SINR Balancing and Beamforming for the MISO Interference Channel

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***Abstract*—In this paper a K user multi-input single-output (MISO) interference channel (IFC) is considered where the interference at each receiver is treated as an additional Gaussian noise contribution (*Noisy IFC*). We address the MISO downlink (DL) beamformer design and power allocation for maximizing the minimum SINR with per base station power constraints and imposing a minimum quality of service (QoS) requirement for each receiver. We study a distributed iterative algorithm for solving the given beamforming problem based on a combination of duality principles and the property that $\max\min SINR$ problem is strictly related to the total power minimization problem. Finally we show that it is possible to characterize the entire Pareto boundary of the SINR (Rate) region for a K-user MISO IFC solving a sequence of $\max\min SINR$ imposing different set of QoS constraints.**

I. Introduction

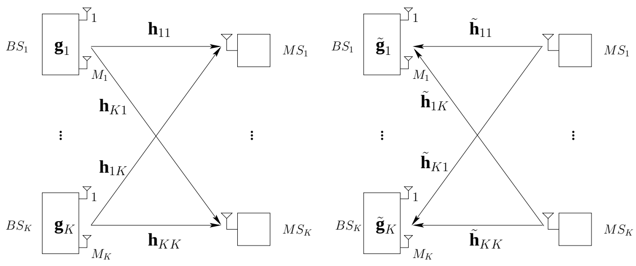
In modern cellular systems a frequency reuse factor of 1 is considered to optimally exploit all spectral resources across the network. The throughput of such systems however is seriously affected by the inter-cell interference that is commonly identified as the major bottleneck of modern wireless communication systems. This consideration has led to intense research, one outcome of which is to somehow curtail interference in cell edge areas. Network operators and manufacturers have lately pushed for coordination and interference management techniques as policing strategies for cell-edge spectrum use. Other techniques where multiple cell signals are used to serve cell-edge users are also being studied. These methods often relying on collaboration between cell towers through backhaul links and joint processing of signals change the nature of cellular communications and represents a significant paradigm shift.

For the purposes of this work we consider cooperation between cell towers up to the point of beamformer design. The underlying problem remains that of inter-cell interference and is mathematically described as a K-user interference channel (IFC) where pairs of users want to communicate between each other without exchanging (data) information with non-intended pairs. Interference at each user is treated as additional Gaussian noise contribution and hence linear beamforming processing is optimal. This, in the information theoretic sense, is the *noisy* interference channel.

This paper addresses the $\max\min$ SINR problem. This beamforming problem formulation satisfies a fairness requirement because at the optimum all the SINRs are equal, for this reason it is also called SINR balancing problem. Balancing the SINR implies that the system performances are limited by the weak users causing a reduction of the overall sum rate. This problem has been extensively studied in [1] for single cell Broadcast (BC) channel under the sum power constraint using the well-established tool of UL-DL duality [2]. In [3] the authors propose a similar algorithm to solve the same problem as in [1]. The multicell problem, that we call the IFC in this paper is more complex to handle due to the per-user (per BS) power constraints. [4] addresses duality in a similar setting which the authors call the multicell setting where previous results on interpretation of UL-DL duality as Lagrangian duality are exploited. [4] then solves the power minimization problem subject to Quality of Service (QoS) constraints and per base station power constraints formulated as weighted total transmit power. The SINR balancing problem in the MISO IFC has been studied, under general power constraints, in a recent paper [5] where only power optimization has been considered. In [6] the authors studied the beamforming design problem for SINR balancing in the MISO IFC under per base station power constraints proposing an iterative algorithm that solves the problem in a centralized fashion. In this paper we are interested in the SINR balancing problem for a MISO IFC with individual power constraints and we propose an iterative algorithm that solves the problem in a decentralized manner. Our solution is based on the relation between the SINR balancing problem and the power minimization problem underlined in [6]. We solve the $\max\min SINR$ problem using a sequence of power minimization problems where the QoS constraints in the beamforming problem are increased gradually until an infeasible point is found. Then, using bisection method, the optimal solution is determined. In the MISO IFC with per user power constraints, a subset of users always transmits with full power according to the antenna and user distribution in the system. We propose an iterative algorithm that solves the max min SINR problem for systems where only one user transmits with full power. In systems where the MISO IFC is separable, it can be shown that all users transmitting with full powers maximizes the minimum SINR.

