analysis of the antennas and streams distribution as a function of the coherence time can be developed for the use of output FB instead of channel FB. These results are not provided here due to lack of space.

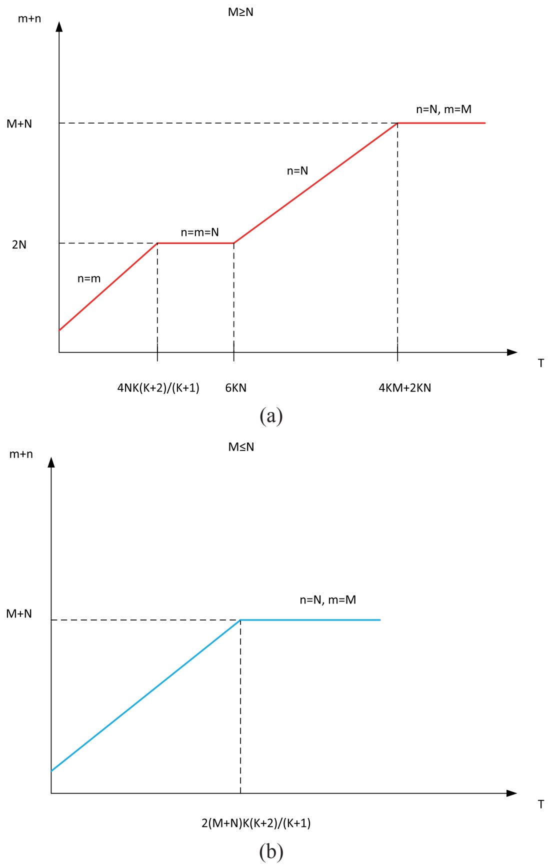

Fig. 4: Behavior of the optimized antennas distribution

6. CONCLUDING REMARKS

We optimized the sum rate of the MIMO IFC under investigation by focusing in particular on the resulting degrees of freedom. We showed that the optimal number of streams should vary as a function of the channel coherence time, just like in the single-user MIMO channel.

We have considered here the use of output FB for the purpose of CSI acquisition. One may wonder whether the more general use of such Shannon feedback would allow to increase the DoF. A recent discussion on this can be found in [14] which appears to indicate that in the case of perfect CSIT, it is unlikely that output FB can help the DoF for the general MIMO IFC. However, [14] shows that in the case of delayed CSIT, (additional) output FB may allow to increase the DoF, depending on the number of antennas in the $K = 2$ MIMO IFC.

Another aspect is that in the approach considered here, we focus on obtaining CSIT in an initial portion of the coherence interval, after which we apply coherent IA transmission. Some improvement can be obtained by adding retrospective IA during the training period. This has been explored for the $K = 2$ MISO IFC in [15], where an optimal combination of delayed CSIT and coherent transmission is proposed. One should note though that the improvements brought about by such further sophistication are only important when the coherence time becomes very short and/or the CSI FB delay becomes large. The approach of [15] (just like delayed CSIT in general) appears to be motivated by considering a substantial FB delay (time unit), e.g. corresponding to a slot in current wireless communication standards. The picture changes though if one considers much shorter FB delay as we do here, and as is possible in principle in FDD systems.

7. REFERENCES

[1] R. H. Etkin, D. N. C. Tse, and Hua Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. on Inform. Theory*, , no. 12, 2008.

[2] V.R. Cadambe and S.A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. on Inform. Theory*, Aug. 2008.

[3] S.A. Jafar and M.J. Fakhereddin, "Degrees of freedom for the MIMO interference channel," *Information Theory, IEEE Transactions on*, July 2007.

[4] T. Gou and S.A. Jafar, "Degrees of freedom of the K user MxN MIMO interference channel," 2008, http://arxiv.org/abs/0809.0099.

[5] H. Bolcskei and I.J. Thukral, "Interference alignment with limited feedback," in *Proc. IEEE Int'l Symp. on Information Theory (ISIT)*, July 2009.

[6] R.T. Krishnamachari and M.K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," in *Proc. IEEE Int'l Symp. Information Theory (ISIT)*, June 2010.

[7] S.A. Jafar, "Exploiting Channel Correlations - Simple Interference Alignment Schemes with No CSIT," in *Proc. IEEE Global Telecommunications Conf. (Globecom)*, Dec. 2010.

[8] O. El Ayach and R. W. Jr. Heath, "Interference alignment with analog channel state feedback," in *Proc. MILCOM*, Oct. 2010, http://arxiv.org/abs/1010.2787.

[9] F. Negro, U. Salim, I. Ghauri, and D.T.M. Slock, "The noisy MIMO interference channel with distributed CSI acquisition and filte computation," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacifi Grove, CA, USA, Nov. 2011.

[10] F. Negro, I. Ghauri, and D.T.M. Slock, "Deterministic annealing design and analysis of the Noisy MIMO Interference Channel," in *Proc. Information Theory and Applications Workshop (ITA)*, feb. 2011.

[11] F. Negro, S. P. Shenoy, I. Ghauri, and D.T.M. Slock, "Interference Alignment Feasibility in Constant Coefficient MIMO Interference Channel," in *Proc. 11th IEEE Int'l Workshop on Signal Processing Advances in Wireless Comm's (SPAWC)*, June 2010.

[12] G. Bresler, D. Cartwright, and D. Tse, "Settling the feasibility of interference alignment for the MIMO interference channel: the symmetric square case," 2011, http://arxiv.org/abs/1104.0888.

[13] G. Bresler, D. Cartwright, and D. Tse, "Geometry of the 3-user MIMO interference channel," 2011, http://arxiv.org/abs/1110.5092.

[14] C.S. Vaze and M.K. Varanasi, "The Degrees of Freedom Region of the MIMO Interference Channel with Shannon Feedback," *Trans. Information Theory, subm.*, Oct. 2011, arXiv:1109.5779v2.

[15] X. Yi, D. Gesbert, S. Yang, and M. Kobayashi, "The DoF Region of the Multiple-Antenna Time Correlated Interference Channel with Delayed CSIT," *Subm. to IEEE Trans. on Information Theory*, Apr. 2012, http://arxiv.org/pdf/1204.3046.pdf.