USER SELECTION IN THE MIMO BC

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ABSTRACT

It is well-known that user selection not only leads to multi-user diversity but also to decreased suboptimality of simple beamforming (BF) techniques compared to optimal Dirty Paper Coding (DPC) approaches in the Broadcast Channel (BC), otherwise called the multi-user (MU) downlink, in a cell with a base station and mobile terminals equipped with multiple antennas (MU-MIMO). User selection by exhaustive search can be simplified to greedy approaches, in which one user gets added at a time. In this paper, we review an approximate criterion for MISO BF-style selection. For a sufficient amount of users, multiple receive antennas do not lead to increased spatial multiplexing, but we indicate how they affect the high SNR rate offset. The resulting added diversity can be exploited at the cost of more involved user selection and transceiver design. We thus propose a novel receiver design for BF-style MU-MIMO stream selection.

Index Terms— Multiple-input multiple-output, broadcast channel, greedy user selection, sum rate.

1. INTRODUCTION

The multi-user MIMO Broadcast Channel (MU-MIMO BC) is one of the most investigated subjects in the literature on wireless communications due to the high potential it offers in improving the system throughput. Information theory has shown that the capacity of MU-MIMO channels could be achieved through dirty-paper coding (DPC) [1–3]. However, DPC is difficult to implement and computationally complex. Some suboptimal linear beamforming algorithms exist and can be divided into two main families: iterative [4–8] and closed form (CF) solutions [9–13].

These solutions can then be differentiated according to the number of streams allocated per user. In fact, there are precoders that can not support more than one stream per user even if the system is not fully charged. Such precoders have been proposed and widely studied in [6, 7, 9–12].

Some multi-stream precoding solutions have nevertheless been proposed such as in [13, 14]. To the best of our knowledge, the best linear CF precoder present in the literature is the so called ZFDPC-SUS (zero forcing DPC with successive user selection) that has been proposed in [13, 15]. This precoding technique is based on the selection of semi-orthogonal users based on the SVD of their respective channels. Another interesting multi-stream technique is the one presented in [14] based on the Signal to Leakage plus Noise Ratio (SLNR) maximization. This technique offers some advantages, e.g. the channel knowledge can be limited to only covariance matrix information. On the other hand, this solution requires the prefixing of the stream distribution.

In reality, d_k streams can be allocated to user k respecting two main constraints: $d_k \leq \min(N_k, N_t)$ which constrains the maximum number of streams per user, and $d = \sum_{k=1}^{K} d_k \leq \min(\sum_{k=1}^{K} N_k, N_t)$ which constrains the total number of streams allocated by the base station (BS). The allocation of these streams could be done to maximize the total sum-rate (SR), along with a crucial point in SR maximization: finding the optimal power distribution over the selected streams.

2. SYSTEM MODEL

We consider a MIMO BC (Multi-User MIMO downlink) with N_t transmit antennas and K users with N_k receiving antennas. We assume perfect CSI and Rayleigh fading. The transmit power constraint is P, the white noise variance is $\sigma^2 = 1$ at all receivers. $\mathbf{H}_k$, $\mathbf{G}_k$, $\mathbf{F}_k$ denotes the MIMO channel, the transmitter (Tx) and receiver (Rx) filters for user k. $\mathbf{H} = [\mathbf{H}_1^T, \cdots, \mathbf{H}_K^T]^T$, $(\cdot)^T$ and $(\cdot)^H$ respectively denote the transpose and the conjugate transpose operations. The received signal is given by $\mathbf{y}_k = \mathbf{H}_k\mathbf{x} + \mathbf{z}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{G}_i\mathbf{s}_i + \mathbf{z}_k$ or

