Relay-Aided Interference Neutralization for the Multiuser Uplink-Downlink Asymmetric Setting

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***Abstract*—In the context of multiuser relay-aided multi-way communications, we identify and meet the optimal degrees of freedom (DOF) for different multiuser uplink-downlink settings of practical importance. Under the imposed constraint of using simple linear techniques, the proposed solutions draw from interference-neutralization (IN) methods which linearly manipulate signals in time and space, and manage to reduce the effect of multiuser interference and of the half-duplex constraint. Focus is placed on asymmetric settings where the connectivity, size and rate of the uplink and downlink groups may vary.**

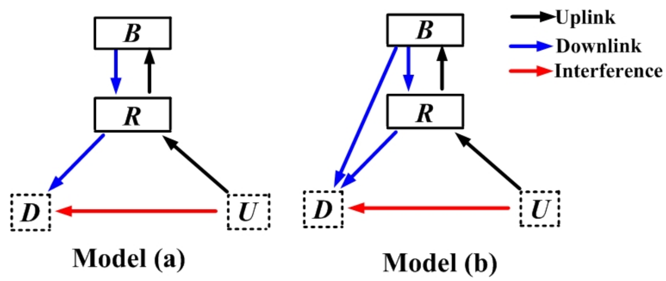

Fig. 1. Considered multiuser uplink-downlink settings.

I. Introduction

Interference and the half-duplex constraint are two limiting bottlenecks in networks having multiple users and nodes that wish to both transmit and receive information (cf. [1]–[4]). In such networks, which appear for example in uplink-downlink cellular systems, the two aforementioned limitations often accept solutions drawing from network coding techniques [5], such as different interference neutralization methods [6], [7], which seek to properly combine signals arriving from different paths in such a way that the interfering signals are canceled while the desired signals are preserved. While such techniques offer implementation simplicity by consisting only of straightforward linear operations across the spatial dimension, their usefulness is limited to specific network topologies which conveniently allow for interference patterns that are treatable in a linear manner. This class of topologies may often involve relays which can be used to properly redirect interfering signals to be then encoded over the spatial (and here also over the temporal) dimension.

We here identify different scenarios which accept such linear treatment in a manner that can be optimal with respect to DOF, focusing on pertinent scenarios of practical importance in the context of uplink-downlink half-duplex constrained cellular systems. This involves identifying the outer bounds of the DOF for these networks, and then constructively meeting these bounds by proposing simple interference neutralization solutions which redirect signals both in spatial and, as proposed here, temporal dimensions. With time encoding in place, special attention is placed so that the linear solutions are also causal. Establishing the optimality of the proposed solutions is completed with an analysis of the signal-attenuating and noise-accentuating effects of the IN schemes.

A. Summary and Notations

Section II describes the pertinent scenarios and the corresponding channel and signal model. Section III describes the outer bound on the DOF region for different scenarios, and then presents the proposed IN solutions that meet part or the entirety of these regions. Section IV concludes by mentioning some salient features of the proposed IN solutions, and finally the Appendix presents some of the proofs.

In this work, $(\bullet)^{-1}$, $(\bullet)^{T}$, $(\bullet)^{\dagger}$ and $||\bullet||_F$ denote the inverse, transpose, conjugate transpose and Frobenius norm of matrix respectively, $(\bullet)^{*}$ denotes the complex conjugate, $||\bullet||$ denotes Euclidean norm, $[\bullet]_j$ denotes j-th row of the matrix or column vector in the argument, and $[\bullet]_{i,j}$ denotes the matrix element in the ith row and jth column. We use $\doteq$ to denote exponential equality, i.e., $f(\rho) \doteq \rho^d$ denotes $\lim\limits_{\rho\to\infty} \dfrac{\log f(\rho)}{\log \rho} = d$ and $\dot{\leq}$, $\dot{\geq}$, $\dot{<}$, $\dot{>}$ are similarly defined.

II. Signal and Channel Model

We consider a setting where a node B with K antennas wants to receive from a set U of J single-antenna users, and transmit to a set D of Q single-antenna users, with the help of a K-antenna relay R, in the presence of the half-duplex constraint, and in the presence of unlimited channel state information at all transmitters and receivers (CSITR). In the context of cellular systems, B may play the role of a base station, and U and D the sets of uplink and downlink users respectively. Let $\mathbf{r}^{(t)}, \mathbf{a}^{(t)}$ respectively denote the transmitted vectors sent at time t by R and by all of U, and let $\mathbf{b}^{(t)}$ denote the information vector of B at time t. Furthermore let $\mathbf{y}_B^{(t)}, \mathbf{y}_R^{(t)}, \mathbf{y}_D^{(t)}, \mathbf{z}_B^{(t)}, \mathbf{z}_R^{(t)}, \mathbf{z}_D^{(t)}$ respectively denote the received signal and noise vectors at B, R and at all of D. In the following we will ignore the time index if no ambiguity is caused. In the scale of interest we consider a uniform power constraint where $\mathbb{E}||\mathbf{a}||^2 \doteq \mathbb{E}||\mathbf{b}||^2 \doteq \mathbb{E}||\mathbf{r}||^2 \dot{\leq} \rho$, where ρ takes the role of the signal-to-noise ratio (SNR).

