Outage Performance of Double Stratospheric Platforms Diversity Systems

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Abstract— **High Altitude Platform Networks (HAPNs) comprise an emerging communication solution promising to exploit many of the best aspects of terrestrial and satellite-based systems. Nevertheless, while offering advantageous propagation characteristics, HAP networks are still subject to large fades due to atmospheric precipitation and especially due to rain, for the allocated high frequency bands, necessary though to deliver high data rates. Multi-HAP or orbital HAP diversity is an effective technique to reduce the large fade margins required to assure the specified quality of service (QoS). Received signals from multiple HAPs can be combined at the terminal station with the use of various signal processing techniques. The subject of this paper is to present physical statistical models for the evaluation of the outage probability of a dual-HAP orbital diversity system using either SC (selection combining) or MRC (maximal ratio combining) techniques. The analytical probabilistic models are interesting since we have rain attenuation correlated fading channels. Extended numerical results are presented in the final section of the paper comparing MRC and SC reception techniques. Some significant conclusions are drawn.**

Index Terms— *High Altitude Platform Network (HAPN); orbital diversity; rain attenuation; log-normal distribution; maximal ratio combining; selection combining ;orbital diversity.*

I. INTRODUCTION

High altitude/stratospheric platforms (HAPs) are either airships or planes that operate in the stratosphere, 17-22km above the ground [1]. Such platforms have the potential capability of being deployed rapidly, using considerably less communications infrastructure than that required if delivered by a terrestrial network. Very recently, ITU-R has allocated several frequency bands for communication services for HAP networks, L and S bands are allocated for mobile communication services and for fixed communication services, at 47/49GHz, and at 28/31GHz [2] .

At millimeter wave bands, the dominant fading mechanism is rain attenuation [2]. Many fade mitigation techniques have been proposed to increase the availability time of the millimeter wave radio systems [2] but the most efficient are the path diversity techniques. Their main drawbacks are that they are cost ineffective and need double capacity for their implementation. Nevertheless, in this paper, we examine a double diversiy scheme of a HAP Network (HAPN) from a propagation point of view. At the ground station we investigate the performance of either the Selection

Combining (SC) reception technique or the Maximal Ratio Combining (MRC) reception technique. In [4], the error performance of a selection combining site divesity configuration has been investigated. In this paper, an accurate propagation prediction model is presented for the outage probability taking into account the convective raincell model. Including the nonraining time, the lognormal distribution is adopted for the distribution of the unconditional point rainfall rate and the induced rain attenuation on the HAP links similarly with [4]. The bivariate lognormal distribution, is also assumed for the rain induced attenuation on the the two HAP link configuration. The correlation coefficient is calculated through a well-behaved physical model [5].

Finally, we present extented numerical results comparing the two techniques and investigate the impact of the system geometry on the outage probability.

II. CALCULATION OF THE OUTAGE PROBABILITIES

A. Geometrical Calculations

The system consists of two HAPs as shown in Fig.1 located at points A and B. These points are located at a height H above the Earth surface and their terrestrial projections' distance is s (length of the arc $A'B'$). The user is located at a distance d from the center O of the projections and at an angle ω which lays on the tangent surface of the point O.
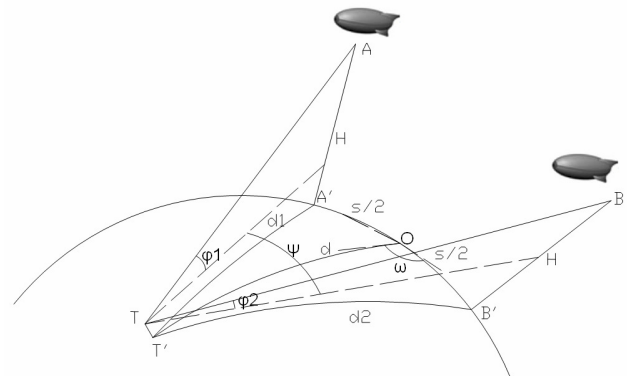

