Maximizing Diversity in Coded Slow Frequency-Hopped Communications

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Abstract

This paper examines the performance of radio systems employing error-control coding in conjunction with frequency-hopping to combat the effects of multipath fading. We show for *Rayleigh fading* that the achievable diversity is bounded by a maximum value which depends on the number of available frequencies, the code rate and the size of the modulation. Two key results are that the maximum diversity can be reached with fairly simple convolutional codes, and that constellation expansion can increase diversity. With the aid of computer simulations we show that added complexity yields non-negligible coding gain in the frame error-rate performance but may have less of an effect on the bit error-rate performance.

1. Introduction

GSM and its derivatives DCS 1800 and PCS 1900 [1,2] are prime examples of systems which combine error-control coding and frequency-hopping effectively to achieve diversity over fading channels. The use of frequency-diversity is especially attractive in such systems since the diversity level is invariant to the speed of channel variation. This is not the case in time-diversity systems such as IS-54 and IS-95 which achieve little diversity from coding at low mobile speeds. Wideband systems like IS-95 do, however, exploit some frequency-diversity by using a RAKE receiver prior to decoding. It should be mentioned that GSM exploits frequency-diversity to some extent with equalization prior to decoding.

It is reasonable to expect that third generation systems may employ similar coding/frequency-hopping techniques, and quite possibly more complex ones. It is for this reason that we examine the interaction between coding and frequency-hopping. More precisely, we are interested in determining the maximum achievable diversity given the code rate, decoding complexity and number of available and independent carrier frequencies. We also examine the possible gains in employing simple coded-modulation schemes which expand the constellation. The approach take here is analytic from the outset and computer simulations are performed in the end to verify the analysis.

2. System Description and Diversity Measure

Let us assume that a block of bits is encoded/modulated into F blocks of N symbols with an information rate of R bits/symbol. In order to spread the information evenly across the F blocks, the coded symbols are interleaved such that after deinterleaving, any F *adjacent* received symbols came from a different block. Frequency-hopping assures that each block is transmitted on a different carrier frequency, and we will assume in this work that the carrier spacing is larger than the coherence bandwidth. As a result, any set of F adjacent received symbols are affected by uncorrelated channel realizations. This is much more restrictive than ideal interleaving models which assume that *all* received symbols are affected by uncorrelated channel realizations. This restriction must be taken into account in the code design process which is considered in Section 3. This issue is treated to a large extent in [3] for a simplified fading channel model.

Denoting the NF interleaved coded symbols belonging to the symbol alphabet (constellation) S by the codevector

$$\mathbf{c} = \left[c_{0,0}\ c_{0,1}\ \cdots\ c_{0,N-1}\ c_{1,0}\ \cdots\ c_{F-1,N-1}\right],$$

we consider the discrete-time model for the system

$$r_{f,k} = \sqrt{E_s\alpha_f}c_{f,k} + n_{f,k},$$

where $r_{f,k}$ is the k^{th} received symbol from the f^{th} block, E_s is the symbol energy, α_f is the attenuation of the f^{th} block and $n_{f,k}$ is a Gaussian random variable with variance $N_0/2$. We must explain the practical validity of this model. It assumes first of all that the channel is stationary during the duration of a block, which is more or less true for systems like GSM when the mobile station is traveling at low speeds. We take the continuous-time channel response to be of the form

$$h_f(\tau) = \sum_{l=0}^{L-1}\sigma_{f,l}\delta(\tau - d_{f,l}),$$

which corresponds to an L-path multipath fading channel. Using a Rayleigh fading model, the path gains $\sigma_{f,l}$ are zero-mean unit variance Gaussian random variables. For convenience, we assume that the average path strengths are equal in each block and are normalized as

$$\sum_{l=0}^{L-1}\overline{\sigma_l^2} = 1,$$

so that the true average attenuation is included in the transmitted signal energy E_s. For narrowband systems (i.e. with Nyquist pulse shapes) without multipath induced *intersymbol interference* (ISI) the multipath channel cannot be resolved and α_f in the discrete-time model is exponentially distributed. For very wideband systems, the multipath can be completely resolved using a RAKE receiver [4] which combines the different multipath components so that α_f has a characteristic function [4]

$$\phi_\alpha(s) = \prod_{l=0}^{L-1}\frac{1}{1-\overline{\sigma_l^2}s}.$$

In medium-band systems like GSM, the multipath is partially resolved using an equalizer and the performance lies between the two limits we have just described depending on the bandwidth of the signalling pulse. Moreover, the diversity factor offered by coding is the same in all cases, so as far as the comparison of coded systems is concerned, there is no loss in generality by assuming that we have an unresolved multipath situation with no ISI (i.e. $L = 1$).

