On the Degrees of Freedom of time correlated MISO broadcast channel with delayed CSIT

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***Abstract*— We consider the time correlated MISO broadcast channel where the transmitter has partial knowledge on the current channel state, in addition to delayed channel state information (CSI). Rather than exploiting only the current CSI, as the zero-forcing precoding, or only the delayed CSI, as the Maddah-Ali-Tse (MAT) scheme, we propose a seamless strategy that takes advantage of both. The achievable degrees of freedom of the proposed scheme is characterized in terms of the quality of the current channel knowledge.**

I. Introduction

In most practical scenarios, perfect channel state information at transmitter (CSIT) may not be available due to the time-varying nature of wireless channels as well as the limited resource for channel estimation. However, many wireless applications must guarantee high-data rate and reliable communication in the presence of channel uncertainty. In this paper, we consider such scenario in the context of the two-user MISO broadcast channel, where the transmitter equipped with m antennas wishes to send two private messages to two receivers each with a single antenna. The discrete time baseband signal model is given by

$$y_t = \boldsymbol{h}_t^{\mathsf{T}} \boldsymbol{x}_t + e_t \tag{1a}$$

$$z_t = \boldsymbol{g}_t^{\mathsf{T}} \boldsymbol{x}_t + b_t, \tag{1b}$$

for any time instant t, where $\boldsymbol{h}_t, \boldsymbol{g}_t \in \mathbb{C}^{m\times 1}$ are the channel vectors for user 1 and 2, respectively; $e_t, b_t \sim \mathcal{N}_{\mathbb{C}}(0,1)$ are normalized additive white Gaussian noise (AWGN) at the respective receivers; the input signal $\boldsymbol{x}_t$ is subject to the power constraint $\mathbb{E}\left(\|\boldsymbol{x}_t\|^2\right) \le P$, $\forall\, t$. For the case of perfect CSIT, the optimal multiplexing gain of the channel at hand is two achieved by linear strategies such as zero-forcing (ZF) beamforming. It is also well known that the full multiplexing gain can be maintained under imperfect CSIT if the error in CSIT decreases at the rate P^{-1} as P grows [4], [5]. Further, in the realistic case where the fading process is correlated with a maximum Doppler frequency shift $0 \le F < \frac{1}{2}$, ZF can achieve a fraction $2(1-2F)$ of the optimal multiplexing gain [4]. This result somehow reveals the bottleneck of a family of precoding schemes relying only on instantaneous CSIT as the fading speed increases ($F \to \frac{1}{2}$). Recently, a breakthrough has been made in order to overcome precisely such a problem. In [1], Maddah-Ali and Tse showed a surprising result that even completely outdated CSIT can be very useful in terms of multiplexing gain. For a system with $m = 2$ antennas and two users, the proposed scheme, hereafter called MAT, achieves the multiplexing gain of $\frac{4}{3}$, irrespectively of the fading speed. This work shifts the paradigm of broadcast precoding from space-only to space-time alignment. The role of delayed CSIT can then be re-interpreted as a "feedback" of the past signal/interference heard by the receiver. This side information enables the transmitter to perform "retrospective" alignment in the space *and* time domain, as demonstrated in different multiuser network systems (see e.g. [2]). Although it exhibits optimal rate scaling behavior, the MAT algorithm is designed based on the worst case scenario where the delayed channel feedback provides no information about the current one. This assumption is over pessimistic as most practical channels exhibit some form of temporal correlation. It should be noticed that MAT does not exploit any current CSIT whereas ZF builds only on the current CSIT. In fact with a simple selection strategy between ZF and MAT, a multiplexing gain of $\max\{2(1-2F), \frac{4}{3}\}$ is achievable. For either very slowly or very rapidly varying channels, a scheme selection approach is reasonable. Yet, for intermediate ranges of temporal correlation, a question arises as how to best exploit both past channel samples and an estimate of the current one, obtained through a linear prediction.

In order to model the quality of the current CSIT, we introduce a parameter α which indicates the rate of decay of the channel estimation error when the transmitted power grows. Thus $\alpha = 0, \infty$ corresponds to no and perfect CSIT respectively. We propose a seamless scheme which bridges smoothly between the two extremal schemes ZF and MAT and we characterize the achievable degrees of freedom. As it will be shown later, the proposed scheme combines the ZF and MAT principles into a single multi-slotted protocol which relies on the retransmission and alignement of the *residual* interference caused by the ZF precoder due to the imperfectness of current channel state information.

