On Greedy Stream Selection in MIMO BC

Yohan Lejosne*†, Dirk Slock†, Yi Yuan-Wu*
*Orange Labs, RESA/WASA
38-40 Rue du General Leclerc 92794 Issy Moulineaux Cedex 9, FRANCE
Email: {yohan.lejosne, yi.yuan}@orange.com
†EURECOM
2229 route des Crêtes, BP 193 06904 Sophia Antipolis Cedex, FRANCE
Email: dirk.slock@eurecom.fr

***Abstract*—We consider the Broadcast Channel (BC), or in other words the multi-user (MU) downlink in a cell with a base station equipped with multiple antennas and mobile terminals equipped with a single antenna (MU-MISO case). It is well-known that user selection not only leads to multi-user diversity but also to decreased suboptimality of simple beamforming (BF) techniques compared to optimal Dirty Paper Coding (DPC) approaches. User selection by exhaustive search can be simplified to greedy approaches, in which one user gets added at a time. We review DPC-style and BF-style greedy user selection (GUS), which are developed with DPC or BF transmitters in mind. We introduce a new interpretation of the BF-style user selection criterion, which corresponds to a new approximate criterion that becomes more accurate for a large user pool. This new criterion allows interpretation and comparison of the respective roles of channel strength and orthogonality in DPC-style and BF-style selection.**

I. Introduction

The Multiuser MIMO Broadcast Channel (MU-MIMO BC) has been one of the most investigated subjects in the literature on wireless communications due to the high potential it offers in improving the system throughput. Information theory has shown that the capacity of MU-MIMO channels could be achieved through dirty-paper coding (DPC) [1]–[3]. However, DPC is difficult to implement and computationally complex. Some suboptimal linear beamforming algorithms exist and can be divided into two main families: the iterative [4]–[8] and the closed form (CF) solutions [9]–[13].

These solutions can also be differentiated according to the number of streams allocated per user. In fact, there are precoders that can not support more than one stream per user even if the system is not fully charged. Such precoders have been proposed and widely studied in [6], [7], [9]–[12].

Some multi-stream precoding solutions have nevertheless been proposed such as in [13], [14]. To the best of our knowledge, the best linear CF precoder present in the literature is the so called ZFDPC-SUS (zero forcing DPC with successive user selection) that has been proposed in [13], [15]. This precoding technique is based on the selection of semi-orthogonal users based on the SVD of their respective channels. Another interesting multi-stream technique is the one presented in [14] based on the Signal to Leakage plus Noise Ratio (SLNR) maximization. This technique offers some advantages as the channel knowledge can be relaxed to only covariance matrix information. On the other hand the solution proposed in [14] imposes prefixing the stream distribution.

EURECOMs research is partially supported by its industrial members: ORANGE, BMW Group, Swisscom, Cisco, SFR, ST Ericsson, Thales, Symantec, SAP, Monaco Telecom, and also by the EU FP7 projects CROWN, SACRA and WHERE2.

In reality, d_k streams can be allocated to user k respecting two main constraints: $d_k \leq \min(N_k, N_t)$ constraining the maximum number of streams per user (1 in the MISO case), and $d = \sum_{k=1}^{K} d_k \leq \min(\sum_{k=1}^{K} N_k, N_t)$ constraining the total number of streams allocated by the base station (BS). The allocation of these streams could be done such as to maximize the total sum-rate (SR). A second crucial point in SR maximization is finding the optimal power distribution over the selected streams.

