

Near Capacity Linear Closed Form Precoders Design with Recursive Stream Selection for MU-MIMO Broadcast Channels

Mustapha Amara^{†*}, Dirk Slock[†] and Yi Yuan-Wu^{*}

[†]Eurecom, Mobile Communications Department
2229 route des crêtes BP 193
06904 Sophia Antipolis Cedex, France
Email: {mustapha.amara, dirk.slock}@eurecom.fr

^{*} Orange Labs, RESA/WASA
38-40 Rue du General Leclerc
92794 Issy Moulineaux Cedex 9, France
Email: yi.yuan@orange-ftgroup.com

Abstract—This paper proposes a novel closed form (CF) precoders design for multi-user multiple input multiple output broadcast channel (MU-MIMO BC). The proposed algorithm constructs stream-wisely the precoders in a recursive manner. It consists in two steps: The first step is a recursive selection of the best streams minimizing the interference between users. The selection is based on a null space criteria combined with the eigenvalues for each available stream. In a second step, the precoders are calculated according to the chosen precoding technique with eventually a power distribution optimization.

The proposed algorithm is then compared to some among the best algorithms presented in the literature such as an all user SVD based solution coupled with a zero forcing beamforming (ZFBF-SUS) or with a zero forcing DPC precoders (ZFDPC-SUS). The obtained results demonstrate better performances for the proposed algorithm and provides performances closer to the channel capacity region.

Index Terms—Multi-user, MIMO, SDMA, broadcast channel, capacity, closed form precoder.

I. INTRODUCTION

Multiuser MIMO Broadcast Channel (MU-MIMO BC) is one of the most importantly investigated subjects in the literature of wireless communications. This important interest, comes from the high potential it offers in improving not only the reliability but also the throughput of the system. This intuitive idea has been proven through theoretical studies. In fact Information theory has shown that the capacity of this kind of channels could be achieved through dirty-paper coding (DPC) [1]–[3] algorithms. Nevertheless, a DPC precoding is difficult to implement and is high resource consuming. Some suboptimal linear algorithms with lower implementation costs exist and can be divided into two main families: the iterative [4]–[8] and the closed form solutions [9]–[13].

The practical solutions for precoder design can also be differentiated according to the number of streams allowed per user. In fact, there are precoders that can only support at maximum one stream per user even if the system is not fully charged. Such precoders have been proposed and widely studied in [6], [7], [9]–[12], [14]. Some multi-stream precoding solutions have nevertheless been proposed such as in [13], [15]. But, the solution proposed in [15] must impose the number of streams per user and thus does not perform a stream selection to optimize the system sum-rate (SR).

The multiple streams can be allocated to the same user

respecting two main constraints $Q_k \leq \min(N_{R_k}, N_T)$ representing the maximum number of streams per user and $Q = \sum_{k=1}^K Q_k \leq \min(\sum_{k=1}^K N_{R_k}, N_T)$ representing the total number of streams allocated by the base station (BS). The allocation of these streams is done such as it maximizes the total sum-rate. A second crucial point in SR maximization is defining the best power distribution over the selected streams.

In this paper we are going to focus on the linear closed form precoders in a multi-stream configuration. We propose a novel precoder design strategy and we compare the performances based on the total achieved sum-rate.

In next section, the model for the considered system is presented, followed by a detailed description of the best existing CF multi-stream precoding algorithms. In Section IV, the new precoding algorithm is explained and different forms are exposed. The last section contains some simulation results, demonstrates the performances obtained by the proposed algorithm and compares it to the one proposed in [13].

II. SYSTEM MODEL

The model considered in this paper is a MU-MIMO system with N_T transmission antennas at the base station and K users with N_{R_k} receiving antennas for user k .

We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmitted streams for the same user. In our paper we are interested in the case of multiple streams per user $Q_k \leq \min(N_{R_k}, N_T)$. The total number of streams must not exceed the maximum number that can be supported by the system and defined as $Q_{max} = \min(\sum_{k=1}^K N_{R_k}, N_T) \geq Q$.

The total transmit power at the base station is supposed to be constant and equal to P_T . The noise variance is noted σ^2 . For the channel part, \mathbf{H}_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix. Each element composing the channel matrix is considered to be a complex Gaussian random variable with unit variance and zero mean.