In the following, after a brief presentation of the assumptions on the CSI and fading process, we present the proposed scheme as an extension the MAT principle. The achievable degrees of freedom (DoF) of the proposed scheme are analyzed afterward. Finally, we interpret the obbtained DoF in a practical temporally correlated fading channel scenario where α can be related to the maximum Doppler shift over the time varying channel.

Throughout the paper, we will use the following notations.

Matrix transpose, Hermitian transpose, inverse, and determinant are denoted by $\boldsymbol{A}^{\mathsf{T}}$, $\boldsymbol{A}^{\mathsf{H}}$, $\boldsymbol{A}^{-1}$, and $\det(\boldsymbol{A})$, respectively. For any real number x, $[x]$ means $\lfloor x \rfloor + \frac{1}{2}$.

II. System Model

For convenience, we provide the following definition on the channel states.

Definition 1 (channel states): The channel vectors $\boldsymbol{h}_t$ and $\boldsymbol{g}_t$ are called the states of the channel at instant t. For simplicity, we also define the state matrix $\boldsymbol{S}_t$ as $\boldsymbol{S}_t \triangleq \begin{bmatrix} \boldsymbol{h}_t^{\mathsf{T}} \\ \boldsymbol{g}_t^{\mathsf{T}} \end{bmatrix}$.

The assumptions on the fading process and the knowledge of the channel states are summarized as follows.

Assumption 1 (mutually independent fading): At any given time instant t, the channel vectors for the two users $\boldsymbol{h}_t, \boldsymbol{g}_t$ are mutually independent and identically distributed (i.i.d.) with zero mean and covariance matrix $\mathbf{I}_m$. Moreover, we assume that $\operatorname{rank}(\boldsymbol{S}_t) = 2$ with probability 1.

Assumption 2 (perfect delayed and imperfect current CSI): At each time instant t, the transmitter knows the delayed channel states up to instant $t-1$. In addition, the transmitter can somehow obtain an estimation $\hat{\boldsymbol{S}}_t$ of the current channel state $\boldsymbol{S}_t$, i.e., $\hat{\boldsymbol{h}}_t$ and $\hat{\boldsymbol{g}}_t$ are available to the transmitter with

$$\boldsymbol{h}_t = \hat{\boldsymbol{h}}_t + \boldsymbol{\delta}_t \tag{2a}$$

$$\boldsymbol{g}_t = \hat{\boldsymbol{g}}_t + \boldsymbol{\varepsilon}_t \tag{2b}$$

where the estimate $\hat{\boldsymbol{h}}_t$ (also $\hat{\boldsymbol{g}}_t$) and estimation error $\boldsymbol{\delta}_t$ (also $\boldsymbol{\varepsilon}_t$) are uncorrelated and both assumed to be zero mean with covariance $(1-\sigma^2)\mathbf{I}_m$ and $\sigma^2\mathbf{I}_m$, respectively, with $\sigma^2 \leq 1$. The receivers knows perfectly $\boldsymbol{S}_t$ and $\hat{\boldsymbol{S}}_t$ without delay.

Without loss of generality, we can introduce a parameter $\alpha_P \geq 0$ as the power exponent of the estimation error

$$\alpha_P \triangleq -\frac{\log(\sigma^2)}{\log P}. \tag{3}$$

The parameter α can be regarded as the quality of the current CSI in the high SNR regime. Note that $\alpha_P = 0$ corresponds to the case with no current CSIT at all while $\alpha_P \to \infty$ corresponds to the case with perfect current CSIT.

III. Proposed Scheme

In this section, we propose a novel scheme that combines ZF exploiting some estimated current CSIT and MAT exploiting delayed CSIT. We start by briefly reviewing the MAT scheme.

A. MAT Alignment Revisited

In the two-user MISO case, the original MAT is a three-slot scheme, described by the following equations

$$\boldsymbol{x}_1 = \boldsymbol{u} \qquad \boldsymbol{x}_2 = \boldsymbol{v} \qquad \boldsymbol{x}_3 = [\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} + \boldsymbol{h}_2^{\mathsf{T}}\boldsymbol{v} \quad 0]^{\mathsf{T}} \tag{4a}$$

$$y_1 = \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{u} \qquad y_2 = \boldsymbol{h}_2^{\mathsf{T}}\boldsymbol{v} \qquad y_3 = h_{31}(\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} + \boldsymbol{h}_2^{\mathsf{T}}\boldsymbol{v}) \tag{4b}$$

$$z_1 = \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} \qquad z_2 = \boldsymbol{g}_2^{\mathsf{T}}\boldsymbol{v} \qquad z_3 = g_{31}(\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} + \boldsymbol{h}_2^{\mathsf{T}}\boldsymbol{v}) \tag{4c}$$

where $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{C}^{m\times 1}$ are useful signals to user 1 and user 2, respectively; for simplicity, we omit the noise in the received signals. The idea of the MAT scheme is to use the delayed CSIT to align the mutual interference into a reduced subspace with only one dimension ($\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}$ for user 1 and $\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u}$ for user 2). And importantly, the reduction in interference is done without sacrificing the dimension of the useful signals. Specifically, a two-dimensional interference-free observation of $\boldsymbol{u}$ (resp. $\boldsymbol{v}$) is obtained at receiver 1 (resp. receiver 2).