Finally we show that is possible to characterize the entire Pareto boundary of the SINR region for a general K-user MISO IFC solving a sequence of *Weighted SINR* (WSINR) problems. Thanks to the one to one logarithmic relation between SINR and Rate we can then characterize the Pareto boundary of the Rate region for a general K-user MISO IFC. The basis of this characterization has been studied in [7] for a single-input single-output (SISO) IFC. Here we extend their results to the MISO IFC.

II. IFC System Model



a) MISO DL b) SIMO UL

Fig. 1: System Model

Fig. 1 depicts a K-user MISO IFC with K transmitter-receiver (TX-RX) pairs. This setting is relevant in the case of a network of femtocell base stations (BS) where each femtocell BS is serving a single user in the time-frequency unit of interest. The k-th BS is equipped with M_k transmitter antennas and k-th mobile station (MS) is a single antenna node. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the received signal y_k at the k-th receiver can be represented as

$$y_k = \mathbf{h}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{h}_{kl}\mathbf{x}_l + n_k \quad (1)$$

where $\mathbf{h}_{kl} \in \mathbb{C}^{1\times M_l}$ represents the channel vector between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k\times 1}$ transmit signal vector of the k-th transmitter and n_k represents (temporally white) AWGN with zero mean and variance σ_k^2. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution.

We denote by $\mathbf{g}_k$, the $\mathbb{C}^{M_k\times 1}$ beamforming (BF) vector of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{g}_k s_k$, where s_k represents the independent symbol for the k-th user pair. We assume s_k to have a temporally white Gaussian distribution with zero mean and unit variance. In the SIMO UL channel the k-th BS applies a receiver $\tilde{\mathbf{g}}_k$ to suppress interference and retrieves its desired symbol. The output of such a receive filter is then given by

$$\tilde{r}_k = \tilde{\mathbf{g}}_k\tilde{\mathbf{h}}_{kk}\tilde{s}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \tilde{\mathbf{g}}_k\tilde{\mathbf{h}}_{kl}\tilde{s}_l + \tilde{\mathbf{g}}_k\tilde{n}_k$$

where we denoted with $\tilde{(.)}$ all the quantities that appear in the UL in order to differentiate with the same quantities in the DL.

III. Max-Min SINR in the MISO IFC per-user power constraints

In this section we consider a MISO IFC in which each signal link has an individual SINR priority $\gamma_i, \forall i = 1, \ldots, K$. Fairness then leads to a max min WSINR cost function.

$$\begin{aligned} &\max_{\mathbf{g}_1,\ldots,\mathbf{g}_K} \min_{k=1,\ldots,K} \frac{SINR_k}{\gamma_k} \\ &\text{s.t.} \quad \mathbf{g}_k^H\mathbf{g}_k \leq P_k, \ \ \forall k = 1, \ldots, K \end{aligned} \quad (2)$$

where P_k represents the maximum available power at transmitter number k. This problem, under a sum power constraint, was already discussed in [8].

The optimal solution to SINR balancing occurs when all the weighted SINRs are equal, thus the commonly used term SINR balancing. As stated also in [5] and [7] we can claim that for fixed beamforming direction at the balanced point in the MISO IFC, at least one user transmits with full power, i.e., at least one power constraint is satisfied with equality. This is easy to see for the SISO IFC or the MISO case with fixed BF vectors because the user with the worse equivalent channel coefficient, cascade of channel vector and BF, to maximize its SINR tends to use all its available power while the other users will adjust their power in order to equate all the SINRs.
Different is the situation when the beamforming design comes into the problem.

When the MISO IFC is separable, meaning that each user has a number of antenna greater than or equal to the number of users $M_k \geq K$, the following proposition describes the SINR balancing behavior.

