Bayesian Semi-Blind FIR Channel Estimation Algorithms in SIMO Systems

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Abstract—**When the transmission scenario includes a training sequence or pilots, semi-blind channel estimation techniques have shown a tendency to fully exploit the information available from the received signal if they are correctly implemented. This feature leads semi-blind channel estimation performance to exceed that of the schemes based on the blind part or the training sequence only. Moreover, in some situations they can estimate the channel when the other techniques fail. Semi-blind channel estimation techniques were developed and usually evaluated for a given channel realization, i.e. with a deterministic channel model. On the other hand, in wireless communications the channel is typically modeled as Rayleigh fading, i.e. with a Gaussian (prior) distribution expressing variances of and correlations between channel coefficients. In recent years, such prior information on the channel has started to get exploited in pilot-based channel estimation, since often the pure pilot-based (deterministic) channel estimate is of limited quality due to limited pilots. In this paper we explore a Bayesian approach to semi-blind channel estimation, exploiting a priori information on fading channels. We mainly focus on semi-blind joint ML/MAP estimation of channels and symbols on one hand, and on semi-blind ML/MAP estimation of channels with elimination of symbols on the other hand. As a consequence, a unified framework along with three novel semi-blind Bayesian estimators are introduced whose performance is compared by simulations to three, one extended and another two already existing semi-blind non-Bayesian estimators.**

I. Introduction

Traditionally, the transmitter sends some known information to the receiver to aid the latter in estimating the channel. However, in wireless communication the channel varies rapidly with time and as a consequence more training sequence/pilots are required. This process wastes a lot of bandwidth as a result of augmenting the transmission rate to maintain the throughput. In the last two decades a new branch of channel estimation has emerged focusing on accomplishing this task blindly i.e. without the need for a training sequence. Nevertheless, most wireless standards that have evolved during that period are still relying on the training sequence/pilots to estimate the channel. This is due probably to the unsatisfactory results of the blind channel estimation algorithms. On the other hand, there are some powerful channel estimation algorithms that take the advantage of both aforementioned techniques have been also developed during the same era. These are known as semi-blind where a superior performance is achieved although few training sequence/pilots are transmitted [1], [2], [3]. We will focus in this paper on the semi-blind maximum likelihood (ML) and/or maximum a posteriori methods (MAP) [4],[5]. Two approaches exist in the literature on how to tackle the problem. The first approach is based on the fact that the unknown symbols are considered as deterministic to be jointly estimated with the channel. Such algorithm is called Semi-Blind Deterministic (or conditional) Maximum Likelihood (SB-DML) method [6]. The second approach is based on treating the unknown symbols as random quantities with known prior information to be eliminated or jointly estimated. When the unknown symbols are eliminated, the method is called Semi-Blind Gaussian Maximum Likelihood (SB-GML) [7], see also [8]. While when they are jointly estimated, we call this method SB-GMAP-ML. This is because we use semi-blind maximum a posteriori (MAP) for unknown symbols and semi-blind ML for channels and noise variance. The corresponding blind algorithm appeared first in [9]. Furthermore, in all these approaches the channel was considered as deterministic unknown however, in wireless communications the channel is typically modeled as Rayleigh fading, i.e. with a Gaussian (prior) distribution expressing variances of and correlations between channel coefficients. The concept of Bayesian blind channel estimation was introduced in [10], with in particular some considerations on identifiability issues. However, there were no algorithms/estimators proposed on how to implement this concept. However, in [11] we discussed briefly some classical Bayesian algorithms and introduced the concept of variational Bayesian in the context of MIMO OFDM. Apart from the variational Bayesian techniques, we have developed in [12] classical Bayesian blind channel estimation algorithms in the context of SIMO transmission systems. In this paper, we extend the work done in [12] to the case of semi-blind. Once the channel is treated as random, we are within the framework of Bayesian semi-blind channel estimation and there are are three cases to be handled. In the first case, the unknown symbols are considered as deterministic to be jointly estimated with channel. We call this method as SB-ML-GMAP, for a similar reasoning discussed above. In the second case, the unknown symbols are again to be jointly estimated with the channel but this time they are considered as random with known prior Gaussian distribution. We call this method SB-GMAP-GMAP. In the third case, the unknown symbols are again random with known prior Gaussian distribution but they are going to be eliminated rather than estimated. We call this method SB-GMAP-Elm-GMAP where Elm stands for elimination of symbols to distinguish it from SB-GMAP-

GMAP where both the unknown symbols and the channel are jointly estimated. Consequently, in section III we revisit two already existing deterministic estimators and develop novel ones, one deterministic and three Bayesian. Therefore, with the introduction of the Bayesian semi-blind channel estimation algorithms, the picture is broadened considerably and to sum up we depict the current picture in Table 1.