Assuming a *maximum-likelihood* decoding rule with perfect knowledge of the α_f, which is implemented in practice with the *Viterbi Algorithm* [5] the pairwise probability of error between two codevectors $\mathbf{c}^{(a)}$ and $\mathbf{c}^{(b)}$ is given by

$$\Pr(\mathbf{c}^{(a)} \to \mathbf{c}^{(b)} | \alpha_f) = Q\left(\sqrt{\frac{d^2(a,b)E_s}{2N_0}}\right),$$

where $d^2(a, b)$ is the *Euclidean distance* between the codevectors given by

$$d^2(a,b) = \sum_{f=0}^{F-1} \alpha_f d^2(\mathbf{c}_f^{(a)}, \mathbf{c}_f^{(b)}),$$

and $\mathbf{c}_f$ is the component of the codevector belonging to block f. It is not difficult to show that when the pairwise probability of error is averaged over the unit-mean exponentially distributed random variables α_f we obtain the upper-bound,

$$\Pr(\mathbf{c}^{(a)} \to \mathbf{c}^{(b)}|) \le 0.5 \prod_{f=0}^{F-1} \phi_\alpha(E_s d^2(\mathbf{c}_f^{(a)}, \mathbf{c}_f^{(b)})/4N_0) < \left(\frac{4N_0}{\chi E_s}\right)^{d_H^F},$$

where d_H^F is the number of non-zero $d^2(\mathbf{c}_f^{(a)}, \mathbf{c}_f^{(b)})$, or equivalently, the *Hamming distance* between $\mathbf{c}^{(a)}$ and $\mathbf{c}^{(b)}$ with the symbols taken to be the subvectors $\mathbf{c}_f$. This distance defines the diversity between the pair of codewords, which is simply the slope of the probability of error vs. E_s/N_0 curve on a log-log scale. The other parameter, χ, is the geometric mean of the $d^2(\mathbf{c}_f^{(a)}, \mathbf{c}_f^{(b)})$. These two quantities sufficiently characterize the pairwise probability of error and can be used to design good codes.

We now determine the maximum d_H^F for a given code rate R bits/symbol, number of carriers F and the size of the constellation $|S|$. This situation can be seen as a non-binary coding scheme of block length F and symbol alphabet S^N, and the *Singleton bound* [6] can be used to bound d_H^F as

$$d_H^F \le 1 + \left\lfloor F\left(1 - \frac{R}{\log_2(|S|)}\right)\right\rfloor.$$

As an example take the case of half-rate GSM where $F = 4$, $R = 0.5$ bits/symbol and $|S| = 2$. We have that the maximum diversity is 3. Suppose we now use an expanded constellation with $|S| = 4$ keeping the same information rate R, we see that the maximum diversity is increased to 4. In general we see that by doubling the size of the constellation we can come close to the absolute maximum diversity F. In the following section we address the necessary code complexity to achieve the maximum diversity bound.

3. Convolutional Code Search and Simulation Results

Using the results of the previous section we performed a search for rate 1/2 binary convolutional codes achieving maximal diversity (d_H^F) for various choices of F (2-16 frequencies). The results of the search are shown in Table 1. The generators are expressed in octal notation according to the standard convention described [5], and were chosen first to maximize the diversity and then χ. The codes which appear in the shaded region do not achieve maximum diversity, since their constraint lengths are not long enough. For $F = 14$ and 16, 128 and 256 states are required respectively to achieve the maximum diversities of 8 and 9.

Table 1: Rate 1/2 bits/symbol convolutional codes

States	$F = 2$			$F = 4$			$F = 6$			$F = 8$		
	d_H^2	χ_{min}	gen.	d_H^4	χ_{min}	gen.	d_H^6	χ_{min}	gen.	d_H^8	χ_{min}	gen.
4	2	9.8	5,7	3	6.4	5,7	4	5.7	5,7	4	5.7	5,7
8	2	12.0	64,54	3	10.1	64,54	4	6.3	64,74	5	4.0	44,64
16	2	12.7	62,72	3	13.2	62,46	4	8.2	62,56	5	5.3	46,66
32	2	16.0	71,73	3	14.5	75,57	4	11.3	21,75	5	8.2	51,66
64	2	17.9	704, 564	3	17.9	724, 534	4	14.7	664, 754	5	10.9	444, 774

States	$F = 10$			$F = 12$			$F = 14$			$F = 16$		
	d_H^{10}	χ_{min}	gen.	d_H^{12}	χ_{min}	gen.	d_H^{14}	χ_{min}	gen.	d_H^{16}	χ_{min}	gen.
4	5	4.0	5,7	5	4.0	5,7	5	4.0	5,7	5	4.0	5,7
8	5	5.3	64,74	6	4.0	64,54	6	4.0	64,54	6	4.0	64,54
16	5	7.6	62,56	6	4.0	41,51	6	5.0	62,66	7	4.0	62,66
32	6	5.0	61,75	7	4.4	51,67	7	5.4	51,67	8	4.0	75,57
64	6	7.3	644, 534	7	6.3	724, 534	7	6.3	604, 634	8	4.76	704, 564