Proposition 1 *At the balanced point in the separable MISO IFC, all users transmit with full power*

Proof: To prove the above statement consider, without loss of generality, a $K = 2$ user MISO IFC with $M_k \geq 2$. Assume that the optimal solution of the SINR balancing problem is given for $\mathbf{g}_1^\star$ and $\mathbf{g}_2^\star$ where only transmitter 1 transmits with full power, $\|\mathbf{g}_1^\star\|^2 = P_1, \ \|\mathbf{g}_2^\star\|^2 < P_2$. Because TX_2 has an excess of power the BF of user 1 can be modified s.t.:

$$\|\bar{\mathbf{g}}_1\|^2 = \|\mathbf{g}_1^\star\|^2 \ ; \quad |\mathbf{h}_{11}\bar{\mathbf{g}}_1|^2 > |\mathbf{h}_{11}\mathbf{g}_1^\star|^2.$$

This new choice of BF for user 1 increases its SINR but at the same time causes a reduction of the SINR of the other user: $SINR_{1,2}(\mathbf{g}_1^\star, \mathbf{g}_2^\star) < SINR_1(\bar{\mathbf{g}}_1, \mathbf{g}_2^\star) > SINR_2(\bar{\mathbf{g}}_1, \mathbf{g}_2^\star)$. TX_2 to compensate for the additional interference caused by the new BF $\bar{\mathbf{g}}_1$ has to increase the transmitted power using a BF of the form:

$$\begin{gathered} \bar{\mathbf{g}}_2 = \mathbf{g}_2^\star + \delta\mathbf{h}_{12}^\perp \\ \|\bar{\mathbf{g}}_2\|^2 > \|\mathbf{g}_2^\star\|^2 \ ; \quad |\mathbf{h}_{22}\bar{\mathbf{g}}_2|^2 > |\mathbf{h}_{22}\mathbf{g}_2^\star|^2 \end{gathered}$$

where $\mathbf{h}_{12}^\perp$ is any vector that belongs to the orthogonal complement of $\mathbf{h}_{12}$ and δ is a complex scaling factor. The choice of δ should be s.t. $SINR_1(\bar{\mathbf{g}}_1, \bar{\mathbf{g}}_2) = SINR_2(\bar{\mathbf{g}}_1, \bar{\mathbf{g}}_2)$. With this choice of $\bar{\mathbf{g}}_2$ we can rise the useful signal power for user 2 without increasing the interference caused to the non intended receiver.

With the new set of beamformers both the SINRs are increased $SINR_{1,2}(\bar{\mathbf{g}}_1, \bar{\mathbf{g}}_2) > SINR_{1,2}(\mathbf{g}_1^\star, \mathbf{g}_2^\star)$. This means that the original BF vectors were not optimal hence both users should transmit with full power. ■

Different is the situation in low SNR regime. Here we can state that the optimal transmission strategy for each user is to maximize the useful signal component. No matter how strong interference becomes, noise remains the dominant impairment. Hence the optimum transmission strategy is to beamform to match the direct link (maximum ratio BF) at each TX. In this case the user with the worse direct link channel transmits with full power to maximize its SINR, which is also the systemwide worst SINR. This is true also for separable MISO channel, regardless the number of transmitting antennas.

A. DL power allocation optimization

For cases where a zero forcing solution is not possible ($M_k < K,\ \forall k$) only one user has its power constraint active. In this case for fixed BF vectors the corresponding power allocation vector can be found solving an eigenvalue problem [1] imposing only one power constraint to be active. At the optimum all the weighted SINRs are equal. Denoting with τ the optimal value of the ratio SINR over target QoS we can write:

$$\frac{1}{\tau}\mathbf{p} = \mathbf{D}\mathbf{\Phi}\mathbf{p} + \mathbf{D}\boldsymbol{\sigma} \tag{3}$$

where matrices $\mathbf{D}$ and $\mathbf{\Phi}$ are defined as:

$$[\mathbf{\Phi}]_{ij} = \begin{cases} \mathbf{g}_j^H \mathbf{h}_{ij}^H \mathbf{h}_{ij} \mathbf{g}_j, & j \neq i \\ 0, & j = i \end{cases} \tag{4}$$

$$\mathbf{D} = \mathrm{diag}\{\frac{\gamma_1}{\mathbf{g}_1^H \mathbf{h}_{11}^H \mathbf{h}_{11} \mathbf{g}_1}, \ldots, \frac{\gamma_K}{\mathbf{g}_K^H \mathbf{h}_{KK}^H \mathbf{h}_{KK} \mathbf{g}_K}\}. \tag{5}$$