Interestingly, the alignment can be done in a different manner.

$$\boldsymbol{x}_1 = \boldsymbol{u} + \boldsymbol{v} \qquad \boldsymbol{x}_2 = [\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v} \quad 0]^{\mathsf{T}} \qquad \boldsymbol{x}_3 = [\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} \quad 0]^{\mathsf{T}} \tag{5a}$$

$$y_1 = \boldsymbol{h}_1^{\mathsf{T}}(\boldsymbol{u}+\boldsymbol{v}) \qquad y_2 = h_{21}\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v} \qquad y_3 = h_{31}\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} \tag{5b}$$

$$z_1 = \boldsymbol{g}_1^{\mathsf{T}}(\boldsymbol{u}+\boldsymbol{v}) \qquad z_2 = g_{21}\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v} \qquad z_3 = g_{31}\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} \tag{5c}$$

In the first slot, the transmitter sends the mixed signal to both users. In the second slot, the transmitter sends the interference seen by receiver 1 in the first slot. The role of this stage is two-fold: *resolving interference for user 1 and reinforcing signal for user 2*. In the third slot, the transmitter sends the interference seen by user 2 to help the users the other way around. Therefore, this variant of the MAT alignment is composed of two phases: i) broadcasting the mixed signal, and ii) multicasting the mutual interference $(\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}, \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u})$. At the end of three slots, the observations at the receivers are given by

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{h}_1^{\mathsf{T}} \\ 0 \\ h_{31}\boldsymbol{g}_1^{\mathsf{T}} \end{bmatrix}}_{\text{rank}=2} \boldsymbol{u} + \underbrace{\begin{bmatrix} \boldsymbol{h}_1^{\mathsf{T}} \\ h_{21}\boldsymbol{h}_1^{\mathsf{T}} \\ 0 \end{bmatrix}}_{\text{rank}=1} \boldsymbol{v}, \tag{6}$$

and

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{g}_1^{\mathsf{T}} \\ g_{21}\boldsymbol{h}_1^{\mathsf{T}} \\ 0 \end{bmatrix}}_{\text{rank}=2} \boldsymbol{v} + \underbrace{\begin{bmatrix} \boldsymbol{g}_1^{\mathsf{T}} \\ 0 \\ g_{31}\boldsymbol{g}_1^{\mathsf{T}} \end{bmatrix}}_{\text{rank}=1} \boldsymbol{u}. \tag{7}$$

For each user, the useful signal lies in a two-dimensional subspace while the interference is aligned in a one-dimensional subspace. Since the latter is not completely included in the signal subspace, it is readily shown that two degrees of freedom are achievable in the three-dimensional time space, yielding $\frac{2}{3}$ as the average degrees of freedom. This variant, although trivial from the original MAT scheme, is crucial to the integration of the current CSI, if there is any.

B. Integrating the Imperfect Current CSI

Based on the above variant of the MAT scheme, we propose the following two-stage scheme that integrates the estimates of the current CSI.

Phase 1 - Precoding and broadcasting the mixed signals: As in the above MAT variant, we first mix the two signals as $\boldsymbol{x}_1 = \boldsymbol{u} + \boldsymbol{v}$, except that $\boldsymbol{u}$ and $\boldsymbol{v}$ are precoded beforehand

$$\boldsymbol{u} = \boldsymbol{W}\tilde{\boldsymbol{u}}, \quad \boldsymbol{v} = \boldsymbol{Q}\tilde{\boldsymbol{v}} \tag{8}$$

