Interference Alignment for Achieving both Full DoF and Full Diversity in the Broadcast Channel with Delayed CSIT

Jinyuan Chen, Raymond Knopp and Petros Elia
Mobile Communications Department
EURECOM, Sophia Antipolis, France
{chenji, knopp, elia}@eurecom.fr

Abstract—Maddah-Ali and Tse have recently shown that delayed transmitter channel state information (CSIT) can still be useful in increasing the degrees-of-freedom (DoF) over the MIMO broadcast channel. This was achieved by constructing a scheme that, in the presence of two transmit antennas, of two single-antenna receivers, and of CSIT that is delayed by one coherence time, manages to provide each user with 2/3 DoF, improving upon the 1/2 DoF corresponding to no CSIT. This same scheme though, as well as all subsequent schemes pertinent schemes, achieve DoF gains by suppressing the inherent diversity of the broadcast parallel channel. The current work proposes a novel broadcast scheme which, over the above described setting of the delayed CSIT broadcast channel, employs a form of interference alignment to achieve both full DoF as well as full diversity.

I. INTRODUCTION

Many multiuser wireless communications settings are known to benefit greatly from the use of CSIT feedback. It is the case though that such feedback is often hard to obtain sufficiently fast, and as a result, efforts have been made to find ways to utilize delayed CSIT. One such case involves communication over the broadcast channel (BC), where recent advances by Maddah-Ali and Tse [1] have shown that stale CSIT can still allow for improvements in the channel's degrees-of-freedom (DoF) region. This same work in [1] developed a novel scheme that, in the specific setting of the multiple-input single-output (MISO) BC with 2 transmit antennas and 2 users each with a single receive antenna, can offer 2/3 DoF to each user, even when the CSIT is delayed by a coherence period.

While achieving the optimal DoF, this novel scheme, as well as subsequent pertinent techniques ([2]–[7]), neglect diversity considerations, thus resulting in substantial suboptimality with respect to diversity. This suboptimality naturally contributes to substantial performance degradation, and thus brings to the fore the need for novel designs that can combine the signal manipulations that allow for full diversity, with the signal manipulations that utilize stale CSIT to give full DoF. We here propose a novel design which employs interference alignment [8] techniques to provide, in the aforementioned two-user MISO BC with stale CSIT, full DoF and full diversity.

A. Notation

Throughout this paper, $(\bullet)^{-1}$, $(\bullet)^{\top}$, $(\bullet)^{\dagger}$, $\mathrm{tr}(\bullet)$, $\det(\bullet)$ and $\mathrm{rank}(\bullet)$ denote the inverse, transpose, conjugate transpose,

trace, determinant and rank of matrix respectively, and $|| \bullet ||$ denotes the Euclidean norm. $| \bullet |$ denotes either the magnitude of a scalar or the cardinality of a set. Logarithms are of base 2. $(\bullet)^{+}$ denotes $\max(0, \bullet)$. $\mathrm{diag}(\bullet)$ denotes a diagonal matrix. We use $\doteq$ to denote exponential equality, i.e., $f(\rho) \doteq \rho^d$ denotes $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = d$ and $\dot{\leq}, \dot{\geq}, \dot{<}, \dot{>}$ are similarly defined.

B. Outline

After describing the system model and briefly recalling the original MAT scheme, Section II describes the new DoF-optimal design, and Section III shows that the scheme achieves full diversity. As a side result, the interested reader can find in Appendix VI an upper bound on the diversity-multiplexing tradeoff (DMT) of the MAT scheme.

C. System model

As stated, we focus on the frequency-flat MISO BC with two transmit antennas and two users with a single receive antenna. We consider a coherence period of T channel uses, during which the channel remains the same. Specifically we consider communication over L coherence periods, or equivalently L phases, where each phase coincides with a coherence period. During phase l ($l = 1, \cdots, L$), the first user's channel is denoted as $\boldsymbol{h}_l = [h_{l,1} \ h_{l,2}]^{\top} \in \mathbb{C}^2$, and the second user's as $\boldsymbol{g}_l = [g_{l,1} \ g_{l,2}]^{\top} \in \mathbb{C}^2$. For $\boldsymbol{x}_{l,t}$ denoting the transmitted signals during timeslot t of phase l, then the corresponding received signals at the first and second user respectively, take the form

$$y_{l,t}^{(1)} = \boldsymbol{h}_l^{\top} \boldsymbol{x}_{l,t} + z_{l,t}^{(1)}, \tag{1}$$

$$y_{l,t}^{(2)} = \boldsymbol{g}_l^{\top} \boldsymbol{x}_{l,t} + z_{l,t}^{(2)}, \tag{2}$$