Algorithm	**Unknown Sym**	**Channel**	**Elm of Sym**	**Novel**
SB-ML-ML	Det	Det	No	No
SB-GMAP-ML	Gauss	Det	No	Yes
SB-GMAP-Elm-ML	Gauss	Det	Yes	No
SB-ML-GMAP	Det	Gauss	No	Yes
SB-GMAP-GMAP	Gauss	Gauss	No	Yes
SB-GMAP-Elm-GMAP	Gauss	Gauss	Yes	Yes

TABLE I
SUMMARY OF ALL ALGORITHMS

II. SIMO FIR TX SYSTEM MODEL

In (semi-)blind channel identification, a multichannel framework can be obtained from oversampling a received signal and leads to a Single Input Multiple Output (SIMO) vector channel representation. The multiple FIR channels we obtain in this representation can also be obtained from multiple received signals from an array of antennas (in the context of mobile digital communications [13] or from a combination of both. To further develop the case of oversampling, consider a linear digital modulation over a linear channel with additive noise so that the received signal $y(t)$ has the following form

$$y(t) \;=\; \sum_k h(t-kT)a(k) \;+\; v(t). \tag{1}$$

In (1) $a(k)$ are the transmitted symbols, T is the symbol period and $h(t)$ is the channel impulse response. The channel is assumed to be FIR with length NT. If the received signal is oversampled at the rate $\frac{m}{T}$ (or if m different samples of the received signal are captured by m sensors every T seconds, or a combination of both), the discrete input-output relationship can be written as:

$$\boldsymbol{y}(k) = \sum_{i=0}^{N-1} \boldsymbol{h}(i)a(k{-}i) + \boldsymbol{v}(k) = \boldsymbol{H}A_N(k) + \boldsymbol{v}(k) \tag{2}$$

where $\boldsymbol{y}(k) = [y_1^H(k)\cdots y_m^H(k)]^H, \boldsymbol{h}(i) = \left[h_1^H(i)\cdots h_m^H(i)\right]^H$, $\boldsymbol{v}(k) = [v_1^H(k)\cdots v_m^H(k)]^H$, $\boldsymbol{H} = [\boldsymbol{h}(N{-}1)\cdots\boldsymbol{h}(0)]$, $A_N(k) = \left[a(k{-}N{+}1)^H\cdots a(k)^H\right]^H$ and superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1}\boldsymbol{h}(i)z^{-i} = [\mathrm{H}_1^H(z)\cdots\mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function, and $\boldsymbol{h} = \left[\boldsymbol{h}^H(N{-}1)\cdots\boldsymbol{h}^H(0)\right]^H$. Consider additive independent white Gaussian circular noise $\boldsymbol{v}(k)$ with $r_{\boldsymbol{vv}}(k{-}i) = \mathrm{E}\,\boldsymbol{v}(k)\boldsymbol{v}(i)^H = \sigma_v^2 I_m\,\delta_{ki}$. Assume we receive M samples:

$$\boldsymbol{Y}_M(k) \;=\; \mathcal{T}_M(\boldsymbol{h})\,A_{M+N-1}(k) + \boldsymbol{V}_M(k) \tag{3}$$

where $\boldsymbol{Y}_M(k) = [\boldsymbol{y}^H(k{-}M{+}1)\cdots\boldsymbol{y}^H(k)]^H$ and similarly for $\boldsymbol{V}_M(k)$, and $\mathcal{T}_M(\boldsymbol{h})$ is a block Toepltiz matrix with M block rows and $[\boldsymbol{H}\;\; 0_{m\times(M-1)}]$ as first block row. We shall simplify the notation in (3) with $k = M{-}1$ to

$$\begin{aligned}\boldsymbol{Y} \;=\; \mathcal{T}(\boldsymbol{h})\,A + \boldsymbol{V} \;&=\; \mathcal{T}_K(\boldsymbol{h})\,A_K + \mathcal{T}_U(\boldsymbol{h})\,A_U + \boldsymbol{V}\\ &=\; \mathcal{A}_K\boldsymbol{h} + \mathcal{A}_U\boldsymbol{h} + \boldsymbol{V}\;.\end{aligned} \tag{4}$$