Figure 1. Double HAP System's geometry

From the spherical triangles OT'B' and OT'A' we can evaluate the distances d1 and d2:

$$d_1 = R_e \cdot arccos\left[cos(\frac{d}{R_e})cos(\frac{s}{2R_e}) + \right.$$
$$\left. + sin(\frac{d}{R_e})sin(\frac{s}{2R_e})cos(\pi - \omega)\right] \quad (1)$$

$$d_2 = R_e \cdot arccos\left[cos(\frac{d}{R_e})cos(\frac{s}{2R_e}) + \right.$$
$$\left. + sin(\frac{d}{R_e})sin(\frac{s}{2R_e})cos\omega\right] \quad (2)$$

From the spherical triangle TA'B' we can evaluate the angle Ψ formed by the projections of the radiopaths on the horizontal level at the user's location.

$$\Psi = arccos\left(\frac{cos(\frac{s}{R_e}) - cos(\frac{d_1}{R_e})cos(\frac{d_2}{R_e})}{sin(\frac{d_1}{R_e})sin(\frac{d_2}{R_e})}\right) \quad (3)$$

Given the height H of the HAPs and h of the user and having calculated the distances d_1, d_2 of the radiopaths' projections, we can evaluate the lengths L_1, L_2 of the radiopaths and the elevation angles ϕ_1, ϕ_2 of the radiopaths relative to the horizontal level.

$$L_i^2 = (R_e + h)^2 + (R_e + H)^2 -$$
$$-2(R_e + h)(R_e + H)cos(\frac{d_i}{R_e}), \quad (i = 1, 2) \quad (4)$$

and

$$\phi_i = arcsin\left[\frac{(R_e + H) \cdot cos(\frac{d_i}{R_e}) - (R_e + h)}{L_i}\right], (i = 1, 2) \quad (5)$$

The rainfall height H_r is taken from the ITU-R P.839 Recommendation [6]. Finally the effective radiopath lengths L_{s_1}, L_{s_2} are given:

$$L_{s_i} = \begin{cases} [(R_e + H_r)^2 - (R_e + h)^2 cos\phi_i^2]^{1/2} \\ -(R_e + h)sin\phi_i, \phi_i \leqslant 10° \\ \\ \frac{H_r - h}{sin\phi_i}, \phi_i > 10° \end{cases}, (i = 1, 2) \quad (6)$$

B. General Considerations

The calculation of the outage probability of a specific link i is directly related to the probability that the attenuation due to rain exceeds a certain value.

It is well known that the signal to noise ratio of a link under rain fading conditions is given [3]:

$$SNR_i = SNR_{cs,i} - A_i \text{ (dB scale)} \quad (7)$$

where the subscript denotes the clear sky conditions and A_i(dB) is the rain attenuation induced on a microwave link. We have outage when the received signal to noise ratio SNR_i falls below a certain threshold SNR_{th}, which depends on the transmission and receiver characteristics, as well as the desired QoS. In this case, when $SNR_i \leq SNR_{th}$, we have :

$$A_i = SNR_{cs} - SNR_i \geq SNR_{cs} - SNR_{th} = FM_i \quad (8)$$

where FM_i is the fade margin of the link and corresponds to the value that the received signal under clear sky conditions can be attenuated without causing the link to fail.

C. Diversity Reception with Signal Combining methods.

1) Joint Statistics.

In the case of diversity reception, the signals received by the two HAPs can be combined in various ways, so as to achieve the optimum signal-to-noise ratio and consequently improve the system's outage probability. The total outage probability of the system depends on the joint statistics of the diversity links and on the method that is used to combine the diversity signals. The signal combining methods in use consist of the choice of the most powerful signal (SC: Selection Combining) and the linear combination of the signals received from the two diversity radiopaths by choosing appropriate weighting factors. The last method is further divided depending on the weighting factors selection. MRC (Maximal Ratio Combining) is used when the final signal's power is the sum of the diversity signals' powers, so the final SNR is the sum of the two SNRs.

2) Selection Combining (SC).

