Sparse Precoding in Multicell MIMO Systems

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***Abstract*—In this work[1], we consider the joint precoding across K distant transmitters (TXs) towards K single-antenna receivers (RXs) and we let the TXs have access to perfect Channel State Information (CSI). Instead of considering the conventional method of clustering to allocate the user's data symbols, we focus on determining the most efficient symbol sharing patterns. Consequently, we optimize directly the user's data symbol allocation subject to a constraint on the total number of user's data bits transmitted through the core network. We develop a novel approach whereby *sparse precoders* approximating the true precoders are computed. These precoders require only a fraction of the overall symbol sharing overhead while introducing only limited losses. Thereby, allocating the symbols only to their non-zero coefficients leads to very efficient symbol sharing (or *routing*) algorithms. Furthermore, these algorithms have a much lower complexity that conventional approaches. By simulations, we show that our approach outperforms clustering-based multicell MIMO methods from the literature and that the routing obtained is mainly dependent on the pathloss structure and can be applied using only long term CSI with reduced losses.**

I. Introduction

Network or Multicell MIMO methods (or *CoMP* in the 3GPP terminology), whereby multiple interfering transmitters (TXs) share user's messages and allow for joint precoding, are currently considered for next generation wireless networks [1]. With perfect message and Channel State Information (CSI) sharing, the different TXs can be seen as a unique virtual multiple-antenna array serving all receivers (RXs), in a multiple-antenna broadcast channel (BC) fashion.

Yet, the sharing of the data symbols and the CSI to the cooperating TXs impose huge requirements on the architecture, particularly as the number of cooperating TXs increases. The common solution is to use disjoint clusters of small size to reduce the amount of data to be shared. Conventional architectures consist in static clusters [2], [3] but finding efficient clustering methods has recently received more attention following the growing interest on cooperation between the TXs in interference limited wireless networks. In [4] a framework is developed to optimize how the users are served by the TXs inside the clusters. An optimized greedy clustering algorithm is derived in [5] while a decentralized approach based on cooperation between neighbors is proposed in [6].

Yet, forming disjoints clusters limits severely the performance. In [7], a scheme is presented where each RX chooses the set of TXs serving him such that overlapping clusters are formed. Yet, the design of the clusters is not optimized and the set of TXs is selected based only on simple heuristics. Finally, in [8]–[10], the impact of partially sharing the user's data is analytically studied for one dimensional Networks.

Instead, we circumvents the drawbacks of clustering by optimizing directly the user's data allocation subject to a constraint on the total number of user's data symbols routed [11]. Note that the knowledge of the CSI for the whole multiuser channel is also necessary at the TXs to apply joint precoding. This represents a strong requirement for large cooperation areas and a method reducing the CSI sharing necessary for joint precoding is proposed in a companion paper [12].

In this work, we exploit the natural *approximate sparsity* property of the total multiuser channel matrix to obtain a *sparse approximate inverse* (SPAI). This SPAI can then be used to transmit the signal and its sparse structure means that only a limited amount of sharing of data symbols across the TXs is required, thus providing a routing solution which outperforms alternative solution from the literature.

II. System Model

A. Multicell MIMO Channel

We consider a joint downlink transmission from K TXs to K RXs using joint linear precoding and single user decoding. In this work, the TXs and the RXs have only one antenna so that the transmission can be mathematically described as

$$\begin{bmatrix} y_1 \\ \vdots \\ y_K \end{bmatrix} = \mathbf{H}\boldsymbol{x} + \boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{x} \\ \vdots \\ \boldsymbol{h}_K^{\mathrm{H}}\boldsymbol{x} \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_K \end{bmatrix} \tag{1}$$

where y_i is the signal received at the i-th RX and $\boldsymbol{h}_i^{\mathrm{H}} \in \mathbb{C}^{1\times K}$ the channel from the K TXs to RX i. The noise is represented by $\boldsymbol{\eta} \triangleq [\eta_1, \ldots, \eta_K]^{\mathrm{T}}$ and is zero mean i.i.d. complex circularly symmetric Gaussian of variance σ^2 ($\mathcal{CN}(0, \sigma^2)$).