($t = 1, 2, \cdots, T$), where $z_{l,t}^{(1)}, z_{l,t}^{(2)}$ denotes the AWGN noise. The fading coefficients are assumed to be i.i.d. circularly symmetric complex Gaussian $\mathbb{CN}(0, 1)$ distributed, and are assumed to remain fixed during a phase and change independently from phase to phase. We let ρ denote the signal-to-noise ration (SNR), and we consider a short-term power constraint where $\mathbb{E}||\boldsymbol{x}^{\ell}(t)||^2 \leq \rho$. We also consider a communication rate of R bits per channel use, which is here taken to be the same for both users. We recall that the corresponding DoF takes

the form $r = \lim_{\rho \to \infty} \frac{R_i}{\log \rho}$, where r is also referred to as the multiplexing gain. Finally, for P_e denoting the probability that at least one user has decoded erroneously, we recall the notion of diversity to be $d = -\lim_{\rho \to \infty} \frac{\log P_e}{\log \rho}$ (cf. [9]).

Regarding knowledge of the channel state, we assume perfect channel state information at the receivers (perfect CSIR), but we only allow the transmitter perfect knowledge of CSIT with a delay of a single phase (single coherence time), and provide no knowledge of current CSIT.

We consider the minimum delay case where communication, just as in [1], takes place over $L = 3$ phases, and we note that naturally the achieved optimal DoF performance cannot further benefit from using $L > 3$ phases. The proposed design is presented here for simplicity only for the minimum phase-delay case ($L = 3$), for which it achieves the optimal diversity of 6 (3 coherence intervals, 2 transmit antennas). We note though that this design can be readily extended to the case where L is a multiple of 3, to again achieve the optimal diversity order of $2L$. We will henceforth consider that $L = 3$.

D. Original MAT scheme

The MAT scheme [1] applies irrespective of the coherence duration T, and it considers communication over $L = 3$ phases. Without consideration for the time index, in describing the scheme, we denote by $\{a_1, a_2\}$ the two symbols intended for the first user, and by $\{b_1, b_2\}$ the symbols for the second user. During the first, second and third phase, the transmitter sequentially sends $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3 \in \mathbb{C}^2$ where

$$\boldsymbol{x}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \ \boldsymbol{x}_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \ \boldsymbol{x}_3 = \begin{bmatrix} \boldsymbol{h}_2^\top \boldsymbol{x}_2 + \boldsymbol{g}_1^\top \boldsymbol{x}_1 \\ 0 \end{bmatrix}. \tag{3}$$

Consequently the resulting input-output relationship seen by the first user, takes the form

$$\mathbf{y}^{(1)} = \begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_3^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1^\top \\ \boldsymbol{0} \\ h_{3,1}\boldsymbol{g}_1^\top \end{bmatrix} \boldsymbol{x}_1 + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{h}_2^\top \\ h_{3,1}\boldsymbol{h}_2^\top \end{bmatrix} \boldsymbol{x}_2 + \begin{bmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{bmatrix}$$

which is converted to the equivalent form

$$\tilde{\mathbf{y}}^{(1)} \triangleq \begin{bmatrix} y_1^{(1)} \\ y_3^{(1)} - h_{3,1}y_2^{(1)} \end{bmatrix} = \tilde{\boldsymbol{H}}\boldsymbol{x}_1 + \tilde{\boldsymbol{z}}^{(1)}, \tag{4}$$

where

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{3,1}g_{1,1} & h_{3,1}g_{1,2} \end{bmatrix}, \ \tilde{\boldsymbol{z}}^{(1)} = \begin{bmatrix} z_1^{(1)} \\ z_3^{(1)} - h_{3,1}z_2^{(1)} \end{bmatrix}. \tag{5}$$

Noting that $\tilde{\boldsymbol{H}}$ is almost surely of full rank, allows us to conclude that user 1 can achieve 2/3 DoF. Due to symmetry, the same holds for the second user.

