

Multiband Time-of-Arrival Positioning Technique for Cognitive Radio Systems

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Abstract—Accurate information regarding a cognitive radio user's location and environment can enhance the adaptive and spectral awareness capabilities of cognitive radio systems. In this paper, a single-path multiband Time-of-Arrival (TOA) positioning technique for cognitive radio is proposed and the performance evaluated using maximum-likelihood (ML) location estimation for a typical rural scenario where signal line-of-sight (LOS) between a transmitter and receiver is prevalent. The multiband Cramer Rao Lower Bound (CRLB) time-delay and channel fading coefficient estimates are derived followed by an estimation combining technique which involves selecting an overall optimum estimate using all the utilized bands. It is observed that the improvement in positioning accuracy for the multiband system depends primarily on the number of utilized bands as well as the (signal-to-noise ratio) SNR of these bands.

Keywords— Cognitive radio, Cramer Rao Lower Bound, Location estimation, Multiband positioning, Time-of-Arrival

I. INTRODUCTION

There has been a growing active interest among the wireless communications research community, within the area of dynamic spectrum access networks as well cognitive radio systems in an effort to tackle the problem of inefficient spectrum usage. Various empirical measurements conducted by regulatory bodies and research institutions indicate that large percentages of the radio frequency band are under-utilized and in certain other cases, extremely utilized [1], [2]. Adaptivity and awareness are certain key characteristics associated with cognitive radio [3]. One such issue which has become the focus of attention is the location and environment awareness capability of cognitive radio [4], [5]. Receiver location information has become an integral part of the IEEE 802.22 working standard for cognitive radio Wireless Regional Area Networks (WRAN) [6]. A key feature of this standard involves dynamic channel allocation which is performed by combining spectrum sensing functions and geo-location information with an operational service database. In addition to this, knowledge of a cognitive radio user's location data also plays an important role in setting up transmission protocols for spectrum efficient communications [4]. The Cognitive Positioning System (CPS) proposed in [7] utilizes a combination of adaptive bandwidth selection as well as dynamic spectrum allocation techniques

to address the location awareness requirements of cognitive radio. Time-based ranging schemes such as Time-of-Arrival (TOA) are particularly suited for such localization systems employing Ultra-wideband (UWB) in cognitive radio since the bandwidth and SNR of the signal play an important role in the positional accuracy of the receiver [8]. Previous works [9], [10] have shown that conventional TOA ranging is a suitable technique for single bands especially with respect to UWB systems. It has also been shown that optimal two-step time-delay estimation can be conducted simultaneously on dispersed bands for cognitive radio systems using a variety of combining schemes, each with different degrees of performance under various modulation schemes [11], [12].

In this paper, an analytic approach for performing two dimensional (2D) location estimation over multiple (unoccupied) bands in the context of opportunistic spectrum access for cognitive radio networks is proposed. The best achievable positioning accuracy is achieved through the generalized derivation of the CRLB time delay and channel coefficient estimates for multiple bands. A combining technique using the computed estimates for each band is then derived to obtain an optimal time delay and channel coefficient estimate which results in improved user location estimation when compared to a single band system. This particular multiband positioning technique is then evaluated using a modified maximum-likelihood estimation algorithm for rural scenarios in which the dominant signal components are LOS. This proposed positioning technique has been developed within the context of future application in various multicarrier communication standards, most of which, are fundamentally based on orthogonal frequency division multiplexing (OFDM) such as DVB-T, LTE, etc.[13].

The rest of the paper is organized as follows: In Section II we provide an overview of the multiband signal model. Section III, provides the derived CRLB estimate of the time delay and channel coefficients for multiple bands. The combining technique is presented in IV and simulation results are provided in Section V. Section VI draws the main conclusions of this study.