II. System Model

We consider a MISO BC (Multi-User MISO downlink) with N_t transmit antennas, K users with $N_k = 1$ receiving antennas and assume perfect channel state information (CSI). The transmit power constraint is P, the white noise variance is $\sigma^2 = 1$ at all receivers. $\mathbf{H}_k$, $\mathbf{G}_k$, $\mathbf{F}_k$ denotes the MIMO channel, the transmitter (Tx) and receiver (Rx) filters for user k, respectively. The received signal is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{G}_i \mathbf{s}_i + \mathbf{z}_k \tag{1}$$

or

$$\underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{y}_k}_{N_k \times 1} = \underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{H}_k}_{N_k \times N_t} \sum_{i=1}^{K} \underbrace{\mathbf{G}_i}_{N_t \times d_i} \underbrace{\mathbf{s}_i}_{d_i \times 1} + \underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{z}_k}_{N_k \times 1} .$$

Then

$$\mathbf{F}_k \mathbf{y}_k = \underbrace{\mathbf{F}_k \mathbf{H}_k \mathbf{G}_k \mathbf{s}_k}_{\text{useful signal}} + \underbrace{\sum_{i=1, i \neq k}^{K} \mathbf{F}_k \mathbf{H}_k \mathbf{G}_i \mathbf{s}_i}_{\text{inter-user interference}} + \underbrace{\mathbf{F}_k \mathbf{z}_k}_{\text{noise}} .$$

Christensen et al [16] showed that the use of linear receivers in MIMO BC is not suboptimal (full CSIR, as in SU MIMO): prefiltering $\mathbf{G}_k$ with a $N_t \times d_k$ unitary matrix makes the interference plus noise prewhitened channel matrix - precoder cascade of user k orthogonal (columns).

III. MOTIVATION

Optimal MIMO BC design requires DPC, which is significantly more complicated than BF. User selection allows

- improvement of the rates of DPC,
- the rates of BF to be closer to those of DPC.

Optimal user or stream selection requires selection of the optimal combination of N_t streams among K users or KN_k and is often overly complex. Greedy user or greedy stream selection (GUS or GSS), selecting one stream at a time, allows a complexity that is approximately N_t times the complexity of selecting one stream ($K \gg N_t$).

Now consider ZF designs for BF and DPC.
ZF-BF:

$$\begin{aligned}\mathbf{F}_{1:i}\mathbf{H}_{1:i}\mathbf{G}_{1:i} &= \begin{bmatrix}\mathbf{F}_1 \, 0 \cdots 0 \\ 0 \, \mathbf{F}_2 \ddots \vdots \\ \vdots \quad \ddots 0 \\ 0 \cdots 0 \, \mathbf{F}_i\end{bmatrix}\begin{bmatrix}\mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_i\end{bmatrix}[\mathbf{G}_1 \; \mathbf{G}_2 \cdots \mathbf{G}_i] \\ &= \begin{bmatrix}\mathbf{F}_1\mathbf{H}_1\mathbf{G}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{F}_2\mathbf{H}_2\mathbf{G}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{F}_i\mathbf{H}_i\mathbf{G}_i\end{bmatrix}\end{aligned}$$

ZF-DPC (modulo reordering issues):

$$\begin{aligned}\mathbf{F}_{1:i}\mathbf{H}_{1:i}\mathbf{G}_{1:i} &= \begin{bmatrix}\mathbf{F}_1 \, 0 \cdots 0 \\ 0 \, \mathbf{F}_2 \ddots \vdots \\ \vdots \quad \ddots 0 \\ 0 \cdots 0 \, \mathbf{F}_i\end{bmatrix}\begin{bmatrix}\mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_i\end{bmatrix}[\mathbf{G}_1 \; \mathbf{G}_2 \cdots \mathbf{G}_i] \\ &= \begin{bmatrix}\mathbf{F}_1\mathbf{H}_1\mathbf{G}_1 & 0 & \cdots & 0 \\ * & \mathbf{F}_2\mathbf{H}_2\mathbf{G}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ * & \cdots & * & \mathbf{F}_i\mathbf{H}_i\mathbf{G}_i\end{bmatrix}\end{aligned}$$

where $*$ denotes an arbitrary non-zero entry. BF-style selection assumes that the selected streams are going to be used in BF, and likewise for DPC-style selection.