Where $\mathcal{T}_K(\boldsymbol{h})$ and $\mathcal{T}_U(\boldsymbol{h})$ represent respectively the portions of $\mathcal{T}(\boldsymbol{h})$ that correspond to A_k (M_K known symbols) and A_U (M_U unknown symbols), see Figure 1. On the other hand, $\mathcal{A}$ is a block Toeplitz matrix filled with the elements of A while $\mathcal{A}_K$ and $\mathcal{A}_U$ are block Toeplitz matrices filled with the elements of A_K and A_U respectively. It is worthy to note that the way we split the received data, as in Figure 1, doesn't permit to fully exploit the information available, nevertheless it is still capable of showing the superiority of semi-blind on one hand, and allows for pursuing an analytical performance analysis. On the other hand, though the formulation here is explained for the time domain, it is actual general enough to allow handling also OFDM, SC-CP, SC-ZP etc. And in the case of OFDM, $\boldsymbol{Y}$ is composed of $\boldsymbol{Y}_K$ and $\boldsymbol{Y}_U$ only.

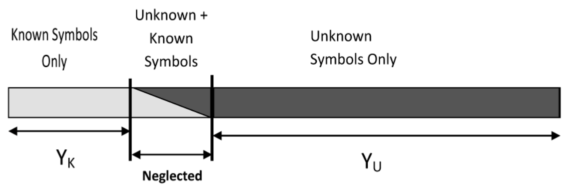

Fig. 1. Splitting the received signal into two parts containing only pure known and unknown symbols.

III. A UNIFIED FRAMEWORK FOR DIFFERENT ALGORITHMS

As we have shown in Table 1 there are six possible estimators that can be classified into two categories. In the first category the subject of the estimators is to estimate the channel and the unknown symbols jointly by making some assumptions on the channel and the unknowns symbols. If we deonte by θ the unknown parameters to be estimated then it is given by:

$$\theta = [A_U^H, \boldsymbol{h}^H]^H \tag{5}$$

The likelihood function is given by:

$$f(Y,\theta) = f(Y/\theta)f(\theta) \tag{6}$$

Where $f(\theta)$ stands for the probability density function (pdf) of θ, $f(Y,\theta)$ stands for the joint probability density function of Y and θ and $f(Y/\theta)$ stands for the pdf of Y conditioned on θ is given or known. Once we substitute θ in (6) by its elements we get:

$$f(Y, A_U, \boldsymbol{h}) = f(Y/A_U, \boldsymbol{h})f(A_U)f(\boldsymbol{h}) \tag{7}$$

Since the symbols and the channel are independent of each other we can write $f(\theta) = f(A_U)f(\boldsymbol{h})$. Of course on the basis of how we treat the symbols and the channel both $f(A_U)$ and $f(\boldsymbol{h})$ differs from one estimator to another as we shall see in the sequel. Knowing that the cost function of the estimator is derived by maximizing the joint probability density function, hence we apply the log function on both sides of (7) to get:

$$\ln[f(Y, A_U, \boldsymbol{h})] = \ln[f(Y/A_U, \boldsymbol{h})] + \ln[f(A_U)] + \ln[f(\boldsymbol{h})] \quad (8)$$

However, in the second category the subject of the estimators is to estimate the channel and the noise variance only while the symbols are supposed to be eliminated during the estimation process. Thus

$$\theta = [\boldsymbol{h}^H, \sigma_v^2]^H \quad (9)$$

Once we substitute θ in (6) by its elements we get:

$$f(Y, \boldsymbol{h}, \sigma_v^2) = f(Y/\boldsymbol{h}, \sigma_v^2) f(\boldsymbol{h}) f(\sigma_v^2) \quad (10)$$

Again, since the cost function for the estimator is derived by maximizing the joint probability density function, hence we apply the log function on both sides of (10) to get:

$$\ln[f(Y, \boldsymbol{h}, \sigma_v^2)] = \ln[f(Y/\boldsymbol{h}, \sigma_v^2)] + \ln[f(\boldsymbol{h})] + \ln[f(\sigma_v^2)] \quad (11)$$

We will develop in the following sections the cost functions of all the estimators that belong to both categories and provide a closed form formula for both the estimated channel and symbols where it is possible. It is worthy to note here that since the channel is treated as random rather than deterministic in some of the above mentioned estimators (SB-ML-GMAP, SB-GMAP-GMAP, SB-GMAP-Elm-GMAP) in both categories, these estimators are considered as an example of the Bayesian semi-blind channel estimation.