Regarding the diversity of the MAT scheme, we have the following.

Proposition 1: The MAT scheme gives diversity that is upper bounded by 3.

This is shown in Appendix VI.

II. INTERFERENCE ALIGNMENT FOR ACHIEVING BOTH FULL DoF AND FULL DIVERSITY

We briefly note that the maximum achievable diversity that we can hope for is 6, simply because the transmitter has 2 antennas and because communication takes place over $L = 3$ statistically independent channel realizations ($L = 3$ phases).

The scheme applies to the case where the coherence period T is no less than 8. Without loss of generality, we will assume that $T = 8$, and that the entire coding duration is $LT = 24$ channel uses, spanning three different phases of 8 channel uses each. For $l = 1, \cdots, L$, $t = 1, \cdots, \frac{T}{2} = 4$, we denote by $\boldsymbol{x}_{l,2t-1}$ the vectors transmitted during odd timeslots of phase l (i.e., during timeslots $1, 3, 5, 7$ of phase l), and we denote by $\boldsymbol{x}_{l,2t}$ ($l = 1, \cdots, L$, $t = 1, \cdots, 4$) the vectors transmitted during even timeslots of phase l.

We assign 16 information symbols $\{a_{1,t}, a_{2,t}, a_{3,t}, a_{4,t}\}_{t=1}^{4}$ to the first user, and 16 symbols $\{b_{1,t}, b_{2,t}, b_{3,t}, b_{4,t}\}_{t=1}^{4}$ to the second user. Consequently for any given $t = 1, 2, 3, 4$, the transmitted signal vectors are designed as follows

$$\boldsymbol{x}_{1,2t-1} = [a_{1,t} + b_{1,t} \quad a_{3,t} + b_{3,t}]^\top$$

$$\boldsymbol{x}_{2,2t-1} = \begin{bmatrix} \gamma_3\sum_{j=1}^{2}g_{1,j}a_{2j-1,t} + \gamma_1\sum_{j=1}^{2}h_{1,j}b_{2j-1,t} \\ 0 \end{bmatrix}$$

$$\boldsymbol{x}_{3,2t-1} = \begin{bmatrix} \gamma_4\sum_{j=1}^{2}g_{1,j}a_{2j-1,t} + \gamma_2\sum_{j=1}^{2}h_{1,j}b_{2j-1,t} \\ 0 \end{bmatrix}$$

$$\boldsymbol{x}_{1,2t} = [a_{2,t} + b_{2,t} \quad a_{4,t} + b_{4,t}]^\top$$

$$\boldsymbol{x}_{2,2t} = \begin{bmatrix} 0 \\ \gamma_3\sum_{j=1}^{2}g_{1,j}a_{2j,t} + \gamma_1\sum_{j=1}^{2}h_{1,j}b_{2j,t} \end{bmatrix}$$

$$\boldsymbol{x}_{3,2t} = \begin{bmatrix} 0 \\ \gamma_4\sum_{j=1}^{2}g_{1,j}a_{2j,t} + \gamma_2\sum_{j=1}^{2}h_{1,j}b_{2j,t} \end{bmatrix} \tag{6}$$

where

$$\gamma_i = \frac{1}{\phi_i + \sum_{j=1}^{2}|h_{1,j}|^2}, \ i = 1, 2 \tag{7}$$

and

$$\gamma_i = \frac{1}{\phi_i + \sum_{j=1}^{2}|g_{1,j}|^2}, \ i = 3, 4, \tag{8}$$

for some positive constants $\{\phi_1, \phi_2, \phi_3, \phi_4\}$ that are specifically designed later on.

