

A Combined Spectrum Sensing and Terminals Localization Technique for Cognitive Radio Networks

Wael Guibène* Dirk Slock

Mobile Communications Department, EURECOM
2229 Route des Crêtes, P.O. Box 193, 06904, Sophia Antipolis, France

*Corresponding author: Wael.Guibene@eurecom.fr

Abstract—Cognitive radio is a smart wireless communication concept that is able to promote the efficiency of the spectrum usage by exploiting its free frequency bands, namely spectrum holes. Detection of spectrum holes is one of the first steps of implementing a cognitive radio system. Another step towards the feasibility and a real implementation of a cognitive radio network is the problem of location awareness. This problem arises when we do consider a realistic scenario in hybrid overlay/underlay systems, when these spectrum opportunities permit cognitive radios to transmit below the primary users tolerance threshold. In this case, the cognitive radio, have to estimate robustly the primary users locations in the network in order to adjust its transmission power function of the estimated location in the network. Adding to this the fact that in wideband radio one may not be able to acquire signals at the Nyquist sampling rate due to the current limitations in Analog-to-Digital Converter (ADC) technology, we end up with a system that should, at a sub-Nyquist rate, properly recover the bands over which the primary users transmit and estimate their location in the network. In this paper ¹, we proposed to analyze all these arisen problems. During the problem formulation and when analyzing more deeply the equations related to each question apart, we will make the link between the formulation of spectrum sensing, location awareness and the hardware limitation by describing those problems in a unique compressed sensing formalism. Via the proposed framework, we made it possible to overcome a challenging postulate of fixed frequency spectrum allocation by also estimating the spectrum usage boundaries in a blind way.

Index Terms—collaborative spectrum sensing; compressed sensing; primary users localization

I. INTRODUCTION

During the last decades, we have witnessed a great progress and an increasing need for wireless communications systems. This need for flexibility and more "mobile" devices led to needs to afford the spectral resources that shall be able to satisfy costumers need for mobility. But, as wide as spectrum seems to be, all those demands made it a scarce resource and highly misused. Measurements lead by the FCC (Federal Communication Commission) in the USA have shown that in some regions and/or at some day intervals up to 70 percent of the statically allocated spectrum is left idle [2]. Facing this inefficient usage of spectrum, the FCC recommends deploying

unlicensed users in the wireless networks. These unlicensed users, also called secondary user (SU), are allowed to use those idle wireless resources only when the licensed users, also called primary user (PU), is not using them so they do not interfere with their transmissions. Cognitive Radio (CR) as introduced by Mitola [1], is one of those possible devices that could be deployed as SU equipments and systems in wireless networks. As originally defined, a CR is a self aware and "intelligent" device that can adapt itself to the wireless environment changes. Such device is able to detect the changes in Wireless network to which it is connected and adapt its radio parameters to the new opportunities that are detected. This constant track of the environment change is called the "spectrum sensing" function of a cognitive radio device.

Thus, spectrum sensing in CR aims in finding the holes in the PU transmission which are the best opportunities to be used by the SU. Many statistical approaches already exist. The easiest to implement and the reference detector in terms of complexity is still the Energy Detector (ED). Nevertheless, the ED is highly sensitive to noise and does not perform well in low Signal to Noise Ratio (SNR). Other advanced techniques based on signals modulations and exploiting some of the transmitted signals inner properties were also developed. For instance, the detector that exploits the built-in cyclic properties on a given signal is the cyclostationary Features Detector (CFD). The CFD do have a great robustness to noise compared to ED but its high complexity is still a consequent draw back. Some other techniques, exploiting a wavelet approach to efficient spectrum sensing of wideband channels were also developed [5].

Recently, compressed sensing/compressive sampling (CS) has been considered as a promising technique to improve and implement cognitive radio (CR) systems. The increasing demand for spectrum from various wireless devices and networks emerges the technical society to use the radio spectrum more efficiently. In wideband radio one may not be able to acquire a signal at the Nyquist sampling rate due to the current limitations in Analog-to-Digital Converter (ADC) technology [4]. Compressive sensing makes it possible to reconstruct a *sparse* signal by taking less samples than Nyquist sampling, and thus wideband spectrum sensing is doable by CS. An *sparse* signal or a *compressible* signal is a signal that is essentially dependent on a number of degrees of freedom

¹The research work leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement SACRA n249060.

which is smaller than the dimension of the signal sampled at Nyquist rate. In general, signals of practical interest may be only nearly sparse [4]. And typically the wireless signal in open networks are sparse in the frequency domain since depending on location and at some times the percentage of spectrum occupancy is low due to the idle radios [3], [5].

In CS a signal with a sparse representation in some basis can be recovered from a small set of nonadaptive linear measurements [6]. A sensing matrix takes few measurements of the signal, and the original signal can be reconstructed from the incomplete and contaminated observations accurately and sometimes exactly by solving a simple convex optimization problem [4], [7]. In [8] and [9] conditions on this sensing matrix are introduced which are sufficient in order to recover the original signal stably. And remarkably, a random matrix fulfills the conditions with high probability and performs an effective sensing [6], [10].

For overlay CR technology, a binary hypothesis is enough to characterize the spectrum hole, called here *white space*. This spectrum hole is either vacant or already occupied by a PU. But, in hybrid overlay/underlay network, the hypothesis of binary state is no more enough to characterize the spectrum opportunities. In this case, the spectrum opportunities are called *gray spectrum areas* where the PU is already communicating, but there is still an opportunity for the CR to communicate. The CR has then to satisfy some constraints on power (secondary transmit power as seen from a primary receiver have to go below the ceil of PU interference tolerance level and when talking about transmit power, it is directly related to the PU positions in the CRN).

In this paper, we will present a joint spectrum sensing and PU localization algorithm for CRN. We will show how localization in CRN could be viewed as a CS problem and formulated in terms of CS equations. This algorithm is presented as a CS approach to both problems. We will use a modified framework of the orthogonal matching pursuit algorithm (OMP) that we feed with some *a priori* knowledge of the CR spectrum usage and thus derive a more appropriate OMP algorithm for CS and the problem of localization.