II. SIGNAL MODEL

For this model, it has been assumed that the mobile terminal (MT) is synchronized with the different base stations, each of which are transmitting single-path signals over multiple bands. These multibands are opportunisticly utilized for TOA positioning. Furthermore, prior information about the state of the signal path (LOS) is also assumed to be known. Figure 1 displays the proposed receiver model for the multiband TOA positioning system where β_1, \dots, β_N represents the bandwidths of each band. The time delay and channel coefficient are estimated for the overall multiband system. The baseband representation of the each received signal is given as:

$$r_i(t) = \alpha_i s_i(t - \tau) + n_i(t), \quad 0 \leq t \leq T, \quad i \in [1..N], \quad (1)$$

where α_i is the channel fading coefficient of each band, $s_i(t - \tau)$ is the delayed transmitted signal occupying a specific bandwidth (β_i) and $n_i(t)$ is zero mean white Gaussian noise with spectral density σ_i^2 .

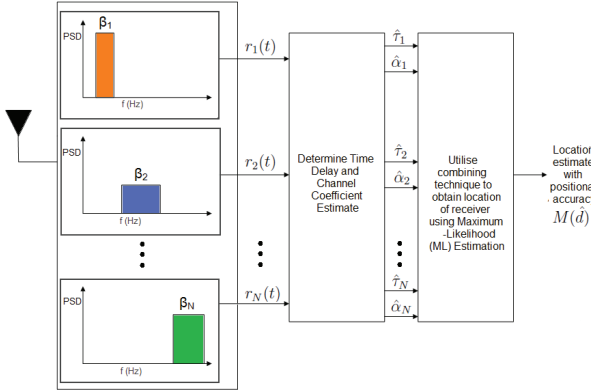


Figure 1. MT Receiver model for multiband positioning systems

A time delay and channel coefficient estimate for each band is obtained through parallel signal processing. In this case the key advantage is that the large bandwidths offered by the network are exploited to obtain an improved location estimate by considering multiple time delays as opposed to a single time delay estimate. The combining technique considers the CRLB estimate of each band and thereafter computes the mean estimate for all N bands to obtain an overall optimal time delay estimate for TOA ranging.

III. CRLB OF TIME-DELAY AND CHANNEL COEFFICIENT ESTIMATES

A vector of unbiased signal parameters are to be estimated:

$$\Theta = [\tau \quad \alpha], \quad (2)$$

where τ is the signal time delay and $\alpha = [\alpha_1 \dots \alpha_N]$ represents the vector of complex channel coefficients corresponding to N bands which describe the fading of each received signal. The assumption is made that over the interval between 0 and T_s (symbol time), the transmitted pulse given by $s(t)$ is non-zero

and band-limited to B Hz. As a result the observation interval encompasses the symbol time and maximum time delay which can be also shown as follows: $T = T_s + \tau_{max}$. The CRLB of a vector of unknown parameters is given by the first row and first column of an inverse matrix and represented as follows [14]:

$$var(\hat{\Theta}_i) \geq [\mathbf{I}^{-1}(\Theta)]_{ii}, \quad (3)$$

where $\mathbf{I}(\Theta)$ is a $q \times q$ Fisher Information Matrix (FIM) and q is defined by the number of unknown parameters to estimate. In this case, $\Theta_1 = \tau$ and $\Theta_2 = \alpha$. The elements of the 2×2 FIM have been determined for a general Gaussian case for a discrete received signal using [14]:

$$\mathbf{I}(\Theta) = \sum_{i=1}^N \frac{1}{\sigma_i^2} \sum_{k=0}^{K-1} \frac{\partial s_i[k; \Theta]}{\partial \Theta_i} \frac{\partial s_i[k; \Theta]}{\partial \Theta_j} \quad (4)$$

where $i = 1, 2, \dots, q$ and $j = 1, 2, \dots, q$ and N is the total number of bands. The FIM is therefore represented as:

$$\mathbf{I}(\Theta) = \begin{bmatrix} I_{\tau\tau} & \mathbf{I}_{\tau\alpha} \\ \mathbf{I}_{\alpha\tau} & \mathbf{I}_{\alpha\alpha} \end{bmatrix} \quad (5)$$