Due to symmetry, we can focus only on the first user. The received signals, accumulated at the first receiver, can be rearranged to take the form

$$\bar{\mathbf{y}}_t^{(1)} = \boldsymbol{H}_{AA}\boldsymbol{a}_t + \boldsymbol{H}_{AB}\boldsymbol{b}_t + \bar{\boldsymbol{z}}_t^{(1)}, \ t = 1, \cdots, T/2, \tag{9}$$

where

$$\begin{aligned} \bar{\mathbf{y}}_t^{(1)} &= \begin{bmatrix} y_{1,2t-1}^{(1)} & y_{2,2t-1}^{(1)} & y_{3,2t-1}^{(1)} & y_{1,2t}^{(1)} & y_{2,2t}^{(1)} & y_{3,2t}^{(1)} \end{bmatrix}^\top \\ \bar{\boldsymbol{z}}_t^{(1)} &= \begin{bmatrix} z_{1,2t-1}^{(1)} & z_{2,2t-1}^{(1)} & z_{3,2t-1}^{(1)} & z_{1,2t}^{(1)} & z_{2,2t}^{(1)} & z_{3,2t}^{(1)} \end{bmatrix}^\top \\ \boldsymbol{a}_t &= [a_{1,t} \ a_{3,t} \ a_{2,t} \ a_{4,t}]^\top, \\ \boldsymbol{b}_t &= [b_{1,t} \ b_{3,t} \ b_{2,t} \ b_{4,t}]^\top, \end{aligned} \tag{10}$$

and where

$$
\boldsymbol{H}_{AA} =
\begin{bmatrix}
h_{1,1} & h_{1,2} & 0 & 0 \\
h_{2,1}\gamma_3 g_{1,1} & h_{2,1}\gamma_3 g_{1,2} & 0 & 0 \\
h_{3,1}\gamma_4 g_{1,1} & h_{3,1}\gamma_4 g_{1,2} & 0 & 0 \\
0 & 0 & h_{1,1} & h_{1,2} \\
0 & 0 & h_{2,2}\gamma_3 g_{1,1} & h_{2,2}\gamma_3 g_{1,2} \\
0 & 0 & h_{3,2}\gamma_4 g_{1,1} & h_{3,2}\gamma_4 g_{1,2}
\end{bmatrix}
$$

$$
\boldsymbol{H}_{AB} =
\begin{bmatrix}
h_{1,1} & h_{1,2} & 0 & 0 \\
h_{2,1}\gamma_1 h_{1,1} & h_{2,1}\gamma_1 h_{1,2} & 0 & 0 \\
h_{3,1}\gamma_2 h_{1,1} & h_{3,1}\gamma_2 h_{1,2} & 0 & 0 \\
0 & 0 & h_{1,1} & h_{1,2} \\
0 & 0 & h_{2,2}\gamma_1 h_{1,1} & h_{2,2}\gamma_1 h_{1,2} \\
0 & 0 & h_{3,2}\gamma_2 h_{1,1} & h_{3,2}\gamma_2 h_{1,2}
\end{bmatrix} .
$$

Rewriting (9) we now get

$$
\bar{\boldsymbol{y}}_t^{(1)} = [\boldsymbol{H}_{AA}\ \boldsymbol{H}_{AB}]
\begin{bmatrix} \boldsymbol{a}_t \\ \boldsymbol{b}_t \end{bmatrix} + \bar{\boldsymbol{z}}_t^{(1)}. \tag{11}
$$

Note that $\boldsymbol{H}_{AA} \in \mathbb{C}^{6\times4}$, $\boldsymbol{H}_{AB} \in \mathbb{C}^{6\times4}$, and that the rank of $[\boldsymbol{H}_{AA}\ \boldsymbol{H}_{AB}] \in \mathbb{C}^{6\times8}$ can generally not support decoding of $[\boldsymbol{a}_t^\top \boldsymbol{b}_t^\top]^\top \in \mathbb{C}^8$. This problem is bypassed by the structure of the designed scheme which allows for aligning some of the interference at the first receiver (cf. Fig. 1), such that

$$
\boldsymbol{H}_{AB}\boldsymbol{b}_t = \bar{\boldsymbol{H}}_{AB}\bar{\boldsymbol{b}}_t \tag{12}
$$

where

$$
\bar{\boldsymbol{H}}_{AB}=
\begin{bmatrix}
h_{1,1} & 0 \\
h_{2,1}\gamma_1 h_{1,1} & 0 \\
h_{3,1}\gamma_2 h_{1,1} & 0 \\
0 & h_{1,1} \\
0 & h_{2,2}\gamma_1 h_{1,1} \\
0 & h_{3,2}\gamma_2 h_{1,1}
\end{bmatrix},\
\bar{\boldsymbol{b}}_t =
\begin{bmatrix}
b_{1,t} + \frac{h_{1,2}}{h_{1,1}} b_{3,t} \\
b_{2,t} + \frac{h_{1,2}}{h_{1,1}} b_{4,t}
\end{bmatrix}.
$$