The rest of the paper is organized as following: in Section II we will give the system model used through this paper. In order to make the paper easier to read and to apprehend, in Section III we start by giving an overview of what will be done at the level of each CR individually and still in Section III we will derive the CS algorithm to be deployed. In IV, we will make the link between location estimation and spectrum reconstruction. In Section V, we will go through the analysis of the proposed technique and derive its performances. Finally, Section VI will conclude about the present work.

II. SYSTEM MODEL

In the considered system model, we will suppose that we do dispose of N_{ch} available channels in a wideband wireless network. Over a large geographic area, let N_p be the number of deployed primary users using N_p different channels. In this large area, we disperse N_c cognitive radios that will

operate and detect all these channels and their states. The measures made by these cognitive terminals will then be sent to the fusion center. In order to enable CRs transmissions, the secondary network have to be aware of the availability and the state of each channel in the sense of hybrid underlay/overlay scheme. Thus, secondary users have to estimate which channels are occupied and to identify the PUs transmission powers and locations.

Adopting the path loss model, we end up with a loss of:

$$L(f, d) = P_0 + 2\lg(f) + 10n\lg(d) \quad [dB] \quad (1)$$

where: P_0 is a constant related to antennas gain; f is the channel frequency; n is the path loss exponent; d is the distance separating the transmitting and receiving nodes and $\lg(\cdot) = \log_{10}(\cdot)$

In our case, we dispose of N_{ch} channels, thus f would be assumed the central frequency of each band, i.e $f \in \{f_0, f_1, \dots, f_{N_{ch}-1}\}$.

Let's keep in mind that the path loss is related to the unknown channel and location of the PU. The received signal power is a combination of the unknown transmit power with the path loss expressed in Eq(1).

Our task is to infer from the received signal at the cognitive terminals all these unknown, but valuable, information about the primary users.

First of all we will describe what is exactly done at the level of each terminal separately in the section III. Then, starting from IV, we proceed with this system model.

III. SINGLE NODE SPECTRUM SENSING BASED ON COMPRESSED SENSING

A. Discrete Spectrum Model

As initiated in [12], the CS algorithms suppose that we do dispose of B Hz of total bandwidth for the CRN. In discrete notation, let's denote by \vec{f} the $N \times 1$ discrete spectrum vector containing the sampled values over B .

$$\vec{f} = [f_1 \ f_2 \ \dots \ f_N]^T \quad (2)$$

where T is the transpose operation and $\{f_i\}$ are the signal values uniformly sampled over B by a B/N resolution and $\{i\}$ is the subset relate the frequencies locations. It is then trivial that in noise free context, a frequency i is said to be *vacant* or *free* if $|f_i|^2 = 0$.

The $N \times N$ normalized discrete Fourier Transform (DFT) matrix, \mathbf{F} , gives the relationship between the frequency samples vector \vec{f} and the time domain samples vector \vec{t} , by the relation:

$$\vec{t} = \mathbf{F}^{-1} \vec{f} \quad (3)$$

In [12], CR spectrum usage was summarized in three main categories:

- 1) Spectrum bands with fixed boundaries to which the PUs are always accessing such as local TV and radio broadcasters.

- 2) Spectrum bands with fixed boundaries to which the PUs rarely access like TVWS.
- 3) Spectrum bands with fixed boundaries which are partially and randomly accessed like cellphone signals, LTE...

And in these three main CR spectrum usage scenarios, spectrum boundaries are fixed and *a priori* known.

This strong assumption of knowledge of boundaries can be overcome by processing as following:

B. Blind Spectrum Boundaries Estimation

In [13]–[15], Guibene *et al.*, developed a spectrum sensing technique based on frequency edge location and exploiting spectrum discontinuities detection. Inspired from the already developed framework, we derive our edge location algorithm.

First we do suppose that the frequency range available in the wireless network is B Hz; so B could be expressed as $B = [f_0, f_K]$. Saying that this wireless network is cognitive, means that it supports heterogeneous wireless devices that may adopt different wireless technologies for transmissions over different bands in the frequency range. A CR at a particular place and time needs to sense the wireless environment in order to identify spectrum holes for opportunistic use. Suppose that the radio signal received by the CR occupies N spectrum bands, whose frequency locations and PSD levels are to be detected and identified. These spectrum bands lie within $[f_1, f_K]$ consecutively, with their frequency boundaries located at $f_1 < f_2 < \dots < f_K$. The n -th band is thus defined by: $B_n : \{f \in B_n : f_{n-1} < f < f_n, n = 2, 3, \dots, K\}$. The following basic assumptions are adopted:

- 1) The frequency boundaries f_1 and $f_K = f_1 + B$ are known to the CR. Even though the actual received signal may occupy a larger band, this CR regards $[f_1, f_K]$ as the wide band of interest and seeks white spaces only within this spectrum range.
- 2) The number of bands N and the locations f_2, \dots, f_{K-1} are unknown to the CR. They remain unchanged within a time burst, but may vary from burst to burst in the presence of slow fading.
- 3) The PSD within each band B_n is smooth and almost flat, but exhibits discontinuities from its neighboring bands B_{n-1} and B_{n+1} . As such, irregularities in PSD appear at and only at the edges of the K bands.
- 4) The corrupting noise is additive white and zero mean.

The PSD structure of a wide-band signal is illustrated in Fig. 1

The input signal is the amplitude spectrum of the received noisy signal. We assume that its mathematical representation is a piecewise regular signal:

$$Y(f) = \sum_{i=1}^K \chi_i[f_{i-1}, f_i](f) p_i(f - f_{i-1}) + n(f) \quad (4)$$

where: $\chi_i[f_{i-1}, f_i]$: the characteristic function of the interval $[f_{i-1}, f_i]$, $(p_i)_{i \in [1, K]}$: an N^{th} order polynomials series,

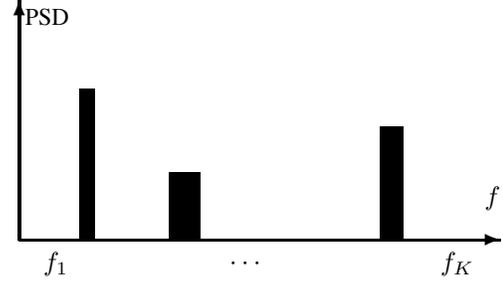


Fig. 1. K frequency bands with piecewise smooth PSD.