The transmitted signal $\boldsymbol{x} \in \mathbb{C}^{K\times 1}$ is obtained from the vector of transmit user's data symbols $\boldsymbol{s} \triangleq [s_1, \ldots, s_K]^{\mathrm{T}} \in \mathbb{C}^{K\times 1}$ (whose entries are assumed to be independent $\mathcal{CN}(0,1)$) as

$$\boldsymbol{x} = \mathbf{T}\boldsymbol{s} = \begin{bmatrix} \boldsymbol{t}_1 & \ldots & \boldsymbol{t}_K \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} \tag{2}$$

where $\mathbf{T} \in \mathbb{C}^{K\times K}$ is the precoder and $\boldsymbol{t}_i \in \mathbb{C}^{K\times 1}$ is the beamforming vector transmitting s_i. Noting that in a statistically symmetric isotropic network, fulfilling a sum power constraint will lead to an equal power used per TX

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and therefore the fulfillment of the induced per TX power constraint, we consider for simplicity a sum power constraint. We also assume that all data streams are allocated with an equal amount of power so that $\|\boldsymbol{t}_i\|^2 = P$.

The channel is block fading and models a Rayleigh fading scenario with a long term pathloss corresponding to a cellular setting. Thus, the entries of the channel matrix $\mathbf{H}$ read as $\{\mathbf{H}\}_{ij} = \gamma_{ij} G_{ij}$ where G_{ij} is i.i.d. $\mathcal{CN}(0,1)$ to model the short term fading, and γ_{ij} is a positive real number modeling the long term attenuation. We consider large networks with many TX/RX pairs distributed over a large area so that many elements of the multiuser channel are very small due to the decay of received power with the distance (i.e., many long term attenuation coefficients γ_{ij} will be very small).

The presence of many small coefficients can be interpreted as a form of *approximate sparsity* in $\mathbf{H}$. In fact, if a thresholding operator is applied to the channel matrix, many coefficients corresponding to interference links originating from distant TXs could be approximated as zeros, leading to a truly sparse representation of the channel. Although the sparsity in $\mathbf{H}$ can give an advantage in itself by suggesting a simple feedback reduction scheme (zeros are simply not reported), a more challenging question is whether sparsity in the channel domain can lead to a sparse behavior for the precoding domain, as sparsity in the precoding domain has a direct impact in terms of reducing the symbol sharing overhead over the backhaul. This is precisely the focus of this paper.

B. System Performance Model

In this work, we aim at achieving the maximum sum throughput, with the throughput for RX i equal to

$$R_i \triangleq \log_2\left(1 + \frac{|\boldsymbol{h}_{\boldsymbol{i}}^{\mathrm{H}} \boldsymbol{t}_i|^2}{\sigma^2 + \sum_{\ell \neq i} |\boldsymbol{h}_{\boldsymbol{i}}^{\mathrm{H}} \boldsymbol{t}_\ell|^2}\right). \tag{3}$$

Intercell interference is recognized as one of the main challenges for the future wireless networks which will be designed to work at intermediate-high SNR. Thus, the optimal precoders should emit little interference, and we can consider Zero-Forcing (ZF) precoding schemes with a reasonably small suboptimality. ZF becomes also optimal at high SNR.

Thus, we consider the ZF criterion and we replace in our optimization problem the maximization of the sum rate by the minimization of the emitted interference, i.e., the Frobenius norm $\|\mathbf{HT} - \mathbf{I}_K\|_{\mathrm{F}}$. This leads to a more simple optimization for which we will be able to derive efficient solutions.

C. Routing Matrix

In this work, we consider that the CSI for the multiuser channel is perfectly known at all TXs and RXs, and we focus on the problem of maximizing the system performance while routing the minimum numbers of user's data symbols to the TXs. In previous approaches, the reduction of the data symbol sharing overhead was based on the concept of clustering where different groups of TXs are determined in order to serve groups of users. A fundamental issue in clustering comes from the edge effects causing inter-cluster interference to edge-of-cluster users. To circumvent the edge effects, we optimize the user's data symbol allocation under a realistic constraint on the total number of symbols shared, which we denote by r^*, independently of any pre-determined cluster concept.