A. SB-ML-ML (SB-DML)

We start with SB-ML-ML or what is called SB-DML in the literature [6]. In this case both the unknown symbols and the channel are considered as deterministic unknowns to be estimated. Hence it belongs to the first category and consequently the joint probability density function is given by (8). Moreover, since both are deterministic we have $f(\boldsymbol{h}) = \boldsymbol{h}^o \delta(\boldsymbol{h} - \boldsymbol{h}^o)$ and $f(A_U) = A_U^o \delta(A_U - A_U^o)$. where $\boldsymbol{h}^o$ and A^o represent respectively the true values of the channel and the symbols. It is obvious that the the pdfs of both the unknown symbols and the channel have no influence on the maximization of (8). Hence, we can derive the cost function by maximizing $\ln[f(Y/A_U, \boldsymbol{h})]$ directly where $f(Y/A_U, \boldsymbol{h}) = \frac{1}{(\pi\sigma_v^2)^{Mm}} exp[-\frac{1}{\sigma_v^2}(Y - \mathcal{T}(h)A)^H(Y - \mathcal{T}(h)A)]$. Thus, the cost function is given by:

$$\min_{A_U, \boldsymbol{h}} \; ||Y - \mathcal{T}(h)A||^2 = \min_{A_U, \boldsymbol{h}} \; ||Y - \mathcal{T}_K(h)A_K - \mathcal{T}_U(h)A_U||^2 \quad (12)$$

However, our model allows for a simplification of this cost function because Y_K and Y_U are decoupled in terms of noise. On the other hand, the drawback of this model as we indicated before is that it leads to a suboptimal solution because we are neglecting the part that contains both known and unknown symbols. Hence, (12) can be written as:

$$\min_{A_U, \boldsymbol{h}} \; \left\{||Y_K - \mathcal{T}_K(h)A_K||^2 + ||Y_U - \mathcal{T}_U(h)A_U||^2\right\} \quad (13)$$

The joint optimization of this cost function in both the channel ($\boldsymbol{h}$) and the symbols (A_U) is difficult. Fortunately, the observation is linear in both the channel and the symbols. In other words, we have a separable nonlinear LS problem, which allows us to reduce the complexity considerably. The nonlinear LS optimization can be done by iterating between minimization with respect to A_U and $\boldsymbol{h}$. By doing so, we get the following estimates:

$$\widehat{\boldsymbol{h}} = (\mathcal{A}_K^H \mathcal{A}_K + \mathcal{A}_U^H \mathcal{A}_U)^{-1} (\mathcal{A}_K^H \boldsymbol{Y}_K + \mathcal{A}_U^H \boldsymbol{Y}_U) \quad (14)$$

$$\widehat{A_U} = (\mathcal{T}_U^H(\boldsymbol{h}) \mathcal{T}_U(\boldsymbol{h}))^{-1} \mathcal{T}_U^H(\boldsymbol{h}) \boldsymbol{Y}_U \quad (15)$$

B. SB-GMAP-ML

This is the first novel estimator we introduce in this paper. It is considered as an extension of the corresponding blind one proposed in [9], [14]. In this estimator we treat the unknown symbols as random with Gaussian distribution while the channel is considered deterministic to be jointly estimated with the unknown symbols. This estimator also belongs to the first category, thus the joint probability density function is given by (8). Moreover, $f(A_U) = \frac{1}{(\pi\sigma_a^2)^{M_U}} exp[-\frac{A_U^H A_U}{\sigma_a^2}]$ and $f(\boldsymbol{h}) = \boldsymbol{h}^o \delta(\boldsymbol{h} - \boldsymbol{h}^o)$. It is obvious here that $\ln[f(\boldsymbol{h})]$ can be omitted without affecting the maximization of the joint probability density function in (8). Hence, the cost function is given by:

$$\min_{A_U, \boldsymbol{h}} \frac{1}{\sigma_v^2} ||Y_K - \mathcal{T}_K(h)A_K||^2 + \frac{1}{\sigma_v^2} ||Y_U - \mathcal{T}_U(h)A_U||^2 + \frac{||A_U||^2}{\sigma_a^2} \quad (16)$$