$(f_i)_{i \in [1, K]}$: the discontinuity points resulting from multiplying each p_i by a χ_i and $n(f)$: the additive corrupting noise. Now, let $X(f)$ the clean version of the received signal given by:

$$X(f) = \sum_{i=1}^K \chi_i[f_{i-1}, f_i](f) p_i(f - f_{i-1}) \quad (5)$$

And let b , the frequency band, given such as in each interval $I_b = [f_{i-1}, f_i] = [\nu, \nu + b]$, $\nu \geq 0$ maximally one change point occurs in the interval I_b .

Now denoting $X_\nu(f) = X(f + \nu)$, $f \in [0, b]$ for the restriction of the signal in the interval I_b and redefine the change point which characterizes the distribution discontinuity relatively to I_b say f_ν given by:

$$y_n = \begin{cases} f_\nu = 0 & \text{if } X_\nu \text{ is continuous} \\ 0 < f_\nu \leq b & \text{otherwise} \end{cases}$$

Now, in order to emphasize the spectrum discontinuity behavior, we decide to use the N^{th} derivative of $X_\nu(f)$, which in the sense of Distributions Theory is given by:

$$\frac{d^N}{df^N} X_\nu(f) = [X_\nu(f)]^{(N)} + \sum_{k=1}^N \mu_{N-k} \delta(f - f_\nu)^{(k-1)} \quad (6)$$

where: μ_k is the jump of the k^{th} order derivative at the unique assumed change point: f_ν

$$\mu_k = X_\nu^{(k)}(f_\nu^+) - X_\nu^{(k)}(f_\nu^-)$$

with: $\begin{cases} \mu_k = 0 \text{]}_{k=1..N} & \text{if there is no change point.} \\ \mu_k \neq 0 \text{]}_{k=1..N} & \text{if the change point is in } I_b. \end{cases}$

$[X_\nu(f)]^{(N)}$ is the regular derivative part of the N^{th} derivative of the signal.

The spectrum sensing problem is now casted as a change point f_ν detection problem. In a matter of reducing the complexity of the frequency direct resolution, the equations are transposed to the operational domain, using the Laplace transform:

$$\begin{aligned} L(X_\nu(f))^{(N)} &= s^N \widehat{X}_\nu(s) - \sum_{m=0}^{N-1} s^{N-m-1} \frac{d^m}{df^m} X_\nu(f) \Big|_{f=0} \\ &= e^{-s f_\nu} (\mu_{N-1} + s \mu_{N-2} + \dots + s^{N-1} \mu_0) \end{aligned} \quad (7)$$

Given the fact that the initial conditions and the jumps of the derivatives of $X_\nu(f)$ are unknown parameters to the problem, in a first time we are going to annihilate the jump values $\mu_0, \mu_1, \dots, \mu_{N-1}$ then the initial conditions as fully detailed

in [14]. After some calculations detailed in [14], we finally obtain:

$$\sum_{k=0}^{N-1} \binom{N}{k} \cdot f_\nu^{N-k} \cdot (s^N \widehat{X}_\nu(s))^{(N+k)} = 0 \quad (8)$$

In the actual context, the noisy observation of the amplitude spectrum $Y(f)$ is taken instead of $X_\nu(f)$. As taking derivative in the operational domain is equivalent to high-pass filtering in frequency domain, which may help amplifying the noise effect. It is suggested to divide the whole equation 11 by s^l which in the frequency domain will be equivalent to an integration if $l > 2N$, we thus obtain:

$$\sum_{k=0}^{N-1} \binom{N}{k} \cdot f_\nu^{N-k} \cdot \frac{(s^N \widehat{X}_\nu(s))^{(N+k)}}{s^l} = 0 \quad (9)$$

Since there is no unknown variables anymore, the equations are now transformed back to the frequency domain, we obtain the polynomial to be solved on each sensed sub-band:

$$\sum_{k=0}^{N-1} \binom{N}{k} \cdot f_\nu^{N-k} \cdot L^{-1} \left[\frac{(s^N \widehat{X}_\nu(s))^{(N+k)}}{s^l} \right] = 0 \quad (10)$$

And denoting:

$$\varphi_{k+1} = L^{-1} \left[\frac{(s^N \widehat{X}_\nu(s))^{(N+k)}}{s^l} \right] = \int_0^{+\infty} h_{k+1}(f) \cdot X(\nu-f) \cdot df \quad (11)$$

$$\text{where: } h_{k+1}(f) = \begin{cases} \frac{(f^l (b-f)^{N+k})^{(k)}}{(l-1)!} & 0 < f < b \\ 0 & \text{otherwise} \end{cases}$$

In [11], it was shown that edge detection and estimation is analyzed based on forming multiscale point-wise products of smoothed gradient estimators. This approach is intended to enhance multiscale peaks due to edges, while suppressing noise. Adopting this technique to our spectrum sensing problem and restricting to dyadic scales, we construct the multiscale product of $N+1$ filters (corresponding to Continuous Wavelet Transform in [11]), given by:

$$Df = \left\| \prod_{k=0}^N \varphi_{k+1}(f_\nu) \right\| \quad (12)$$

Thus, the changes points, or also called frequency boundaries obey to equation 15 or equivalently equation 12.

So now on, the only assumptions are the ones we introduced from [14], [15] and not the ones describes in [12] (describes by the previous three use cases) which gives to the algorithm lighter assumption and gives it some blind processing properties. So finally, we adopt the following assumptions:

- Only the two boundaries of the whole band of interest are known
- The total number of used sub-bands is unknown and to be determined
- The PSD of the signals is almost flat in the used sub-bands
- Noise is white gaussian with zero mean

Then, spectrum usage categories is still valid at the CR level but after estimating the boundaries.