Consequently we can rewrite (9) as

$$
\bar{\boldsymbol{y}}_t^{(1)} = \bar{\boldsymbol{H}}_A \bar{\boldsymbol{x}}_t + \bar{\boldsymbol{z}}_t^{(1)}, \tag{13}
$$

where

$$
\bar{\boldsymbol{H}}_A = [\boldsymbol{H}_{AA}\ \bar{\boldsymbol{H}}_{AB}],\ \bar{\boldsymbol{x}}_t = \begin{bmatrix} \boldsymbol{a}_t \\ \bar{\boldsymbol{b}}_t \end{bmatrix}, \tag{14}
$$

and where $\bar{\boldsymbol{H}}_A \in \mathbb{C}^{6\times6}$, $\bar{\boldsymbol{x}}_A \in \mathbb{C}^6$.

At this point we randomly pick $\phi_1, \phi_2, \phi_3, \phi_4$ from the set of all possible numbers that guarantee

$$
\frac{\gamma_3}{\gamma_4} \neq \frac{\gamma_1}{\gamma_2}
$$

as well as guarantee that

$$
\begin{aligned}
\phi_3 &\doteq \phi_4 \doteq \max(|g_{1,1}|^2, |g_{1,2}|^2), \\
\phi_1 &\doteq \phi_2 \doteq \max(|h_{1,1}|^2, |h_{1,2}|^2).
\end{aligned} \tag{15}
$$

It is then easy to show that this random choice of $\phi_1, \phi_2, \phi_3, \phi_4$, while satisfying the power constraints, also guarantees that the rank of $\bar{\boldsymbol{H}}_A$ is full with probability 1. Simple zero-forcing (ZF) decoding guarantees the 2/3 DoF for both users.
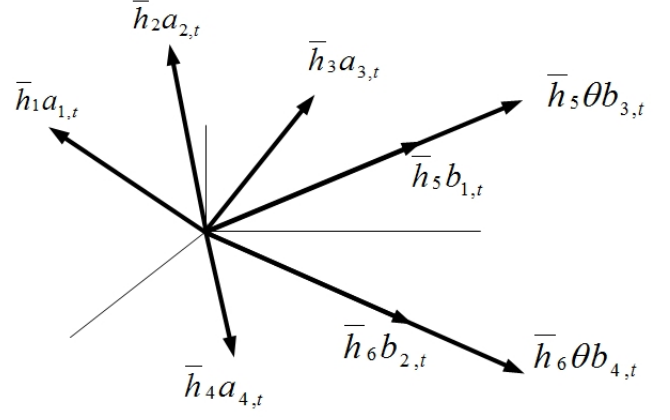

Fig. 1. Received signal space at user 1 after interference alignment. $\bar{\boldsymbol{h}}_i$ denotes the ith column of $\bar{\boldsymbol{H}}_A$, and $\theta = \frac{h_{1,2}}{h_{1,1}}$.

III. DIVERSITY ANALYSIS OF THE PROPOSED SCHEME

We again focus, without loss of generality, on the first user, and consider joint ML decoding for the MAC channel corresponding to (13). We remind the reader that we are interested only in establishing the diversity of the scheme, i.e., we are interested in the case of $r = 0$ (R is fixed). Directly from [10] we know that the probability of error in this lightly loaded regime (cf. [10]) is dominated by the outage event

$$
\mathcal{O} \triangleq \left\{ \bar{\boldsymbol{H}}_A : \frac{1}{6} I\left(\boldsymbol{a}_t ; \bar{\boldsymbol{y}}_t^{(1)} | \bar{\boldsymbol{b}}_t, \bar{\boldsymbol{H}}_A \right) < R \right\}, \tag{16}
$$