Some of these codes were simulated by computer for $F = 4, 8$ which have maximum diversities of 3 and 5 respectively. The results are shown in Figures 1 and 2 indicating the frame and bit error rates respectively, where we have used frames of length 100 input bits. These are plotted versus the signal-to-noise ratio per information bit E_b/N_0 ($E_s = RE_b$) expressed in decibels. We have assumed binary antipodal modulation for simplicity. We also show simulation of some simple coded modulation schemes employing rate 1/4 convolutional codes and 4-AM modulation (i.e. R is still.5 bits/symbol) to demonstrate the diversity improvement due to constellation expansion (maximum diversities of 4 and 7 for $F = 4$ and 8 resp.) The codes chosen in the GSM standard were also simulated. As a general remark we see that increasing complexity beyond the minimum required to achieve maximum diversity can have a significant effect on the frame error-rate performance, but has less in terms of bit error-rate. The simple coded modulation can provide significant performance enhancement, especially in the case of frequency hopping over $F = 8$ carriers.

In terms of the choice of code in GSM, we see that in both the case of full and half-rate, the complexity is more than that necessary to achieve maximum diversity. For full-rate, we can achieve comparable performance with and 8 state code. A potential drawback of this lower complexity code is

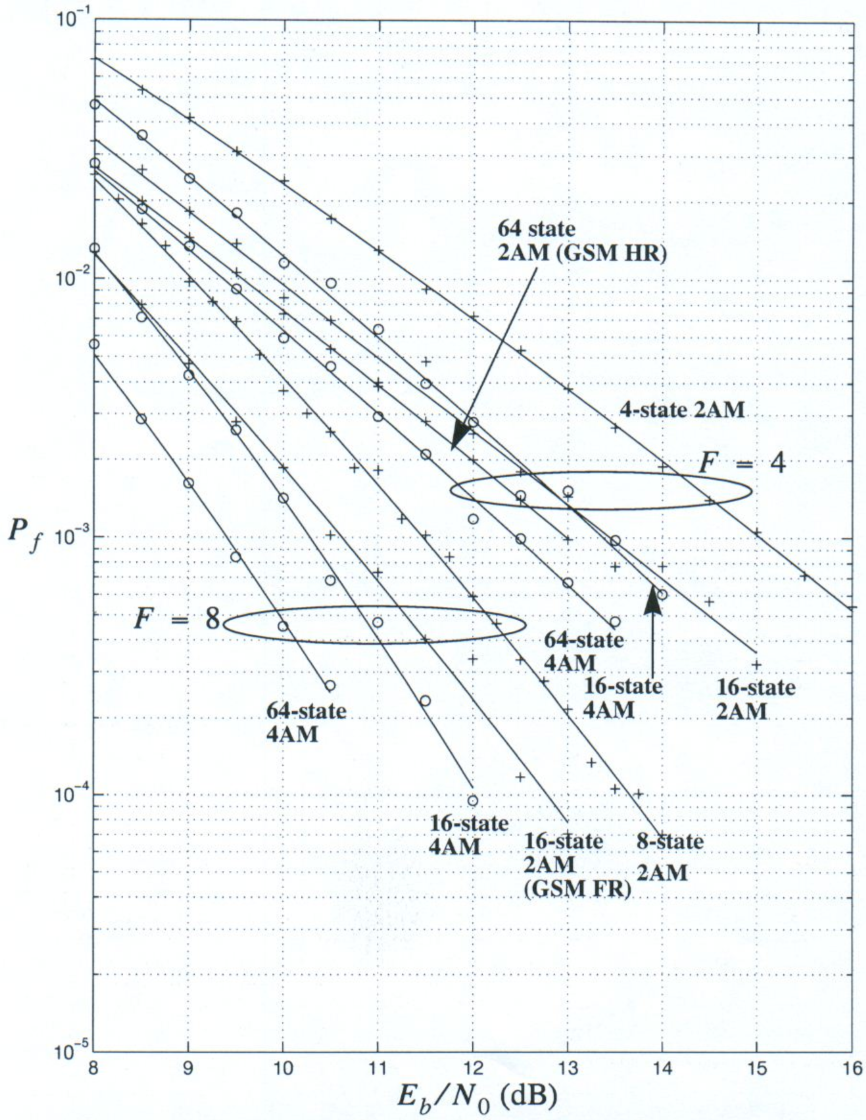

Figure 1: Simulated Frame-Error Rate (.5 bits/symbol)

that is does not achieve maximum diversity for $F = 6$, whereas the code used in GSM does.