where $\boldsymbol{W} \triangleq [\boldsymbol{w}_1 \quad \boldsymbol{w}_2] \in \mathbb{C}^{m\times 2}$ and $\boldsymbol{Q} \triangleq [\boldsymbol{q}_1 \quad \boldsymbol{q}_2] \in \mathbb{C}^{m\times 2}$ are the precoding matrices; $\tilde{\boldsymbol{u}} \triangleq [\tilde{u}_1 \quad \tilde{u}_2]^{\mathsf{T}}$ and $\tilde{\boldsymbol{v}} \triangleq [\tilde{v}_1 \quad \tilde{v}_2]^{\mathsf{T}}$ are input signals of dimension 2 for user 1 and user 2, respectively. Furthermore, we suppose that $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{v}}$ are mutually independent. In this paper, we restrict ourselves to orthogonal precoders, i.e., $\boldsymbol{W}^{\mathsf{H}}\boldsymbol{W} = \mathbf{I}$ and $\boldsymbol{Q}^{\mathsf{H}}\boldsymbol{Q} = \mathbf{I}$. In particular, we align $\boldsymbol{w}_2$ and $\boldsymbol{q}_2$ with the estimated channels $\hat{\boldsymbol{g}}_1$ and $\hat{\boldsymbol{h}}_1$, respectively. That is,

$$\boldsymbol{w}_1 \in \text{null}(\hat{\boldsymbol{g}}_1), \quad \boldsymbol{w}_2 \in \text{span}(\hat{\boldsymbol{g}}_1) \tag{9a}$$
$$\boldsymbol{q}_1 \in \text{null}(\hat{\boldsymbol{h}}_1), \quad \boldsymbol{q}_2 \in \text{span}(\hat{\boldsymbol{h}}_1) \tag{9b}$$

Let us define the covariance matrices $\boldsymbol{\Lambda} \triangleq \mathbb{E}(\tilde{\boldsymbol{u}}\tilde{\boldsymbol{u}}^{\mathsf{H}})$ and $\boldsymbol{\Phi} \triangleq \mathbb{E}(\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}^{\mathsf{H}})$. Without loss of generality, we can assume that both $\boldsymbol{\Lambda}$ and $\boldsymbol{\Phi}$ are diagonal. Hence, the power constraint is simply

$$\lambda_1 + \lambda_2 + \phi_1 + \phi_2 \leq P. \tag{10}$$

In other words, for each user, we send two streams in two orthogonal directions: one aligned with the estimated channel while the other one perpendicular to it.

Phase 2 - Quantizing and multicasting the mutual interference : As the second phase of the MAT variant, the objective of this phase is, by sending the mutual interferences $(\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}, \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u})$ seen at the receivers, to resolve the interference *and* to reinforce the useful signal at the same time. However, unlike the original MAT scheme where the interferences $(\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}, \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u})$ is transmitted in an analog form, we will quantize it and then transmit the digital version. The rationale behind this choice is as follows. With (imperfect) CSI on the current channel, the transmitter can use the precoding to align the signals and allocate the transmit power in such a way that the mutual interferences have a reduced power, without sacrificing too much the received signal power.[1] As a result, we should be able to save the resource needed to multicast the interferences, which increases the average rate. The reduction can be significant when the current CSI is good enough. In this case, the analog transmission is not suitable any more, due to the mismatch of the source power and available transmit power. Therefore, a good alternative is to quantize the interferences before transmission. The number of quantization bits depends naturally on the interference power, which means that the multicasting can be done efficiently.

Let us look into the interferences by taking into account the precoding. We start by examining the interference $\eta_1 \triangleq \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}$ seen by user 1. It can be rewritten as

$$\begin{aligned} \eta_1 &= \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{Q}\tilde{\boldsymbol{v}} && (11)\\ &= (\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{q}_1)\tilde{v}_1 + (\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{q}_2)\tilde{v}_2 && (12)\\ &= (\boldsymbol{\delta}_1^{\mathsf{T}}\boldsymbol{q}_1)\tilde{v}_1 + (\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{q}_2)\tilde{v}_2 && (13)\end{aligned}$$

where $\boldsymbol{\delta}_1^{\mathsf{T}}\boldsymbol{q}_1$ and $\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{q}_2$ are known at the end of the first slot to both receivers, according to Assumption 2. Therefore, the average power of η_1 is $\sigma_{\eta_1}^2 \triangleq \mathbb{E}(|\eta_1|^2)$, i.e.,

$$\sigma_{\eta_1}^2 = |\boldsymbol{\delta}_1^{\mathsf{T}}\boldsymbol{q}_1|^2\phi_1 + |\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{q}_2|^2\phi_2. \tag{14}$$

Similarly, for the interference seen by user 2 during the first slot $\eta_2 \triangleq \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u}$, the average power is

$$\sigma_{\eta_2}^2 = |\boldsymbol{\varepsilon}_1^{\mathsf{T}}\boldsymbol{w}_1|^2\lambda_1 + |\boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{w}_2|^2\lambda_2. \tag{15}$$

[1] With no CSIT on the current channel, the only way to reduce the interference power is to reduce the transmit power, therefore the received signal power.