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hence

$$\underbrace{\mathbf{F}_k}_{d_k\times N_k}\underbrace{\mathbf{y}_k}_{N_k\times 1} = \underbrace{\mathbf{F}_k}_{d_k\times N_k}\underbrace{\mathbf{H}_k}_{N_k\times N_t}\sum_{i=1}^{K}\underbrace{\mathbf{G}_i}_{N_t\times d_i}\underbrace{\mathbf{s}_i}_{d_i\times 1} + \underbrace{\mathbf{F}_k}_{d_k\times N_k}\underbrace{\mathbf{z}_k}_{N_k\times 1}.$$

Then

$$\mathbf{F}_k\mathbf{y}_k = \underbrace{\mathbf{F}_k\mathbf{H}_k\mathbf{G}_k\mathbf{s}_k}_{\text{useful signal}} + \underbrace{\sum_{i=1,i\neq k}^{K}\mathbf{F}_k\mathbf{H}_k\mathbf{G}_i\mathbf{s}_i}_{\text{inter-user interference}} + \underbrace{\mathbf{F}_k\mathbf{z}_k}_{\text{noise}}. \tag{1}$$

Christensen et al [16] showed that the use of linear receivers in MIMO BC is not suboptimal (full CSIR, as in SU MIMO): prefiltering $\mathbf{G}_k$ with a $N_t \times d_k$ unitary matrix makes the interference plus noise prewhitened channel matrix - precoder cascade of user k orthogonal (columns). For the user selection we will use the following notations: $h_k = \mathbf{H}_k^H$, k_i is the user selected at stage i, $H_i = h_{k_{1:i}}^H$, $P_{h_{k_{1:i}}}^{\perp}$ is the projector onto the orthogonal complement of the subspace spanned by $h_{k_{1:i}}$ and ϕ_i denote the angle between h_{k_i} and $h_{k_{1:i-1}}$.

3. MOTIVATION

Optimal MIMO BC design requires DPC, which is significantly more complicated than BF. User selection allows to improve the rates of DPC and bring the rates of BF close to those of DPC.Optimal user or stream selection requires selection of the optimal combination of N_t users or streams among K users or KN_k streams and is often overly complex. Greedy user or greedy stream selection (GUS or GSS), selecting one stream at a time, results in a complexity that is approximately N_t times the complexity of selecting a single stream ($K \gg N_t$). Multiple receive antennas cannot improve the sum rate prelog (spatial multiplexing gain) but they can be used to cancel interference from other transmitters (spatially colored noise), however, this is not in the scope of this paper.

At high SNR, using optimized (MMSE style) filters instead of ZF filters or using optimized power allocation instead of uniform power allocation only leads to $\frac{1}{\text{SNR}}$ terms in rates. At high SNR, the form of the sum rate is $N_t \log(\text{SNR}/N_t)$ plus the constant $\sum_i \log\det(\mathbf{F}_i\mathbf{H}_i\mathbf{G}_i)$ for properly normalized ZF Rx $\mathbf{F}_i$ and ZF Tx $\mathbf{G}_i$.

4. STATE OF THE ART IN GUS/GSS IN THE MIMO BC

In [13] the authors transform the MIMO channel into a MISO channel and carry an analysis as in [17], using pseudo-BF-style GUS (SUS) and analysis that can be used in DPC and BF. However, this only shows effect of antennas in higher-order terms. In [18] the focus is on single stream MIMO BC and the use of Rx antennas to minimize quantization error (for feedback) on resulting virtual channel particularly for partial CSIT (and CSIR) with (G)US. In [19] the authors obtain the high SNR SR offset between BF and DPC without user selection. They extend the analysis of [20] from MISO to MIMO. It is also done in [21]. In [22] SESAM is introduced: proper DPC-style GUS for MIMO case (extension of [23] from MISO to MIMO). In [24] the authors propose a BF-style GUS for MIMO-BC-BF. In the style of predecessors, they only adapt the Rx of the new stream to be added. They replace the proper geometric average of the stream channel powers by its harmonic average: $1/\text{tr}\{diag((HH^H)^{-1})\}$, which leads to a generalized eigenvector solution for the Rx filter (min-Frob algo). It can be simplified to a classical eigenvector problem: the LISA algorithm is equivalent to the SESAM algorithm. In [25] the same greedy approaches are proposed now for max WSR, without user selection. In [16] the authors prove that working per stream is equivalent to working per user.