We let $h_{i,j}$ denote the channel fading coefficient between the ith user in U and the receiver of the jth user in D, let $\mathbf{h}_{U,j}$ denote the vector of fading coefficients between the entire set U and the receiver of the jth user in D, and let $\mathbf{H}_{U,D}, \mathbf{H}_{U,R}, \mathbf{H}_{R,D}, \mathbf{H}_{R,B}, \mathbf{H}_{B,R}, \mathbf{H}_{B,D}$ denote the U-to-D, U-to-R, R-to-D, R-to-B, B-to-R, and B-to-D collective channel fading matrices. Fading and additive noise coefficients are considered to be i.i.d. complex Gaussian $\mathcal{CN}(0,1)$, and the fading is assumed to remain constant during the coherence period and to change independently afterwards. Regarding the different settings (a) and (b) addressed here (cf. Figure 1), the channel model in (a) is distinguished by the fact that $\mathbf{H}_{B,D} = \mathbf{H}_{U,B} = \mathbf{0}$, as the channel model in (b) by the fact that $\mathbf{H}_{U,B} = \mathbf{0}$. We are interested in analyzing the degrees of freedom, so a rate of $\mathcal{R}$ bits per channel use (bpcu), corresponds to $d = \lim_{\rho\to\infty} \frac{\mathcal{R}}{\log \rho}$ degrees of freedom.

III. Proposed IN schemes and DOF analysis

Before proceeding to establish and meet the DOF limits for the different scenarios shown in Figure 1, we provide the following necessary lemmas whose proofs are found in the Appendix, or due to lack of space are left to appear as part of a larger, journal version of this work [11].

Lemma 1: In a two-hop setting where the first (second) hop from node S to node R (from node R to node D) is over a link with d_{SR} (d_{RD}) DOF, and given a short-term power constraint, then the S-to-D link allows for a total of $d_{SD} = \frac{d_{SR}d_{RD}}{d_{SR}+d_{RD}}$ DOF.

Lemma 2: Let $\mathbf{E} \triangleq \sum_{i=1}^{N}\prod_{j=1}^{M_i}\mathbf{H}_{i,j}^{\tau_{i,j}},\ \ \tau_{i,j} \in \{-1,1\}$, where the $\mathbf{H}_{i,j}$ are independently drawn square random matrices each consisting of i.i.d. $\mathcal{CN}(0,1)$ entries. Then for any $\epsilon > 0$ there exists a constant $d > 0$ such that $P(||\mathbf{E}^{-1}||_F^2 \geq \rho^{\epsilon}) \dot{\leq} \rho^{-\epsilon/d}$.

Lemma 3: Let $\mathbf{E} \triangleq \prod_{i=1}^{M}\mathbf{H}_i^{\tau_i}$, $\tau_i \in \{-1,1\}$, where the $\mathbf{H}_i$ are *independently* drawn square random matrices each consisting of i.i.d. $\mathcal{CN}(0,1)$ entries. Then $P(||\mathbf{E}^{-1}||_F^2 \geq \rho^{\epsilon}) \dot{\leq} \rho^{-\epsilon}$ for any $\epsilon > 0$.

We henceforth adopt the rate uniformity assumption where all users in set U have the same rate $\mathcal{R}_U$, and all users in D have the same rate $\mathcal{R}_D$, but where $\mathcal{R}_U$ and $\mathcal{R}_D$ are not necessarily equal. For simplicity we also limit exposition to the case where $J \leq Q = K$.

A. Outer Bound

To derive the DOF outer bound we apply the cut-set theorem (cf. [8]), in the ergodic setting, as illustrated in Fig. 2. For the U-to-B communication under model (a), the pertinent cuts $\{U,D\} \to \{R,B\}$ and cut $\{U,R,D\} \to \{B\}$, together with Lemma 1 give that each user in U (under the rate uniformity assumption) can achieve no more than $d_U^{(a)} \leq \frac{K}{J+K}$ DOF. A similar approach shows the same for the U-to-B communication under model (b) where again each user in U can achieve at most $d_U^{(b)} \leq \frac{K}{J+K}$ DOF.

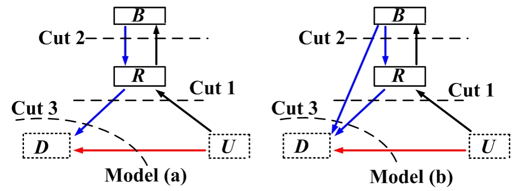

Fig. 2. Cut sets for models (a) and (b).

For the B-to-D communication under model (a), the pertinent cuts $\{B\} \to \{U,R,D\}$ and $\{U,B,R\} \to \{D\}$ together with Lemma 1 give that each user in D (under the rate uniformity assumption) can achieve at most $d_D^{(a)} \leq \frac{1}{2}$ DOF, and for the B-to-D communication under model (b), the pertinent cut $\{U,B,R\} \to \{D\}$ implies that each user in D has at most $d_D^{(b)} \leq 1$ DOF. Towards tightening the outer bound of the DOF region we consider, for model (b), the two-directional uplink-downlink cut $\{U,D,R\} \leftrightarrow \{B\}$, after which it is not difficult to show that in a half-duplex uplink-downlink setting, the sum DOF is equal to the maximum of the DOF of the uplink and of the downlink, which in turn implies that $Jd_U^{(b)} + Kd_D^{(b)} \leq K$. The equivalent cut for model (a) does not yield any tightening, and is thus ignored. The above is summarized in the following Lemma and in Table I.

