A Compressive Sampling Approach for Spectrum Sensing and Terminals Localization in Cognitive Radio Networks

Wael Guibène Dirk Slock
Mobile Communications Department, EURECOM
2229 Route des Crêtes, P.O. Box 193, 06904, Sophia Antipolis, France
Wael.Guibene@eurecom.fr Dirk.Slock@eurecom.fr

***Abstract*—In this paper[1], we propose to analyze and combine two of the main enabling features of cognitive radio: location awareness and spectrum sensing with taking into account one of the most challenging hardware limitation that cognitive radio may suffer from: signal acquisition at a Nyquist rate. During the problem formulation and when analyzing more deeply the equations related to each question apart, we will make the link between the formulation of spectrum sensing, location awareness and the hardware limitation by describing those problems in a unique compressed sensing formalism. Via the proposed framework, and compared to what has already been proposed, we made it possible to overcame another challenging postulate of fixed frequency spectrum allocation by also estimating the spectrum usage boundaries dynamically and in a fully blind way.**

***Index Terms*—collaborative spectrum sensing; compressive sampling; primary users localization**

I. Introduction

Cognitive Radio (CR) as introduced by Mitola [1], is one of those possible devices that could be deployed as SU (secondary user) equipments and systems in wireless networks. As originally defined, a CR is a self aware and "intelligent" device that can adapt itself to the wireless environment changes. Such device is able to detect the changes in the wireless network to which it is connected and adapt its radio parameters to the new opportunities that are detected. This constant track of the environment change is called the "spectrum sensing" function of a cognitive radio device.

Recently, compressed sensing/compressive sampling (CS) has been considered as a promising technique to improve and implement cognitive radio (CR) systems. As,in wideband radio one may not be able to acquire a signal at the Nyquist sampling rate due to the current limitations in Analog-to-Digital Converter (ADC) technology [3]. Compressive sensing makes it possible to reconstruct a *sparse* signal by taking less samples than Nyquist sampling . In general, signals of practical interest may be only nearly sparse [3] and typically the wireless signal in open networks are sparse in the frequency domain since depending on location and at duration the percentage of spectrum occupancy is low due to the idle radios [2], [4].

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In CS a signal with a sparse representation in some basis can be recovered from a small set of nonadaptive linear measurements [5]. A sensing matrix takes few measurements of the signal, and the original signal can be reconstructed from the incomplete and contaminated observations accurately and sometimes exactly by solving a convex optimization problem [3]. In [6] and [7] conditions on this sensing matrix are introduced which are sufficient in order to recover the original signal stably. In this paper, we will present a joint spectrum sensing and PU localization algorithm for CRN (CR networks). We will show how localization in CRN could be viewed as a CS problem and formulated in terms of CS equations. This algorithm is presented as a CS approach to both problems. We will use a modified framework of the orthogonal matching pursuit algorithm (OMP) that we feed with some *apriori* knowledge of the CR spectrum usage and thus derive a more appropriate OMP algorithm for CS and the problem of localization. The rest of the paper is organized as following: in Section II we will give the system model used through this paper. In order to make the paper easier to read and to apprehend, in Section III we start by giving an overview of what will be done at the level of each CR individually and still in Section III we will derive the CS algorithm to be deployed. In IV, we will make the link between location estimation and spectrum reconstruction. In Section V, we will go through the analysis of the proposed technique and derive its performances. Finally, Section VI will conclude about the present work.

II. System Model

In the considered system model, we will suppose that we do dispose of N_{ch} available channels in a wideband wireless network. Over a large geographic area, let N_p be the number of deployed primary users using N_p different channels. In this large area, we disperse N_c cognitive radios that will operate and detect all these channels and their states. The measures made by these cognitive terminals will then be sent to the fusion center. In order to enable CRs transmissions, the secondary network have to be aware of the availability and the state of each channel in the sense of hybrid underlay/overlay scheme. Thus, secondary users have to estimate which channels are occupied and to identify the PUs (primary users) transmission powers and locations.

Adopting the path loss model, we end up with a loss of:

$$L(f,d) = P_0 + 20\lg(f) + 10n\lg(d) \qquad [dB] \qquad (1)$$

where: P_0 is a constant related to antennas gain; f is the channel frequency; n is the path loss exponent; d is the distance separating the transmitting and receiving nodes and $\lg(.) = \log_{10}(.)$

In our case, we dispose of N_{ch} channels, thus f would be assumed the central frequency of each band, i.e $f \in \{f_0, f_1..., f_{N_{ch}-1}\}$.