C. Spectrum Sensing based on Compressive Sampling

In the CS framework, we do consider the sampling of $N \times 1$ signal $\vec{x} = \Psi \vec{s}$, where \vec{s} is an $N \times 1$ sparse source vector with L non-zero components s_i , so $L \ll N$ and Ψ is an $N \times N$ dictionary matrix. In literature [16], [17] it was shown that M samples of \vec{x} can recover the whole vector, by projecting \vec{x} by an $M \times N$ observation matrix, say Φ . This matrix has to satisfy two conditions: $L < M < N$ and the rows of the sensing matrix Φ should be incoherent with the columns of Ψ . Finally we obtain the $M \times 1$ measurement vector \vec{y} given by:

$$\vec{y} = \Phi \vec{x} = \Phi \Psi \vec{s} \quad (13)$$

\vec{s} can be fully reconstructed by adopting the basis pursuit algorithm as shown in [18]. Its reconstruction is subject to a convex optimization problem as shown in Eq(14):

$$\widehat{\vec{s}} = \arg \min_{\vec{s}} \|\vec{s}\|_{l_1} \text{ subject to } \Phi \Psi \vec{s} = \vec{y} \quad (14)$$

where l_p is the p-norm for $p \geq 1$ given by:

$$\|\vec{s}\|_{l_1} = \left(\sum |s_i|^p \right)^{\frac{1}{p}}$$

Another way to reconstruct the signal could be the matching pursuit algorithm (MP) derivatives as will be shown in next paragraph.

Through a deeper look into equations Eq(5) and Eq(13), one can intuitively say that the time domain vector \vec{t} can be viewed as \vec{x} and the inverse DFT matrix \mathbf{F} could be seen as the matrix substituting the dictionary matrix Ψ and \vec{f} is no more than the sparse vector \vec{s} .

With this new formalism, if we can properly design a measurement matrix Φ satisfying then incoherence constraint with \mathbf{F}^{-1} , than we would be able to use the CS formalism as a spectrum sensing technique and sub-Nyquist sampling rate could be recovered by CS algorithms as well. Given the work lead in [19], the use of $M \times N$ Gaussian random matrix as a measurement matrix Φ would guarantee good reconstruction performance. Back now to the spectrum sensing model, which in noise free environment is formulated as following:

$$\vec{y} = \Phi \vec{f} \quad (15)$$

and as results \vec{f} reconstruction is solution of:

$$\widehat{\vec{f}} = \arg \min_{\vec{f}} \|\vec{f}\|_{l_1} \text{ subject to } \Phi \vec{f} = \vec{y} \quad (16)$$

and in a general additive white gaussian noise environment (AWGN), the sensing model becomes:

$$\vec{y} = \Phi \vec{f} + \vec{w} \quad (17)$$

and as results \vec{f} reconstruction is solution of:

$$\widehat{\vec{f}} = \arg \min_{\vec{f}} \frac{1}{2} \|\vec{y} - \Phi \vec{f}\|_{l_2} + \gamma \|\vec{f}\|_{l_1} \quad (18)$$

where \vec{w} is an $M \times 1$ noise vector with a normal distribution and γ is determined by the noise level.

D. Matching Pursuit Varieties for Compressed Sensing

1) *Basis Pursuit based Reconstruction Algorithms*: In order to stick to the three knowledge categories of spectrum usage we previously introduced, we need to reformulate the use of l_1 norm in Eq (18). In [20], mixed norm l_1/l_2 denoising operator uses the frequency boundaries as *a priori* known. \vec{f} is then subdivided according to the estimated $\{b_i\}$ into K blocks as following:

$$\vec{f} = \underbrace{[f_1 \dots f_{b_1} \dots]}_{\vec{f}_1^T} \underbrace{[f_{b_{K-1}+1} \dots f_{b_K}]}_{\vec{f}_K^T}^T \quad (19)$$

$$\vec{f} = [\vec{f}_1^T \vec{f}_2^T \dots \vec{f}_K^T]^T$$

In [20], the authors then reformulate the mixed noise denoising operator to a new convex optimization formalism with known boundary information:

$$\widehat{\vec{f}} = \arg \min_{\vec{f}} (\|\vec{f}_1\|_{l_2} + \dots + \|\vec{f}_K\|_{l_2}) \quad (20)$$

$$\text{subject to } : \|\vec{y} - \Phi \vec{f}\|_{l_2} \leq \eta$$

where η is dependent of the noise variance.

2) *Modified Blind Orthogonal Matching Pursuit Algorithm*: The original OMP (orthogonal matching pursuit) algorithm is a greedy algorithm based on the basis pursuit algorithm that reconstructs iteratively the original signal by the search of non zero indices and performs least square estimation of the values on the non zero indices.

The estimated frequency boundaries, $\{\nu_i, i \in [0..K]\}$, do actually separate the spectrum in K consecutive sub-bands. Keeping in mind, that in one hand B is actually divided into K sub-bands and the fact that the frequency indices we were using is of length N , this means that the indices set we are using in frequency domain is actually divided into K consecutive subsets. Let $\{b_i\}$ denote these indices in each frequency boundary, i.e, $\nu_0 \triangleq 1$, ν_1 occurs at the frequency index b_1 and so on until $\nu_K \triangleq N$.