Lemma 4: For model (a) and (b), the outer bound on the DOF is respectively given by

$$\begin{aligned} d_U^{(a)} &\leq K/(K+J),\ d_D^{(a)} \leq 1/2, \\ d_U^{(b)} &\leq K/(K+J),\ d_D^{(b)} \leq 1,\ Jd_U^{(b)} + Kd_D^{(b)} \leq K. \end{aligned}$$

TABLE I
Upper bound on DOF for models (a) and (b)

$U \to B$	$B \to D$	total
$d_U^{(a)} \leq K/(K+J)$	$d_D^{(a)} \leq 1/2$	−
$d_U^{(b)} \leq K/(K+J)$	$d_D^{(b)} \leq 1$	$Jd_U^{(b)} + Kd_D^{(b)} \leq K$

B. Achievability and IN schemes

In what follows we describe two IN schemes $\mathcal{X}_1$ and $\mathcal{X}_2$ where $\mathcal{X}_1$ corresponds to the general case of $J < K$ and applies with minor modifications to models (a) and (b), and where $\mathcal{X}_2$ is for $J = K$ and applies with minor modifications to models (a) and (b). Before describing the actual schemes $\mathcal{X}_1, \mathcal{X}_2$ in different settings, we hasten to summarize their performance in what follows.

Proposition 1: Scheme $\mathcal{X}_1$ achieves $\left(d_U^{(a)} = K/(K+J)\ ,\ d_D^{(a)} = 1/(K+J)\right)$ and $\left(d_U^{(b)} = K/(K+J)\ ,\ d_D^{(b)} = 1/(K+J)\right)$ where $d_U^{(a)} = K/(K+J)$ and $d_U^{(b)} = K/(K+J)$ are optimal. Scheme $\mathcal{X}_2$ achieves $\left(d_U^{(a)} = 1/2\ ,\ d_D^{(a)} = 1/2\right)$ and achieves $\left(d_U^{(b)} + d_D^{(b)} = 1,\ d_U^{(b)} \leq 1/2\right)$ which are optimal for the corresponding setting of $J = K$ [1].

[1]The optimal $d_D^{(b)} = 1$ can be trivially met by silencing U and R.

TABLE II
ACHIEVABLE DOF FOR $\mathcal{X}_1$ AND $\mathcal{X}_2$. SOLUTION $\mathcal{X}_2'$ INVOLVES TIME-SHARING OF $\mathcal{X}_2$ WITH SILENCING U.

	applicability	achievable DOF
$\mathcal{X}_1$	(a)&(b), $J \le K$	$(\underbrace{d_U = \frac{K}{K+J}}_{\text{optimal}}, d_D = \frac{1}{K+J})$
$\mathcal{X}_2$	(a), $J < K$	$(d_U^{(a)} = \frac{1}{2}, \underbrace{d_D^{(a)} = \frac{1}{2}}_{\text{optimal}})$
$\mathcal{X}_2$	(a), $J = K$	$\underbrace{(d_U^{(a)} = \frac{K}{K+J} = \frac{1}{2},\ d_D^{(a)} = \frac{K}{K+J} = \frac{1}{2})}_{\text{optimal}}$
$\mathcal{X}_2'$	(b), $J = K$	$\underbrace{d_U^{(b)} + d_D^{(b)} = 1,\ d_U^{(b)} \le 1/2}_{\text{optimal}}$

We will first present scheme $\mathcal{X}_2$ simply because the derivation of the DOF, and specifically the analysis of the effects of signal attenuation and noise accentuation introduced by the IN solution, can be described in a more concise manner. The same techniques can then be applied with minor modifications to $\mathcal{X}_1$.

1) Scheme $\mathcal{X}_2$ - description and DOF analysis: For the case of $J = K$, we introduce scheme $\mathcal{X}_2$ and analyze its DOF performance for model (b), with model (a) handled similarly.

In the proposed two-phase scheme, each phase has duration equal to one channel use. In the first phase, U collectively transmit $\mathbf{a}$, B transmits $\beta \mathbf{V}_B \mathbf{b}$ with $\beta = \left(||\mathbf{V}_B||_F^2\right)^{-1/2}$ ensuring the power constraint $\mathbb{E}||\mathbf{b}||^2 \le \rho$, where by construction we ask that $[\mathbf{b}]_j$ be destined for the jth downlink user in D. The relay and set D then respectively receive

$$\begin{aligned} \mathbf{y}_R &= \beta \mathbf{H}_{B,R} \mathbf{V}_B \mathbf{b} + \mathbf{H}_{U,R} \mathbf{a} + \mathbf{z}_R, && (1) \\ \mathbf{y}_D^{(1)} &= \beta \mathbf{H}_{B,D} \mathbf{V}_B \mathbf{b} + \mathbf{H}_{U,D} \mathbf{a} + \mathbf{z}_D^{(1)}. && (2) \end{aligned}$$