To represent the allocation of user's data to the TXs, we define the *Routing Matrix* $\mathbf{D} \in \{0,1\}^{K \times K}$ as the matrix whose (i,j)-th element is 1 if symbol s_j is allocated to TX i and 0 otherwise. We also define $\mathbf{D}_{(\cdot,i)}$ to denote the i-th column of $\mathbf{D}$ (i.e., the allocation corresponding to s_i). The constraint on the number of user's data symbol shared can then be seen to be $\|\mathbf{D}\|_{\mathrm{F}}^2 \leq r^*$. The goal of this paper is to find the routing matrices which lead to the best performance while still fulfilling a predetermined level of sparsity r^*.

The first step of the optimization is then to derive the optimal precoder for given routing matrices, as conventional precoding cannot be used directly.

D. Precoder Optimization for a Given Routing Matrix

We now consider the routing matrix $\mathbf{D}$ to be given and we derive the optimal precoder $\mathbf{T}$ in that case. Note that each beamforming vector can be derived independently due to the ZF constraints and the per-stream power allocation. If one TX does not receive one symbol, it can not participate into the transmission of that symbol and the coefficient used for that beamformer at that TX is then 0. Therefore, the effective precoder is constrained to bear the form of the Hadamard (element wise) product $\mathbf{D} \odot \mathbf{T}$. The beamforming vector $\boldsymbol{t}_i$ transmitting symbol s_i to RX i is then obtained from:

$$\underset{\boldsymbol{t}_i(\mathbf{D}_{(\cdot,i)})}{\text{minimize}} \|\mathbf{H}\left(\mathbf{D}_{(\cdot,i)} \odot \boldsymbol{t}_i(\mathbf{D}_{(\cdot,i)})\right) - \boldsymbol{e}_i\|_2^2. \tag{4}$$

We start by introducing some necessary notations. We denote by $\mathcal{J}$ the set of indices such that $\mathbf{D}_{(\cdot,i)} \odot \boldsymbol{t}_i(\mathbf{D}_{(\cdot,i)}) \neq 0$ and we define the reduced beamforming vector containing only the nonzero elements as $\tilde{\boldsymbol{t}}_i(\mathcal{J}) \in \mathbb{C}^{n_2 \times 1}$ with $n_2 = |\mathcal{J}|$. Then, $\mathcal{I}$ denotes the set of indices i such that $\mathbf{H}(i, \mathcal{J})$ is not identically zero and the resulting reduced channel without the rows and the columns identically zero, is $\mathbf{H}(\mathcal{I}, \mathcal{J}) \in \mathbb{C}^{n_1 \times n_2}$ where $n_1 = |\mathcal{I}|$. We also define $\mathbf{H}(\cdot, \mathcal{J}) \in \mathbb{C}^{K \times n_2}$ to denote the channel with the reduced number of columns but all the rows. Finally, we give the QR decomposition of $\mathbf{H}(\mathcal{I}, \mathcal{J})$:

$$\mathbf{H}(\mathcal{I}, \mathcal{J}) = \mathbf{Q}(\mathcal{I}, \mathcal{J}) \begin{bmatrix} \mathbf{R}(\mathcal{I}, \mathcal{J}) \\ \mathbf{0}_{(n_1 - n_2) \times n_2} \end{bmatrix} \tag{5}$$

where $\mathbf{R}(\mathcal{I}, \mathcal{J}) \in \mathbb{C}^{n_2 \times n_2}$ is an upper triangular matrix and $\mathbf{Q}(\mathcal{I}, \mathcal{J}) \in \mathbb{C}^{n_1 \times n_1}$ is an orthonormal matrix. With these definitions, the objective in (4) can be rewritten as

$$\|\mathbf{H}\boldsymbol{t}_i(\mathbf{D}_{(\cdot,i)}) - \boldsymbol{e}_i\|_2^2 = \|\mathbf{H}(\mathcal{I}, \mathcal{J})\boldsymbol{t}_i(\mathcal{J}) - \boldsymbol{e}_i(\mathcal{I})\|_2^2 \tag{6}$$

which is a regular Least Square problem. The expression for the precoder solving (4) follows then easily and we give it here for the sake of completeness.