In the case of combining the received by the 2 HAPs signals by using the method Selection Combining (SC), the system's outage probability P_{SC} is the probability of the event that the attenuations due to rain A_1, A_2 at the reception locations exceed simultaneously the two links' fade margins FM_1, FM_2. It can be calculated as previously presented, with the use of the joint PDF. The probability $P[A_1 \geqslant FM_1, A_2 \geqslant FM_2]$ for the effective slant radiopaths of length L_{s_1}, L_{s_2}, according to Crane's simplification [7] is equal to the probability $P[A_1 \geqslant FM_1 cos\phi_1, A_2 \geqslant FM_2 cos\phi_2]$ for the projection paths of lengths L_{d_1}, L_{d_2}. Using mathematical manipulations that can be found in [4] and [5], the selection combining outage probability is given from the following final expression:

$$P_{SC} = \frac{1}{2}\int_{u_{FM_2}}^{\infty} f_{U_2}(u_2) \cdot erfc\left(\frac{u_{FM_1} - \rho_{n12}u_2}{\sqrt{2(1 - \rho_{n12}^2)}}\right)du_2$$

(9)

where

$$U_i = \frac{\ln\left(FM_i \cos\phi_i\right) - lnA_{m_i}}{S_{a_i}}, (i=1,2)$$
(10)

In the above equations lnA_{m_i}, S_{a_i} are the the long-term rain attenuation statistical parameters of each link and can be found in terms of the statistical parameters of point rainfall rate that are given for every location in the world from ITU-R rainmaps [8] and in terms of specific attenuation parameters [9] Finally ρ_{n12} is the logarithmic correlation coefficient. Its analytical expression can be found in [5].

3) Maximal Ratio Combining (MRC).

In the case of Maximal Ratio Combining (MRC) for the received signals of the 2 HAPs, the system's outage probability P_{MRC} is the probability that the signal to noise ratio SNR of the final signal is below a certain threshold SNR_{th} that assures reliable communication.

According to the Maximal Ratio Combining method (MRC) the signal-to-noise ratio under rain fades is equal in linear scale to the sum of the signal to noise ratio of the diversity signals, that is

$$\left(\frac{S}{N}\right)_{MRC} = \left(\frac{S}{N}\right)_1 + \left(\frac{S}{N}\right)_2$$
(11)

which in logarothmic scale

$$SNR_{MRC} = 10log\left(10^{SNR_1/10} + 10^{SNR_2/10}\right)$$
(12)

or

$$SNR_{MRC} = 10log\left(10^{\frac{SNR_{cs,1}-A_1}{10}} + 10^{\frac{SNR_{cs,2}-A_2}{10}}\right)$$
(13)

If we apply Crane's simplification [7], if A_i are the rain attenuations in the projection paths then the final SNR at MRC receiver is given by:

$$SNR_{MRC} = 10log\left(10^{\frac{SNR_{cs,1}-A_1/cos\phi_1}{10}} + 10^{\frac{SNR_{cs,2}-A_2/cos\phi_2}{10}}\right)$$
(14)

The outage probability P_{MRC} is the probability

$$P_{MRC} = P(SNR_{MRC} < SNR_{th})$$
(15)

, which can also be evaluated from the joint PDF of rain attenuation variables $f_{A_1A_2}(\alpha_1, \alpha_2)$, that means:

$$P_{MRC} = \iint_A f_{A_1A_2}(\alpha_1, \alpha_2)d\alpha_1 d\alpha_2$$
(16)

whereas A the area for which the condition $SNR_{MRC} < SNR_{th}$ holds. The area is defined by the curve $SNR_{MRC} = C(\alpha_1, \alpha_2) = SNR_{th}$ and if $A_{2\infty}$ and $\alpha_1(\alpha_2)$ are the roots of the equations $C(\infty, \alpha_2) = SNR_{th}$ and $C(\alpha_1(\alpha_2), \alpha_2) = SNR_{th}$ respectively then:

$$P_{MRC} = \int_{A_{2\infty}}^{\infty} \int_{\alpha_1(\alpha_2)}^{\infty} f_{A_1A_2}(\alpha_1, \alpha_2)d\alpha_1 d\alpha_2$$
(17)