During the second phase, node R precodes, attenuates, and forwards the received signal, sending $\mathbf{r} = \gamma \mathbf{V}_R \mathbf{y}_R$ where $\gamma = \left(\frac{||\mathbf{V}_R \mathbf{H}_{U,R}||_F^2}{K} + \frac{||\mathbf{V}_R \mathbf{H}_{B,R} \mathbf{V}_B||_F^2}{||\mathbf{V}_B||_F^2} + \frac{||\mathbf{V}_R||_F^2}{\rho}\right)^{-1/2}$, and B and D respectively receive

$$\begin{aligned} \mathbf{y}_B &= \gamma \mathbf{H}_{R,B} \mathbf{V}_R \mathbf{y}_R + \mathbf{z}_B, \\ \mathbf{y}_D^{(2)} &= \gamma \mathbf{H}_{R,D} \mathbf{V}_R \mathbf{y}_R + \mathbf{z}_D^{(2)}. && (3) \end{aligned}$$

Towards neutralizing the interference experienced by an individual downlink user, where this interference contains (cf. (3)) uplink signals as well as downlink signals intended for other users in D, we combine

$$\begin{aligned} \tilde{\mathbf{y}}_D =& \gamma \mathbf{y}_D^{(1)} + \mathbf{y}_D^{(2)} \\ =& \gamma\beta(\mathbf{H}_{B,D} + \mathbf{H}_{R,D} \mathbf{V}_R \mathbf{H}_{B,R}) \mathbf{V}_B \mathbf{b} + \gamma(\mathbf{H}_{R,D} \mathbf{V}_R \mathbf{H}_{U,R} \\ &+ \mathbf{H}_{U,D}) \mathbf{a} + \gamma \mathbf{H}_{R,D} \mathbf{V}_R \mathbf{z}_R + \gamma \mathbf{z}_D^{(1)} + \mathbf{z}_D^{(2)} && (4) \end{aligned}$$

and remove all interference by setting

$$\begin{aligned} \mathbf{V}_R &= -\mathbf{H}_{R,D}^{-1} \mathbf{H}_{U,D} \mathbf{H}_{U,R}^{-1}, && (5) \\ \mathbf{V}_B &= (\mathbf{H}_{B,D} - \mathbf{H}_{U,D} \mathbf{H}_{U,R}^{-1} \mathbf{H}_{B,R})^{-1} && (6) \end{aligned}$$

to get

$$\tilde{\mathbf{y}}_D = \gamma\beta \mathbf{b} + \gamma \mathbf{H}_{R,D} \mathbf{V}_R \mathbf{z}_R + \gamma \mathbf{z}_D^{(1)} + \mathbf{z}_D^{(2)} \quad (7)$$

thus having the jth downlink user in D observe an interference-free signal

$$\tilde{y}_j = \gamma\beta[\mathbf{b}]_j + \gamma[\mathbf{H}_{R,D} \mathbf{V}_R]_j \mathbf{z}_R + [\gamma \mathbf{z}_D^{(1)} + \mathbf{z}_D^{(2)}]_j, \quad j = 1, \cdots, K, \quad (8)$$

albeit with a possibly attenuated signal $\gamma\beta[\mathbf{b}]_j$ and accentuated noise $\tilde{z}_j \triangleq \gamma[\mathbf{H}_{R,D} \mathbf{V}_R]_j \mathbf{z}_R + [\gamma \mathbf{z}_D^{(1)} + \mathbf{z}_D^{(2)}]_j$, both of which will be analyzed in what follows.

In light of the fact that β, γ and $\mathbb{E}[\tilde{z}_j \tilde{z}_j^*]$ are random variables and functions of the fading, we proceed to identify problematic channel regions. Towards this, for some positive constant ϵ, we consider regions $\mathcal{E}_1 = \{\beta^2 < \rho^{-\epsilon}\}$, $\mathcal{E}_2 = \{\gamma^2 < \rho^{-\epsilon}\}$ that may cause signal attenuation (cf. (8)), and region $\mathcal{E}_3 = \{||\mathbf{H}_{R,D} \mathbf{V}_R||_F^2 > \rho^{\epsilon}\}$ that may cause noise accentuation (cf. (8)). Lemma 2, and [10, Lemma 1] tells us that for some $d_0 > 0$

$$P(\mathcal{E}_1) \doteq P(||(\mathbf{H}_{B,D} - \mathbf{H}_{U,D} \mathbf{H}_{U,R}^{-1} \mathbf{H}_{B,R})^{-1}||_F^2 > \rho^{\epsilon}) \dot{\le} \rho^{-\frac{\epsilon}{d_0}} \quad (9)$$

$$P(\mathcal{E}_2) = P(\gamma^2 < \rho^{-\epsilon}) \dot{\le} \rho^{-\epsilon/2}, \quad (10)$$

and from

$$\mathbb{E}[\tilde{z}_j \tilde{z}_j^*] = \gamma^2 ||[\mathbf{H}_{R,D} \mathbf{V}_R]_j||^2 + \gamma^2 + 1 \le \gamma^2 ||\mathbf{H}_{R,D} \mathbf{V}_R||_F^2 + \gamma^2 + 1 \quad (11)$$

and Lemma 3 we have that

$$P(\mathcal{E}_3) = P(||-\mathbf{H}_{U,D} \mathbf{H}_{U,R}^{-1}||_F^2 > \rho^{\epsilon}) \dot{\le} \rho^{-\epsilon}. \quad (12)$$