Let's denote these subsets by :

$$\begin{aligned} u_1 &= 1, 2, \dots, b_1 \\ u_2 &= b_1 + 1, b_1 + 2, \dots, b_2 \\ &\dots \\ u_K &= b_{K-1} + 1, b_{K-1} + 2, \dots, N \end{aligned} \quad (21)$$

Now, let's define three category sets $\{S_n\}$, according to the following condition:

$$\begin{aligned} S_n &= \{u_i \mid n = 1, 2, 3\} \\ \Omega &= \cup S_n \quad (S_i \cap S_j = \emptyset, \text{ for } i \neq j) \end{aligned} \quad (22)$$

According to the measurement results of spectrum utilization, we assume as in [12] that :

$$\frac{\#\{\vec{f}\}}{N} \leq 10\%$$

where $\#\{\vec{f}\}$ is the number of non-zero values in \vec{f}

Algorithm 1 Proposed Matching Pursuit Algorithm

Require: An $M \times N$ matrix $\Theta = \Phi$

An $M \times 1$ sample vector \vec{y}

Minimum iterations number m

Error tolerance η

- 1: Estimate from the wideband observation the boundaries as stated in III-B and then preselect the sets as in Eq (21) and (22)
- 2: Initialize: $\overrightarrow{res}_0 = \vec{y}$, $\Lambda_0 = S_1$, $\Theta_1 = \Theta_{S_1}$, iteration_index: $t = 1$
- 3: Solve the least-squares problem in Equation:

$$\vec{x}_t = \arg \min_{\vec{x}} (\|\Theta_t \vec{x} - \vec{y}\|_2) \quad (23)$$

- 4: Compute the new residual given by:

$$\overrightarrow{res}_t = \vec{y} - \Theta_t \vec{x}_t \quad (24)$$

- 5: Increment: $t \leftarrow t + 1$
- 6: Find λ_t satisfying:

$$\lambda_t = \arg \max_{j=1..N} |\langle \overrightarrow{res}_{t-1}, \theta_j \rangle| \quad (25)$$

where θ_j is the j^{th} column vector of Θ_t and $\langle \cdot, \cdot \rangle$ is the inner vector product operator.

- 7: Increase the index set $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$
if $\{\lambda_t \in u_i \subset S_2\}$
then $\Lambda_t = \Lambda_{t-1} \cup \{u_i\}$
end
 - 8: Set the atom to: $\Theta_t = \Theta_{\Lambda_t}$
 - 9: Solve the least square problem in Equation (23) and get the new estimate of \vec{x} .
 - 10: Calculate the new residual using Equation (24)
 - 11: **if** $\{t < m \text{ or } \|\overrightarrow{res}_t\|_2 > \eta\}$
then return to step (5)
end
 - 12: Finally: $\vec{f} \leftarrow \vec{x}_t$ and its non zero indices are listed in Λ_t
 - 13: **return** An estimate $N \times 1$ vector \vec{f} of the ideal signal
An index set Λ_t containing t elements from $\{1..N\}$
An $M \times 1$ residual vector \overrightarrow{res}_t
-

The iteration operation gives us the freedom to consider the three already defined categories separately. Since, by construction, S_1 do have at least a non zero value, the initialization output, Λ_0 , could be set to S_1 . This particular initialization guarantees us always counting the occupied indices. Then, during the rest of the iterations, if we do find an index λ_t , satisfying: $\lambda_t \in u_i \subset S_2$, all elements in u_i will be added in Λ_t . This would enable us counting only the $\{u_i\}$ subset. The other case is $\lambda_t \in u_i \subset S_3$, in which only λ_t is added to Λ_t as in formal OMP.

The modified blind orthogonal matching pursuit is fully describes by Algorithm 1.

IV. JOINT SPECTRUM SENSING AND PRIMARY USERS LOCALIZATION BASED ON COMPRESSIVE SAMPLING FOR COGNITIVE RADIO NETWORKS

A. Spectrum Reconstruction

For discrete signals, the time domain samples \vec{t} are used to construct the spectrum in frequency domain as shown before in Eq(5). Thus we obtained:

$$\vec{f} = \mathbf{F}\vec{t} \quad (26)$$

And as sufficiently detailed in Section III, on the level of each node, this problem as formulated in a context of wide-band and involving sparse signals can be casted as a CS problem and spectrum can be reconstructed and spectrum sensing task is thus achieved by all terminals.

B. Primary Users Location Reconstruction

Once spectrum reconstructed and spectrum sensing achieved, more information can be derived while looking deeper into channels occupied by primary users.

Let's assume that in a certain wide area, PUs are located at coordinates (xp_m, yp_n) ; where $xp_m \in \{0, \Delta xp, \dots, (M-1)\Delta xp\}$ are M possible x axis positions (abscissæ) of the PUs²; $yp_n \in \{0, \Delta yp, \dots, (N-1)\Delta yp\}$ are N possible y axis positions (ordinates) of the PUs; Δxp and Δyp are respectively the resolutions over x and y axis. Here, we do impose and suppose to the PU coordinates to be in discrete $M \times N$ dictionary (which, actually, is always true !). It is good to remind at this level that the exact positions of the N_p PUs $\{(xp_i, yp_i) ; i \in [1..N_p]\}$ are unknown to our problem.

The N_c CRs positions in the network are located at positions: $\{(a_i, b_i) ; i \in [1..N_c]\}$ (on which we do not impose being in a finite set, even if they necessarily are).

For the k^{th} CR, sensing the i^{th} channel, the contribution of the PU located at the (xp_m, yp_n) position on the received PSD is:

$$R_{k,i}(m, n) = P(m, n, i) \times 10^{L(f_i, d(m, n, k))/10} \quad (27)$$

$$d(m, n, k) = \sqrt{(xp_m - a_k)^2 + (yp_n - b_k)^2}$$

where $P(m, n, i)$ is the power transmitted by a PU using the i^{th} channel, located at (xp_m, yp_n) ; f_i is the center frequency of the i^{th} channel; $d(m, n, k)$ represents the distance between the k^{th} CR and the PU located at (xp_m, yp_n) .