In the above integral the transformation $U_i = \frac{lnA_i - lnA_{m_i}}{S_{a_i}}, (i=1,2)$ can also be used so as to simplify the calculation to a simple integration.
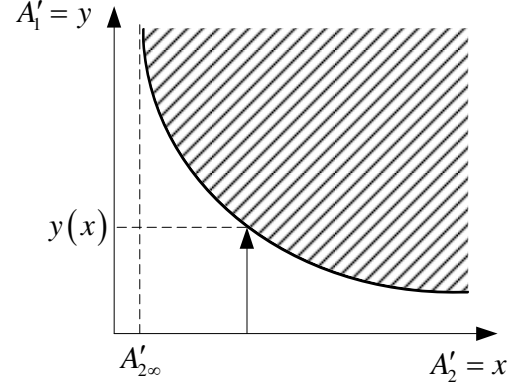

Figure 1 Region of integration in the x-y plane for calculation of outage probability of dual-branch MRC.

The value $A_{2\infty}$ which is the vertical asymptote in Fig. 2 can be easily calculated by a limiting process or algebraically.

In either case in order to evaluate the above probability in terms of the fade margins of the diversity links the transmission parameters have to be considered. The term fade margin FM_i for the individual link i hereby refers to the fade margin that the link would exhibit as if the MRC receiver was used for the link's reception with the absence of the other's branch signal. This way the analytical calculation proceeds as following: The fade margins are given by:

$$FM_i = SNR_{cs,i} - SNR_{th}$$
(18)

and $S_{cs,i}$ is the received signal power at the link i under clear sky conditions. From the link budget equation for the free space loss propagation

$$S_{cs,i} = \frac{P_{ti}G_{ti}G_{ri}\lambda^2}{(4\pi L_i)^2}$$
(19)

where P_{ti} the supply power, G_{ti} the transmission antenna's gain, G_{ri} the reception antenna's gain, λ the wavelength and L_i the radiopath's i length (user - HAP i). Substituting for readability

$$\frac{T_1}{T_2} = \frac{P_{t1}G_{t1}G_{r1}}{P_{t2}G_{t2}G_{r2}}$$
(20)

as the ratio of the transmission specific parameters we obtain

$$\frac{S_{cs,1}}{S_{cs,2}} = \frac{L_2^2}{L_1^2} \cdot \frac{T_1}{T_2}$$
(21)

and

$$\frac{\left(\dfrac{S}{N}\right)_{cs,1}}{\left(\dfrac{S}{N}\right)_{cs,2}} = \frac{L_2^2}{L_1^2} \cdot \frac{N_2}{N_1} \cdot \frac{T_1}{T_2} \qquad (22)$$

where N_1, N_2 the mean noise powers at HAPs 1,2 respectively. It is easily verified that:

$$\frac{10^{FM_1/10}}{10^{FM_2/10}} = \frac{\left(\dfrac{S}{N}\right)_{cs,1}}{\left(\dfrac{S}{N}\right)_{cs,2}} = \frac{L_2^2}{L_1^2} \cdot \frac{N_2}{N_1} \cdot \frac{T_1}{T_2} \qquad (23)$$

and

$$\frac{\left(\dfrac{S}{N}\right)_2}{\left(\dfrac{S}{N}\right)_1} = \frac{\left(\dfrac{S}{N}\right)_{cs,2} \cdot 10^{A_1/10}}{\left(\dfrac{S}{N}\right)_{cs,1} \cdot 10^{A_2/10}} = \frac{L_1^2}{L_2^2} \cdot \frac{N_1}{N_2} \cdot \frac{T_2}{T_1} \cdot \frac{10^{A_1/10}}{10^{A_2/10}} \qquad (24)$$