With the above in place we now seek, as an intermediate step, to achieve DOF point $d_U^{(b)} = d_D^{(b)} = 1/2$. For this we first analyze the error performance of the users in D for rate $\mathcal{R} = (\frac{1}{2} - \epsilon - \delta)\log\rho$ for some positive but arbitrarily small δ. Towards this we calculate the mutual information between $[\mathbf{b}]_j$ and $\tilde{y}_j$ which, in the proposed two-phase protocols, is equal to

$$I \triangleq \frac{1}{2} \log(1 + \frac{\gamma^2 \beta^2 \rho}{K \mathbb{E}[\tilde{z}_j \tilde{z}_j^*]}).$$

For $\mathcal{E} \triangleq \{\mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}_3\}$ and $d_m \triangleq \max\{2, d_0\}$, the corresponding outage probability is then bounded as

$$\begin{aligned} P[I < \mathcal{R}] =& P[I < \mathcal{R} \mid \mathcal{E}] P[\mathcal{E}] + P[I < \mathcal{R} | \mathcal{E}^c] P[\mathcal{E}^c] \\ \overset{(a)}{\le}& \sum_{i=1}^{3} P[\mathcal{E}_i] + P[\frac{1}{2} \log(1 + \frac{\gamma^2 \beta^2 \rho / K}{(\gamma^2 ||\mathbf{H}_{R,D} \mathbf{V}_R||_F^2 + \gamma^2 + 1)}) < \mathcal{R} | \mathcal{E}^c] \\ \overset{(b)}{\dot{\le}}& \rho^{-\frac{\epsilon}{d_m}} + P[\frac{1}{2} \log(1 + \frac{\rho \rho^{-\epsilon}}{\rho^{\epsilon} + 1 + \rho^{-\epsilon}}) < \mathcal{R}] \\ =& \rho^{-\frac{\epsilon}{d_m}} + P[\frac{1}{2}(1 - 2\epsilon)^+ \log\rho < (\frac{1}{2}(1 - 2\epsilon) - \delta)\log\rho] = \rho^{-\frac{\epsilon}{d_m}} \end{aligned}$$

where, (a) follows from the union bound and (11), where (b) follows from (9), (10) and (12), and where $\mathcal{E}^c$ is the complement of $\mathcal{E}$. Similar treatment holds for the uplink case where all interference is self-interference and where the signal attenuation and noise accentuation may be treated exactly as in the downlink case, in which case we conclude that the point $d_U^{(b)} = d_D^{(b)} = 1/2$ is reached. Time-sharing of $\mathcal{X}_2$ with silencing U achieves the optimal DOF region $\left(d_U^{(b)} + d_D^{(b)} = 1,\ d_U^{(b)} \le \frac{1}{2}\right)$.

2) Scheme $\mathcal{X}_1$ - description and DOF analysis: For the case of $J < K$, we proceed to describe scheme $\mathcal{X}_1$ and analyze its DOF performance. We focus on model (a), with (b) handled similarly. We will proceed to show that each uplink user can communicate without interference $\frac{K}{K+J}$ symbols per channel use, and each downlink user can communicate without interference $\frac{1}{K+J}$ symbols per channel use. The signal attenuation and noise accentuation analysis follows directly from the previous subsection regarding $\mathcal{X}_2$ and is hence omitted here.

The proposed scheme has two phases, the first of duration K and the second of duration J. During the first phase, i.e., for $t = 1, \cdots, K$, R and D listen, the jth user in U transmits $a_{j,t}$, while at the same time node B transmits a vector $\beta \mathbf{v}_B b_t$, where symbol b_t is intended for the $(i = t)$th user in D, and where $\beta \mathbf{v}_B$ precodes and handles the power constraint.

The received vectors at R and at the ith user of D are respectively

$$\mathbf{y}_R^{(t)} = \mathbf{H}_{U,R}\mathbf{a}^{(t)} + \beta \mathbf{H}_{B,R}\mathbf{v}_B b_t + \mathbf{z}_R^{(t)}, \quad (13)$$
$$y_i^{(t)} = \mathbf{h}_{U,i}^T \mathbf{a}^{(t)} + z_i^{(t)} \quad (14)$$

where $\mathbf{a}^{(t)} \triangleq [a_{1,t} \cdots a_{J,t}]^T$. The relay then proceeds to calculate

$$\tilde{\mathbf{y}}_R^{(t)} = \mathbf{H}_{U,R}^{+}\mathbf{y}_R^{(t)} = \mathbf{a}^{(t)} + \beta \mathbf{H}_{U,R}^{+}\mathbf{H}_{B,R}\mathbf{v}_B b_t + \mathbf{H}_{U,R}^{+}\mathbf{z}_R^{(t)} \quad (15)$$

where $\mathbf{H}_{U,R}^{+} = (\mathbf{H}_{U,R}^T \mathbf{H}_{U,R})^{-1}\mathbf{H}_{U,R}$. Finally for $\mathbf{v} = [v_1, \cdots, v_J]^T = \beta \mathbf{H}_{U,R}^{+}\mathbf{H}_{B,R}\mathbf{v}_B$ and for $\tilde{\mathbf{z}}_R^{(t)} = [\tilde{z}_{R,1}^{(t)}, \cdots, \tilde{z}_{R,J}^{(t)}]^T = \mathbf{H}_{U,R}^{+}\mathbf{z}_R^{(t)}$ the relay collects the J-length vector

$$\tilde{\mathbf{y}}_R^{(t)} = [a_{1,t} + v_1 b_t + \tilde{z}_{R,1}^{(t)}, \ \cdots \ , a_{J,t} + v_J b_t + \tilde{z}_{R,J}^{(t)}]^T. \quad (16)$$