The following FIM elements can be derived using (4):

$$I_{\tau\tau} = \sum_{i=1}^N \frac{|\alpha_i|^2 \hat{\epsilon}_i}{\sigma_i^2} \quad (6)$$

$$\mathbf{I}_{\tau\alpha} = \mathbf{I}_{\alpha\tau}^T = - \begin{bmatrix} \frac{|\alpha_1| \tilde{\epsilon}_1}{\sigma_1^2}, \dots, \frac{|\alpha_N| \tilde{\epsilon}_N}{\sigma_N^2} \end{bmatrix} \quad (7)$$

$$\mathbf{I}_{\alpha\alpha} = \text{diag} \left[\frac{\epsilon_1}{\sigma_1^2}, \dots, \frac{\epsilon_N}{\sigma_N^2} \right], \quad (8)$$

where $\hat{\epsilon}_i$ and $\tilde{\epsilon}_i$ are respectively given as follows:

$$\hat{\epsilon}_i = \int_0^T |s'_i(t - \tau)|^2 dt, \quad (9)$$

$$\tilde{\epsilon}_i = \int_0^T |s'_i(t - \tau)| |s_i(t - \tau)| dt. \quad (10)$$

The first derivative of the signal energy $s_i(t - \tau)$ is given by $s'_i(t - \tau)$, while the energy (ϵ_i) of the signal $s_i(t - \tau)$ is given as:

$$\epsilon_i = \int_0^T |s_i(t - \tau)|^2 dt. \quad (11)$$

The CRLB of the time delay estimate can be obtained using the following matrix algebraic manipulation:

$$[I_{\tau\tau}]^{-1} = (I_{\tau\tau} - \mathbf{I}_{\tau\alpha} \mathbf{I}_{\alpha\alpha}^{-1} \mathbf{I}_{\alpha\tau})^{-1}, \quad (12)$$

and similarly the channel coefficients can be computed as follows:

$$[\mathbf{I}_{\alpha\alpha}]^{-1} = (\mathbf{I}_{\alpha\alpha} - \mathbf{I}_{\alpha\tau} I_{\tau\tau}^{-1} \mathbf{I}_{\tau\alpha})^{-1}. \quad (13)$$

As a result the time delay estimate of the signal is given by:

$$[I_{\tau\tau}]^{-1} = \sum_{i=1}^N \frac{|\alpha_i|^2}{\sigma_i^2} \left(\hat{\varepsilon}_i - \frac{\tilde{\varepsilon}_i^2}{\varepsilon_i} \right) \quad (14)$$

$$= \text{var}(\hat{\tau})^{-1} \quad (15)$$

It can be noted the overall estimated time-delay is dependent on the channel coefficient for each band. Using (15) it is possible to derive a relationship between the CRLB time-delay estimate and the positional accuracy of receiver (MT):

$$\text{var}(\hat{\tau}) = \frac{1}{c^2 M(\hat{d})}, \quad (16)$$

where c is the speed of light and $M(\hat{d})$ is the positional accuracy of the MT. Using (14), (16) and the bandwidth representation in the Fourier domain which translates to $\hat{\varepsilon}_i = \varepsilon_i \beta_i^2$, the positional accuracy is shown to be:

$$M(\hat{d}) = \frac{1}{c^2} \sum_{i=1}^N \frac{|\alpha_i|^2}{\sigma_i^2} \left(\varepsilon_i \beta_i^2 - \frac{\tilde{\varepsilon}_i^2}{\varepsilon_i} \right). \quad (17)$$

It can be also shown that the CRLB estimate of the channel coefficient for all dispersed bands can be computed using (13) resulting in:

$$[\mathbf{I}_{\alpha\alpha}]^{-1} = \text{diag} \left[\frac{\varepsilon_1}{\sigma_1^2}, \dots, \frac{\varepsilon_N}{\sigma_N^2} \right] - \frac{\sum_{i=1}^N \frac{|\alpha_i|^2 \tilde{\varepsilon}_i^2}{(\sigma_i^2)^2 \varepsilon_i}}{\sum_{i=1}^N \hat{\varepsilon}_i \frac{|\alpha_i|^2}{\sigma_i^2}} \quad (18)$$