By using the above relations and applying simple algebraic manipulations the outage condition for the MRC system is expressed

$$10^{A_1/10} \frac{L_1^2}{L_2^2} \cdot \frac{N_1}{N_2} \cdot \frac{T_2}{T_1} \left[\frac{1}{10^{FM_2/10}} - \frac{1}{10^{A_2/10}} \right] > 1 \qquad (25)$$

The above inequality in order to be true gives $A_2 > FM_2$ (positive term in brackets). This way the outage probability is obtained by

$$A_1 > 10 log\left(\frac{L_2^2 N_2 T_1}{L_1^2 N_1 T_2} \cdot \frac{10^{A_2/10} \cdot 10^{FM_2/10}}{10^{A_2/10} - 10^{FM_2/10}} \right) \qquad (26)$$

and

$$A_2 > FM_2 \qquad (27)$$

If the rain attenuation values at the projection paths are used instead

$$A_1 > 10 log\left(\frac{L_2^2 N_2 T_1}{L_1^2 N_1 T_2} \cdot \frac{10^{A_2/10 cos\phi_2} \cdot 10^{FM_2/10}}{10^{A_2/10 cos\phi_2} - 10^{FM_2/10}} \right) cos\phi_1 \qquad (28)$$

and

$$A_2 > FM_2 cos\phi_2 \qquad (29)$$

Finally we have that the outage condition is equivalent to

$$A_2 \geq (SNR_{cs,2} - SNR_{th}) cos\phi_2 \qquad (30)$$

and

$$A_1 > SNR_{cs,1} cos\phi_1 - $$
$$-10 log\left(10^{\frac{SNR_{th}}{10}} - 10^{\frac{SNR_{cs,2} - A_2/cos\phi_2}{10}} \right) cos\phi_1 \qquad (31)$$

The above calculated lower values for A_2 and A_1 correspond to the values $A_{2\infty}$ and $\alpha_1(\alpha_2)$ previously mentioned. The outage probability can be calculated by means of the joint lognormal distribution with the use of the transformed R.V. U_1, U_2. In detail if we denote $\beta = \dfrac{N_2 T_1}{N_1 T_2}$ the ratio of the transmission and noise parameters (it can be

assumed equal to unity) the probability P_{MRC} is given by:

$$A_2 > FM_2 cos\phi_2 \Rightarrow$$
$$U_2 > \frac{ln(FM_2 cos\phi_2) - lnA_{m2}}{S_{a2}} = u_{y_1} \qquad (32)$$

$$A_1 > 10 log\left(\frac{L_2^2}{L_1^2} \beta \cdot \frac{10^{A_2/10 cos\phi_2} \cdot 10^{FM_2/10}}{10^{A_2/10 cos\phi_2} - 10^{FM_2/10}} \right) cos\phi_1 \Rightarrow$$

$$U_1 > \frac{1}{S_{a1}} \left[ln(A) - lnA_{m1} \right] = g(u_2) \qquad (33)$$

where

$$A = 10 log \frac{\dfrac{L_2^2}{L_1^2} \beta 10^{A_{m2} exp(u_2 S_{a2})/10 cos\phi_2} \cdot 10^{FM_2/10} cos\phi_1}{10^{A_{m2} exp(u_2 S_{a2})/10 cos\phi_2} - 10^{FM_2/10}} \qquad (34)$$

and, finally we have:

$$P_{MRC} = P[U_1 \geqslant g(u_2), U_2 \geqslant u_{y_1}] =$$
$$\frac{1}{2} \int_{u_{y_1}}^{\infty} f_{U_2}(u_2) \cdot erfc\left(\frac{g(u_2) - \rho_n u_2}{\sqrt{2(1-\rho_n^2)}} \right) du_2 \qquad (35)$$

III. NUMERICAL RESULTS AND DISCUSSION

This section will present some numerical evaluations of the final analytical expressions. Table I summarizes the parameters that are used.