A. Stream Selection Criterion from Sum Rate

At high SNR, both optimized (MMSE style) filters vs. ZF filters and optimized vs. uniform power allocation only lead to $\frac{1}{\text{SNR}}$ terms in rates. At high SNR, the sum rate is of the form $N_t \log(\text{SNR}/N_t)$ plus the constant

$$\sum_i \log\det(\mathbf{F}_i\mathbf{H}_i\mathbf{G}_i)$$

for properly normalized ZF Rx $\mathbf{F}_i$ and ZF Tx $\mathbf{G}_i$ (BF or DPC).

B. MIMO BC Greedy Stream Selection (GSS) Criteria

All schemes considered are in a first instance motivated by ZF considerations. For all schemes, we can focus on either

(i) (the constant term in) the sum rate at high SNR,
(ii) the sum rate at any SNR of the associated ZF transceiver designs with uniform power loading,
(iii) the sum rate at any SNR of the associated ZF transceiver designs with waterfilling,
(iv) the sum rate at any SNR of optimized transceiver designs.

At high SNR, (i) is the analysis of interest. More variations could be considered, e.g. regularized ZF as an intermediate between ZF and optimized transceiver designs.

Another dimension is that, regardless whether the selection mechanism is DPC-style or BF-style, the transceiver design could be

(a) DPC with the ordering provided by the selection procedure,
(b) DPC with the optimized ordering,
(c) BF.

For DPC, other orderings could also be considered. Since BF-style selection tends to be more complex than DPC-style selection (2-sided vs.1-sided orthogonalization), in practice it makes sense to consider only DPC-style selection for DPC transceiver design, and either DPC- or BF-style selection for BF transceiver design.

IV. STATE OF THE ART IN GUS IN THE MISO BC

In [17], the Gram-Schmidt channel orthogonalization with pivoting (DPC-style GUS) was introduced. In [18], the authors presented a proper BF-style GUS, a large K analysis for DPC-style GUS, simulations and they used the matrix inversion lemma for bordered matrices, in order to lower the complexity of BF-style GUS. In [15], the BF is analysed, but with pseudo-BF-style GUS: SUS (semi-orthogonal) i.e., DPC-style GUS with inner product constraints limiting the size of pool of users for selection. The authors show that for BF-SUS, as for DPC-SUS,

$$\lim_{K\to\infty} \frac{SR}{N_t \log(1+\frac{P}{N_t}\log K)} = 1$$

and in [19] a refinement of this with more constraints is done. A simplified at finite SNR, but otherwise exact, sum rate expression for MISO BF (regularized ZF style) can be found in [20]. They also propose a suboptimal user selection with complexity of order K^2 and an interesting power loading algorithm, equating the correct SR gradient with that of an equivalent virtual parallel channel and performing WF on the virtual parallel channel.

V. MISO DPC-STYLE GUS

In the MISO case, let $h_k = \mathbf{H}_k^H$, k_i = user selected at stage i, $H_i = h_{k_{1:i}}^H$, $S_i = \{k_1 \cdots k_i\}$ and $A(S_i) = H_i H_i^H$. $h_{k_{1:i-1}\backslash k_j}$ denotes $[h_{k_1} \cdots h_{k_{j-1}} \; h_{k_{j+1}} \cdots h_{k_{i-1}}]$ and $P^{\perp}_{h_{k_{1:i}}}$ is the projector onto the orthogonal complement of the subspace spanned by $h_{k_{1:i}}$. Then $\det(H_i H_i^H) = \prod_{j=1}^{i} \|P^{\perp}_{h_{k_{1:j-1}}} h_{k_j}\|^2$. and at stage i: $k_i = \arg\max_k \|P^{\perp}_{h_{k_{1:i-1}}} h_k\|^2$. However in the case of beamforming, the quantity of interest is

$$(\det(\text{diag}\{(H_i H_i^H)^{-1}\}))^{-1} = \prod_{j=1}^{i} \frac{1}{(A(i)^{-1})_{(j,j)}}$$