and as a result, the corresponding probability of error takes the form

$$
\begin{aligned}
P_e &\doteq P(\mathcal{O}) \\
&= P\left(\frac{1}{6} I(\boldsymbol{a}_t ; \bar{\boldsymbol{y}}_t^{(1)} | \bar{\boldsymbol{b}}_t, \bar{\boldsymbol{H}}_A) < R \right) \\
&\doteq P\left(\frac{1}{6} \log \det(\mathbf{I} + \rho \boldsymbol{H}_{AA} \boldsymbol{H}_{AA}^\dagger) < R \right)
\end{aligned}
$$

where for the above we considered optimal Gaussian distributions for $\boldsymbol{a}_t$ and $\boldsymbol{b}_t$.

For

$$
\boldsymbol{H}_{AA,j} =
\begin{bmatrix}
h_{1,1} & h_{1,2} \\
h_{2,j}\gamma_3 g_{1,1} & h_{2,j}\gamma_3 g_{1,2} \\
h_{3,j}\gamma_4 g_{1,1} & h_{3,j}\gamma_4 g_{1,2}
\end{bmatrix}, \tag{17}
$$

and

$$
\Omega_j = \det\left(\mathbf{I} + \rho \boldsymbol{H}_{AA,j} \boldsymbol{H}_{AA,j}^\dagger \right),\ \text{for } j = 1, 2, \tag{18}
$$

and after considering that $\boldsymbol{H}_{AA}$ has a block-diagonal structure, we see that (17) implies that

$$
P_e \doteq P(\log(\Omega_1 \Omega_2) < 6R). \tag{19}
$$

Using the law of expansion of determinants by diagonal

elements [11], and the Cauchy-Binet equation, we then have

$$\Omega_j = 1 + \sum_{n=1}^{2} \rho^n \sum_{\substack{J \subset \{1,2,3\} \\ |J|=n}} \sum_{\substack{S \subset \{1,2\} \\ |S|=n}} \det \left(([\boldsymbol{H}_{AA,j}]_{J,S})^\dagger [\boldsymbol{H}_{AA,j}]_{J,S} \right)$$

$$= 1 + \rho|h_{1,1}|^2 + \rho|h_{2,j}|^2|\gamma_3|^2|g_{1,1}|^2 + \rho|h_{3,j}|^2|\gamma_4|^2|g_{1,1}|^2$$
$$+ \rho|h_{1,2}|^2 + \rho|h_{2,j}|^2|\gamma_3|^2|g_{1,2}|^2 + \rho|h_{3,j}|^2|\gamma_4|^2|g_{1,2}|^2$$
$$+ \rho^2|h_{2,j}|^2|\gamma_3|^2|h_{1,1}g_{1,2} - h_{1,2}g_{1,1}|^2$$
$$+ \rho^2|h_{3,j}|^2|\gamma_4|^2|h_{1,1}g_{1,2} - h_{1,2}g_{1,1}|^2,$$

where in the above, $[\boldsymbol{E}]_{J,S}$ denotes the submatrix of matrix $\boldsymbol{E}$ that includes the rows of $\boldsymbol{E}$ labeled by the elements of set J, and the columns labeled by the elements of set S. Continuing we get that

$$\Omega_j \overset{(a)}{\doteq} 1 + \rho|h_{1,1}|^2 + \rho|h_{1,2}|^2 + \rho|h_{2,j}|^2 + \rho|h_{3,j}|^2$$
$$+ \rho^2(|h_{2,j}|^2 + |h_{3,j}|^2)|\gamma_B|^2|h_{1,1}g_{1,2} - h_{1,2}g_{1,1}|^2$$
$$\geq 1 + \rho|h_{1,1}|^2 + \rho|h_{1,2}|^2 + \rho|h_{2,j}|^2 + \rho|h_{3,j}|^2 \quad (20)$$

where in (a) we used that

$$\gamma_A \triangleq \frac{1}{|h_{1,1}|^2 + |h_{1,2}|^2}, \quad \gamma_B \triangleq \frac{1}{|g_{1,1}|^2 + |g_{1,2}|^2}, \quad (21)$$

and that (cf. (7),(15))

$$\gamma_A \doteq \gamma_1 \doteq \gamma_2, \ \gamma_B \doteq \gamma_3 \doteq \gamma_4.$$