As a result, let $[\mathbf{I}_{\alpha\alpha}]^{-1}$ be a matrix such that:

$$[\mathbf{I}_{\alpha\alpha}]^{-1} = \text{var}(\hat{\alpha}) = D_{i,j} \Big|_{\substack{i=1..N \\ j=1..N}}, \quad (19)$$

where N represents the total number of bands and each element $D_{i,j}$ is given by:

$$D_{i,j} = \begin{cases} \frac{\varepsilon_i}{\sigma_i^2} - \frac{\sum_{i=1}^N \frac{|\alpha_i|^2 \tilde{\varepsilon}_i^2}{(\sigma_i^2)^2 \varepsilon_i}}{\sum_{i=1}^N \hat{\varepsilon}_i \frac{|\alpha_i|^2}{\sigma_i^2}} & \text{if } i = j \\ -\frac{\sum_{i=1}^N \frac{|\alpha_i|^2 \tilde{\varepsilon}_i^2}{(\sigma_i^2)^2 \varepsilon_i}}{\sum_{i=1}^N \hat{\varepsilon}_i \frac{|\alpha_i|^2}{\sigma_i^2}} & \text{if } i \neq j \end{cases} \quad (20)$$

Location accuracy adaptation is dependent on the available bandwidth in the spectrum. According to (17), it is inherently impossible to extract the required bandwidth for a specific positional accuracy. Therefore an alternative combining estimation technique has been proposed which allows the extraction of the estimated required bandwidth which in turn, enhances the location accuracy adaptivity of cognitive radio.

IV. TIME-DELAY AND CHANNEL COEFFICIENTS COMBINING ESTIMATION TECHNIQUE

The previous section provided the CRLB time delay estimates and channel coefficient estimates for the overall multi-band system. Using similar methods, it is possible to derive the estimates for each band, and combine each estimate to obtain an overall optimal solution. The estimation parameters for each band can be represented in vector form as follows:

$$\Theta_i = [\tau_i \ \alpha_i] \quad (21)$$

This results in a 2×2 FIM given by:

$$\mathbf{I}(\Theta_i) = \begin{bmatrix} I_{\tau_i \tau_i} & I_{\tau_i \alpha_i} \\ I_{\alpha_i \tau_i} & I_{\alpha_i \alpha_i} \end{bmatrix} \quad (22)$$

In a similar fashion, using (4), each FIM element is given by:

$$I_{\tau_i \tau_i} = \frac{|\alpha_i|^2 \hat{\varepsilon}_i}{\sigma_i^2} \quad (23)$$

$$I_{\tau_i \alpha_i} = I_{\alpha_i \tau_i} = -\frac{|\alpha_i| \tilde{\varepsilon}_i}{\sigma_i^2} \quad (24)$$

$$I_{\alpha_i \alpha_i} = \frac{\varepsilon_i}{\sigma_i^2} \quad (25)$$

The CRLB of the time delay estimate can be computed in a similar way to (12), bearing in mind that none of the elements consist of vectors:

$$[I_{\tau_i \tau_i}]^{-1} = (I_{\tau_i \tau_i} - I_{\tau_i \alpha_i} I_{\alpha_i \alpha_i}^{-1} I_{\alpha_i \tau_i})^{-1} \quad (26)$$

Therefore, the CRLB of the time delay estimate for an individual band is given as:

$$\begin{aligned} [I_{\tau_i \tau_i}]^{-1} &= \frac{|\alpha_i|^2 \hat{\varepsilon}_i}{\sigma_i^2} - \frac{|\alpha_i| \tilde{\varepsilon}_i}{\sigma_i^2} \times \frac{\sigma_i^2}{\varepsilon_i} \times \frac{|\alpha_i| \tilde{\varepsilon}_i}{\sigma_i^2} \\ &= \frac{|\alpha_i|^2}{\sigma_i^2} \left(\hat{\varepsilon}_i - \frac{\tilde{\varepsilon}_i^2}{\varepsilon_i} \right) \\ &= \text{var}(\hat{\tau}_i)^{-1} = c^2 M(\hat{d}) \end{aligned} \quad (27)$$

As mentioned earlier using $\hat{\varepsilon}_i = \varepsilon_i \beta_i^2$, we can obtain a bandwidth determination equation based on the required positional accuracy of the MT which can be adapted based on the availability of the required number of bands:

$$\hat{\beta}_i = \sqrt{\frac{c^2 M(\hat{d}) \sigma_i^2}{|\alpha_i|^2 \varepsilon_i} + \frac{\tilde{\varepsilon}_i^2}{\varepsilon_i^2}}. \quad (28)$$