For $j < i$ in order to compute $A(S_i)^{-1}_{j,j}$ easily, we perform some operations on H_i, we move its jth row to the last position, meaning that we consider $\tilde{H}_i = h^H_{k_{1:j-1,j+1:i,j}}$. We note that for a given subset of i users, the order of the channel vectors does not matter when doing ZF-BF, thus defining $\tilde{A}(S_i) = \tilde{H}_i \tilde{H}_i^H$ yields $A(S_i)^{-1}_{j,j} = \tilde{A}(S_i)^{-1}_{i,i}$. We can decompose $\tilde{A}(S_i)$

$$\tilde{A}(S_i) = \begin{bmatrix} A(S_{i-1} \backslash j) & B \\ B^H & A_r \end{bmatrix}$$

where $A(S_{i-1} \backslash j) = h^H_{k_{1:i-1 \backslash j}} h_{k_{1:i-1 \backslash j}}$, $A_r = h^H_{k_{i,j}} h_{k_{i,j}}$ and $B = h^H_{k_{1:i-1 \backslash j}} h_{k_{i,j}}$. The matrix inversion lemma yields

$$\tilde{A}(S_i)^{-1} = \begin{bmatrix} X & Y \\ Z & U \end{bmatrix}$$

where $U = (A_r - B^H A(S_{i-1} \backslash j)^{-1} B)^{-1}$ hence

$$\begin{aligned} U^{-1} = & \; h^H_{k_{i,j}} h_{k_{i,j}} \\ & -(h^H_{k_{1:i-1 \backslash j}} h_{k_{i,j}})^H (h^H_{k_{1:i-1 \backslash j}} h_{k_{1:i-1 \backslash j}})^{-1} h^H_{k_{1:i-1 \backslash j}} h_{k_{i,j}} \\ = & \begin{bmatrix} h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i} & h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j} \\ h^H_{k_j} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i} & h^H_{k_j} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j} \end{bmatrix} \end{aligned}$$

Finally the 2×2 matrix inversion applied to U^{-1} yields

$$\begin{aligned} \frac{1}{U_{2,2}} &= \frac{\det(U^{-1})}{h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}} \\ &= \frac{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2 \|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2} \\ & \quad - \frac{|h^H_{k_j} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2} \\ &= \|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2 - \frac{|h^H_{k_j} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2} \end{aligned} \quad (2)$$

This gives the value of $\frac{1}{(A(S_i)^{-1})_{(j,j)}}$ for $j < i$. For $j = i$ we apply the matrix inversion lemma to $A(S_i)$ with a different decomposition

$$A(S_i) = \begin{bmatrix} A(S_{i-1}) & h^H_{k_{1:i-1}} h_{k_i} \\ h^H_{k_i} h_{k_{1:i-1}} & h^H_{k_i} h_{k_i} \end{bmatrix}$$

yielding

$$\begin{aligned} \frac{1}{A(S_i)^{-1}_{i,i}} &= h^H_{k_i} h_{k_i} - h^H_{k_i} h_{k_{1:i-1}} A(S_{i-1})^{-1} h^H_{k_{1:i-1}} h_{k_i} \\ &= h^H_{k_i} (I - h_{k_{1:i-1}} A(S_{i-1})^{-1} h^H_{k_{1:i-1}}) h_{k_i} \\ &= \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 \end{aligned} \quad (3)$$

Combining (2) and (3) leads to $\det(\text{diag}\{(H_i H_i^H)^{-1}\}) =$

$$\|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 \prod_{j=1}^{i-1} (\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2 - \frac{|h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2}).$$

A similar reasoning, moving the jth row of H_i to the last position and applying the matrix inversion lemma with A_r reduced to only one term, yields

$$\det(\text{diag}\{(H_{i-1} H^H_{i-1})^{-1}\}) = \prod_{j=1}^{i-1} \|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2.$$

These formulations allows us to have the gain due to the selection of user i decomposed into the DPC gain and the loss due to the BF:

$$\begin{aligned} & \frac{\det(\text{diag}\{(H_i H_i^H)^{-1}\})^{-1}}{\det(\text{diag}\{(H_{i-1} H^H_{i-1})^{-1}\})^{-1}} \\ &= \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 \prod_{j=1}^{i-1} (1 - \frac{|h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2 \|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2}) \\ &= \underbrace{\|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2}_{\text{DPC gain}} \underbrace{\prod_{j=1}^{i-1} \sin^2 \phi_{ij}}_{\text{further BF loss}} \end{aligned}$$

where ϕ_{ij} is the angle between $P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}$ and $P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}$.