5. ZF-BF AND ZF-DPC LOSSES

It was shown in [26], for channel vectors that are close to be mutually othogonal, that compared to DPC in the case of orthogonal channels the rate offset loss due to ZF-BF was twice that of ZF-DPC. We derive here this result differently both at the sum rate level and at the user level. Let $\mathbf{D} = \text{diag}\left(\mathbf{H}\mathbf{H}^H\right)$ and $\mathbf{D}^{-1/2}\mathbf{H}\mathbf{H}^H\mathbf{D}^{-1/2} = I + \mathbf{\Delta}$. Hence $\mathbf{H}\mathbf{H}^H = \mathbf{D}^{1/2}(I + \mathbf{\Delta})\mathbf{D}^{1/2}$. Note that $\text{diag}(\mathbf{\Delta}) = 0$. We shall express rate here in nats, so that $\log$ represents $\ln$ in fact. At high SNR, the channel dependent term in the sum rate (SR) constant (offset) for DPC is

$$SR_o^{DPC} = \log\det(\mathbf{H}\mathbf{H}^H) = \log\det(\mathbf{D}) + \underbrace{\log\det(I+\mathbf{\Delta})}_{\Delta SR_o^{DPC}}$$

where the first term in the last expression corresponds to the SR offset in the case of orthogonal channels and the second term represents the difference of the DPC SR offset compared to the orthogonal case. For BF we get

$$\begin{aligned} SR_o^{BF} &= -\log\det(\text{diag}((\mathbf{H}\mathbf{H}^H)^{-1})) \\ &= \log\det(\mathbf{D}) \underbrace{-\log\det(\mathbf{D}\,\text{diag}((\mathbf{H}\mathbf{H}^H)^{-1}))}_{\Delta SR_o^{BF}}. \end{aligned}$$

Now consider the case of small $||\mathbf{\Delta}||_F$. Using $\log\det(\mathbf{A} + \mathbf{\Delta}) = \log\det(\mathbf{A}) + \text{tr}\{\mathbf{A}^{-1}\mathbf{\Delta}\} - \frac{1}{2}\text{tr}\{(\mathbf{A}^{-1}\mathbf{\Delta})^2\} + \cdots$, we get up to second order

$$\begin{aligned} \Delta SR_o^{DPC} &= \log\det(I) + \text{tr}\{\mathbf{\Delta}\} - \frac{1}{2}\text{tr}\{(\mathbf{\Delta})^2\} \\ &= -\frac{1}{2}\text{tr}\{(\mathbf{\Delta})^2\} \end{aligned}$$

which is appropriately negative since it represents a SR loss. For the BF case we get

$$(\mathbf{H}\mathbf{H}^H)^{-1} = \mathbf{D}^{-1/2}(I+\mathbf{\Delta})^{-1}\mathbf{D}^{-1/2} \tag{2}$$

hence

$$\Delta SR_o^{BF} = -\log\det(\text{diag}((I+\mathbf{\Delta})^{-1}))\ . \quad (3)$$

Now,

$$\text{diag}((I+\mathbf{\Delta})^{-1}) = \text{diag}(I-\mathbf{\Delta}+\mathbf{\Delta}^2) = I+\text{diag}(\mathbf{\Delta}^2) \quad (4)$$

hence

$$\Delta SR_o^{BF} = -\log\det(I+\text{diag}(\mathbf{\Delta}^2)) = -\text{tr}\{\mathbf{\Delta}^2\}\,. \quad (5)$$