TABLE I.
DUAL-BRANCH DIVERSITY PARAMETERS

Location	Parameters			
	Fading Characteristics	Long-term Lognormal Rainfall Rate Parameters		System Frequency	Ground distance of HAPs
Athens, Greece	Joint lognormal fading distrib.	R_m, S_r	0.0315, 1.6877	31 GHz	100km

Figure 3 depicts the SC and MRC performance in terms of outage probability versus SNR threshold for a receiver located in Athens, Greece. The curves correspond to a ground receiver located in a direction vertical to the direction of the 2 HAPs at a distance of 100km. The curve for single branch (single HAP system) is also given for comparison. It can be seen that a MRC receiver outperforms the single-branch receiver by around 5 dB for low threshold values and at about 3dB for higher threshold values (low fade margins). When we compared to a SC receiver, the MRC technique has an advantage varying from 1dB to 3dB depending on the threshold. In links with low fade margin the efficiency of MRC is significantly better than that of SC, as the last one converges to that of the single branch. It can thus be concluded that, the MRC technique has a significant advantage over both SC and single-branch receivers.

Figure 4 investigates the impact of the distance between the 2 HAPs in the case of MRC. As it is expected, the greater

the angular opening between the two branches the better the efficiency of the diversity scheme. This applies for SC as well. This factor should be considered in the decision of the spacing between HAPs in a HAPN, as well as any other parameters that relate to the selected architecture, such as coverage area and the minimum elevation angle at the user's terminal.
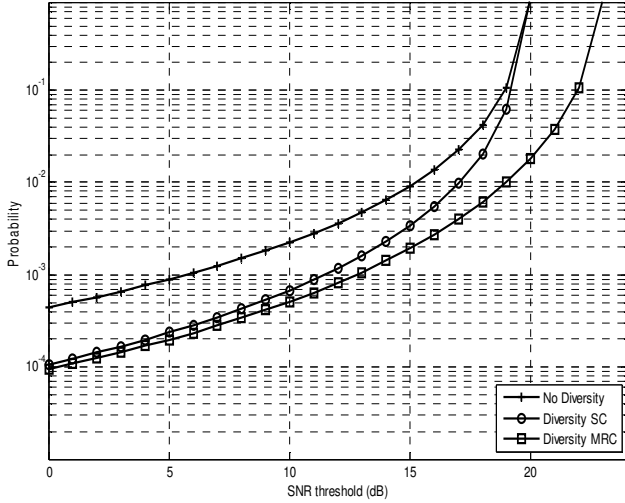

Fig. 3 MRC vs. SC and Single branch for f= 31GHz, ω: 90° (direction vertical to HAPs connecting line) and ground distance 100km
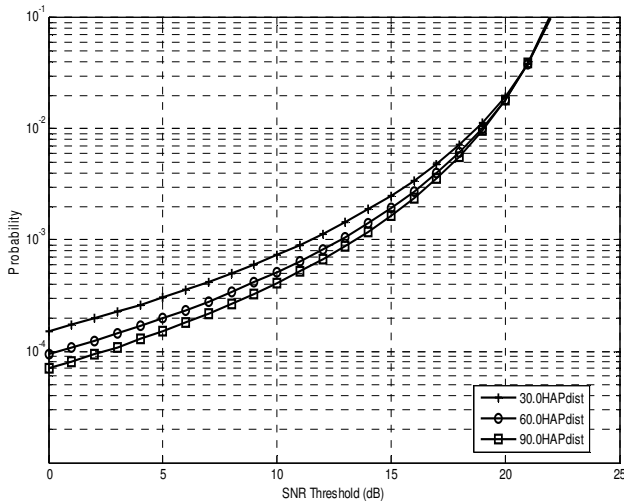

Fig. 4 MRC outage probabilities for HAP ground distances of 30, 60 and 90km

IV. CONCLUSION AND FUTURE WORK

HAP Networks are a potential solution to several communication scenarios providing global connectivity. Techno-economic reasons make them more suitable for applying diversity schemes not only at site but also orbital. In this paper we have presented analytical models for the calculation MRC and SC outage probabilities. The double stratospheric system under consideration may be seen as a 2x2 MIMO system and important capacity calculations can be presented. This is a subject of future work. MRC superiority has been shown and the performance sensitivity in terms of the separation of the two HAPs has also been investigated.

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