Let's keep in mind that the path loss is related to the unknown channel and location of the PU. The received signal power is a combination of the unknown transmit power with the path loss expressed in Eq(1).

Our task is to infer from the received signal at the cognitive terminals all these unknown, but valuable, information about the primary users.

First of all we will describe what is exactly done at the level of each terminal separately in the section III. Then, starting from IV, we proceed with this system model.

III. Single Node Spectrum Sensing based on Compressed Sensing

A. Discrete Spectrum Model

In cognitive radio networks, spectrum usage was summarized in [8] in three main categories:

1) Spectrum bands with fixed boundaries to which the PUs are always accessing such as local TV and radio broadcasters.
2) Spectrum bands with fixed boundaries to which the PUs rarely access like TVWS (TV White Space).
3) Spectrum bands with fixed boundaries which are partially and randomly accessed like cellphone signals, LTE...

In discrete notation, let's denote by $\overrightarrow{f}$ the $N \times 1$ discrete spectrum vector containing the sampled values over B, represented by:

$$\overrightarrow{f} = [f_1 \quad f_2 \quad ... \quad f_N]^T \qquad (2)$$

where T is the transpose operation and $\{f_i\}$ are the signal values uniformly sampled over B by a B/N resolution and $\{i\}$ is the subset relate the frequencies locations. It is then trivial that in noise free context, a frequency i is said to be *vacant* or *free* if $|f_i|^2 = 0$.
The $N \times N$ normalized discrete Fourier Transform (DFT) matrix, **F**, gives the relationship between the frequency samples vector $\overrightarrow{f}$ and the time domain samples vector $\overrightarrow{t}$, by the relation:

$$\overrightarrow{t} = \mathbf{F}^{-1}\overrightarrow{f} \qquad (3)$$

And in these three main CR spectrum usage scenarios, spectrum boundaries are fixed and *a priori* known.
This strong assumption of knowledge of boundaries can be overcame by processing as following:

B. Blind Spectrum Boundaries Estimation

In [9], [10], Guibene *et al*, developed a spectrum sensing technique based on frequency edge location and exploiting spectrum discontinuities detection. Inspired from the already developed framework, we derive our edge location algorithm.

First we do suppose that the frequency range available in the wireless network is B Hz; so B could be expressed as $B = [f_0, f_K]$. Suppose that the radio signal received by the CR occupies N spectrum bands, whose frequency locations and PSD levels are to be detected and identified. These spectrum bands lie within $[f_1, f_K]$ consecutively, with their frequency boundaries located at $f_1 < f_2 < ... < f_K$. The n-th band is thus defined by: $B_n \; : \; \{f \in B_n : f_{n-1} < f < f_n, \; n = 2, 3, ..., K\}$. We do suppose that only f_1 and $f_K = f_1 + B$ are known to the CR and $f_2, ..., f_{K-1}$ (frequencies boundaries) are unknown and to be determined by the CR.

The input signal for boundaries estimation is the amplitude spectrum of the received noisy signal. We assume that its mathematical representation is a piecewise regular signal:

$$X(f) = \sum_{i=1}^{K} \chi_i[f_{i-1}, f_i](f) p_i(f - f_{i-1}) + n(f) \qquad (4)$$

where: $\chi_i[f_{i-1}, f_i]$: the characteristic function of the interval $[f_{i-1}, f_i]$, $(p_i)_{i\in[1,K]}$: an N^{th} order polynomials series, $(f_i)_{i\in[1,K]}$: the discontinuity points resulting from multiplying each $p_i$by a χ_i and $n(f)$:the additive corrupting noise.
Using the exact framework as in [9], [10], the frequency boundaries estimation is casted as a change point f_ν detection problem. After the calculation steps detailed in [9], [10], we end up with f_ν verifying :

$$\sum_{k=0}^{N-1} \binom{N}{k} . f_\nu^{N-k} . \varphi_{k+1} = 0 \qquad (5)$$

where: $$\varphi_{k+1} = \int_0^{+\infty} h_{k+1}(f) . X(\nu - f) . df \qquad (6)$$

where: $h_{k+1}(f) = \begin{cases} \frac{(f^l(b-f)^{N+k})^{(k)}}{(l-1)!} & 0 < f < b \\ 0 & otherwise \end{cases}$ and b is the filter h_{k+1} bandwidth.
So now on, the only assumptions are the ones we introduced from [9], [10] and not the ones given in [8] which gives to the algorithm lighter assumption and gives it some blind processing properties. So finally, we adopt the following assumptions:

- Only the two delimiting boundaries of the whole band of interest are known and the rest are to be determined
- The PSD of the signals is almost flat in the used subbands
- Noise is white gaussian with zero mean

Then, spectrum usage categories is still valid at the CR level but after estimating the boundaries.

C. Spectrum Sensing based on Compressive Sampling

In the CS framework, we do consider the sampling of $N \times 1$ signal $\overrightarrow{x} = \Psi \overrightarrow{s}$, where $\overrightarrow{s}$ is an $N \times 1$ sparse source vector with L non-zero components s_i, so $L << N$ and $\mathbf{\Psi}$ is an $N \times$

N dictionary matrix. In literature [11], [12] it was shown that M samples of $\overrightarrow{x}$ can recover the whole vector, by projecting $\overrightarrow{x}$ by an $M \times N$ observation matrix, say $\mathbf{\Phi}$. This matrix has to satisfy two conditions: $L < M < N$ and the rows of the sensing matrix $\mathbf{\Phi}$ should be incoherent with the columns of $\mathbf{\Psi}$. Finally we obtain the $M \times 1$ measurement vector $\overrightarrow{y}$ given by:

$$\overrightarrow{y} = \mathbf{\Phi}\ \overrightarrow{x} = \mathbf{\Phi\Psi}\ \overrightarrow{s} \tag{7}$$

$\overrightarrow{s}$ can be fully reconstructed by adopting the basis pursuit algorithm as shown in [13]. Its reconstruction is subject to a convex optimization problem as shown in Eq(8):

$$\widehat{\overrightarrow{s}} = \arg\min_{\overrightarrow{s}} ||\overrightarrow{s}||_{l_1} \text{ subject to } \Phi\Psi\overrightarrow{s} = \overrightarrow{y} \tag{8}$$

where l_p is the p-norm for $p \geqslant 1$ given by $:||\overrightarrow{s}||_{l_1} = (\sum |s_i|^p)^{\frac{1}{p}}$.

Another way to reconstruct the signal could be the matching pursuit algorithm (MP) derivatives as will be shown in next paragraph.

Through a deeper look into equations Eq(3) and Eq(7), one can intuitively say that the time domain vector $\overrightarrow{t}$ can be viewed as $\overrightarrow{x}$ and the inverse DFT matrix $\mathbf{F}$ could be seen as the matrix substituting the dictionary matrix $\mathbf{\Psi}$ and $\overrightarrow{f}$ is no more than the sparse vector $\overrightarrow{s}$.

With this new formalism, if we can properly design a measurement matrix $\mathbf{\Phi}$ satisfying then incoherence constraint with $\mathbf{F^{-1}}$, than we would be able to use the CS formalism as a spectrum sensing technique and sub-Nyquist sampling rate could be recovered by CS algorithms as well. Given the work lead in [14], the use of $M \times N$ Gaussian random matrix as a measurement matrix $\mathbf{\Phi}$ would guarantee good reconstruction performance. Back now to the spectrum sensing model, which in noise free environment is formulated as following:

$$\overrightarrow{y} = \mathbf{\Phi}\overrightarrow{f} \tag{9}$$

and as results $\overrightarrow{f}$ reconstruction is solution of:

$$\widehat{\overrightarrow{f}} = \arg\min_{\overrightarrow{f}} ||\overrightarrow{f}||_{l_1} \text{ subject to } \Phi\overrightarrow{f} = \overrightarrow{y} \tag{10}$$

and in a general additive white gaussian noise environment (AWGN), the sensing model becomes:

$$\overrightarrow{y} = \mathbf{\Phi}\overrightarrow{f} + \overrightarrow{w} \tag{11}$$

and as results $\overrightarrow{f}$ reconstruction is solution of:

$$\widehat{\overrightarrow{f}} = \arg\min_{\overrightarrow{f}} \frac{1}{2}||\overrightarrow{y} - \mathbf{\Phi}\overrightarrow{f}||_{l_1} + \gamma||\overrightarrow{f}||_{l_1} \tag{12}$$

where $\overrightarrow{w}$ is an $M \times 1$ noise vector with a normal distribution and γ is determined by the noise level.