Yielding a BF sum rate offset loss that is twice the DPC sum rate offset loss:

$$\Delta SR_o^{BF} = 2\,\Delta SR_o^{DPC}\,. \quad (6)$$

We will now prove that this results also holds per user. Note that for the BF case, per user means that we analyze jointly the rate loss that the non-orthogonality between a given user and the others causes to the user itself, and to the other users, so it is more a contribution per user to the sum rate loss rather than a rate loss peruser that we consider. In BF, when considering the current user as the last of the users treated so far, we have

$$\begin{array}{l} (H_i H_i^H)^{-1} = \begin{bmatrix} \mathbf{C} & \mathbf{b} \\ \mathbf{b}^H & a \end{bmatrix}^{-1} \\ = \begin{bmatrix} (\mathbf{C}-\mathbf{b}a^{-1}\mathbf{b}^H)^{-1} & * \\ * & (a-\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b})^{-1} \end{bmatrix} \end{array} \quad (7)$$

where we use the tradiational notations of the matrix lemma inversion for sake of simplicity, and a corresponds to $\|h_{k_i}\|^2$, $a - \mathbf{b}^H\mathbf{C}^{-1}\mathbf{b}$ corresponds to $\|P^{\perp}_{h_{k_{1:i-1}}} h_{k_i}\|^2 = \|h_{k_i}\|^2 \sin^2\phi_i$, etc. Due to user selection, $H_i H_i^H$ will be very diagonally dominant and $\mathbf{C}$ will have small off-diagonal elements. For BF, we are interested in $-\log\det(\text{diag}((\mathbf{HH}^H)^{-1}))$ and the non-orthogonality effect on the user itself is

$$(a-\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b})^{-1} = a^{-1}(1-a^{-1}\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b})^{-1} \quad (8)$$

and hence for the rate effect

$$-\log((1-a^{-1}\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b})^{-1}) \approx -a^{-1}\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b}\ . \quad (9)$$

This is also the rate loss for the user in the DPC approach and it is not surprising since in both ZF-BF and ZF-DPC the user channel is orthogonalized w.r.t. to all the *previous* users hence the same loss due to the orthogonalization of the user itself in both cases. However in case of BF there is a second component to the rate offset loss because the previous users need to be orthogonalized w.r.t. the newly added user. For this second component,using the Matrix Inversion Lemma, we get

$$(\mathbf{C}-\mathbf{b}a^{-1}\mathbf{b}^H)^{-1} = \mathbf{C}^{-1} - \mathbf{C}^{-1}\mathbf{b}(\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b}-a)^{-1}\mathbf{b}^H\mathbf{C}^{-1}\ .$$

Hence we get up to second order in off-diagonal elements

$$\begin{array}{l} \text{diag}((\mathbf{C}-\mathbf{b}a^{-1}\mathbf{b}^H)^{-1}) \\ = \text{diag}(\mathbf{C}^{-1}) + \frac{1}{a}\text{diag}(\mathbf{C}^{-1})\text{diag}(\mathbf{bb}^H)\text{diag}(\mathbf{C}^{-1}) \\ = \text{diag}(\mathbf{C}^{-1})\,\text{diag}(I+\frac{1}{a}\mathbf{bb}^H\mathbf{C}^{-1}) \end{array} \quad (10)$$

Hence we get for the rate effect

$$\begin{array}{l} -\log\det(\text{diag}(I+\frac{1}{a}\mathbf{bb}^H\mathbf{C}^{-1})) = \\ -\log(1+\frac{1}{a}\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b}) = -\frac{1}{a}\mathbf{b}^H\mathbf{C}^{-1}\mathbf{b} \end{array} \quad (11)$$

which is the same as the rate effect in (9). Therefore the overall rate loss, due to a given user, in BF is twice that of DPC, for the case of near orthogonality.