During the second phase, the relay reverts the dimension of time and space with respect to $\tilde{\mathbf{y}}_R^{(t)}$, and for $t = K + 1, \cdots, K + J$ it broadcasts

$$\mathbf{r}^{(t=K+j)} = \gamma \mathbf{V}_R \big[[\tilde{\mathbf{y}}_R^{(1)}]_j, \ \cdots \ , \ [\tilde{\mathbf{y}}_R^{(K)}]_j\big]^T, \ j = 1, \cdots, J,$$

where $\gamma \mathbf{V}_R$ precodes and constrains the power. With $\mathbf{V}_R = \mathbf{H}_{R,D}^{-1}$, at the end of the second phase, the i user in D collects the $J + 1$ observations

$$y_i^{(K+j)} = \gamma\big(a_{j,i} + v_j b_i + \tilde{z}_{R,j}^{(i)}\big) + z_i^{(K+j)}, \quad j = 1, \cdots, J,$$
$$y_i^{(i)} = \sum_{p=1}^{J} h_{p,i} a_{p,i} + z_i^{(i)} \quad (17)$$

received during $t = i$ and $t = K+1, \cdots, K+J$, and calculates the interference free

$$\tilde{y}_i = \gamma y_i^{(i)} - \sum_{j=1}^{J} h_{j,i} y_i^{(K+j)} = g_i b_i + \tilde{z}_i, \quad (18)$$

where $g_i = -\gamma \sum_{j=1}^{J} h_{j,i} v_j$ and $\tilde{z}_i = -\sum_{j=1}^{J} h_{j,i}\big(\gamma \tilde{z}_{R,j}^{(i)} + z_i^{(K+j)}\big) + \gamma z_i^{(i)}$ describe the signal attenuation and noise accentuation, and which are analyzed similar as in the case of $\mathcal{X}_2$, so that we can conclude that each downlink user achieves $1/(K+J)$ degrees of freedom. The case of uplink is simple as all interference is self-interference, and the number of unknown symbols per channel use equals the dimension of the channel. This combined with noise accentuation analysis as in the case of $\mathcal{X}_2$, gives us the achievable region of $\big(d_U^{(a)} = \frac{K}{K+J}, d_D^{(a)} = \frac{1}{K+J}\big)$, where $d_U^{(a)}$ is optimal.

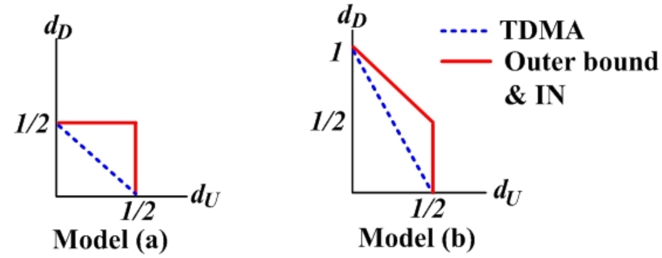

Fig. 3. DOF regions for $J = K$ for different settings. $\mathcal{X}_2$ v.s. TDMA. DOF in (b) is achieved with time-sharing $\mathcal{X}_2$ with silencing A.

IV. Conclusions

The work provided analysis and practical IN solutions that simultaneously reduce the effect of interference and half-duplex in selected multiuser uplink-downlink communications settings. The selected settings corresponded to different practical scenarios such as having differently sized uplink and downlink sets of possibly different rates and different connectivity to the base station. In terms of optimality, under the assumption of equal rate and size $(J = K)$ for the uplink and downlink sets, the solutions met the entire optimal DOF region, whereas in the $(J < K)$ case, the proposed solutions met the outer bound separately for the uplink and downlink, while maintaining a positive (but potentially suboptimal) rate in the opposite direction. This latter rate-asymmetric case brought to the fore the role of multiple-antenna relays in not only handling uplink-downlink interference, but also in further reducing the negative effects of the half-duplex constraint - for example it allowed uplink users without base station connectivity to achieve closer to full-duplex rates.

The near-optimal solutions, at a cost of substantial CSITR, offer simplicity of implementation and reduced delay as compared to interference alignment techniques. A final salient feature is that, even though transmitters encode over time, the solutions are causal, rendering them useful in cases where the different communication phases experience different fading.