The total received power over all the existing PUs, i.e over the $M \times N$ possible positions of the PUs, can be formulated as following:

$$Y_{k,i} = \sum_m \sum_n R_{k,i}(m, n) \quad (28)$$

$$Y_{k,i} = \sum_m \sum_n 10^{L(f_i, d(m, n, k))/10} \times P(m, n, i)$$

$$Y_{k,i} = \vec{L}^T(k, i) \vec{P}(i)$$

²When we say $xp_m \in \{0, \Delta xp, \dots, (M-1)\Delta xp\}$, that does not mean that there are M PUs, but it means that N_p primary users abscissæ (for ordinates as well) do actually have a finite "dictionary"

where $\vec{P}(i)$ is the vector containing the transmission power of the over all $M \times N$ grid over the i^{th} channel; and $\vec{L}(k, i)$ is the path loss vector computed according to Eq(1) from all PU possible positions at the level of the k^{th} CR, on the i^{th} channel.

$$\vec{L}(k, i) = 10^{\vec{L}_{dB}(k, i)/10}$$

and :

$$\vec{L}_{dB}(k, i) = [L(f_i, d(0, 0, k)), L(f_i, d(1, 0, k)), \dots, L(f_i, d(m, n, k)), \dots, L(f_i, d(M, N, k))]^T \quad (29)$$

Let's denote by $\vec{Y}_k = [Y_{k,1} \dots Y_{k,N_{ch}}]^T$, the received signal power vector at the level of the k^{th} CR over the N_{ch} available channels. This according to Eq(28), and adopting the previous notation can be expressed as:

$$\vec{Y}_k = \mathbf{L}_k \vec{P} \quad (30)$$

where \vec{P} is the vector containing the transmission power of the $M \times N$ grid of PU locations over the N_{ch} available channels of the N_C deployed CRs:

$$\vec{P}_k = [\vec{P}^T(i_1), \vec{P}^T(i_2), \dots, \vec{P}^T(i_{N_C})]^T \quad (31)$$

The matrix \mathbf{L}_k , is the fading gain matrix grouping at the level of the k^{th} CR the loss path contributions of the $M \times N$ PU positions. The j^{th} row of \mathbf{L}_k is:

$$\mathbf{L}_k(j) = [\vec{0}, \vec{0}, \dots, \vec{L}^T(k, j), \vec{0}, \dots, \vec{0}] \quad (32)$$

Combining all the equations describing the N_C CR system, we do obtain:

$$\vec{Y} = \mathbf{L} \vec{P} \quad (33)$$

Where $\vec{Y} = [Y_1^T, \dots, Y_{N_C}^T]^T$ and $\mathbf{L} = [\mathbf{L}_1, \dots, \mathbf{L}_{N_C}]$

The equation we ended with in Eq(33), reminds us of the CS formalism we introduced previously: as \vec{P} is an unknown but sparse vector because over the $M \times N$ area we've been considering, only N_P PUs are deployed in this area.

Since the two stages, spectrum sensing and localization, seem to be attached to the same CS framework we've introduced before, it is easy then to combine both of them in only one process.

V. SIMULATIONS AND RESULTS

In this section we propose to investigate the performances of the proposed technique in terms of spectrum sensing and PUs location.

In order to cover the whole aspects of the proposed work, three parts will be simulated: first of all, the boundaries estimation, then the compressed sensing alone and finally the over all system simulation.

First of all, we consider a frequency band in the range of [50, 250]MHz, in order to compare the compressive sensing using the algebraic method and the wavelet approach introduced in [11]. The signal is fully described in [11]. During the observed burst of transmissions in the network, there 6 bands, with frequency boundaries at

$n_{\nu_{n=0}}^6 = [50, 120, 170, 200, 220, 224, 250]$ MHz. In figure 2, we show how accurate the proposed blind boundaries algorithm is and how sensitive it is to PSD level change. Comparing with the wavelet approach, in the algebraic detection technique change points are detected only in one shot, while in the wavelets approach, many detections have to be conducted and fused to make a final decision.

We propose then the following scenario: PUs do dispose of 10 channels that they will randomly select in the range of $50 + [1, 2.., 10]$ MHz. We propose deploying 2 primary users in a 15×15 ($M = N = 15$) unit area with a resolution of $\Delta xp = \Delta yp = 0.1$ having randomly generated power transmits. In this area we also deploy 5 CRs that will achieve the sensing and the localization task. Figures 3 and 4 do report the MSE of the spectrum reconstruction and PU positions reconstruction using the suggested technique. These figures report how efficient the recovery of the sparse spectrum and position model is.

Finally, we suggest the over all system power recovery and spectrum recovery from the sparse observation. Figures 5 and 5 show an example of the spectrum and power reconstruction at -10dB for a 2 PU and 5 CRs scenario using the proposed technique.

VI. CONCLUSION

This paper presents a first look towards a combined spectrum sensing and localization task. These two tasks are fundamental in order to really enable cognition in wireless networks. With the combination of the two tasks, we also considered a realistic data acquisition constraint, which is sparsity due to the ADC technology limits. In order to make the algorithm and the over all system a real stand-alone one and blindly operating, we suggested removing the apriori knowledge of the channels and spectrum use by blindly and accurately estimating the frequencies boundaries. Simulation results of the proposed technique show promising and interesting results.

REFERENCES

- [1] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *Personal Communications, IEEE*, vol. 6, no. 4, pp. 13–18, 1999.
- [2] FCC, "Spectrum policy task force report," tech. rep., Nov. 2002.
- [3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201–220, 2005.
- [4] E. J. Cands, "Compressive sampling," in *Proceedings of the International Congress of Mathematicians*, (Madrid, Spain), 2006.
- [5] Z. Tian and G. Giannakis, "Compressed sensing for wideband cognitive radios," in *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*, vol. 4, pp. IV–1357–IV–1360, 2007.
- [6] M. A. Davenport, M. B. Wakin, and R. G. Baraniuk, "Detection and estimation with compressive measurements," technical, Rice University, 2007.
- [7] E. Candes, J. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *math/0503066*, Mar. 2005.
- [8] E. Candes and T. Tao, "Decoding by linear programming," *math/0502327*, Feb. 2005.
- [9] E. J. Candes and T. Tao, "Near-Optimal signal recovery from random projections: Universal encoding strategies", *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5406–5425, 2006.
- [10] E. Candes and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.