It can be noted that the derivative of the signal energy (constant) is zero which results in $\tilde{\varepsilon} = 0$. The estimated required bandwidth for a specified positioning accuracy is therefore shown as:

$$\hat{\beta}_i = \sqrt{\frac{c^2 M(\hat{d})}{|\alpha_i|^2 \zeta_i}}, \quad (29)$$

where ζ_i is the SNR of the received signal. The CRLB of

the channel coefficient estimate for an individual band is calculated using:

$$[I_{\alpha_i \alpha_i}]^{-1} = (I_{\alpha_i \alpha_i} - I_{\alpha_i \tau_i} I_{\tau_i \tau_i}^{-1} I_{\tau_i \alpha_i})^{-1}. \quad (30)$$

This leads to the following CRLB for the channel coefficient estimate:

$$\begin{aligned} [I_{\alpha_i \alpha_i}]^{-1} &= \frac{1}{\sigma_i^2} \left(\varepsilon_i - \frac{\hat{\varepsilon}_i^2}{\hat{\varepsilon}_i} \right) \\ &= \text{var}(\hat{\alpha}_i)^{-1}. \end{aligned} \quad (31)$$

The SNR estimates of each band can be extracted from (31), provided $\tilde{\varepsilon} = 0$. Each of the time delay and channel coefficient estimates for each band are then averaged over the total number of bands to obtain an optimal estimate. As a result the channel coefficient and SNR of the received signal together with the total number of bands, are important parameters which affect the TOA location estimate.

V. NUMERICAL RESULTS

A. Simulation Environment

The CRLB is much harder to achieve practically since it represents the absolute lower bound on the variance of an unbiased estimator and as a result various techniques have been developed which closely approximate the CRLB. Hence, the performance of the multiband TOA positioning model for cognitive radio was analyzed using a two-step maximum-likelihood (ML) ranging location estimation algorithm. This particular iterative approximation to the ML location estimation algorithm is based on determining a MTs 2D position using intersecting hyperbolic curves. As a result the two-step ML estimator (θ) can be found by maximizing the following probability density function (pdf) [15]:

$$P(\hat{\mathbf{r}}|\theta) = \left(\sqrt{(2\pi)^N \det\{\mathbf{Q}\}} \right)^{-1} \exp\left\{-\frac{\mathbf{J}}{2}\right\}, \quad (32)$$

where \mathbf{Q} represents the noise covariance matrix and \mathbf{J} is given as:

$$\mathbf{J} = [\hat{\mathbf{r}} - \mathbf{r}(\theta)]^T \mathbf{Q}^{-1} [\hat{\mathbf{r}} - \mathbf{r}(\theta)]. \quad (33)$$

The solution can therefore be determined by calculating the θ that will minimize \mathbf{J} . As a result two ML equations are derived from (33) which have to be simultaneously solved [15]:

$$\sum_{i=1}^M \frac{(r_i - \hat{r}_i)(x - x_i)}{r_i} = 0, \quad (34)$$

$$\sum_{i=1}^M \frac{(r_i - \hat{r}_i)(y - y_i)}{r_i} = 0, \quad (35)$$

where M is the number of base stations, r is the true distances and \hat{r}_i is the estimated distances between the base station and MT, x and y are the 2D coordinates of the MT and x_i and y_i are the 2D coordinates of base station i .

The scenario under investigation is one in which the LOS is a dominant component as typically exhibited by a rural environment. It is assumed that prior information about the channel coefficients are known and therefore modeled as random variables having a Rician distribution with a K-factor of 4 dB. In addition, four base stations with fixed co-ordinates were utilized to perform localization of the mobile terminal. A NLOS bias was added to the signal components of one of the base stations to model a reasonably realistic rural scenario.[16].

B. Results and Analysis

The performance of the single band and multiband systems are compared to quantify the improvement in terms of positional accuracy for three different bands (3, 6 and 9 MHz). Figure 2 represents the MTs Root Mean Square Error (RMSE) of the location estimate as a function of the SNR. An improvement in the location estimate can be noted when estimating the MT's location using more than one band. This is due to the fact that each band is characterized by a different time delay and channel fading coefficient and can be optimally combined to yield an improved estimate. At lower SNRs the improvement in accuracy is much more pronounced while at higher SNRs the location estimates tend to converge according to the number of bands utilized to perform TOA.