Proposition 1. *The reduced beamforming vector associated with the sparse beamformer solving* (4) *reads as*

$$\boldsymbol{t}_i(\mathcal{J}) = \sqrt{P}\frac{\left[\mathbf{R}^{-1}(\mathcal{I},\mathcal{J}) \quad \mathbf{0}_{n_2\times(n_1-n_2)}\right]\mathbf{Q}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\boldsymbol{e}_i(\mathcal{I})}{\left\|\left[\mathbf{R}^{-1}(\mathcal{I},\mathcal{J}) \quad \mathbf{0}_{n_2\times(n_1-n_2)}\right]\mathbf{Q}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\boldsymbol{e}_i(\mathcal{I})\right\|}.$$

E. Optimization of the User's Data Allocation

We can now formulate the optimization problem of interest:

$$\underset{\mathbf{D}\in\{0,1\}^{K\times K}}{\text{minimize}} \ \|\mathbf{H}\left(\mathbf{D}\odot\mathbf{T}(\mathbf{D})\right)-\mathbf{I}_K\|_{\mathrm{F}}^2, \text{ s.t. } \|\mathbf{D}\|_{\mathrm{F}}^2 \leq r^* \quad (7)$$

with r^* the maximal number of user's data symbol shared and the function $\mathbf{T}(\mathbf{D})$ which gives the precoding matrix for given routing matrix as described in Subsection II-D.

This optimization problem is combinatorial and hence very hard to solve optimally. However, this optimization consists in fact in finding a sparse inverse of the approximately sparse channel matrix and we will be able to exploit this sparsity.

III. Theory of Sparse Approximate Inverses

The inverse of a sparse matrix is a priori not sparse and it is proved in [13] that the inverse is full (without considering coincidental cancellations) if and only if the channel matrix is irreducible. This will be the case in all practical cases and it is hence necessary to share all the symbols to obtain a *perfect* inversion of the channel. Yet, it is known that the inverse of a sparse matrix has many weak elements. This is shown analytically for band limited matrices in [14] and is expected to hold for all sparse matrices. This theoretical analysis means in practice that it should be possible to find sparse precoding matrices which approximate accurately the ZF precoder. These SParse Approximate Inverse (SPAI) matrices could then be used as precoders achieving close to perfect ZF with only a fraction of the data symbol sharing overhead, thus providing efficient solutions to the optimization problem (7).

A. Thresholding of the Channel Inverse

The simplest approach to introduce some sparsity in the channel inverse consists in thresholding the inverse to remove the weakest coefficients until the required degree of sparsity is reached. This method has a extremely low complexity as it requires to invert the channel matrix only once and then do a bisection search over the value of the threshold until the required sparsity is reached. Note that the thresholding of the inverse provides a routing solution, but the precoder is then obtained by computing a new inverse as described in Subsection II-D for the sparsity pattern obtained. This improves the performance of the precoder as the precoding coefficients are then more adapted to the final sparsity pattern.

This intuitive approach brings some compromise between the sparsity of the inverse and the quality of the channel inversion. However, it is natural to ask oneself: *Is it possible to improve on the naive thresholding solution and optimize the routing matrix and the precoder at the same time?*

B. SPAI-Algorithm

We will make use of an existing mathematical literature deriving *Sparse Approximate Inverse*. Indeed, the problem of finding SPAI to large sparse matrices has been studied to obtain preconditionners for iterative algorithms [15]–[17].

We use in the following the *the SParse Approximate Inverse (SPAI)* algorithm [17] which has the advantage of having a version available online with an interface to MATLAB and to be efficient on matrices of relatively small size.

Note that the algorithm deals with real matrices, so that we need to consider the canonical isomorphism between the complex field and the real field. The transmission is then :

$$\begin{bmatrix}\Re(\boldsymbol{y})\\ \Im(\boldsymbol{y})\end{bmatrix} = \begin{bmatrix}\Re(\mathbf{H}) & -\Im(\mathbf{H})\\ \Im(\mathbf{H}) & \Re(\mathbf{H})\end{bmatrix}\begin{bmatrix}\Re(\mathbf{T}) & -\Im(\mathbf{T})\\ \Im(\mathbf{T}) & \Re(\mathbf{T})\end{bmatrix}\begin{bmatrix}\Re(\boldsymbol{s})\\ \Im(\boldsymbol{s})\end{bmatrix} + \begin{bmatrix}\Re(\boldsymbol{\eta})\\ \Im(\boldsymbol{\eta})\end{bmatrix}.$$

We describe now the algorithm for the problem of finding the vector $\boldsymbol{t}_i$ minimizing $\|\mathbf{H}\boldsymbol{t}_i - \boldsymbol{e}_i\|$ where the matrix $\mathbf{H}$ is real valued. We do the abuse of notation of keeping the same notation for both the complex and the real versions.