6. NEW MIMO BF-STYLE GUS CRITERION

In [26] we proposed an approximation of the greedy algorithm from [27] that proved to be very accurate, our criterion for the GUS in the MISO BC is at stage i: $k_i = \arg\max_k \|P^{\perp}_{h_{k_{1:i-1}}} h_k\|^4/\|h_k\|^2$. There is a straightforward extension to the MIMO case in which, for GUS, only the Rx for each candidate user will be adapted to optimize the BF rate offset (although once k_i has been identified, it is useful while remaining at acceptable cost to reoptimize the Rx filters for the various streams by alternating the following Rx filter optimization over the various streams). In the MIMO case, we have the virtual channels $h_{k_i}^H = f_i\,H_{k_i}$ (receiver-channel cascade per stream) and in order to evaluate the expected contribution of a user to the sum rate we must optimize its receive filter:

$$\max_{f_i} \|P^{\perp}_{h_{k_{1:i-1}}} h_{k_i}\|^4/\|h_{k_i}\|^2 \quad \text{s.t. } f^H f = 1 \quad (12)$$

This problem can be seen as a generalized eigenvalue problem and can be solved iteratively via

$$f_i^H = V_{max}(H_{k_i} P^{\perp}_{h_{k_{1:i-1}}} H^H_{k_i}, H_{k_i} H^H_{k_i} + ||f_i\,H_{k_i}||^2 I) \quad (13)$$

where $V_{max}(A,B)$ is the generalized eigenvector of matrices A and B corresponding to the maximum eigenvalue.

Our algorithm performs the greedy user selection and the receive filter optimization, at stage *i* for each candidate user it optimizes the corresponding receive filter iteratively according to (13) and evaluates its approximate contribution (13). It adds the one yielding the largest contribution to the selection only if it increases the total sum rate, otherwise the algorithm stops. When a user is added all the receive filters j are reoptimized one or multiple times with the same approach:

$$f_j^H = V_{max}(H_{k_j} P^{\perp}_{h_{k_{1:i\setminus j}}} H^H_{k_j}, H_{k_j} H^H_{k_j} + ||f_j\,H_{k_j}||^2 I). \quad (14)$$

We initialize with $f_i = \frac{[1\cdots 1]}{\sqrt{N_r}}$ since we observed that different initializations yield almost the same results. Initializing with the minFrob of [24] increases the complexity but offers little improvement. An overview of the algorithm is given in **Table 1**.

Table 1. MIMO IT

Initialization:

- $G_i = []\ \forall i,\ H_{comp} = [],\ i = 2$
- $k_1 = \arg\max\limits_k \|\text{Vmax}(H_k)H_k\|^2$
- $H_{comp}^H = [\text{Vmax}(H_{k_1})H_{k_1}]$

while $i \leq N_t$: • Find the user with the largest approximated contribution to the sum rate:

- $k_i = \arg\max\limits_k \|P_{h_{k_{1:i-1}}}^{\perp} h_k\|^4/\|h_k\|^2$

where $h_k^H = f_k H_k$ and f_k is iteratively approached using (13) and $P_{h_{k_{1:i-1}}}^{\perp}$ is the projection on the orthogonal complement of $h_{k_{1:i-1}}$.

• Update if there is an actual sum rate increase:

if SR(H_{comp}^H) $<$ SR($[H_{comp}^H, f_{k_i} H_{k_i}]$) **then**

- $H_{comp}^H = [H_{comp}^H, f_{k_i} H_{k_i}]$
- $G_i = f_{k_i}$
- $i = i + 1$

Receive filter reoptimization cycles:

repeat N_{cycle} times

- $j = 1$

while $j \leq i$

- $H_{reopt} = H_{comp,\setminus j}$ (H_{comp} without its jth row)
- Reoptimize the receive filter f_{k_j} by iterating:
  - $f_{k_j}^H = V_{max}(H_{k_j} P_{h_{reopt}}^{\perp} H_{k_j}^H, H_{k_j} H_{k_j}^H + \|f_{k_j} H_{k_j}\|^2 I)$
- $H_{comp,j} = f_{k_j} H_{k_j}$ (Update of H_{comp}'s jth row)
- $G_j = f_{k_j}$
- $j = j + 1$

end while

end repeat

else

- break

end while

7. SIMULATION RESULTS

The simulation generates 5000 independent realizations of a Rayleigh fading channel for each user.