VI. New MISO BF-style GUS Criterion

Let ϕ_i be the angle between h_{k_i} and $h_{k_{1:i-1}}$, then we can write $\|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 = \|h_{k_i}\|^2 \sin^2 \phi_i$. For a sufficiently large K, the BF-style user selection process will lead to the selection of channel vectors that are close to being mutually orthogonal. We can then write up to first order

$$\frac{|h^H_{k_i} P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}|^2}{\|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_i}\|^2 \|P^\perp_{h_{k_{1:i-1 \backslash k_j}}} h_{k_j}\|^2} \approx \frac{|h^H_{k_i} h_{k_j}|^2}{\|h_{k_i}\|^2 \|h_{k_j}\|^2}$$

and also

$$\begin{aligned} \prod_{j=1}^{i-1} \sin^2 \phi_{ij} &= \prod_{j=1}^{i-1} (1 - \cos^2 \phi_{ij}) \\ &\approx 1 - \sum_{j=1}^{i-1} \cos^2 \phi_{ij} \\ &\approx 1 - \sum_{j=1}^{i-1} \frac{|h^H_{k_i} h_{k_j}|^2}{\|h_{k_i}\|^2 \|h_{k_j}\|^2} \\ &\approx 1 - \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 / \|h_{k_i}\|^2 \\ &= \sin^2 \phi_i \end{aligned}$$

As a result the contribution of stream i to the sum rate offset can be approximated by

$$\begin{aligned} & \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 \prod_{j=1}^{i-1} \sin^2 \phi_{ij} \\ & \approx \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 \sin^2 \phi_i \\ & = \|h_{k_i}\|^2 \sin^4 \phi_i = \|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^4 / \|h_{k_i}\|^2 . \end{aligned} \quad (4)$$

The DPC offset is $\|P^\perp_{h_{k_{1:i-1}}} h_{k_i}\|^2 = \|h_{k_i}\|^2 \sin^2 \phi_i$ which represents a certain compromise between max $\|h_{k_i}\|^2$ and min $\cos^2 \phi_i$. In the case of BF, $\|h_{k_i}\|^2 \sin^4 \phi_i$ leads to a similar compromise, but with more emphasis on orthogonality.

The BF rate offset expression (4) is not exact when evaluated for arbitrary candidate channels h_k. However, its optimization over sufficiently many candidates K should lead to fairly orthogonal choices, in which case (4) becomes an arbitrarily good approximation of the BF rate offset.

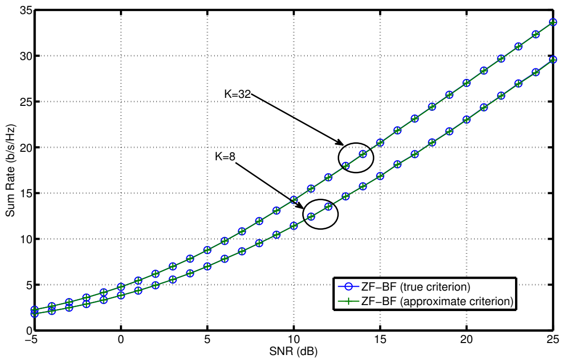

Fig. 1. MISO ZF-BF-GUS: true versus approximate criterion for $N_t = 4$, $K \in \{8, 32\}$.