Assuming now that the $j-$th power constraint is the only one satisfied with equality and multiplying both sides of the previous equation by $\mathbf{x}_j^T = \frac{1}{P_j}\mathbf{e}_j$, where $\mathbf{e}_j$ is a vector with 1 only in position j, we get:

$$\frac{1}{\tau} = \mathbf{x}_j^T \mathbf{D}\mathbf{\Phi}\mathbf{p} + \mathbf{x}_j^T \mathbf{D}\boldsymbol{\sigma} \tag{6}$$

Introducing the compound matrix:

$$\mathbf{\Delta} = \begin{bmatrix} \mathbf{D}\mathbf{\Phi} & \mathbf{D}\boldsymbol{\sigma} \\ \mathbf{x}_j^T \mathbf{D}\mathbf{\Phi} & \mathbf{x}_j^T \mathbf{D}\boldsymbol{\sigma} \end{bmatrix} \tag{7}$$

and the extended vector $\overline{\mathbf{p}} = [\mathbf{p}\ \ 1]^T$, using the results from the nonnegative matrix framework [9] the solution of the WSINR balancing problem w.r.t. the power optimization is given by: $\tau = \frac{1}{\lambda_{max}(\mathbf{\Delta})}$ and the power vector is the corresponding positive eigenvector with the $(K+1)-$th entry normalized to one. This approach that allows to extend the known result from SIR balancing to SINR balancing is called *Bordering Method*, it was introduced by [9] and then used in [1]. A different approach to handle noise in the SINR balancing problem is to transform (3) into an homogeneous system of linear equations. This method is based on considering a rank one modification of the matrix $\mathbf{D}\mathbf{\Phi}$ that leads to the same solution obtained using the bordering method. The fact that the $j-$th power constraint is active: $\mathbf{x}_l^T \mathbf{p} = 1$ allows us to modify WSINR balancing problem in order to obtain an unconstraint optimization problem in terms of powers. Introducing a reparametrization of the Tx power vector:

$$\mathbf{p} = \frac{1}{\mathbf{x}_j^T \tilde{\mathbf{p}}}\tilde{\mathbf{p}} \tag{8}$$

we can rewrite (3) as

$$\frac{1}{\tau}\tilde{\mathbf{p}} = (\mathbf{D}\mathbf{\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_j^T)\tilde{\mathbf{p}}. \tag{9}$$

Also in this case the solution of the problem is given by the positive eigenvalue $\tau = \frac{1}{\lambda_{max}(\mathbf{D}\mathbf{\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_j^T)}$ and the associated positive eigenvector is the optimal power vector. At this point a question arises: Which power constraint is the only one satisfied with equality? It is possible to show that the only feasible constraint is given by $\mathbf{x}_{j^\star} = \arg\max_{\mathbf{x}_j} \lambda_{max}(\mathbf{B})$ [10], where $\mathbf{B}$ can be the rank 1 modified matrix or matrix $\mathbf{\Delta}$ in (7).

To solve the problem when only one power constraint is active and none of the users can do ZF BF we can determine the following algorithm which solves K different optimization problems, imposing only one power constraint to be active, and finally we choose the optimal solution. The problem can be mathematically expressed as:

$$\begin{aligned} &\max_{\{p_i\},\tau_j}\ \tau_j \\ &\text{s.t.}\quad \mathbf{e}_j \mathbf{p} \le P_j \\ &\quad SINR_k^{DL} = \frac{1}{\gamma_k}\frac{p_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l\neq k} p_l \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \ge \tau_j \quad \forall k \end{aligned} \tag{10}$$