Following the same methodology used in SB-ML-ML estimator we get:

$$\widehat{\boldsymbol{h}} = (\mathcal{A}_K^H \mathcal{A}_K + \mathcal{A}_U^H \mathcal{A}_U)^{-1} (\mathcal{A}_K^H \boldsymbol{Y}_K + \mathcal{A}_U^H \boldsymbol{Y}_U) \quad (17)$$

$$\widehat{A_U} = (\mathcal{T}_U^H(\boldsymbol{h}) \mathcal{T}_U(\boldsymbol{h}) + \frac{\sigma_v^2}{\sigma_a^2} I_{M+N-1-M_K})^{-1} \mathcal{T}_U^H(\boldsymbol{h}) \boldsymbol{Y}_U \quad (18)$$

C. SB-GMAP-Elm-ML (SB-GML)

This estimator belongs to the second category [7], hence we are interested in estimating the channel and the variance of the noise only while the unknown symbols are supposed to be eliminated during the estimation process. Furthermore, the joint probability density function is given by (11) where we consider the channel and the noise variance to be deterministic while the unknown symbols have a Gaussian distribution. Here again, $\ln[f(\boldsymbol{h})]$ and $\ln[f(\sigma_v^2)]$ have no influence on maximizing (11). Substituting $f(Y_U/\boldsymbol{h}, \sigma_v^2) = \frac{1}{(\pi)^{(M-M_K)m} |R_{Y_U Y_U}|} exp[-Y_U^H R_{Y_U Y_U}^{-1} Y_U)]$ Where $R_{Y_U Y_U} = E \;\; \boldsymbol{Y}_U \boldsymbol{Y}_U^H = \sigma_a^2 \mathcal{T}_U(\boldsymbol{h}) \mathcal{T}_U(\boldsymbol{h})^H + \sigma_v^2 I_{(M-M_K)m}$ in (11) after omitting $\ln[f(\boldsymbol{h})]$ and $\ln[f(\sigma_v^2)]$ we get:

$$\min_{\boldsymbol{h}, \sigma_v^2} \; \frac{1}{\sigma_v^2} ||Y_K - \mathcal{T}_K(h)A_K||^2 + \ln|R_{Y_U Y_U}| + \mathrm{tr}\,(R_{Y_U Y_U}^{-1} \hat{R}_{Y_U Y_U}) \quad (19)$$

This cost function can be minimized by resorting to the method of scoring [15]. This method consists in an approximation of the Newton-Raphson algorithm which finds an estimate $\theta(i)$ at iteration i from $\theta(i-1)$, the estimate at iteration $i-1$, as:

$$\theta^{(i)} = \theta^{(i-1)} - \mu \left[\mathcal{F}^{''}|_{\theta^{(i-1)}} \right]^{-1} \mathcal{F}^{'}|_{\theta^{(i-1)}} \quad (20)$$

where $\mathcal{F}(\theta)$ is the cost function in (19), $\mathcal{F}^{''}$ is the hessian, $\mathcal{F}^{'}$ is the gradient of the cost function nad μ is the step length that should be appropriately chosen to guarantee convergence to a local minimum. The method of scoring approximates the Hessian by its expected value, which is here the Gaussian Fisher Information Matrix (FIM). This approximation is justified by the law of large numbers as the number of data is generally large.

D. SB-ML-GMAP

This estimator is novel where we introduce the concept of semi-blind Bayesian channel estimation by treating the channel as random with Gaussian distribution $f(\boldsymbol{h}) = \frac{1}{(\pi)^{mN}|C_h^o|} exp[-\boldsymbol{h}^H C_h^{o-1} \boldsymbol{h}]$. However, the unknown symbols are considered as deterministic to be jointly estimated with the channel hence, this estimator belongs to the first category where the joint probability density function is given by (8). Moreover, here again $\ln[f(A_U)]$ has no effect on maximizing (8) so it can be omitted. Therefore, the cost function is given by:

$$\min_{A_U, \boldsymbol{h}} \frac{1}{\sigma_v^2} ||Y_K - \mathcal{T}_K(h) A_K||^2 + \frac{1}{\sigma_v^2} ||Y_U - \mathcal{T}_U(h) A_U||^2 + \boldsymbol{h}^H C_h^{o-1} \boldsymbol{h} \quad (21)$$