The SPAI-algorithm is an iterative greedy algorithm in which new non-zero elements are added to the approximate inverse at each step, so as to minimize the objective norm. We give a brief description and we refer to [17] for more details.

To initialize the algorithm, we take the diagonal elements as the only nonzero elements. This is meaningful as it means that, for each RX, at least the TX in the cell of the RX receives the symbol and that all the users have to be served.

For a given routing matrix $\mathbf{D}$, the precoder can be derived as described in Subsection II-D. We can then compute the residual error which represents the interference created at the other RXs for the given beamforming vector. The residual error is denoted by $\boldsymbol{r}_i$ and is given by

$$\boldsymbol{r}_i \triangleq \mathbf{H}(\cdot,\mathcal{J})\boldsymbol{t}_i(\mathcal{J}) - \boldsymbol{e}_i. \quad (8)$$

The routing matrix is then increased so as to minimize the residual error $\boldsymbol{r}_i$. We denote by $\mathcal{L}$ the set of nonzero elements of $\boldsymbol{r}_i$. To this set $\mathcal{L}$ corresponds a column set with nonzero elements in these rows which we denote by $\mathcal{N}_{\mathcal{L}}$. The set $\mathcal{N}_{\mathcal{L}}$ contains all the indices which can potentially lead to a reduction of the residual error. For each element j in that set, we consider then the one dimensional problem

$$\underset{\mu_j}{\text{minimize}} \ \|\boldsymbol{r_i} + \mu_j\mathbf{H}\boldsymbol{e}_j\|_2 \quad (9)$$

from which the solution is

$$\mu_j = -\frac{\boldsymbol{r}_i^{\mathrm{T}}\mathbf{H}\boldsymbol{e}_j}{\|\mathbf{H}\boldsymbol{e}_j\|_2^2} \quad (10)$$

and which leads to the new residual error

$$\rho_j^2 = \|\boldsymbol{r}_i\|_2^2 - \frac{(\boldsymbol{r}_i^{\mathrm{T}}\mathbf{H}\boldsymbol{e}_j)^2}{\|\mathbf{H}\boldsymbol{e}_j\|_2^2}. \quad (11)$$

The indices bringing the largest reduction in the residual error are kept and added to the sparsity pattern of the inverse. The residual error can then be again obtained after having computed the precoder for the increased sparsity pattern according to Subsection II-D. The algorithm continues until the sparsity constraint is reached.

This algorithm makes use of the sparsity pattern to reduce the complexity of computing the inverse. Indeed, the method described needs to be done for each column and requires computing at most s Least Square problems where s is the number of nonzero elements per column in the channel matrix, such that the complexity is of the order of $O(Ks^3)$ when the iterative update of the QR decomposition is used to reduce the complexity [17]. In the practical cellular settings considered, the matrices are of relatively small sizes so that thresholding the channel is not necessary, however this could be considered for larger matrices to reduce the complexity as well as potentially even improve the performance [16].

IV. Adaptation to Precoding in Wireless Networks

We have provided two approaches to derive a sparse precoder inverting the channel. Yet, we need to adapt these methods to the constraints of practical transmission settings.

A. SPAI Based on Long Term Information

Routing the user's data symbol based on instantaneous channel realizations is extremely demanding for the backhaul architecture. Indeed, the routing of the user's data symbols is made in the core network which introduces significant delay (sharing of the CSI, determination of the routing, sharing of the symbols,...). Thus, determining the routing based on the long term statistics of the channel appears more realistic.

Consequently, we consider a routing using SPAI based on long term CSI where the routing matrix using the SPAI-algorithm is computed only once for each random generation of a user, i.e., it depends only on the user's positions but not on the fast fading realization. The routing matrix obtained is then kept for all the following fast fading realizations while the precoder is obtained based on the instantaneous CSI available at the TX, as described in Subsection II-D.