where we assume that the BFs are unit norm and for the moment they are not optimization variables, they are fixed. The Lagrange dual of the optimization problem can be transformed into an equivalent dual UL problem:

$$\begin{aligned} &\min_{\mu}\ \max_{\{\lambda_i\},\tau_j}\ \tau_j \\ &\text{s.t.}\quad \textstyle\sum_i \lambda_i \sigma_i^2 \le P_j,\ \mu \le 1 \\ &\quad SINR_k^{UL} = \frac{1}{\gamma_k}\frac{\lambda_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l\neq k} \lambda_l \mathbf{g}_k^H \mathbf{h}_{lk}^H \mathbf{h}_{lk} \mathbf{g}_k + \mu e_{j,k}} \ge \tau_j \quad \forall k \end{aligned} \tag{11}$$

where λ_i represents the Lagrange multiplier associated to the i-th SINR constraint and μ is introduced to handle the power constraint. Those quantities represent the dual UL Tx power and the UL dual noise power respectively. Because we need to minimize the SINRs w.r.t. μ this variable should be large so it will assume its maximum value at the optimum: $\mu = 1$. The UL max min WSINR problem can be solved w.r.t. to the UL power using one of the method described before, for example solving the following:

$$\tfrac{1}{\tau_l}\tilde{\boldsymbol{\lambda}} = (\mathbf{D}\mathbf{\Phi}^T + \mathbf{D}\mathbf{e}_j\boldsymbol{\sigma}^T)\tilde{\boldsymbol{\lambda}}; \quad \boldsymbol{\lambda} = \tfrac{P_j}{\boldsymbol{\sigma}^T\tilde{\boldsymbol{\lambda}}}\tilde{\boldsymbol{\lambda}} \tag{12}$$

From the SINR constraint in the UL problem (11) we can see that the BF vector plays the role of RX filter. The optimal $\mathbf{g}_k$ is the one that maximizes the SINR in UL and the solution for this problem is the well known generalized eigenvector solution that for rank one channels has the following close form solution:

$$\mathbf{g}_k = (\sum_{l \neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I})^{-1} \mathbf{h}_{kk}^H \tag{13}$$

where η_k represents the dual noise power, in this case $\eta = e_{j,k}$. Finally the DL power allocation can be determined using equation (9). Once the K optimization problems have been solved the optimal solution that satisfies all the power constraints at the same time is obtained looking at the solution that has the minimum $l^\star = \arg\min_j \tau_j$. In the corresponding DL power vector the $j^\star$-th user transmits with full power and at the same time all the other power constraints are inactive.

For a more general system configuration the max min WSINR problem below:

$$\begin{array}{lll} \max\limits_{\mathbf{g}_1,\ldots,\mathbf{g}_K} & \tau & \\ \text{s.t.} & \mathbf{g}_k^H \mathbf{g}_k \leq P_k & \forall k \\ SINR_k^{DL} = & \frac{1}{\gamma_k} \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \neq k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \geq \tau & \forall k \end{array} \tag{14}$$

can be solved as in [6] using UL-DL duality.

B. SINR Region Characterization

The beamforming problem in terms of max min WSINR described in (2) and further refined in (14) can be interpreted as exploring the SINR region along the ray with direction $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]$. Solving the max min WSINR problem allows us to find the maximum values of SINR on the direction given by $\boldsymbol{\gamma}$. Then the optimal point is given by the intersection of the straight line described by $\boldsymbol{\gamma}$ and the Pareto boundary of the SINR region. This result was claimed for a SISO IFC in [7], here is extended to the MISO case. The Pareto boundary of the SINR region is commonly defined as follows:

A SINR tuple $(S_1, \ldots, S_K)$ belongs to the Pareto boundary if there is no other tuple $(\hat{S}_1, \ldots, \hat{S}_K)$ with $(\hat{S}_1, \ldots, \hat{S}_K) \geq (S_1, \ldots, S_K)$ and $(\hat{S}_1, \ldots, \hat{S}_K) \neq (S_1, \ldots, S_K)$.