D. Modified Blind Orthogonal Matching Pursuit Algorithm

The original OMP (orthogonal matching pursuit) algorithm is a greedy algorithm based on the basis pursuit algorithm that reconstructs iteratively the original signal by the search of non zero indices and performs least square estimation of the values on the non zero indices.

The estimated frequency boundaries, $\{\nu_i,\ i \in [0..K]\}$, do actually separate the spectrum in K consecutive sub-bands. Keeping in mind, that in one hand B is actually divided into K sub-bands and the fact that the frequency indices we were using is of length N, this means that the indices set we are using in frequency domain is actually divided into K consecutive subsets. Let $\{b_i\}$ denote these indices in each frequency boundary, i.e, $\nu_0 \triangleq 1$, ν_1 occurs at the frequency index b_1 and so on until $\nu_K \triangleq N$.

Let's denote these subsets by :

$$\begin{aligned} &u_1 = 1, 2, .., b_1 \\ &u_2 = b_1 + 1, b_1 + 2, .., b2 \\ &... \\ &u_K = b_{K-1} + 1, b_{K-1} + 2, .., N \end{aligned} \tag{13}$$

Now, let's define three category sets $\{S_n\}$, according to the following condition:

$$\begin{aligned} &S_n = \{u_i \mid n = 1, 2, 3\} \\ &\Omega = \cup S_n \quad (S_i \cap S_j = \emptyset, \text{for } i \neq j) \end{aligned} \tag{14}$$

According to the measurement results of spectrum utilization, we assume as in [8] that :

$$\frac{\#\{\overrightarrow{f}\}}{N} \leq 10\%$$

where $\#\{\overrightarrow{f}\}$ is the number of non-zero values in $\overrightarrow{f}$

The iteration operation gives us the freedom to consider the three already defined categories separately. Since, by construction, S_1 do have at least a non zero value, the initialization output, Λ_0, could be set to S_1. This particular initialization guarantees us always counting the occupied indices. Then, during the rest of the iterations, if we do find an index λ_t, satisfying: $\lambda_t \in u_i \subset S_2$, all elements in u_i will be added in Λ_t. This would enable us counting only the $\{u_i\}$ subset. The other case is $\lambda_t \in u_i \subset S_3$, in which only λ_t is added to Λ_t as in formal OMP.

The modified blind orthogonal matching pursuit is fully describes by Algorithm 1.

IV. Joint Spectrum Sensing and Primary Users Localization based on Compressive Sampling for Cognitive Radio Networks

A. Spectrum Reconstruction

For discrete signals, the time domain samples $\overrightarrow{t}$ are used to construct the spectrum in frequency domain as shown before in Eq(3). Thus we obtained:

$$\overrightarrow{f} = \mathbf{F}\overrightarrow{t} \tag{18}$$

And as sufficiently detailed in Section III, on the level of each node, this problem as formulated in a context of wide-band and involving sparse signals can be casted as a CS problem and spectrum can be reconstructed and spectrum sensing task is thus achieved by all terminals.

Algorithm 1 Proposed Matching Pursuit Algorithm

Require: An $M \times N$ matrix $\mathbf{\Theta} = \mathbf{\Phi}$
An $M \times 1$ sample vector $\overrightarrow{y}$
Minimum iterations number m
Error tolerance η

1: Estimate from the wideband observation the boundaries as stated in III-B and then preselect the sets as in Eq (13) and (14)
2: Initialize: $\overrightarrow{res_0} = \overrightarrow{y}$, $\Lambda_0 = S_1$, $\mathbf{\Theta_1} = \mathbf{\Theta_{S_1}}$, iteration_index: t = 1
3: Solve the least-squares problem in Equation:

$$\overrightarrow{x_t} = \arg\min_{\overrightarrow{x}}(||\mathbf{\Theta_t}\overrightarrow{x} - \overrightarrow{y}||_2) \tag{15}$$