We will show by simulations that the SPAI routing using only long term information still performs close to the SPAI routing using instantaneous CSI. This is due to the fact that the structure of the inverse is mainly dependent on the sparsity structure of the channel and the amplitude of its coefficients.

B. Regularized Zero-Forcing

In Section III, we have considered ZF precoders inverting the channel because this allowed us to use the well developed mathematical literature on the subject. Yet, conventional ZF is optimal only at high SNR and performs well at finite SNR only when the matrix is well conditioned. Indeed, the effective channel gains are equal to the inverse of the Frobenius norm of the channel inverse:

$$\mathbf{H}\mathbf{T} = \frac{KP}{\|\mathbf{H}^{-1}\|_{\mathrm{F}}}\mathbf{I}_K = KP\left(\sqrt{\sum_{k=1}^{K}\frac{1}{\sigma_k^2}}\right)^{-1} \tag{12}$$

where the σ_k's are the singular values of the channel matrix. When the channel matrix grows large, it is well known that the minimal eigenvalue of a square channel in Rayleigh fading tends to zero, thus the ZF scheme will achieve a vanishing rate per user as the size of the network increases. To avoid this behavior, we consider the conventional regularized ZF precoder, also called *Transmit Wiener filter* which minimizes the mean square error and reads as [18]:

$$\mathbf{W} = \frac{\sqrt{KP}}{\|\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}+\frac{\sigma^2}{P}\mathbf{I}\right)^{-1}\mathbf{H}^{\mathrm{H}}\|_{\mathrm{F}}}\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}+\frac{\sigma^2}{P}\mathbf{I}\right)^{-1}\mathbf{H}^{\mathrm{H}}. \tag{13}$$

Adapting the SPAI-algorithm to regularized ZF requires modifying significantly the algorithm but seems tractable and could be done in future works. Yet, this is not in the scope of this work and we consider a much easier solution which consists in simply modifying the precoder which is applied once the routing matrix is given.

Keeping the notations of Subsection II-D, we define the *extended channel* $\bar{\mathbf{H}}(\mathcal{I},\mathcal{J})$ and its *QR decomposition* as

$$\bar{\mathbf{H}}(\mathcal{I},\mathcal{J}) = \begin{bmatrix}\mathbf{H}(\mathcal{I},\mathcal{J})\\ \sqrt{\frac{\sigma^2}{P}}\end{bmatrix} = \bar{\mathbf{Q}}(\mathcal{I},\mathcal{J})\begin{bmatrix}\bar{\mathbf{R}}(\mathcal{I},\mathcal{J})\\ \mathbf{0}\end{bmatrix} \tag{14}$$

such that

$$\mathbf{H}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\mathbf{H}(\mathcal{I},\mathcal{J}) + \frac{\sigma^2}{P}\mathbf{I} = \bar{\mathbf{R}}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\bar{\mathbf{R}}(\mathcal{I},\mathcal{J}). \tag{15}$$

Using the definition of the extended channel, the regularized ZF precoder can be obtained following the same method as for the conventional ZF precoder in Proposition 1.

Proposition 2. *The generalized ZF beamformer $\boldsymbol{t}_i(\mathbf{D}_{(\cdot,i)})$ for a routing matrix $\mathbf{D}$ has its non-zeros elements indexed by the set $\mathcal{J}$ given by*

$$\boldsymbol{t}_i(\mathcal{J}) = \sqrt{P}\frac{\begin{bmatrix}\bar{\mathbf{R}}^{-1}(\mathcal{I},\mathcal{J}) & \mathbf{0}_{n_2\times(n_1-n_2)}\end{bmatrix}\bar{\mathbf{Q}}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\boldsymbol{e}_i(\mathcal{I})}{\left\|\begin{bmatrix}\bar{\mathbf{R}}^{-1}(\mathcal{I},\mathcal{J}) & \mathbf{0}_{n_2\times(n_1-n_2)}\end{bmatrix}\bar{\mathbf{Q}}^{\mathrm{H}}(\mathcal{I},\mathcal{J})\boldsymbol{e}_i(\mathcal{I})\right\|}.$$

V. Simulations

We simulate a multicell networks with K single antenna TXs and RXs. We consider the propagation parameters of the LTE cellular network for the Hata urban scenario path loss model and we average over 20 uniform random generations of the users in the cells and 20 fast fading channels realizations. Our parameters give an attenuation between a TX and a RX located at a distance d equal to $-114.5 - 37.19\log_{10}(d)$ dB with d in km and the antennas gains already taken into account. The transmit power per TX is $P = 43$ dBm, the noise power is $\sigma^2 = -101$ dBm, and the radius of the cell $R = 1.5$ kms, which translates to a SNR without short term fading for an edge user equal to 25 dB.