Consequently (20) directly gives that

$$P_e \overset{.}{\leq} P(\log(1 + \rho|h_{1,1}|^2 + \rho|h_{1,2}|^2 + \rho|h_{2,1}|^2$$
$$+ \rho|h_{3,1}|^2 + \rho|h_{2,2}|^2 + \rho|h_{3,2}|^2) < 6R),$$

which directly shows that the diversity of the scheme is 6 (again for $r = 0$). At this point we also note that the above design can be readily extended to the case where L is a multiple of 3. The proof for this is simple and it is omitted. The result is summarized in the following.

Proposition 2: In the setting of the described two-user MISO BC with delayed CSIT, the proposed interference alignment based precoding scheme achieves full DoF and full diversity.

IV. CONCLUSIONS

In the setting of the two-user MISO broadcast channel with delayed CSIT, we designed the first scheme to achieve full DoF as well as full diversity. The scheme borrows from the techniques of interference alignment, which allow for combining the signal manipulations that increase the DoF with the signal manipulations that allow for full diversity. Future work can extend the result by analyzing the entire DMT behavior of the scheme, as well as extend the result to other BC settings.

VI. APPENDIX - PROOF OF PROPOSITION 1

In deriving the diversity achieved by the MAT scheme, we again focus, without loss of generality, on the performance of the first user.

From (4) we first recall that decoding is based on

$$\tilde{\mathbf{y}}^{(1)} = \tilde{\boldsymbol{H}}\boldsymbol{x}_1 + \tilde{\boldsymbol{z}}^{(1)}.$$

Consequently, in the presence of Gaussian input $\boldsymbol{x}_1 = [a_1 \ a_2]^\top$ (cf. (3)), and noting that the noise term $\tilde{\boldsymbol{z}}^{(1)}$ in (4) has zero mean and covariance

$$\mathbb{E}[\tilde{\boldsymbol{z}}^{(1)}(\tilde{\boldsymbol{z}}^{(1)})^\dagger] = \text{diag}(\ 1\ ,\ 1 + |h_{3,1}|^2),$$

we calculate the probability of outage to take the form

$$P_{\text{out}}(r) = P\left(I(\boldsymbol{x}_1; \tilde{\mathbf{y}}^{(1)} | \tilde{\boldsymbol{H}}) < 3R \right) \quad (22)$$
$$\doteq P\left(\log \det(\mathbf{I} + \rho \tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^\dagger) < 3R \right)$$
$$= P(\log \Phi < 3r \log \rho), \quad (23)$$

where we used that

$$\Phi = \det\left(\mathbf{I} + \rho \tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^\dagger \right),$$

and that

$$P(|h_{3,1}|^2 \geq \rho^\epsilon) = \exp(-\rho^\epsilon) \doteq 0,$$

for any positive ϵ. Consequently expanding the determinant and applying the Cauchy-Binet rule, gives

$$\Phi = 1 + \sum_{n=1}^{2} \rho^n \sum_{\substack{J \subset \{1,2\} \\ |J|=n}} \sum_{\substack{S \subset \{1,2\} \\ |S|=n}} \det\left([\tilde{\boldsymbol{H}}]_{J,S}([\tilde{\boldsymbol{H}}]_{J,S})^\dagger \right)$$
$$= 1 + \rho|h_{1,1}|^2 + \rho|h_{1,2}|^2 + \rho|h_{3,1}|^2|g_{1,1}|^2 + \rho|h_{3,1}|^2|g_{1,2}|^2$$
$$+ \rho^2|h_{3,1}|^2|h_{1,1}g_{1,2} - h_{1,2}g_{1,1}|^2. \quad (24)$$