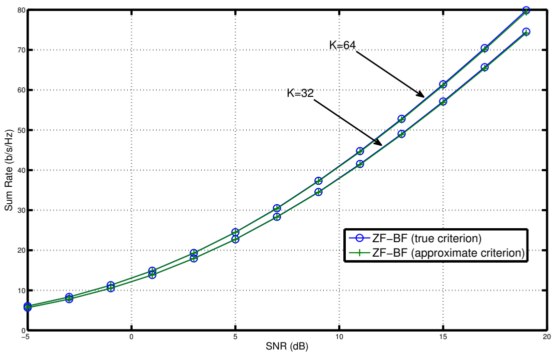

Fig. 2. MISO ZF-BF-GUS: true versus approximate criterion for $N_t = 16$, $K \in \{32, 64\}$.

This analysis also shows that when the channels vectors are close to being mutually orthogonal, such as resulting from user selection, then for a given user selection, the rate offset loss of BF compared to DPC is equal to the rate offset loss of DPC itself compared to DPC for the case of orthogonal channels (orthogonal hypothesis), since $\log \sin^4 \phi_i = 2 \log \sin^2 \phi_i$.

We can use (4) to select the user with the largest contribution to the rate, which according to the approximation is at stage i:

$$k_i = \arg\max_k \|P^{\perp}_{h_{k_{1:i-1}}} h_k\|^4 / \|h_k\|^2 \quad (5)$$

As opposed to ZF-DPC, the optimal stream subset for ZF-BF may be of cardinality less than N_t, therefore we add a stream, chosen according to (5), to the subset of previously selected streams only if this does not decrease the sum rate, otherwise the selection process is stopped.

VII. Complexity

In [18] a thorough complexity analysis of the ZF-BF GUS true criterion and of the ZF-DPC GUS is done. The evaluation of the complexity of the ZF-BF GUS with the approximate criterion can easily be deduced from their analysis.

Our algorithm will perform a maximum of N_t rate evaluations in order to determine whether to stop or continue selecting users. The evaluation of the rate is proven to be $O(N_t^2)$ in [18]. Finding the arg max in (5) requires K vector-matrix multiplication, for which complexity is $O(N_t^2)$. This is to be done at each stage, therefore the complexity of our algorithm is $O(N_t^3) + O(N_t^3 K) = O(N_t^3 K)$, which is the same complexity as the original criterion found in [18].

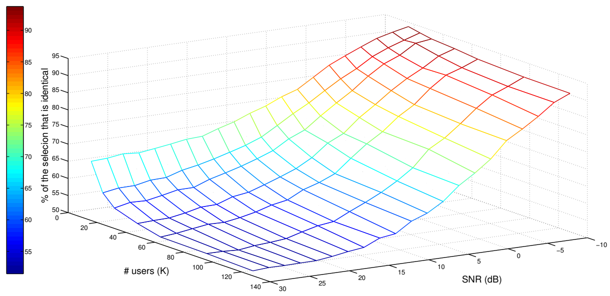

Fig. 3. Similarity between the user subsets selected by the true and by the approximate criterion

VIII. Simulation Results

The comparison of throughputs of the ZF-BF algorithm with true and approximate criterion for the MISO GUS is presented in Fig. 1 and Fig. 2. The simulation generates 10000 independent channel realizations for each user. Once the user subset is selected the ZF-BF and the ZF-DPC are performed as described in [2], with waterfilling for the power loading. We observe that the use of the approximate criterion yields almost the same performances as the true criterion.

At high SNR and with a large pool of users the algorithm can select users that are close to being mutually orthogonal, therefore the approximation is accurate. However due to the large number of possible choices the user subset selected can differ from what would be selected by the exact criterion. For example, in Fig. 3 we can see that the similarity between the selections decreases when K or the SNR increases, but since the approximation is accurate these different user subsets yield similar performances. When the user pool is small or at low SNR, the selection is smaller, therefore even though the approximation is less accurate we can see in Fig. 3 that the same users are selected by both criteria resulting in more similar performances for a small K or at low SNR.