In this paper \mathbf{X}^H stands for the transpose conjugate of \mathbf{X} and $tr(\mathbf{X})$ for the trace of \mathbf{X} .

III. STATE OF THE ART

To the best of the authors' knowledge, the best linear CF precoder present in the literature is the so called ZFBF-SUS (zero forcing beam forming with successive user selection) that has been proposed in [13], [16]. This precoding technique is based on the selection of semi-orthogonal users. In reality the selection is not done on users but on their available streams. To select the best candidate, the SVD of all users are considered. Based on these, the best users are then determined.

The selection process proposed in [13] is a recursive solution where at each recursion the user that suffers the least interference generated by the previously selected ones are chosen. Once the selection procedure accomplished, the authors apply either a ZFBF (zero forcing beam forming) or a DPC precoding on the selected streams. This eliminates the residual part of the interference.

To evaluate the performances of the system, they take the achieved sum-rate (SR) given by formulas [13, (22)] and [13, (9)] respectively for ZFBF-SUS and ZFDPC-SUS.

Another interesting CF precoding technique is the one presented in [15]. For this solution, the authors base their algorithm on the generalized eigen space of the interference part and the desired signals. In fact, this represents the maximization of the signal to leakage and noise ratio. The leakage is defined as the power transmitted over one stream but received by the undesired ones.

This technique offers some advantages as only the covariance matrices are required and not the entire channel knowledge. Nevertheless, the solution described in [15] must impose the number of streams per user and thus underestimates the real achievable sum-rates and limits the possible applications to real scenarios with greater number of users.

IV. NULL SPACE ZERO FORCING

In this section, the proposed solution is detailed. The objective is to maximize the total sum-rate of a MU-MIMO broadcast channel allowing users to get more than one stream without the use of an iterative solution. Iterative solutions are known to have some problems for convergence [5].

We constructed therefore a recursive algorithm by designing the precoders to be used at the base station.

A. Main Algorithm

1) *Stream Selection*: The proposed algorithm is based on the selection of streams from the null spaces (NS) generated from the previously selected ones by means of orthogonal projections. This constructs a set of beamformers for streams that does not generate any interference with the previously selected ones. These streams can be considered to find the new stream orthogonal to the previously selected directions (i.e. streams). The selection process from this set is done based on the quality of the directions (i.e. their respective eigenvalues).

So the selection procedure proposed here cancels out all the Upper Interference (UI) part defined as the interferences generated by $\{f_j\}_{j>i}$ to the stream $\{f_i\}$.

2) *Precoder construction*: The aim of the precoder construction part is to apply a power distributions and to get rid of the remaining parts of the interference that we call lower interference (LI). This is the interferences generated by the streams $\{f_j\}_{j<i}$ to the stream $\{f_i\}$.

To construct the precoders corresponding to the selected streams, different approaches can be considered. In fact, an MMSE precoder (1) can be applied to the \mathbf{H}_S matrix.

$$\mathbf{t}_{u_i, f_i} = \left[\beta \mathbf{H}_S^H (\mathbf{H}_S \mathbf{H}_S^H + \sigma^2 \mathbf{I})^{-1} \right]_i, \forall i \in [1 \dots Q] \quad (1)$$

where β is a scalar normalization factor to respect the total transmit power constraint $\sum_{i=1}^Q \text{tr}(\mathbf{t}_{u_i, f_i} \mathbf{t}_{u_i, f_i}^H) = P_T$ and $[\mathbf{X}]_i$ represents the i^{th} column of matrix \mathbf{X}

It must be noted that, with this MMSE precoder, no power optimization is required. Nevertheless the Lower Interference part is not completely eliminated. Another kind of precoder that can be applied here is a ZF precoder such as it inverses the matrix \mathbf{H}_S of selected streams. The precoders are then given by (2)

$$\mathbf{t}_{u_i, f_i} = \sqrt{P_i} \frac{\mathbf{h}_i^\dagger}{\|\mathbf{h}_i^\dagger\|}, \forall i \in [1 \dots Q] \quad (2)$$

where \mathbf{h}_i^\dagger is the i^{th} column of $\mathbf{H}_S^\dagger = \mathbf{H}_S^H (\mathbf{H}_S \mathbf{H}_S^H)^{-1}$.