Obviously, the interference powers $\sigma_{\eta_1}^2$ and $\sigma_{\eta_2}^2$ depend on the both the precoder and the power allocation at the transmitter. The power allocation issue will be discussed in the next section.

The first step is to quantize $(\boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{v}, \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u})$. Although it is possible to apply directly a 2-dimensional quantizer, we choose to quantize both signals individually for simplicty of demonstration. Let us assume that an R_k-bits scalar quantizer is used for η_k, $k = 1, 2$. Hence, we have

$$\eta_k = \hat{\eta}_k + \xi_{\Delta,k}, \quad \hat{\eta}_k \in \mathcal{C}_k \tag{16}$$

where $\mathcal{C}_k$, $k = 1, 2$, is a quantization codebook of size 2^{R_k}; $\hat{\eta}_k$ and $\xi_{\Delta,k}$ are the quantized value and the quantization noise, respectively. The indices of both $\hat{\eta}_1$ and $\hat{\eta}_2$, represented in $R_1 + R_2$ bits, are then multicast to both users in κ channel uses. As will be specified in the next section, we choose κ such that the indices can be recovered with high probability.

At the receivers' side, each user first tries to recover $(\hat{\eta}_1, \hat{\eta}_2)$. If this step is done successfully, then receiver 1 has

$$\begin{aligned} y_1 &= \boldsymbol{h}_1^{\mathsf{T}}\boldsymbol{u} + \eta_1 + e_1 && (17)\\ \hat{\eta}_1 &= \eta_1 - \xi_{\Delta,1} && (18)\\ \hat{\eta}_2 &= \eta_2 - \xi_{\Delta,2} = \boldsymbol{g}_1^{\mathsf{T}}\boldsymbol{u} - \xi_{\Delta,2} && (19)\end{aligned}$$

from which an equivalent 2×2 MIMO channel is obtained

$$\tilde{\boldsymbol{y}} \triangleq \begin{bmatrix} y_1 - \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix} = \boldsymbol{S}_1\boldsymbol{W}\tilde{\boldsymbol{u}} + \begin{bmatrix} e_1 + \xi_{\Delta,1} \\ -\xi_{\Delta,2} \end{bmatrix} \tag{20}$$

where the noise $\tilde{\boldsymbol{n}} \triangleq [e_1 + \xi_{\Delta,1} \quad -\xi_{\Delta,2}]^{\mathsf{T}}$ is not Gaussian and can depend on the signal in general; the equivalent channel matrix is $\boldsymbol{F} \triangleq \boldsymbol{S}_1\boldsymbol{W} \in \mathbb{C}^{2\times 2}$. Similarly, if receiver 2 can recover $(\hat{\eta}_1, \hat{\eta}_2)$ correctly, then the following term is available

$$\tilde{\boldsymbol{z}} \triangleq \begin{bmatrix} \hat{\eta}_1 \\ z_1 - \hat{\eta}_2 \end{bmatrix} = \boldsymbol{S}_1\boldsymbol{Q}\tilde{\boldsymbol{v}} + \begin{bmatrix} -\xi_{\Delta,1} \\ b_1 + \xi_{\Delta,2} \end{bmatrix}. \tag{21}$$

In order to finally recover the message, each user performs the MIMO decoding of the above equivalent channel.

IV. Achievable Degrees of Freedom

In this section, we analyze the achievable rate of the proposed scheme in the high SNR regime. In particular, we are interested in the pre-log factor of the achievable rate, the so-called degrees of freedom (DoF). However, since we do not assume ergodic fading process in this work, we do not use directly ergodic capacity as our performance measure. Instead, following the definition of multiplexing gain in [3], we define the achievable degrees of freedom as follows.

Definition 2 (achievable degrees of freedom): For a family of codes $\{\mathcal{X}(P)\}$ of length L and rate $R(P)$ bits per channel use, we let $P_e(P)$ be the average probability of error and define

$$r \triangleq \lim_{P\to\infty} \frac{R(P)}{\log P}. \tag{22}$$

Then, the achievable degrees of freedom of $\mathcal{X}$ is defined as

$$\mathsf{DoF} \triangleq \sup\left\{r : \lim_{P\to\infty} P_e(P) = 0\right\}. \tag{23}$$

In other words, the DoF defined in this work is the maximum pre-log factor of the rate of a coding scheme for a reliable communication in the high SNR regime. Note that the code length L here is fixed, which avoids the involvement of the whole fading process.