In Fig. 1 we compare the performances in terms of sum rate of our MIMO BF-style GUS criterion with those of the

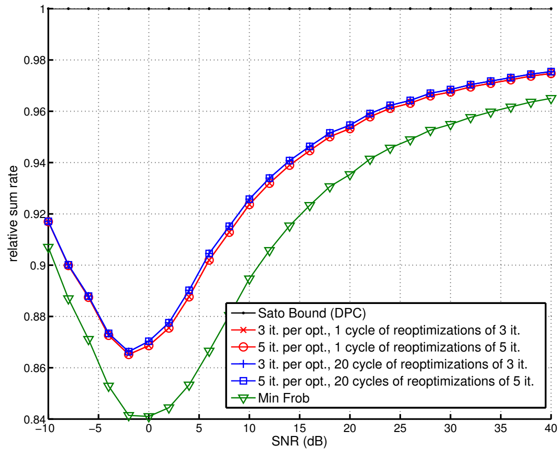

Fig. 1. MIMO ZF-BF-GUS: Min Frob versus our iterative algorithm normalized to the Sato bound for $N_t = 8$, $N_r = 4$ and $K = 30$.

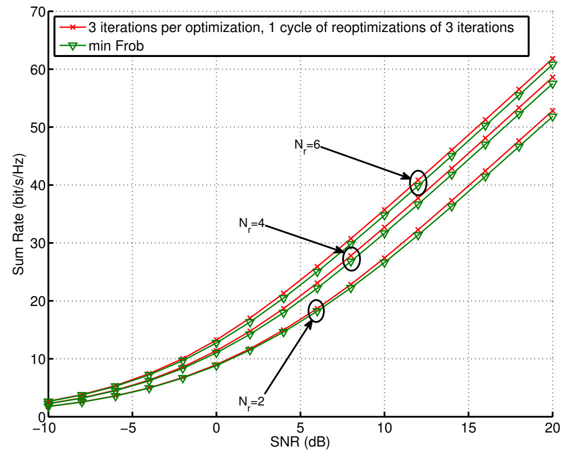

Fig. 2. MIMO ZF-BF-GUS: Min Frob versus our iterative algorithm for $N_t = 8$, $K = 30$ and $N_r \in \{2, 4, 6\}$.

Min Frob from [24]. For better differentiation, we plot the relative sum rates yielded by the algorithm normalized to the Sato bound, the sum rate that would be achieved by DPC, computed according to [3]. We observe some gain with our iterative algorithm and as we plot the curves for different number of iterations we notice that for $N_t = 8$, $N_r = 4$ and $K = 30$, 3 iterations per optimization are sufficient, and concerning the number of cycles of reoptimization going from 1 to 20 offers little improvement in regards to the increased complexity.

In Fig. 2 we plot the sum rates yielded by Min Frob and by our iterative algorithm for $N_t = 8$, $K = 30$ as well as for different values of N_r, $N_r \in \{2, 4, 6\}$. We observe that our algorithm performs better for the different values of N_r even with few iterations and only one reoptimization cycle.

8. CONCLUDING REMARKS

Starting from the MISO BF-style GUS criterion in [26], we developed a MIMO BF-style GUS criterion with iterative receive filter optimization. The algorithm we propose proves to have better performances than the Min Frob algorithm. Our algorithm is iterative but did not show any convergence problems during the simulations. Being iterative also calls for a compromise between complexity and performance. Empirically, we found the tradeoff to be very good as the performances converge quickly. In fact, few iterations per optimization of receive filter are needed and few cycles of reoptimization are enough to obtain a gain in the sum rates.

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