The fact that

$$|h_{1,1}g_{1,2} - h_{1,2}g_{1,1}|^2 \leq |h_{1,1}|^2|g_{1,2}|^2 + |h_{1,2}|^2|g_{1,1}|^2$$
$$+ 2|h_{1,1}||g_{1,2}||h_{1,2}||g_{1,1}|,$$

together with a change of variables where

$$h_{l,j} = \rho^{-\alpha_{l,j}}, \quad g_{l,j} = \rho^{-\beta_{l,j}},$$

and along with the fact that

$$P(\alpha_{l,j}) \doteq \begin{cases} \rho^{-\infty}, & \text{for } \alpha_{l,j} < 0 \\ \rho^{-\alpha_{l,j}}, & \text{for } \alpha_{l,j} \geq 0 \end{cases}, \quad (25)$$

gives that the diversity $d(r)$ of the MAT scheme is upper bounded as

$$d(r) \leq d_M(r) \triangleq \inf_{\mathcal{O}^M(r)} (\alpha_{1,1} + \alpha_{1,2} + \beta_{1,1} + \beta_{1,2} + \alpha_{3,1}) \quad (26)$$
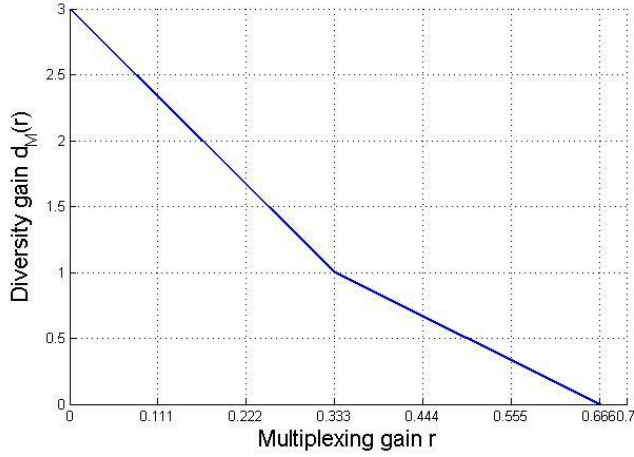
Fig. 2. DMT upper bound for the MAT scheme.

where

$$
\mathcal{O}^M(r) = \left\{ \begin{array}{c} (1 - \alpha_{1,1})^+ \le 3r, \\ (1 - \alpha_{1,2})^+ \le 3r, \\ (1 - \alpha_{3,1} - \beta_{1,1})^+ \le 3r, \\ (1 - \alpha_{3,1} - \beta_{1,2})^+ \le 3r, \\ (2 - \alpha_{3,1} - \alpha_{1,1} - \beta_{1,2})^+ \le 3r, \\ (2 - \alpha_{3,1} - \alpha_{1,2} - \beta_{1,1})^+ \le 3r, \\ (2 - \alpha_{3,1} - 0.5 \sum_{j=1}^{2} (\alpha_{1,j} + \beta_{1,j}))^+ \le 3r \end{array} \right\}.
$$

At this point it is easy to see that the diversity is upper bounded by 3 (see also Fig. 2). $\qquad\square$

REFERENCES

[1] M. A. Maddah-Ali and D. N. C. Tse, "Completely stale transmitter channel state information is still very useful," Sep. 2011, submitted to *IEEE Trans. Inf. Theory*, available on arXiv:1010.1499v2.

[2] H. Maleki, S. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 228–240, Mar. 2012.

[3] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "On the degrees of freedom of X channel with delayed CSIT," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Jul. 2011.

[4] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region of two-user and certain three-user MIMO broadcast channel with delayed CSI," Dec. 2010, submitted to *IEEE Trans. Inf. Theory*, available on arXiv:1101.0306v2.

[5] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Jul. 2011.

[6] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the MIMO interference channel with delayed local CSIT," Fer. 2011, available on arXiv:1102.5673v1.

[7] J. Xu, J. G. Andrews, and S. A. Jafar, "Broadcast channels with delayed finite-rate feedback: Predict or observe?" May 2011, submitted to *IEEE Trans. on Wireless Communications*, available on arXiv:1105.3686v1.

[8] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.

[9] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[10] D. N. C. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.

[11] A. C. Aitken, *Determinants and Matrices*, 8th ed. Edinburgh: Oliver and Boyd, 1954.