In the following, we focus on the symmetrical case where the two users have the same data rate. The whole achievable region is straightforward following the same lines. In addition, we assume that $\lim_{P\to\infty} \alpha_P$ exists and define

$$\alpha \triangleq \lim_{P\to\infty} \alpha_P. \tag{24}$$

The main result is stated in the following theorem.

Theorem 1: In the two-user MISO broadcast channel with delayed perfect CSIT and imperfect current CSIT (Assumption 2), the following DoF is achievable for each user

$$d = \begin{cases} \dfrac{2-\alpha}{3-2\alpha}, & \alpha \in [0,1] \\ 1, & \alpha > 1. \end{cases} \tag{25}$$

Note that when α is close to 0, the estimation of current CSIT is bad and therefore useless. In this case, the optimal scheme is MAT [1], achieving DoF of $\frac{2}{3}$ for each user. On the other hand, when $\alpha \geq 1$, the estimation is good and the interference at the receivers due to the imperfect estimation is below the noise level and thus can be neglected as far as the DoF is concerned. In this case, ZF with the estimated current CSI is asymptotically optimal, achieving degrees of freedom 1 for each user. Interestingly, our result (Fig. (1)) reveals that strictly larger DoF than $\max\{\frac{2}{3}, \alpha\}$ can be obtained by exploiting both the imperfect current CSIT and the perfect delayed CSIT in an intermediate regime $\alpha \in (0,1)$. The intuition behind equation (25) is as follows. Decreasing the interference power will reduce the receive power of useful signal, incurring a loss of degrees of freedom. On the other hand, decreasing the interference power will also save the resources needed to communicate the interference *a la* MAT. By smartly aligning the signals and allocating the transmit power, the proposed scheme loses only α (numerator in (25)) degrees of freedom, but reduces 2α channel uses (denominator in (25)).

In the rest of the section, we provide a proof outline of the Theorem. Detailed proofs is reported in [6]. Some important ingredients of the proposed scheme are:

- Two independent Gaussian codebooks $\mathcal{X}_1$ and $\mathcal{X}_2$ with same size 2^R are used for $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{v}}$, respectively.
- Since we are interested in the symmetrical case, same power allocation scheme is applied to both user, i.e., $\phi_l = \lambda_l = P_l$, $l = 1, 2$. Hence, we have $P_1 + P_2 = P/2$.
- Truncated uniform quantization with unit step and truncation value $\bar{\eta} = P^{\frac{1+\zeta}{2}}\sigma$, for some $\zeta > 0$, is used for both the real and imaginary parts of η_1 and η_2, i.e.,

$$\hat{\eta}_k = [\text{trunc}(\text{Re}(\eta_k))] + i\,[\text{trunc}(\text{Im}(\eta_k))] \tag{26}$$

  where $\text{trunc}(x) = x$ if $x \in [-\bar{\eta}, \bar{\eta}]$ and 0 otherwise.
- The double indices of $(\hat{\eta}_1, \hat{\eta}_2)$, represented in

$$4\log(2\lceil\bar{\eta}\rceil) \approx 4 + 2(1+\zeta-\alpha_P)\log P \quad \text{bits}, \tag{27}$$

  are sent with a multicast code.

We define the error event $\mathcal{E}$ as the event that one of the users cannot recover his message correctly. The error event implies one of the following events:

- Quantization range error $\mathcal{E}_\Delta$: the amplitude of real or imaginary parts of the interferences is out of $[-\bar{\eta}, \bar{\eta}]$. It can be shown that, by defining $\beta_P \triangleq \frac{\log P_2}{\log P}$, as long as

$$\lim_{P\to\infty} \beta_P \leq 1 - \alpha, \tag{28}$$

  and that $\zeta > 0$, $\lim_{P\to\infty} \mathbb{P}(\mathcal{E}_\Delta) = 0$. It means that the power P_2 of the stream in the direction of the estimated channel should not scale faster than $P^{1-\alpha}$.
- Multicast error $\mathcal{E}_{\mathsf{mc}}$: one of the users cannot recover the double indices of $(\hat{\eta}_1, \hat{\eta}_2)$ correctly. Note that the number of bits needed to describe the indices is $4 + 2(1+\zeta-\alpha_P)\log P$. From Lemma 1, we know that for any $\delta < 0$, a rate $(1-\delta)\log P$ can be achieved reliably when $P \to \infty$. Therefore, as long as the number of channel uses

$$\kappa \geq \frac{4}{(1-\delta)\log P} + \frac{2(1+\zeta-\alpha_P)}{1-\delta}, \tag{29}$$