This result is important from an information theoretic point of view because solving the simple max min WSINR problem allows us to draw the entire Pareto boundary of the rate region, thanks to the logarithmic relation between SINRs and rates. This result is valid for a general K-user MISO IFC regardless of system parameters. In a recent paper [11] the authors provide a characterization of the Pareto boundary of the Rate region where the BF at each base station is a linear combination of the cross channels directly connected to it. This representation requires $K(K-1)$ complex parameters while the use of max min WSINR only requires $(K-1)$ real values, the fairness constraints γ_k. In [12] the authors propose a similar characterization of the Pareto boundary of the rate region using what they call *rate profile*. That problem can be thought as a rate balancing problem imposing different priority constraints and they state that to solve the problem a centralized solution in necessary. On the other hand for max min WSINR it is possible to develop a distributed algorithm to solve the problem, as shown in the following section, that represents a preferable solution compares to a centralized approach.

IV. Decentralized Iterative algorithm

In this section we describe an iterative algorithm that solves the weighted SINR balancing problem. It is essentially based on the link between the SINR balancing problem and the power minimization under QoS constraints underlined in [6]. The idea behind the proposed algorithm is to solve a sequence of power minimization problems with per base station power constraints incrementing at each step of the algorithm the QoS requirements imposed on the system. When the QoS constraints become not feasible then using bisection method we determine the optimal value of the max min WSINR problem. The advantage of this algorithm is that there exist a distributed solution [4] for TDD systems where UL and DL channel are reciprocals of each other.

The power minimization problem is written as:

$$\begin{array}{c} \min\limits_{\mathbf{g}_1,\ldots,\mathbf{g}_K} \sum_{k=1}^K \mathbf{g}_k^H \mathbf{g}_k \\ \text{s.t.} \quad \mathbf{g}_k^H \mathbf{g}_k \leq P_k; \; k = 1, \ldots, K \\ SINR_k^{DL} = \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \neq k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \geq \gamma_k; \; k = 1, \ldots, K \end{array} \tag{15}$$

where P_k represents the maximum TX power for user k.

The Lagrange dual of the DL beamforming problem (15) can be rewritten as an equivalent UL optimization problem for the RX filter (13) where the dual noise is $\eta_k = \mu_k + 1$. The dual UL problem can be mathematically expressed as:

$$\begin{array}{c} \max\limits_{\{\mu_i\}} \min\limits_{\{\lambda_i\}} \sum_{k=1}^K \lambda_k \sigma_k^2 - \sum_{k=1}^K \mu_k P_k \\ SINR_k^{UL} = \frac{\lambda_k \tilde{\mathbf{g}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{g}}_k}{\tilde{\mathbf{g}}_k^H (\sum_{l \neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \tilde{\mathbf{g}}_k} \geq \gamma_k; \; k = 1, \ldots, K \\ \lambda_k \geq 0; \; \mu_k \geq 0; \; \forall k. \end{array} \tag{16}$$

At the optimum the SINR constraints in the UL and DL problems must be satisfied with equality [4]. Using this property it is possible to derive the UL and DL TX powers. The UL TX power is determined using the following:

$$\lambda_k = \gamma_k \frac{\tilde{\mathbf{g}}_k^H (\sum_{l \neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \tilde{\mathbf{g}}_k}{\tilde{\mathbf{g}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{g}}_k} \stackrel{a}{=} \frac{\gamma_k}{\mathbf{h}_{kk} \tilde{\mathbf{g}}_k} \tag{17}$$

where a is obtained using (13). Because a scaling factor in the receiver filter at the BS does not affect the SINR, the optimal DL BF is $\mathbf{g}_k = \sqrt{p_k} \tilde{\mathbf{g}}_k$ and p_k is such that the WSINR in DL for user k is satisfied with equality. The last quantity that remains to be optimized is the Lagrange multiplier μ_k. On this purpose we use a subgradient method:

$$\mu_k^{(n)} = [\mu_k^{(n-1)} + t(\mathbf{g}_k^H \mathbf{g}_k - P_k)]_+ \tag{18}$$

where t represents the step size.

As stated at the beginning of this section the most important feature of the proposed algorithm is the possibility of distributed implementation that relies on channel reciprocity and few feedback of scalar quantities.