\mathbf{t}_{u_i, f_i} represents the precoding vector for stream f_i sent to user u_i and P_i is the power allocated to this stream. The power distribution can be taken as uniformly distributed or allocated according to a Water Filling (WF) algorithms. To apply the WF algorithm to the obtained precoders, the powers of the virtual channels are computed according to (3)

$$\lambda_i = (\mathbf{h}_i \mathbf{h}_i^H), \forall i \in [1 \dots Q] \quad (3)$$

After that, a WF is applied over the λ_i . This generates the power distribution $\{P_i\}_{i \in [1 \dots Q]}$ for the considered streams respecting the total power constraint $P_T = \sum_{i=1}^Q P_i$.

Another possible precoder that can be applied here is a DPC precoder that will eliminate efficiently all the Lower Interference part. In this case the equivalent precoders are taken such as (4)

$$\mathbf{t}_{u_i, f_i} = \sqrt{P_i} \tilde{\mathbf{t}}_i, \forall i \in [1 \dots Q] \quad (4)$$

The P_i are generated according to the same procedure used for the ZF as no interference is remaining.

The overall algorithm designing the linear precoders consists in two steps and is given in **Algorithm 1** where the interference part $A_{j,l}$ is computed according to (5)

$$A_{j,l} = \sum_{m=1}^{i-1} (\mathbf{u}_{l,j}^\perp)^H \mathbf{H}_l^\perp \tilde{\mathbf{t}}_m \quad (5)$$

for $\forall l \in [1 \dots K]$ and $\forall j \in [1 \dots \text{rank}(\mathbf{H}_l^\perp)]$.

B. ZFBF-NS with Greedy Selection

In this part, an improvement of the algorithm described above is introduced. The idea here is to optimize the performances obtained by applying a ZF precoding to the selected streams. In fact, in some cases and especially at low SNRs where a low

Algorithm 1 Null Space ZF

- 1) Determine the SVDs for all channels
 $\mathbf{H}_k = \mathbf{U}_k \mathbf{S}_k \mathbf{V}_k^H, \forall i \in [1 \cdots K]$.
 - 2) Select the first couple (User, Stream) noted (u_1, f_1) corresponding to the highest eigenvalue, thus the best virtual channel.
 - 3) Construct the corresponding virtual channel
 $\mathbf{h}_1 = \mathbf{u}_{u_1, f_1}^H \mathbf{H}_{u_1}$.
Add this channel to the selection list $\mathbf{H}_S = \mathbf{h}_1$.
And save the corresponding precoder $\tilde{\mathbf{t}}_1 = \mathbf{v}_{u_1, f_1}$.
Set $i = 2$.
 - 4) Compute the orthogonal projection matrix to the space generated by \mathbf{H}_S . This matrix is given by
 $\mathbf{P}_i^\perp = \mathbf{I} - \mathbf{H}_S^H (\mathbf{H}_S \mathbf{H}_S^H)^{-1} \mathbf{H}_S$.
 - 5) Project all channels of all users over the null space $\mathbf{H}_k^\perp = \mathbf{H}_k \mathbf{P}_i^\perp$.
 - 6) Perform an SVD of the obtained projected channels
 $\mathbf{H}_k^\perp = \mathbf{U}_k^\perp \mathbf{S}_k^\perp \mathbf{V}_k^{\perp H}$.
 - 7) At this step a choice between the following a) and b) options is done according to the desired option.
 - a) Compute the interference levels received by all the remaining streams $A_{j,l}$ using (5). Create a set containing the streams getting interference less than a desired threshold δ . If the set is empty then select all the streams.
 - b) Create a set with all available streams.
 - 8) Select among the streams existing in the set obtained in 7) the stream with the largest eigenvalue. The couple of (User, Stream) is noted (u_i, f_i) .
 - 9) Save the corresponding precoder $\tilde{\mathbf{t}}_i = \mathbf{v}_{u_i, f_i}^\perp$ and virtual channel $\mathbf{h}_i = (\mathbf{u}_{u_i, f_i}^\perp)^H \mathbf{H}_{u_i}$.
Update the matrix of selected streams $\mathbf{H}_S = [\mathbf{H}_S^T, \mathbf{h}_i^T]^T$.
Increment the counter $i = i + 1$.
 - 10) Repeat steps 4) to 9) until the maximum number of streams is reached (i.e. $i > \min(\sum_{k=1}^K N_{R_k}, N_T)$).
 - 11) Construct the precoders and define the powers allocation for the streams as described in Section IV.A.2.
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amount of power is available, applying a ZF over Q selected streams may degrade the performances. This is due to the high level of constraints imposed to the calculations of the ZF precoders.