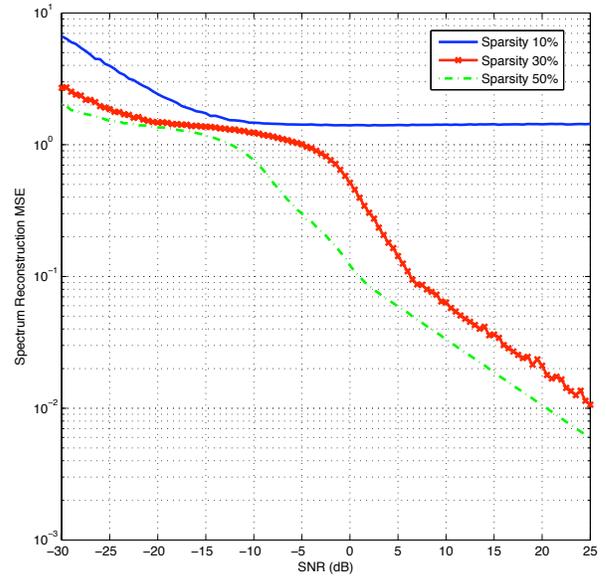


Fig. 3. Spectrum reconstruction MSE

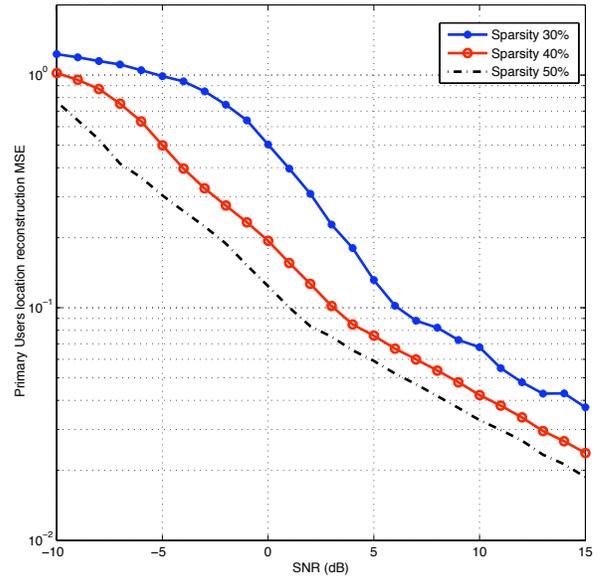


Fig. 4. PU position estimation MSE

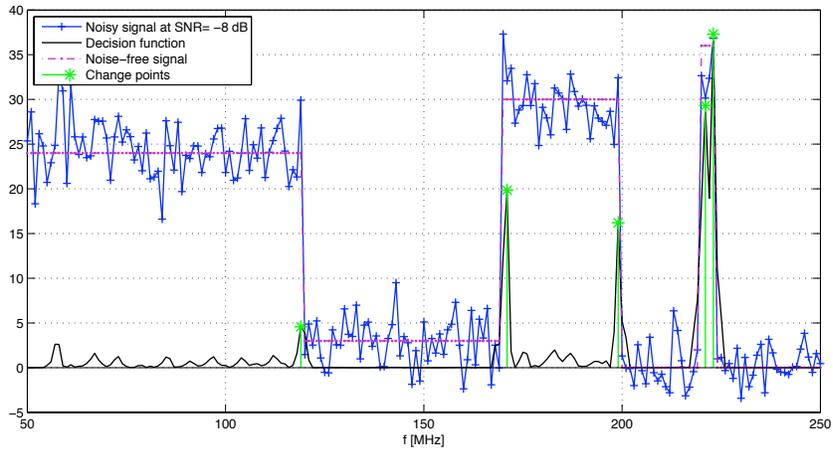


Fig. 2. Edge detection using the algebraic technique. The signal in red is the original signal, the one in blue is the noisy observation with $\text{SNR}=-5\text{dB}$. The black signal is the computed decision function and the green stars are the detected change points.