Fig. 4 illustrates the rate offset loss between ZF-BF compared to ZF-DPC and the rate offset loss between ZF-DPC compared to DPC with orthogonal hypothesis, when the channel vectors are close to orthogonal. For that purpose we find a user subset with the approximate criterion of the ZF-BF selection algorithm. Given this specific user subset we compute the sum rate achieved with the two different algorithms (ZF-BF and ZF-DPC) and the sum rate that DPC would yield if the channel vectors were orthogonal. We observe the expected equality: for large Ks, the rate offset loss between ZF-BF and ZF-DPC approaches the rate offset loss between ZF-DPC and DPC with orthogonal hypothesis.

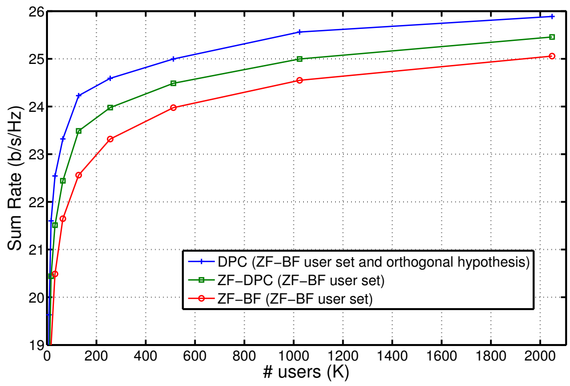

Fig. 4. Offset between DPC (orthogonal hypothesis), ZF-DPC, ZF-BF for a given user subset selected for ZF-BF by the approximate criterion in the MISO case with Nt=4, SNR=15 dB.

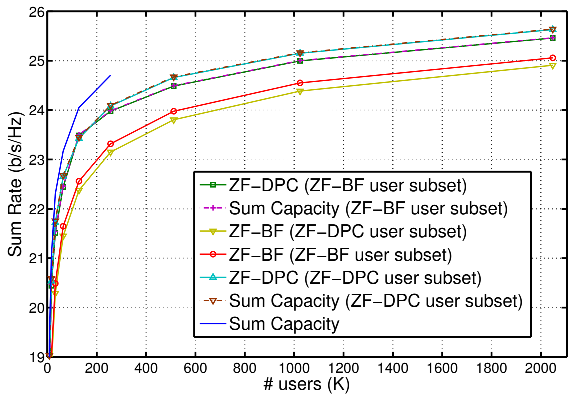

Fig. 5. Offset between Sum Capacity, ZF-DPC, and ZF-BF for user subset selected for ZF-BF by the approximate criterion and ZF-DPC and ZF-BF for a user subset selected for ZF-DPC and the Sum Capacity for all users in the MISO case with Nt=4, SNR=15 dB.

In Fig. 5 we observe that for the user subset selected either for ZF-BF or for ZF-DPC, ZF-DPC almost reaches the sum capacity. This also illustrates the loss one could expect from not matching the selection process and the ZF algorithm, namely performing ZF-BF with a user subset selected with ZF-DPC GUS or performing ZF-DPC with a user subset selected with ZF-BF GUS.

IX. Concluding Remarks

We introduced a new interpretation of the ZF-BF GUS in the MISO BC described in [18] and an approximate version of the selection criterion. For a sufficiently large K, this user selection process leads to a selection of channel vectors that are close to being mutually orthogonal and the contribution of each stream to be added can be approximated using (4). Numerical simulations confirmed that this approximation was accurate enough to result in either the same user selection as the original criteria (small values of K or low SNR) or in the selection of streams that yield a similar sum rate (large values of K and high SNR).

This also shows that for a given, almost mutually orthogonal, user subset, when compared to DPC with orthogonal hypothesis, the rate offset loss induced by the ZF-BF is twice the rate offset loss induced by ZF-DPC.

References

[1] M. Costa, "Writing on dirty paper (corresp.)," *Information Theory, IEEE Transactions on*, vol. 29, no. 3, pp. 439–441, May 1983.