  we can guarantee that $P\{\mathcal{E}_1\} \to 0$ when $P \to \infty$.
- MIMO decoding error $\mathcal{E}_{\mathsf{mimo}}$: based on the received signal and the recovered indices, one of the users cannot recover his original message after performing a MIMO decoding of the equivalent channel (20) or (21). We can show that $\lim_{P\to\infty} \mathbb{P}\left(\mathcal{E}_{\mathsf{mimo}} \cap \bar{\mathcal{E}}_\Delta \cap \bar{\mathcal{E}}_{\mathsf{mc}}\right) = 0$ for any $\epsilon', \epsilon'' > 0$, and

$$r \leq 1 + \beta_P - \epsilon' - \epsilon''. \tag{30}$$

Since $\mathcal{E} \subseteq \mathcal{E}_\Delta \cup \mathcal{E}_{\mathsf{mc}} \cup \mathcal{E}_{\mathsf{mimo}}$, we have

$$P_e \leq \mathbb{P}(\mathcal{E}_\Delta) + \mathbb{P}(\mathcal{E}_{\mathsf{mc}}) + \mathbb{P}\left(\mathcal{E}_{\mathsf{mimo}} \cap \bar{\mathcal{E}}_\Delta \cap \bar{\mathcal{E}}_{\mathsf{mc}}\right). \tag{31}$$

Therefore, by making each term of the right hand side vanish with P, i.e., by letting (28), (29), and (30) be satisfied, the proposed scheme can deliver *reliably*

$$\frac{r}{1+\kappa}\log P \leq \frac{(1-\delta)(1+\beta_P-\epsilon'-\epsilon'')}{1-\delta+\frac{4}{\log P}+2(1+\zeta-\alpha_P)}\log P \tag{32}$$

bits per channel use, when $P \to \infty$. From (32), we can deduce the following achievable pre-log factor

$$\frac{(1-\delta)(1+\lim_{P\to\infty}\beta_P-\epsilon'-\epsilon'')}{1-\delta+2(1+\zeta-\alpha)}. \tag{33}$$

We can maximize (33) over the power exponent β_P under the constraint (28). The maximizing value of $\lim_{P\to\infty}\beta_P$ is $1-\alpha$, i.e., the power attributed to the stream in the direction of estimated channel should scale exactly as $P^{1-\alpha}$. Finally, by making ζ, ϵ', ϵ'', and δ as close to 0 as possible in (33), we prove the achievable DoF for user 1 in (25). Due to the symmetry, same proof applies to finding precisely the same DoF for user 2. ■

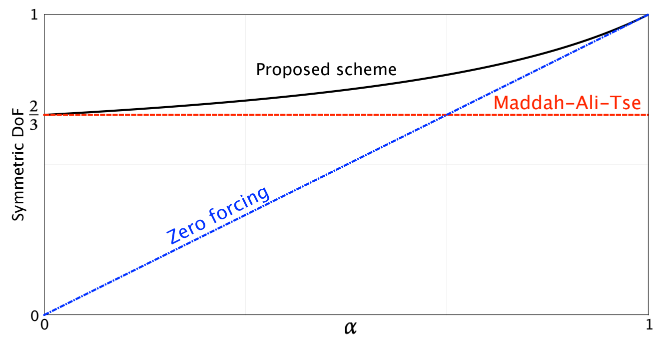

Fig. 1. Comparison of the achievable DoF between the proposed scheme and the zero-forcing and MAT alignment as a function of α.

V. Example: Temporally Correlated Fading

The main result on the achievable DoF has been presented in terms of an artificial parameter α, denoting the speed of decay of the estimation error $\sigma^2 \sim P^{-\alpha}$ in the current CSIT. In this section, we provide an example showing the practical interpretation of this parameter. Focusing on receiver 1 due to symmetry, we describe the fading process, channel estimation, and feedback scheme as follows:

- The channel fading $\boldsymbol{h}_t$ follows a Doppler process with power spectral density $S_h(w)$. The channel coefficients are strictly band-limited to $[-F, F]$ with $F = \frac{v f_c T_f}{c} < \frac{1}{2}$ where v, f_c, T_f, c denotes the mobile speed in m/h, the carrier frequency in Hz, the slot duration in sec, the light speed in m/sec.
- The channel estimation is done at the receivers side with pilot-based downlink training. At slot t, receiver 1 estimates $\boldsymbol{h}_t$ based on a sequence of the noisy observations $\{\boldsymbol{s}_\tau = \sqrt{P\gamma}\boldsymbol{h}_\tau + \boldsymbol{\nu}_\tau\}$ up to t, where a constant $\gamma \geq 1$ denotes the resource factor dedicated to the training and $\boldsymbol{\nu}_t$ is AWGN with zero mean unit covariance. The estimate is denoted by $\tilde{\boldsymbol{h}}_t$ with
$$\boldsymbol{h}_t = \tilde{\boldsymbol{h}}_t + \tilde{\boldsymbol{\delta}}_t \tag{34}$$
Under this model, the estimation error vanishes as $\mathbb{E}\left(\|\tilde{\boldsymbol{\delta}}_t\|^2\right) \sim P^{-1}$.
- At the end of slot t, the noisy observation $\boldsymbol{s}_t$ is sent to the transmitter and receiver 2 over a noise-free channel. At slot $t+1$, based on the noisy observation $\{\boldsymbol{s}_\tau\}$ up to t, the transmitter and receiver 2 acquire the prediction $\hat{\boldsymbol{h}}_{t+1}$ of $\boldsymbol{h}_{t+1}$ and estimation $\tilde{\boldsymbol{h}}_t$ of $\boldsymbol{h}_t$. The corresponding prediction model is
$$\boldsymbol{h}_t = \hat{\boldsymbol{h}}_t + \hat{\boldsymbol{\delta}}_t \tag{35}$$
From [4, Lemma 1], we have $\mathbb{E}\left(\|\hat{\boldsymbol{\delta}}_t\|^2\right) \sim P^{-(1-2F)}$.

In this channel with imperfect delayed CSIT, we can still apply the proposed scheme and analysis in exactly the same way as above except for the following principal changes. First, the known interference becomes $\eta_1 = \tilde{\boldsymbol{h}}_1^\mathsf{T}\boldsymbol{v}$ and $\eta_2 = \tilde{\boldsymbol{g}}_1^\mathsf{T}\boldsymbol{u}$ and the received signal y_1 becomes $y_1 = \tilde{\boldsymbol{h}}_1^\mathsf{T}\boldsymbol{u} + \eta_1 + e_1 + \tilde{\boldsymbol{\delta}}_1^\mathsf{T}\boldsymbol{x}_1$. Second, the precoding is now based the prediction, still given by (9). Last, the parameter α, charactering the estimation error $\boldsymbol{\delta}_1$ of the current channel states in Assumption 2, now characterizes the mismatch between the estimated CSIT and the predicted one. That is, $\mathbb{E}\left(\|\tilde{\boldsymbol{\delta}}_1 - \hat{\boldsymbol{\delta}}_1\|^2\right) \sim P^{-\alpha}$, which means that, from (34) and (35), $\alpha = 1 - 2F$. Consequently, the equivalent MIMO channel (20) becomes
$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{S}}_1 \boldsymbol{W} \tilde{\boldsymbol{u}} + \begin{bmatrix} e_1 + \xi_{\Delta,1} + \tilde{\boldsymbol{\delta}}_1^\mathsf{T}\boldsymbol{x}_1 \\ -\xi_{\Delta,2} \end{bmatrix}. \tag{36}$$
Since it can be shown that $\mathbb{P}\left(|\tilde{\boldsymbol{\delta}}_1^\mathsf{T}\boldsymbol{x}_1|^2 > P^\epsilon\right) < O(P^{-\epsilon})$ from the Chebyshev's inequality, $\tilde{\boldsymbol{\delta}}_1^\mathsf{T}\boldsymbol{x}_1$ can be considered bounded as far as the DoF is concerned and thus does not affect the achievable DoF.

VI. Conclusion

We considered a practical scenario of the time-correlated MISO broadcast channel where the transmitter takes an opportunity to exploit both past (delayed) channel state and an estimate of current channel state. We proposed a novel multi-slotted strategy which enhances the degrees of freedom promised by the MAT scheme according to the quality of the current channel knowlege.

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Appendix

Lemma 1: The DoF of the multicast communication in the considered two-user MISO channel is $\mathsf{DoF}_{\mathsf{mc}} = 1$. That is, for any $\delta > 0$, there exists a code with rate $(1-\delta)\log P$, such that the average error probability goes to 0 when $P \to \infty$.

Proof: Since each receiver has only one antenna, the DoF per user for the multicast communication is upper-bounded by 1. For the lower bound, let us consider a trivial scheme in which only one transmit antenna out of m is used. The MISO BC becomes a SISO BC that is degraded. And the multicast capacity is just that of the worse user, which yields 1 as DoF as well. This can be achieved with a single-letter code. ■

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