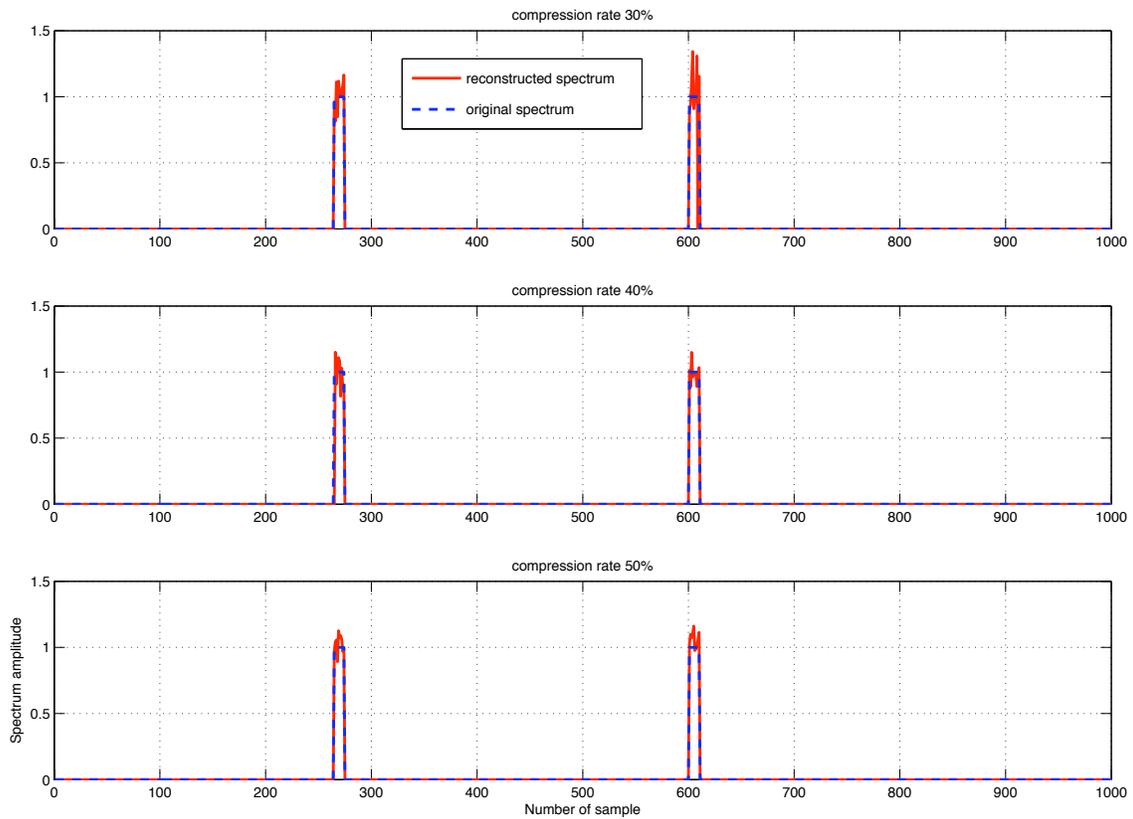


Fig. 5. Spectrum reconstruction example for sparsity of 30%, 40% and 50%

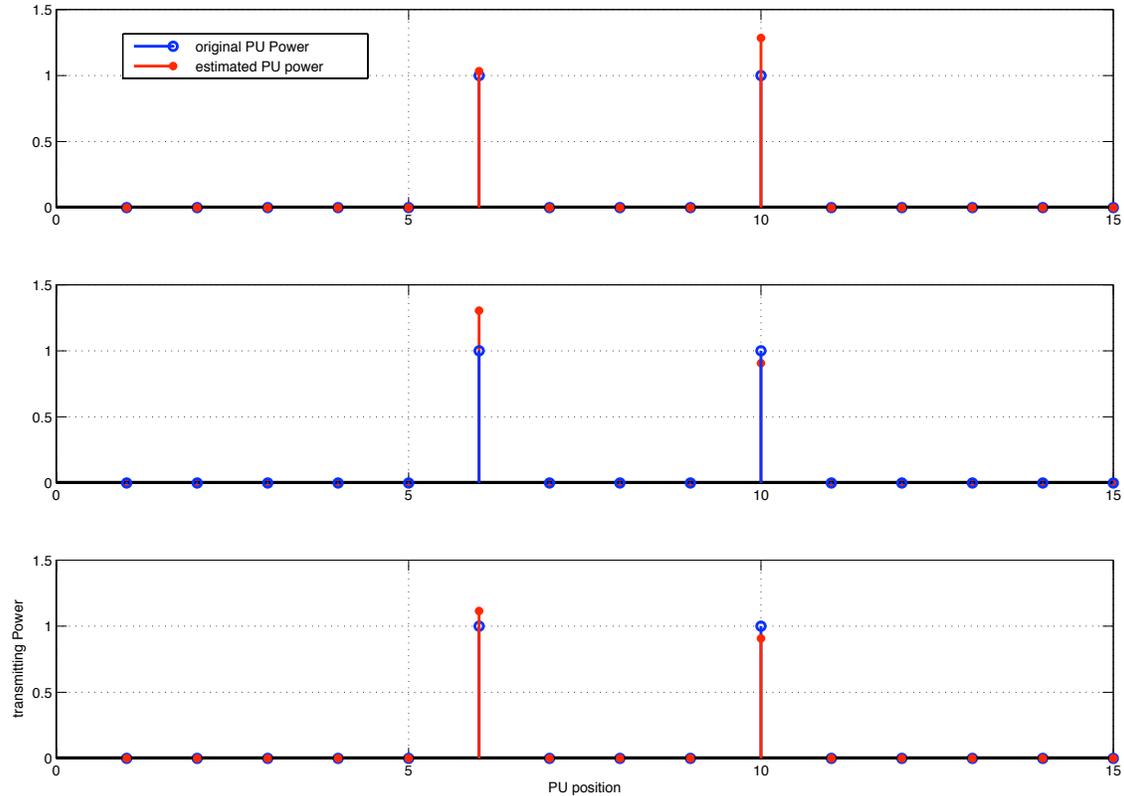


Fig. 6. PU power estimation recovery example for sparsity of 30%, 40% and 50%

- [11] Z. Tian and G. B. Giannakis, *A wavelet approach to wideband spectrum sensing for cognitive radios*. In *Cognitive Radio Oriented Wireless Networks and Communications*, 2006. 1st International Conference on, page 1.5, 2006.
- [12] P. Zhang, R. Qiu, "Modified Orthogonal Matching Pursuit Algorithm for Cognitive Radio Wideband Spectrum Sensing", Cornell University Library, <http://arxiv.org/abs/1102.2881v1>
- [13] W. Guibène, M. Turki, and A. Hayar, "Distribution discontinuities detection using algebraic technique for spectrum sensing cognitive radio networks", *5th International conference on cognitive radio oriented wireless networks and communication*, France, pp.1-5, June 2010.
- [14] W. Guibène, M. Turki, B. Zayen and A. Hayar, "Spectrum sensing for cognitive radio exploiting spectrum sensing discontinuities detection", Published in *EURASIP Journal on Wireless Communication and Networking*, 2012.
- [15] W. Guibène, M. Turki, B. Zayen and A. Hayar, "Spectrum sensing for cognitive radio exploiting spectrum sensing discontinuities detection", Published in *EURASIP Journal on Wireless Communication and Networking*, 2012.
- [16] D. Donoho, Compressed sensing, *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 12891306, 2006.
- [17] E. Candes and T. Tao, Near-optimal signal recovery from random projections: Universal encoding strategies, *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 54065425, 2006
- [18] S. Chen, D. Donoho, and M. Saunders, Atomic decomposition by basis pursuit, *SIAM review*, vol. 43, no. 1, pp. 129159, 2001.
- [19] E. Candes, J. Romberg, and T. Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489509, 2006.
- [20] Y. Liu and Q. Wan, Compressive wideband spectrum sensing for fixed frequency spectrum allocation. Arxiv preprint arXiv:1005.1804, 2010