4: Compute the new residual given by:

$$\overrightarrow{res_t} = \overrightarrow{y} - \mathbf{\Theta_t}\overrightarrow{x_t} \tag{16}$$

5: Increment: $t \longleftarrow t + 1$
6: Find λ_t satisfying:

$$\lambda_t = \arg\max_{j=1...N} |\langle \overrightarrow{res_{t-1}}, \theta_j \rangle| \tag{17}$$

where θ_j is the j^th column vector of $\mathbf{\Theta_t}$ and $< .,. >$ is the inner vector product operator.
7: Increase the index set $\mathbf{\Lambda_t} = \mathbf{\Lambda_{t-1}} \bigcup \{\lambda_t\}$
if $\{\lambda_t \in u_i \subset S_2\}$
then $\mathbf{\Lambda_t} = \mathbf{\Lambda_{t-1}} \bigcup \{u_i\}$
end
8: Set the atom to: $\mathbf{\Theta_t} = \mathbf{\Theta_{\Lambda_t}}$
9: Solve the least square problem in Equation (15) and get the new estimate of $\overrightarrow{x}$.
10: Calculate the new residual using Equation (16)
11: **if** $\{t < m$ or $||\overrightarrow{res_t}||_2 > \eta\}$
then return to step (5)
end
12: Finally: $\overrightarrow{f} \longleftarrow \overrightarrow{x_t}$ and its non zero indices are listed in $\mathbf{\Lambda_t}$
13: **return** An estimate $N \times 1$ vector $\overrightarrow{f}$ of the ideal signal
An index set Λ_t containing t elements from $\{1..N\}$
An $M \times 1$ residual vector $\overrightarrow{res_t}$

B. Primary Users Location Reconstruction

Once spectrum reconstructed and spectrum sensing achieved, more information can be derived while looking deeper into channels occupied by primary users.

Let's assume that in a certain wide area, PUs are located at coordinates (xp_m, yp_n); where $xp_m \in \{0, \Delta xp, ...(M-1)\Delta xp\}$ are M possible x axis positions (abscissæ) of the PUs [2]; $yp_m \in \{0, \Delta yp, ...(N-1)\Delta yp\}$ are N possible y axis positions (ordinates) of the PUs; Δxp and Δyp are respectively the resolutions over x and y axis. Here, we do impose and suppose to the PU coordinates to be in discrete $M \times N$ dictionary (which, actually, is always true !). It is good to remind at this level that the exact positions of the N_p PUs $\{(xp_i, yp_i)\ ; i \in [1..N_p]\}$ are unknown to our problem.

The N_c CRs positions in the network are located at positions: $\{(a_i, b_i)\ ; i \in [1..N_c]\}$ (on which we do not impose being in a finite set, even if they necessarily are).

[2]When we say $xp_m \in \{0, \Delta xp, ...(M-1)\Delta xp\}$, that does not mean that there are M PUs, but it means that N_p primary users abscissæ (for ordinates as well) do actually have a finite "dictionary"

For the k^{th} CR, sensing the i^{th} channel, the contribution of the PU located at the (xp_m, yp_n) position on the received PSD is:

$$\begin{aligned} R_{k,i}(m,n) &= P(m,n,i) \times 10^{L(f_i, d(m,n,k))/10} \\ d(m,n,k) &= \sqrt{(xp_m - a_k)^2 + (yp_n - b_k)^2} \end{aligned} \tag{19}$$

where $P(m,n,i)$ is the power transmitted by a PU using the i^{th} channel, located at (xp_m, yp_n); f_i is the center frequency of the i^{th} channel; $d(m,n,k)$ represents the distance between the k^{th} CR and the the PU located at (xp_m, yp_n).

The total received power over all the existing PUs, i.e over the $M \times N$ possible positions of the PUs, can be formulated as following:

$$\begin{aligned} Y_{k,i} &= \textstyle\sum_m \sum_n R_{k,i}(m,n) \\ Y_{k,i} &= \textstyle\sum_m \sum_n 10^{L(f_i, d(m,n,k))/10} \times P(m,n,i) \\ Y_{k,i} &= \overrightarrow{L}^T(k,i)\overrightarrow{P}(i) \end{aligned} \tag{20}$$

where $\overrightarrow{P}(i)$ is the vector containing the transmission power of the over all $M \times N$ grid over the i^{th} channel; and $\overrightarrow{L}(k,i)$ is the path loss vector computed according to Eq(1) from all PU possible positions at the level of the k^{th} CR, on the i^{th} channel.