[2] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna gaussian broadcast channel," *Information Theory, IEEE Transactions on*, vol. 49, no. 7, pp. 1691–1706, July 2003.

[3] Nihar Jindal, Wonjong Rhee, Syed Jafar, and Goldsmith, "Sum power iterative water-filling for multi-antenna gaussian broadcast channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 1570–1580, 2005.

[4] Hongmei Wang, Xibin Xu, Ming Zhao, Weiling Wu, and Yan Yao, "Robust transmission for multiuser MIMO downlink systems with imperfect csit," in *Proc. WCNC*, 2008, pp. 340–344.

[5] Jinfan Zhang, Yongle Wu, Shidong Zhou, and Jing Wang, "Joint linear transmitter and receiver design for the downlink of multiuser MIMO systems," *Communications Letters, IEEE*, vol. 9, no. 11, pp. 991–993, Nov. 2005.

[6] M. Amara, Y. Yuan-Wu, and D. Slock, "Receiver and transmitter iterative optimization using maximum sum-rate criterion for multi-user MIMO systems," in *ISCCSP 2010. IEEE International Symposium on Communications, Control and Signal Processing*, March 2010.

[7] M. Amara, Y. Yuan-Wu, and D. Slock, "Optimal MU-MIMO precoder with MISO decomposition approach," in *Proc. SPAWC*, June 2010.

[8] M.T. Ivrlac, R.L.U. Choi, R.D. Murch, and J.A. Nossek, "Effective use of long-term transmit channel state information in multi-user MIMO communication systems," in *VTC Fall*, Oct. 2003, vol. 1, pp. 373–377 Vol.1.

[9] Min Lee and Seong Keun Oh, "A per-user successive mmse precoding technique in multiuser MIMO systems," in *Proc. VTC Spring*, April 2007, pp. 2374–2378.

[10] Q.H. Spencer, A.L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *Signal Processing, IEEE Transactions on*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[11] M. Amara, Y. Yuan-Wu, and D. Slock, "Optimized linear receivers and power allocation for two multi-user MIMO downlink schemes with linear precoding," in *Proc. ISCCSP*, March 2010.

[12] M. Stojnic, H. Vikalo, and B. Hassibi, "Rate maximization in multi-antenna broadcast channels with linear preprocessing," *Wireless Communications, IEEE Transactions on*, vol. 5, no. 9, pp. 2338–2342, September 2006.

[13] L Sun and M Mckay, "Eigen-based transceivers for the MIMO broadcast channel with semi-orthogonal user selection," *Signal Processing, IEEE Transactions on*, vol. PP, no. 99, pp. 1 –1, 2010.

[14] M. Sadek, A. Tarighat, and A.H. Sayed, "A leakage-based precoding scheme for downlink multi-user mimo channels," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 5, pp. 1711 –1721, may. 2007.

[15] Taesang Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *Selected Areas in Communications, IEEE Journal on*, vol. 24, no. 3, pp. 528 – 541, mar. 2006.

[16] S.S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. on Wireless Communications*, vol. 7, no. 12, pp. 4792–4799, December 2008.

[17] Z. Tu and R.S. Blum, "Multiuser Diversity for a Dirty Paper Approach," *IEEE Comm. Letters*, Aug. 2003.

[18] G. Dimic and N.D. Sidiropoulos, "On Downlink Beamforming with Greedy User Selection: Performance Analysis and a Simple New Algorithm," *IEEE Trans. Sig. Proc.*, Oct. 2005.

[19] J. Wang, D.J. Love, and M.D. Zoltowski, "User Selection with Zero-Forcig Beamforming Achieves the Asymptotically Optimal Sum Rate," *IEEE Trans. Sig. Proc.*, Aug. 2008.

[20] D.A. Schmidt, M. Joham, R. Hunger, and W. Utschick, "Near Maximum Sum-Rate Non-Zero-Forcing Linear Precoding with Successive User Selection," in *in Proc. Asilomar*, 2006, pp. 2092 –2096.