V. Acknowledgments

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VI. Appendix

A. Proof of Lemma 1

1) Upper Bound: Consider a S-to-R-to-D half-duplex two-hop communication over a large number M of fading realizations $\{\mathbf{H}_{1,i}, \mathbf{H}_{2,i}\}_{i=1}^{M}$ where $\mathbf{H}_{j,i}$ denotes the fading realization during jth hop and ith block. In this case the rate is bounded as

$$R_{SD} \leq \max_{\{\Delta_{1,i}\}} \min\{\frac{1}{M}\sum_{i=1}^{M}\Delta_{1,i}\mathrm{I}_{1,i}, \frac{1}{M}\sum_{i=1}^{M}(1-\Delta_{1,i})\mathrm{I}_{2,i}\}, \quad (19)$$

where $\mathrm{I}_{1,i} = \max_{p(x)} \mathrm{I}(\mathbf{x}_S;\mathbf{y}_R|\mathbf{H}_{1,i})$, $\mathrm{I}_{2,i} = \max_{p(x)} \mathrm{I}(\mathbf{r};\mathbf{y}_D|\mathbf{H}_{2,i})$, where $\Delta_{1,i}$ is the duration of the first phase/hop (R in listening mode) during the ith fading realization, and where $p(x)$ denotes the joint probability distribution of the transmitted signals $\mathbf{x}_S$, $\mathbf{r}$ at S and R. We note that $\Delta_{1,i}, \Delta_{2,i}$ can be functions of the fading realizations.

For $\mathbb{C}_1 \triangleq \frac{1}{M}\sum_{i=1}^{M}\Delta_{1,i}\mathrm{I}_{1,i}, \mathbb{C}_2 \triangleq \frac{1}{M}\sum_{i=1}^{M}(1-\Delta_{1,i})\mathrm{I}_{2,i}$, we rewrite the optimization problem in (19) in its Lagrangian form of maximizing $\Gamma = \mathbb{C}_1 + \varphi(\mathbb{C}_2 - \mathbb{C}_1)$. We also define a function $g(s)$ as $g(s) = 1$ for $s \geq 0$ and $g(s) = 0$ for $s < 0$, and note that since $\frac{\partial\Gamma}{\partial\Delta_{1,i}} = \frac{(\mathrm{I}_{1,i}+\mathrm{I}_{2,i})}{M}(\frac{\mathrm{I}_{1,i}}{(\mathrm{I}_{1,i}+\mathrm{I}_{2,i})} - \varphi), \forall i$, the optimal $\Delta_{1,i}$ takes the form $\Delta_{1,i} = g(\frac{\mathrm{I}_{1,i}}{\mathrm{I}_{1,i}+\mathrm{I}_{2,i}} - \varphi)$. In turn, the maximizing $\varphi = \varphi'$ guarantees that $\mathbb{C}_1 = \mathbb{C}_2$, which implies that $\Delta_{1,i} = g(\frac{\mathrm{I}_{1,i}}{\mathrm{I}_{1,i}+\mathrm{I}_{2,i}} - \varphi'), \forall i$. Defining

$$\mathcal{A}_x = \{\{\mathbf{H}_{1,i}, \mathbf{H}_{2,i}\} \mid g(\frac{\mathrm{I}_{1,i}}{\mathrm{I}_{1,i}+\mathrm{I}_{2,i}} - \varphi') = x\}, \quad x = 0, 1$$

we have that $\mathbb{C}_1 = P(\mathcal{A}_1)\mathbb{E}_{\{\mathbf{H}_{1,i},\mathbf{H}_{2,i}\}\in\mathcal{A}_1}\mathrm{I}_{1,i}$ and $\mathbb{C}_2 = P(\mathcal{A}_0)\mathbb{E}_{\{\mathbf{H}_{1,i},\mathbf{H}_{2,i}\}\in\mathcal{A}_0}\mathrm{I}_{2,i}$. For $\mathbb{C}_{SR}^{'} \triangleq \mathbb{E}_{\{\mathbf{H}_{1,i},\mathbf{H}_{2,i}\}\in\mathcal{A}_1}\mathrm{I}_{1,i}$, $\mathbb{C}_{RD}^{'} \triangleq \mathbb{E}_{\{\mathbf{H}_{1,i},\mathbf{H}_{2,i}\}\in\mathcal{A}_0}\mathrm{I}_{2,i}$, $\eta \triangleq \frac{\mathbb{C}_{SR}^{'}}{\mathbb{C}_{RD}^{'}}$, for the optimal φ', and for the optimal sequence $\{\Delta_{1,i}\}$, then the expression

$$\begin{aligned}\mathbb{C}_{SD} &= \frac{\mathbb{C}_1 + \eta\mathbb{C}_2}{1+\eta} = \frac{P(\mathcal{A}_1)\mathbb{C}_{SR}^{'} + \eta P(\mathcal{A}_0)\mathbb{C}_{RD}^{'}}{1+\eta} \\ &= \frac{\mathbb{C}_{SR}^{'}}{1+\eta} = \frac{\mathbb{C}_{SR}^{'}\mathbb{C}_{RD}^{'}}{\mathbb{C}_{SR}^{'}+\mathbb{C}_{RD}^{'}}, \end{aligned} \quad (20)$$

serves as an upper bound on the rate.