V. Numerical Examples

In this section we present some numerical results in which we study the behaviour of max min WSINR. In Fig. 2 we report the Rate region of a 2-user MISO IFC where each

Algorithm 1 Iterative Algorithm for max min WSINR

Initialize: $i = 0$ and a feasible $\boldsymbol{\gamma}^0 = [\gamma_1^{(0)}, \ldots, \gamma_K^{(0)}]$
repeat
  i=i+1
  Find $\mathbf{g}_k^{(i)}$ solving Power min for $\boldsymbol{\gamma}^{(i)}$
  Set $\boldsymbol{\gamma}_{min} = \boldsymbol{\gamma}^{(i)}$
  Increase $\boldsymbol{\gamma}^{(i+1)} = \alpha\boldsymbol{\gamma}^{(i)}$
until $\boldsymbol{\gamma}^{(i)}$ is feasible
repeat
  Set $\boldsymbol{\gamma}_{max} = \boldsymbol{\gamma}^{(i)}$
  i=i+1
  Set $\boldsymbol{\gamma}^{(i)} = \frac{\boldsymbol{\gamma}_{max}+\boldsymbol{\gamma}_{min}}{2}$
  Find $\mathbf{g}_k^{(i)}$ solving Power min for $\boldsymbol{\gamma}^{(i)}$
  if $\boldsymbol{\gamma}^{(i)}$ is feasible **then**
    Set $\boldsymbol{\gamma}_{min} = \boldsymbol{\gamma}^{(i)}$
  else
    Set $\boldsymbol{\gamma}_{max} = \boldsymbol{\gamma}^{(i)}$
  end if
until $|\boldsymbol{\gamma}_{max} - \boldsymbol{\gamma}_{min}| > \epsilon$

base station has $M_k = 2, \ \forall k$ transmitting antennas for a single channel realization. We plot on the same figure the rate obtained optimizing the max min $WSINR$ for different priority constraints γ_k. The rate region reported is obtained using the BF parametrization proposed in [11] for the 2-user MISO IFC that allows to draw the rate region, and hence the Pareto boundary. As we can see the rates obtained optimizing

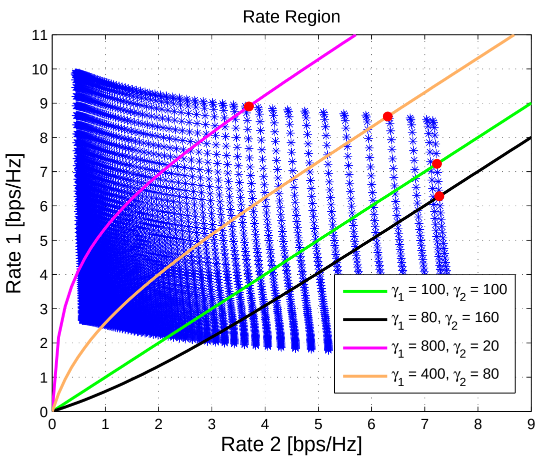


Fig. 2: Rate region for a 2-user MISO IFC for $\sigma_k^2 = 30$ dB

the max min WSINR (red points in the figure) lie always on the boundary of the region. In addition we can see that varying the priority constraint γ_k it is possible to explore different points on the boundary. This figure sustain our statement on the possibility to characterize the entire Pareto boundary of the rate region using max min WSINR. The solid lines drawn on the figure represent the rays with direction given by γ_k. Those curves are straight lines in the SINR region but due to the log relation between SINR and Rate they have a logarithmic behaviour.

VI. Conclusions

In this paper we show that SINR balancing in the MISO IFC leads to a balanced state where at least one user transmits with full power. When the IFC is separable (number of antennas sufficient to zero force), the SINR balanced state is where all users transmit with full powers. We derive an iterative algorithm to solve the given optimization problem based on the equivalence between SINR balancing problem and the power minimization problem with QoS constraints. Finally we show that WSINR balancing problem can be used to characterize the complete Pareto boundary of the SINR (Rate) region.

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