Once again here, following the same methodology used in SB-ML-ML estimator we get:

$$\widehat{\boldsymbol{h}} = (\mathcal{A}_K^H \mathcal{A}_K + \mathcal{A}_U^H \mathcal{A}_U + \sigma_v^2 C_h^{o-1})^{-1} (\mathcal{A}_K^H \boldsymbol{Y}_K + \mathcal{A}_U^H \boldsymbol{Y}_U) \quad (22)$$

$$\widehat{A_U} = (\mathcal{T}_U^H(\boldsymbol{h}) \mathcal{T}_U(\boldsymbol{h}))^{-1} \mathcal{T}_U^H(\boldsymbol{h}) \boldsymbol{Y}_U \quad (23)$$

E. SB-GMAP-GMAP

This estimator is also novel where both the channels and the unknown symbols are assumed random with Gaussian distribution and are supposed to be estimated jointly. Hence, this estimator in its turn belongs to the first category and its joint probability density is given by (8). By substituting the terms in (8) by their corresponding functions we deduce the cost function as follows:

$$\min_{A_U, \boldsymbol{h}} \frac{1}{\sigma_v^2} ||Y_K - \mathcal{T}_K(h) A_K||^2 + \frac{1}{\sigma_v^2} ||Y_U - \mathcal{T}_U(h) A_U||^2 + \frac{||A_U||^2}{\sigma_a^2} + \boldsymbol{h}^H C_h^{o-1} \boldsymbol{h} \quad (24)$$

Also here, following the same methodology used in SB-ML-ML estimator we get:

$$\widehat{\boldsymbol{h}} = (\mathcal{A}_K^H \mathcal{A}_K + \mathcal{A}_U^H \mathcal{A}_U + \sigma_v^2 C_h^{o-1})^{-1} (\mathcal{A}_K^H \boldsymbol{Y}_K + \mathcal{A}_U^H \boldsymbol{Y}_U) \quad (25)$$

$$\widehat{A_U} = (\mathcal{T}_U^H(\boldsymbol{h}) \mathcal{T}_U(\boldsymbol{h}) + \frac{\sigma_v^2}{\sigma_a^2} I_{M_U})^{-1} \mathcal{T}_U^H(\boldsymbol{h}) \boldsymbol{Y}_U \quad (26)$$

F. SB-GMAP-Elm-GMAP

This is the last novel estimator we are going to introduce in this paper. It belongs to the second category since the symbols are supposed to be eliminated. It can be considered as an extension of SB-GMAP-Elm-ML by exploiting the prior information exists about the channel. Its joint probability density function is given by (11) but this time $\ln[f(\boldsymbol{h})]$ can't be omitted. Substituting the terms in (11) by their corresponding functions we get the cost function as follows:

$$\min_{\boldsymbol{h}, \sigma_v^2} \frac{1}{\sigma_v^2} ||Y_K - \mathcal{T}_K(h) A_K||^2 + \ln |R_{Y_U Y_U}| + \mathrm{tr}\,(R_{Y_U Y_U}^{-1} \hat{R}_{Y_U Y_U}) + \boldsymbol{h}^H C_h^{o-1} \boldsymbol{h} \quad (27)$$

This cost function can be minimized using the scoring method discussed in case of SB-GMAP-Elm-ML estimator.

IV. Simulations

In this section we try to shed light by means of Monte-Carlo simulations on the advantages of semi-blind Bayesian compared to the semi-blind non-Bayesian channel estimation. In each MonteCarlo simulation we generate a Rayleigh fading channel with exponentially decaying power delay profile (PDP) for the channel between each transmitting and receiving antenna pair as follows: e^{-wn} where $n = 0 : N-1$ and $w = 1$ except otherwise stated. Hence, $C_{\mathbf{h}}^o$ is the diagonal matrix $C_{\mathbf{h}}^o = I_m \otimes C$ where $C = \mathrm{diag}\{e^{-wn}, n = 0 : N-1\}$. As for the symbols, we generate random 8PSK symbols to reflect the real world case. The performance of the different channel estimators is evaluated by means of the Normalized MSE (NMSE) vs. SNR. The SNR is defined as: $\mathrm{SNR} = \frac{||\mathcal{T}(h)A||^2}{mM\,\sigma_v^2}$. The NMSE is defined as $\frac{avg\,||\boldsymbol{h} - \hat{\boldsymbol{h}}||^2}{avg\,||\boldsymbol{h}||^2}$. All the simulations are initialized by the Subchannel Response Matching (SRM) estimate [16] where the scalar ambiguity of the latter has been fixed by a least squares constraint. In Figure 2 we compare the performance of SB-ML-ML estimator with SB-ML-GMAP, we can notice how the latter exceeds the former by around 5 dB along the medium SNR range while their performances are congruent at high SNR. In Figure 3 we take a look at the considerable gain (7 dB) offered by SB-GMAP-GMAP over SB-GMAP-ML along the medium SNR range. However, in Figure 4 both SB-GMAP-Elm-ML and SB-GMAP-Elm-GMAP are plotted on the same figure. Also in this case, where the symbols are eliminated, we can obviously observe the indispensable role that the prior information about the channel plays in enhancing the estimation quality at the receiver, especially at low ($\approx$ 5 dB gain) and medium SNR.