$$\begin{aligned} &\overrightarrow{L}(k,i) = 10^{\overrightarrow{L}_{dB}(k,i)/10} \\ &\text{and :} \\ &\overrightarrow{L}_{dB}(k,i) = [L(f_i, d(0,0,k)), L(f_i, d(1,0,k)), \\ &..L(f_i, d(m,n,k)), ..L(f_i, d(M,N,k))]^T \end{aligned} \tag{21}$$

Let's denote by $\overrightarrow{Y}_k = [Y_{k,1}..Y_{k,N_{ch}}]^T$, the received signal power vector at the level of the k^{th} CR over the N_{ch} available channels. This according to Eq(20), and adopting the previous notation can be expressed as:

$$\overrightarrow{Y_k} = \mathbf{L_k}\overrightarrow{P} \tag{22}$$

where $\overrightarrow{P}$ is the vector containing the transmission power of the $M \times N$ grid of PU locations over the N_{ch} available channels of the N_C deployed CRs:

$$\overrightarrow{P}_k = [\overrightarrow{P}^T(i_1), \overrightarrow{P}^T(i_2), .., \overrightarrow{P}^T(i_{N_C})]^T \tag{23}$$

The matrix $\mathbf{L_k}$, is the fading gain matrix grouping at the level of the k^{th} CR the loss path contributions of the $M \times N$ PU positions. The j^{th} row of $\mathbf{L_k}$ is:

$$\mathbf{L_k}(j) = [\overrightarrow{0}, \overrightarrow{0}, ..., \overrightarrow{L}^T(k,j), \overrightarrow{0}, .., \overrightarrow{0}] \tag{24}$$

Combining all the equations describing the N_C CR system, we do obtain:

$$\overrightarrow{Y} = \mathbf{L}\overrightarrow{P} \tag{25}$$

Where $\overrightarrow{Y} = [\overrightarrow{Y_1}^T, ..., \overrightarrow{Y_{N_C}}^T]^T$ and $\mathbf{L} = [\mathbf{L_1}, ..., \mathbf{L_{N_C}}]$

The equation we ended with in Eq(25), reminds us of the CS formalism we introduced previously: as $\overrightarrow{P}$ is an unknown but sparse vector because over the $M \times N$ area we've been considering, only N_P PUs are deployed in this area.

Since the two stages, spectrum sensing and localization, seem to be attached to the same CS framework we've introduced before, it is easy then to combine both of them in only one process.

V. Simulations and Results

In this section we propose to investigate the performances of the proposed technique in terms of spectrum sensing and PUs location.

We propose the following evaluation scenario: PUs do dispose of 10 channels that they will randomly select in the range of $50+[1, 2.., 10]$ MHz. We propose deploying 2 primary users in a 15×15 $(M = N = 15)$ unit area with a resolution of $\Delta xp = \Delta yp = 0.1$ having randomly generated power transmits. In this area we also deploy 5 CRs that will achieve the sensing and the localization task.Figures 1 and 2 do report the MSE of the spectrum reconstruction and PU positions reconstruction using the suggested technique. These figures report how efficient the recovery of the sparse spectrum and position model is.