4. Conclusion

In this paper we have approached the problem of designing coding schemes for slow-frequency hopping systems such as GSM and its derivatives using analytical techniques. We have shown that there is a fundamental limit on the achievable code diversity which depends on the number of carriers over which the frequency-hopping is carried out, the desired information rate and the number of points in the signal constellation. We have shown that for $R = 0.5$ bits/symbol a diversity close or equal to F can be achieved using a constellation with 4 points instead of 2, and that relatively simple codes can provide the maximum diversity. The codes employed in the GSM standard are optimum in terms of diversity and their added complexity provides a significant improvement in terms of frame-error rate performance.

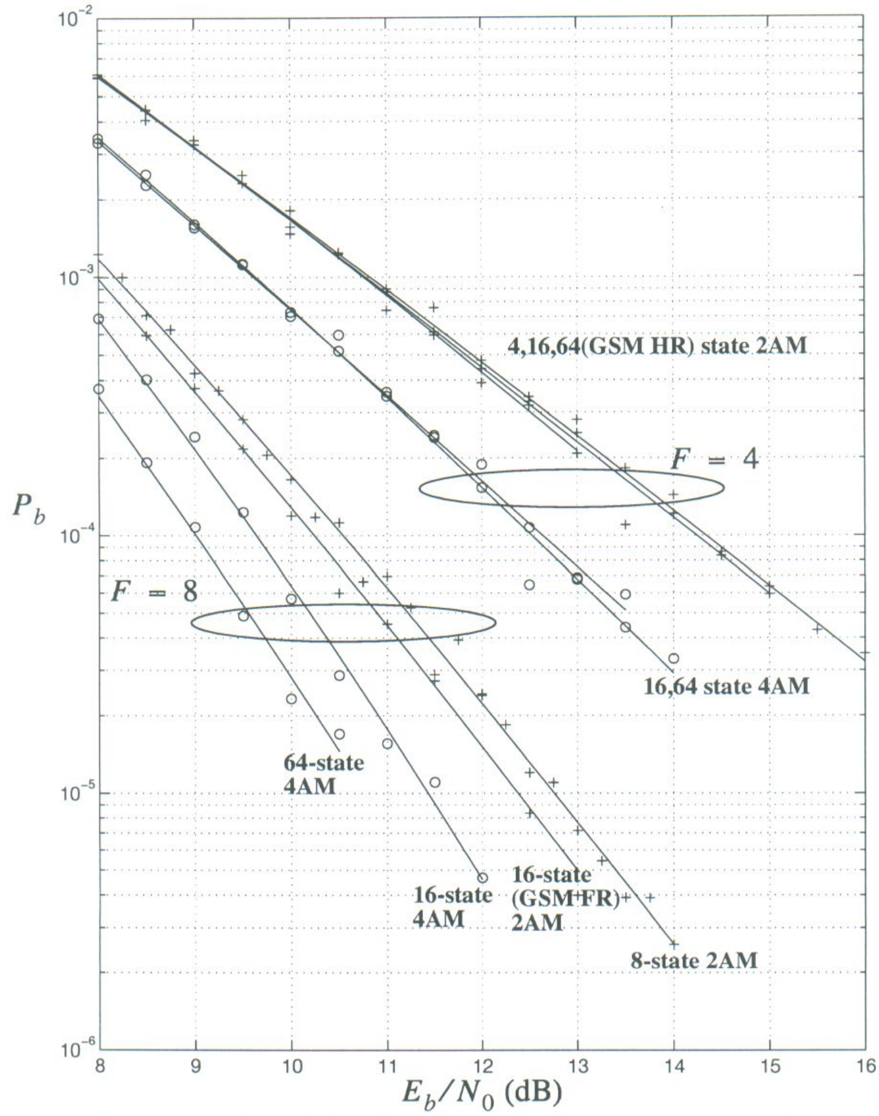

Figure 2: Simulated Bit-Error Rate (.5 bits/symbol)

References

[1] European Digital Cellular Telecommunications System (Phase2): Channel Coding (GSM 05.03), ETSI, 1995.

[2] Steele, R., *Mobile Radio Communications*, Pentech Press, 1992.

[3] Lapidoth, A., "The Performance of Convolutional Codes on the Block Erasure Channel Using Various Finite Interleaving Techniques," *IEEE Trans. on Inform. Theory,* vol. 40, no. 5, Sept. 1994.

[4] Proakis, J.G., *Digital Communications,* 3rd ed., McGraw-Hill 1995.

[5] Lin S., Costello D.J., *Error Control Coding: Fundamentals and Applications*, Prentice-Hall 1983.

[6] MacWilliams F.J., Sloane J.J.A., *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam, 1977.