For d_j being the maximum DOF of the jth hop channel, $j \in \{1,2\}$, it can be shown that in the limit of high ρ, $\mathrm{I}_{j,i}$ is bounded above by $d_j \log\rho$ with probability one, in the sense that for an arbitrary positive scalar $\delta > 0$, then $P[\log\det(I + \rho\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\dagger}) \geq (d_j+\delta)\log\rho] \doteq 0$. As a result at high ρ, $\mathbb{C}_{SR}^{'}$ and $\mathbb{C}_{RD}^{'}$ are respectively upper bounded by $d_1\log\rho$ and $d_2\log\rho$, and consequently the DOF are upper bounded as $\frac{d_1 d_2}{d_1+d_2}$ since $\mathbb{C}_{SD} = \frac{\mathbb{C}_{SR}^{'}\mathbb{C}_{RD}^{'}}{\mathbb{C}_{SR}^{'}+\mathbb{C}_{RD}^{'}} \leq \frac{(d_1\log\rho)(d_2\log\rho)}{d_1\log\rho + d_2\log\rho}$.

2) Achievability: For achievability we show that $\mathbb{C}_{SD} = \frac{\mathbb{C}_{SR}\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}$ is achievable, where $\mathbb{C}_{SR}$ ($\mathbb{C}_{RD}$) is the ergodic capacity of first (second) hop. For achievability we set $\Delta_{1,i} = \frac{\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}} = 1 - \Delta_{2,i}, \forall i$ and divide the total duration of transmission into M_1 blocks, each with $M_2 = M/M_1$ sub-blocks, where each sub-block has two phases of fractional duration of $\Delta_{1,i}$ and $\Delta_{2,i}$. We further ask that both M_1, M_2 are sufficiently large, and that the channel changes independently with each sub-block. During the first phase of each sub-block of the jth block, S transmits while R listens, and then R waits to transmit this decoded message during the second phase of the sub-blocks of the next $(j+1)$th block, during which S is silent. As a result $\mathbb{C}_1 = \frac{\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}\mathbb{E}\mathrm{I}_{1i} = \frac{\mathbb{C}_{SR}\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}$ and $\mathbb{C}_2 = \frac{\mathbb{C}_{SR}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}\mathbb{E}\mathrm{I}_{2i} = \frac{\mathbb{C}_{SR}\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}$ which proves that $\mathbb{C}_{SD} = \frac{\mathbb{C}_{SR}\mathbb{C}_{RD}}{\mathbb{C}_{SR}+\mathbb{C}_{RD}}$ and that $d_{SD} = \frac{d_1 d_2}{d_1+d_2}$ is achievable which completes the proof. □

B. Proof of Lemma 2

We wish to bound the probability that the inverse of $\mathbf{E} = \sum_{i=1}^{N}\prod_{j=1}^{M_i}\mathbf{H}_{i,j}^{\tau_{i,j}}$ has large power. To begin with, split exponent indexes into sets $S_i(z) = \{j|\tau_{i,j} = z\}$ and let $\{x\}$ be the collection of all random variables, entries in any $\mathbf{H}_{i,j}$. Now recalling that $\mathbf{H}_{i,j}^{-1} = \frac{\mathrm{adj}(\mathbf{H}_{i,j})}{\det(\mathbf{H}_{i,j})}$ where $\mathrm{adj}(\mathbf{H}_{i,j})$ denotes the matrix of cofactors of $\mathbf{H}_{i,j}$, and setting $\mathbf{S}_{i,j} = \mathrm{adj}(\mathbf{H}_{i,j})$ when $j \in S_i(-1)$, else setting $\mathbf{S}_{i,j} = \mathbf{H}_{i,j}$, we have that

$$\mathbf{E} = \sum_{i=1}^{N}\frac{\prod_{j=1}^{M_i}\mathbf{S}_{i,j}}{\prod_{j\in S_i(-1)}\det(\mathbf{H}_{i,j})} = \frac{\sum_{i=1}^{N}\prod_{\ell\neq i}\prod_{j\in S_\ell(-1)}\det(\mathbf{H}_{\ell,j})\mathbf{S}_{i,1}\cdots\mathbf{S}_{i,M_i}}{\prod_{i}\prod_{j\in S_i(-1)}\det(\mathbf{H}_{i,j})}.$$

Defining the numerator and denominator in the above equation respectively as $\bar{\mathbf{E}}$ and ζ, we have that $\mathbf{E}^{-1} = (\frac{\bar{\mathbf{E}}}{\zeta})^{-1} = \zeta\frac{\mathrm{adj}(\bar{\mathbf{E}})}{\det(\bar{\mathbf{E}})}$ and that $||\mathbf{E}^{-1}||_F^2 = \frac{|\zeta|^2\sum_{m,n}|[\mathrm{adj}(\bar{\mathbf{E}})]_{m,n}|^2}{|\det(\bar{\mathbf{E}})|^2}$. Defining the numerator and denominator in the above equation respectively as $f_1(\{x\}), f_2(\{x\})$, we note that all the entries of $\bar{\mathbf{E}}$ and ζ, and consequently both $f_1(\{x\})$ and $f_2(\{x\})$ are, with probability 1, positive exponent polynomials in $\{x\}$ without constant terms. Hence the result in [9, Lemma 2.10] applies to tell us that there exists a constant $d > 0$ such that $P(f_2(\{x\}) \dot{\leq} \rho^{-\epsilon}) \dot{\leq} \rho^{-\epsilon/d}$. Combining this with the straightforward fact that $P(|f_2(\{x\})| \dot{>} \rho^0) \doteq 0$ completes the proof. □

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