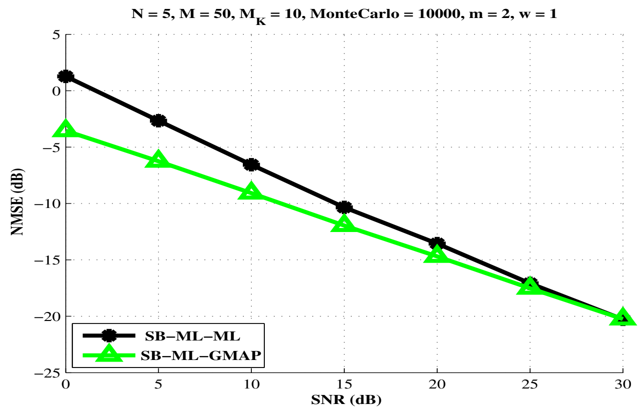

Fig. 2. NMSE vs. SNR for SB-ML-ML and SB-ML-GMAP.

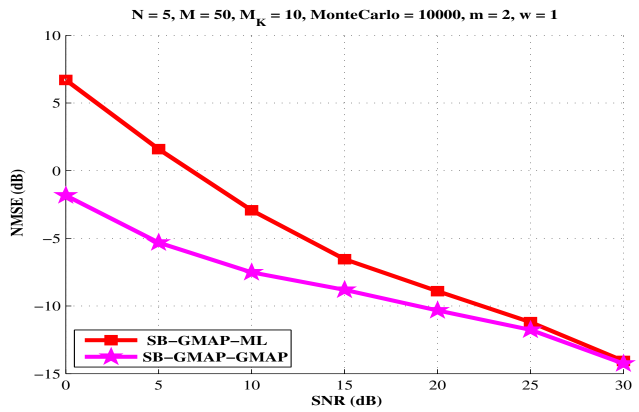

Fig. 3. NMSE vs. SNR for SB-GMAP-ML and SB-GMAP-GMAP.

V. Conclusion

We have shown in this paper that there is still a considerable room to enhance the performance of semi-blind channel estimation. This is true when we take into consideration the a priori information that exists about the channel power delay profile. Hence, we have introduced in this paper the concept of Bayesian semi-blind channel estimation and proposed three new Bayesian semi-blind estimators. On the other hand, we have also extended one existing deterministic blind estimator to the semi-blind case. Furthermore, we have presented a unified framework from which we can derive both deterministic and Bayesian estimators. As our simulations show, the proposed Bayesian estimators have a superior performance compared to the deterministic ones. The main disadvantage of the algorithms introduced in this paper is that they require a large number of iterations to converge. However, we believe that this topic could be further investigated to provide more practical algorithms.

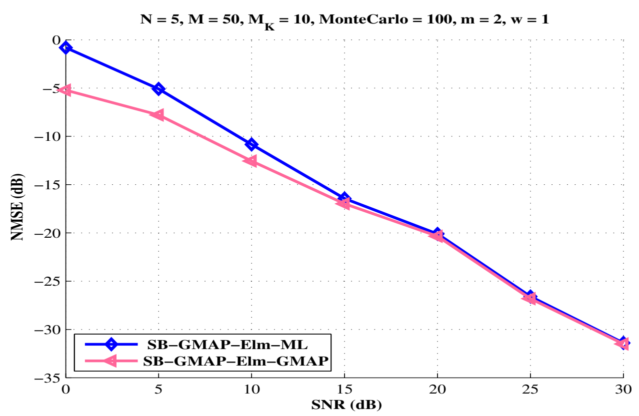

Fig. 4. NMSE vs. SNR for SB-GMAP-Elm-ML and SB-GMAP-Elm-GMAP.

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