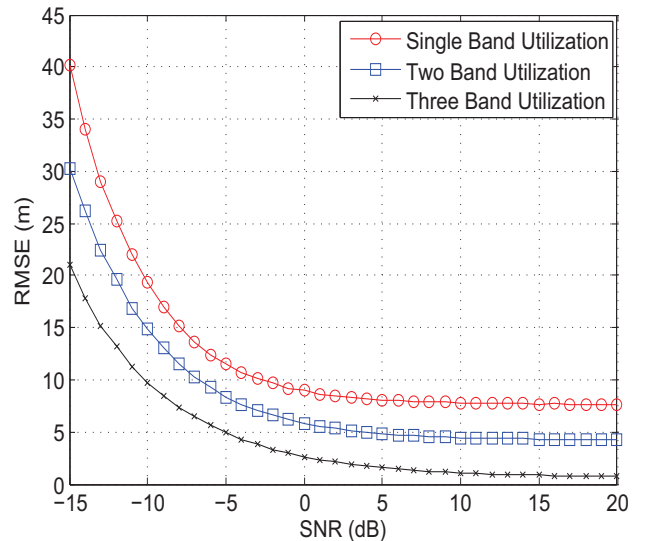


Figure 2. RMSE of the location estimate versus the SNR for rural scenarios for single band, two bands and three bands utilization.

Figure 3 further illustrates the effect of the combining estimation technique on the RMSE error as the number of bands are increased. These results are compared for three different fixed SNR values. For this particular scenario four different bands were chosen, each with a bandwidth of 3 MHz, 6 MHz, 9 MHz and 12 MHz respectively. For the case of single band utilization, the RMSE error for each individual band was separately computed and then this error averaged over all four bands. The overall positioning accuracy for two bands

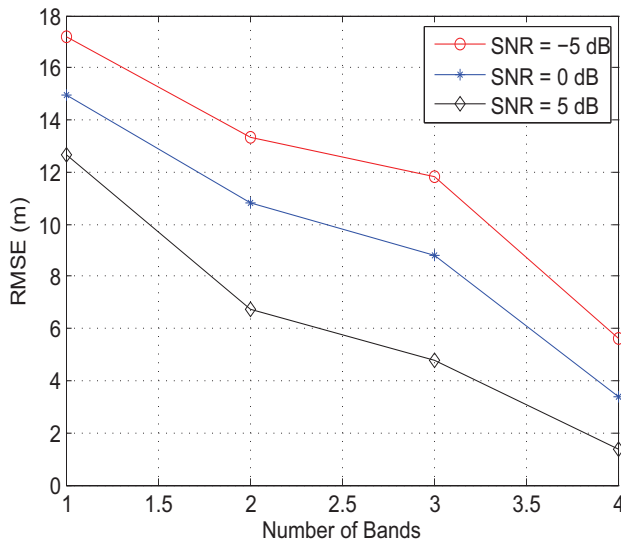


Figure 3. RMSE of the location estimate versus the number of bands for rural scenarios using different SNR (-5 dB, 0 dB and 5 dB).

was computed by taking the mean ML RMSE error over all possible two band permutations (i.e. an error was obtained for utilizing the 3 and 6 MHz bands, another error was obtained for using the 3 and 9 MHz bands, etc. and thereafter the mean RMSE error was computed using the errors from each unique band permutation). In a similar fashion, the positional accuracy for three bands represented the average error for all unique 3 band permutations such as 3, 6 and 9 MHz, 3, 9 and 12 MHz, etc. In the case of four band utilization only one unique permutation exists (3, 6, 9 and 12 MHz) and therefore the mean time delay from all these bands provided the overall RMSE error. An increase in the RMSE error can be observed for each band when comparing Figures 2 and 3. This is due to the fact that since an additional band (i.e. 12 MHz) has been introduced, the number of permutations increase, causing the overall RMSE error to be higher for each band. However, this does not affect the overall improvement in multiband TOA positioning accuracy over single band.