We compute the performance obtained with the naive thresholding [Cf. Subsection III-A], with the SPAI based on instantaneous CSI [cf. Subsection III-B] and with only long term CSI [Cf. Subsection IV-A]. We compare these schemes to the greedy clustering algorithm from [5]. For all the schemes, the Regularized ZF described in Subsection IV-B is used once the routing matrix is obtained.

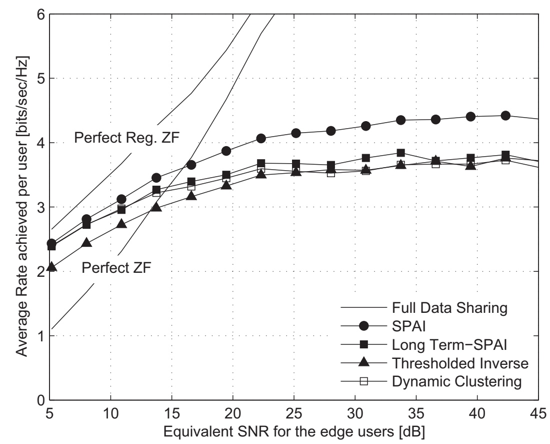

Fig. 1. Average rate per user in terms of the edge-SNR for a percentage of data symbol shared equal to 25% in a network with 19 TX/RX pairs.

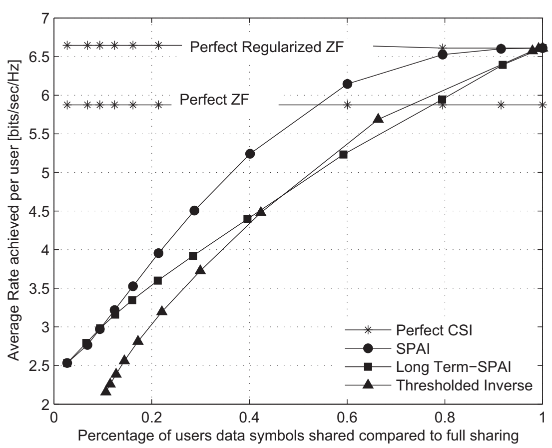

Fig. 2. Average sum rate achieved with full cooperation in terms of the percentage of data symbol shared for a transmit power per TX equal to 43 dBm in a network with 37 TX/RX pairs.

In Fig. 1, we show the average rate per user achieved in terms of the reference SNR for an edge user. The percentage of user's data symbols allocated is equal to 25% of the full sharing consisting of K^2 symbols shared. For dynamic clustering, this is obtained with 4 clusters of 4 and one cluster of 5. The SPAI routing outperforms the other solutions, and particularly the dynamic clustering one. The routings based on long term SPAI and using the thresholded inverse perform similarly to the dynamic clustering. This comes from the relatively small number of cells and the SPAI routings perform better as the size of the network increases.

In Fig. 2, we consider the same setting only with $K = 37$ cells and we plot the average rate per user for each of the schemes in terms of the percentage of user's data allocated. The SPAI-algorithm achieves the best performance over the whole range of sharing, while the version based on long term CSI remains efficient, particularly at low percentage of sharing.

VI. Conclusion

In this work, we have provided an alternative to clustering where the user's data symbol allocation is *directly* optimized instead of allocating the symbols to form disjoint clusters. Particularly, we show that the multiuser channel in large networks could be seen as approximately sparse, and that this could be exploited to derive efficient *sparse* precoders which require therefore only a reduced user's data symbol allocation. We have discussed sparse approximate inverse and the extension to arbitrary *sparse precoding schemes* will be the focus of future works. Furthermore, we believe that sparse precoding could be developed for many other settings as for example with multiple-antenna receivers where Interference Alignment has to be considered.

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