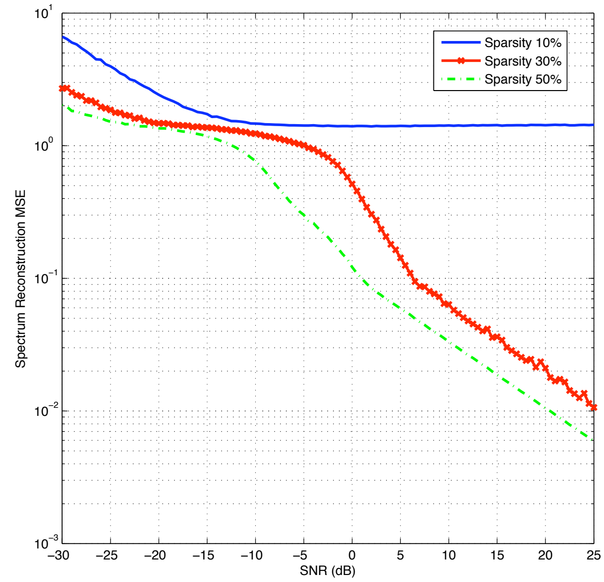

Fig. 1. Spectrum reconstruction MSE

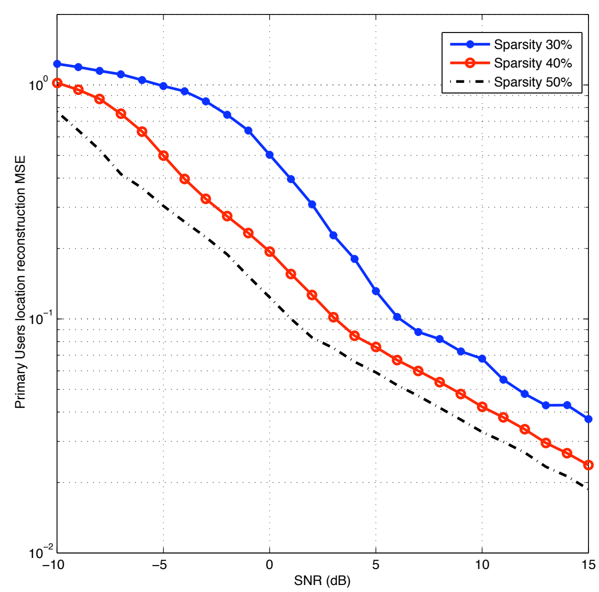

Fig. 2. PU position estimation MSE

VI. Conclusion

This paper presents a first look towards a combined spectrum sensing and localization task. These two tasks are fundamental in order to really enable cognition in wireless networks. With the combination of the two tasks, we also considered a realistic data acquisition constraint, which is sparsity due to the ADC technology limits. In order to make the algorithm and the over all system a real stand-alone one and blindly operating, we suggested removing the apriori knowledge of the channels and spectrum use by blindly and accurately estimating the frequencies boundaries. Simulation results of the proposed technique show promising and interesting results.

References

[1] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *Personal Communications, IEEE*, vol. 6, no. 4, pp. 13–18, 1999.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201–220, 2005.

[3] E. J. Cands, "Compressive sampling," in *Proceedings of the International Congress of Mathematicians*, (Madrid, Spain), 2006.

[4] Z. Tian and G. Giannakis, "Compressed sensing for wideband cognitive radios," in *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*, vol. 4, pp. IV–1357–IV–1360, 2007.

[5] M. A. Davenport, M. B. Wakin, and R. G. Baraniuk, "Detection and estimation with compressive measurements," technical, Rice University, 2007.

[6] E. Candes and T. Tao, "Decoding by linear programming," *math/0502327*, Feb. 2005.

[7] E. J. Candes and T. Tao, "Near-Optimal signal recovery from random projections: Universal encoding strategies", *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5406–5425, 2006.

[8] P. Zhang, R. Qiu, "Modified Orthogonal Matching Pursuit Algorithm for Cognitive Radio Wideband Spectrum Sensing", Cornell University Library, http://arxiv.org/abs/1102.2881v1

[9] W.Guibène, M.Turki, B.Zayen and A.Hayar, "Spectrum sensing for cognitive radio exploiting spectrum sensing discontinuities detection", Published in EURASIP Journal on Wireless Communication and Networking, 2012.

[10] W. Guibene, M. Turki, A. Hayar and D. Slock, *A Complete Framework for Spectrum Sensing based on Spectrum Change Points Detection for Wideband Signals*, 2012 IEEE 75th Vehicular Technology Conference: VTC2012-Spring, 6-9 May 2012, Yokohama, Japan..

[11] D. Donoho, Compressed sensing, IEEE Transactions on Information Theory, vol. 52, no. 4, pp. 12891306, 2006.

[12] E. Candes and T. Tao, Near-optimal signal recovery from random projections: Universal encoding strategies, IEEE Transactions on Information Theory, vol. 52, no. 12, pp. 54065425, 2006

[13] S. Chen, D. Donoho, and M. Saunders, Atomic decomposition by basis pursuit, SIAM review, vol. 43, no. 1, pp. 129159, 2001.

[14] E. Candes, J. Romberg, and T. Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE Transactions on Information Theory, vol. 52, no. 2, pp. 489509, 2006.