VI. CONCLUSIONS AND FUTURE WORK

The localization performance of a multiband positioning system for cognitive radios has been presented for scenarios where LOS is the dominant signal component (rural environments). The simulation results highlight the advantage of exploiting large bandwidths in a dispersed and opportunistic manner at the cognitive radio receiver for lowering the distance estimation error. The key performance advantage involves the utilization of multiple dispersed bands in frequency and time to increase the positioning accuracy of the MT using the mean estimate combining technique.

Suggestions for future work may involve the performance evaluation in a typical cognitive radio environment where data from spectrum sensing provides realistic view on the availability of multiple bands which can be then used to perform TOA localization. Consideration of the synchronization error

between the base station and mobile terminal will also cater for a more practical scenario.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement SACRA n°249060. This work was supported also by the European projects: ACROPOLIS (advanced coexistence technologies for radio optimization and unlicensed spectrum), and CROWN (cognitive radio oriented wireless networks).

REFERENCES

- [1] M. Mueck *et al.*, "ETSI reconfigurable radio systems: status and future directions on software defined radio and cognitive radio standards," *IEEE Commun. Mag.*, vol. 48, no. 9, pp. 78–86, Sept. 2010.
- [2] M. Wellens, J. Wu, and P. Mahonen, "Evaluation of Spectrum Occupancy in Indoor and Outdoor Scenario in the Context of Cognitive Radio," in *Proc. 2nd Intl. Crowncom Conf.*, vol. 4, June 2006, pp. 1658–1663.
- [3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, February 2005.
- [4] H. Arslan and H. Celebi, "Utilisation of Location Information in Cognitive Wireless Networks," *IEEE Wireless Commun. Mag.*, vol. 14, no. 4, pp. 6–13, August 2007.
- [5] H. Celebi and H. Arslan, "Enabling location and environment awareness in cognitive radios," *Comp. Commun.*, vol. 31, no. 4, pp. 1114–1125, January 2008.
- [6] C. Stevenson, G. Chouinard, Z. Lei, W. Hu, S. Shellhammer, and W. Caldwell, "IEEE 802.22: The First Cognitive Radio Wireless Regional Area Network Standard," *IEEE Commun. Mag.*, vol. 47, no. 1, pp. 130–138, January 2009.
- [7] H. Celebi and H. Arslan, "Cognitive Positioning Systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4475–4483, December 2007.
- [8] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, "Localization via Ultra-Wideband Radios," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 70–84, July 2005.
- [9] Y. Qi, H. Kobayashi, and H. Suda, "On Time-of-arrival Positioning in Multipath Environment," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 5, pp. 1516–1526, September 2006.
- [10] L. Mailaender, "Comparing Geo-location bounds for TOA, TDOA and Round trip TOA," in *Proc. 18th IEEE Int. Symp. PIMRC*, Athens, Sept. 2007, pp. 1–5.
- [11] S. Gezici, H. Celebi, H. V. Poor, and H. Arslan, "Fundamental limits on time delay estimation in dispersed spectrum cognitive radio systems," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 1, pp. 78–83, Jan. 2009.
- [12] F. Kocak, H. Celebi, S. Gezici, K. A. Qaraqe, H. Arslan, and H. V. Poor, "Time delay estimation in dispersed spectrum cognitive radio systems," *Eurasip Journal on Advances in Signal Processing*, vol. 2010, pp. 1–10, 2010.
- [13] B. Farhang-Boroujeny and R. Kempter, "Multicarrier communication techniques for spectrum sensing and communication in cognitive radios," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 80–85, April 2008.
- [14] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume 1: Estimation Theory*, 1st ed. Addison Wesley Longman, 2001.
- [15] I. Guvenc and C. Chong, "A Survey on TOA based Wireless Localization and NLOS Mitigation Techniques," *IEEE Commun. Surveys and Tutorials*, vol. 11, no. 3, pp. 107–124, August 2009.
- [16] P. C. Chen, "A non-line-of-sight error mitigation algorithm in location estimation," in *Proc. IEEE WCNC Conf.*, New Orleans, Sept. 1999, pp. 316–320.