The proposed solution to solve this problem consists in relaxing some of the constraints by applying a greedy selection procedure while selecting the different streams. The algorithm evaluates at each recursion the impact of introducing the new selected stream. And according to the result the algorithm decides weather to accept the new stream and carry on with the algorithm (in case there are no losses) or to reject it and stop the algorithm.

Taking into account these observations, the new obtained algorithm is described in **Algorithm 2**.

$$SR_i = \sum_{j=1}^i \log_2 \left(1 + \frac{\mathbf{u}_{u_j, f_j}^H \mathbf{H}_{u_j} \mathbf{t}_{u_j, f_j} \mathbf{t}_{u_j, f_j}^H \mathbf{H}_{u_j}^H \mathbf{u}_{u_j, f_j}^H}{\mathbf{u}_{u_j, f_j}^H (\mathbf{\Gamma}_j + \sigma^2 \mathbf{I}) \mathbf{u}_{u_j, f_j}^H} \right) \quad (6)$$

Algorithm 2 ZFBF-NS-G

- 1) Execute **Algorithm 1** until step 3).
 - 2) Evaluate the SR_1 using expression (6) with \mathbf{t}_{u_1, f_1} obtained from the construction of the precoders defined in Section IV.A.2.
Save \mathbf{t}_{u_1, f_1} .
Set $i = 2$.
 - 3) Execute steps 4) to 8) of **Algorithm 1**.
 - 4) Save the corresponding precoder $\tilde{\mathbf{t}}_i = \mathbf{v}_{u_i, f_i}^\perp$ and virtual channel $\mathbf{h}_i = (\mathbf{u}_{u_i, f_i}^\perp)^H \mathbf{H}_{u_i}$.
Update the matrix of selected streams $\mathbf{H}_S = [\mathbf{H}_S^T, \mathbf{h}_i^T]^T$.
 - 5) Construct the precoders $\mathbf{t}_{u_j, f_j}; j \in [1, \dots, i]$ as defined in Section IV.A.2.
Save these Precoders.
 - 6) Evaluate the new SR_i using equation (6).
 - 7) *if* $SR_i < SR_{i-1}$ get $\mathbf{t}_{u_j, f_j}, j = i - 1$ and jump to 9).
 - 8) If the maximum number of streams is reached (i.e. $i = Q_{max}$) get precoders $\mathbf{t}_{u_j, f_j}, j = i$ and jump to 9).
Increment the counter $i = i + 1$ and repeat steps 3) to 8)
 - 9) End
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where $\mathbf{\Gamma}_j = \mathbf{H}_{u_j} \sum_{m=1, m \neq j}^i \mathbf{t}_{u_m, f_m} \mathbf{t}_{u_m, f_m}^H \mathbf{H}_{u_j}$ is the remaining interference.

In addition to the improvements in the obtained sum-rate, these modifications give us the possibility to reduce the complexity of the system by avoiding extra useless computations.

V. SIMULATIONS AND RESULTS

In all our simulations, we consider that the number of receiving antennas is the same for all users $N_{R_k} = N_R$.

The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR . The channel coefficients $(h_{i,j}^k)_{1 \leq i \leq N_R, 1 \leq j \leq N_T}$ are generated such as $E\|h_{i,j}^k\|^2 = 1$. In the presented plots, the SNR is taken as P_T/σ^2 .

Simulation results are presented and performances of the proposed solution are compared to some existing techniques.

Unless otherwise mentioned, the system configuration considered here is a base station with $N_T = 4$ transmitting antennas and K users with $N_R = 4$ receiving antennas for each user.

Figure 1 represents a comparison of some existing precoding technique for $N_R = 3, N_T = 4$ and $K = 2$. We represented here the SLNR algorithm from [15] for different stream configurations as well as an exhaustive search over the possible stream sets noted SLNR Max. We must also say that simulations have been carried out using an equal power distribution over the streams and not among users as mentioned in [15]. This enhances the obtained performances. These curves are compared to the ZFBF-SUS from [13] and the ZF version from [10]. The number of receiving antennas is $N_R = 3$ for $K = 2$ users to respect the constraints for the ZF algorithm. Through these curves we can clearly see the dominance of the ZFBF-SUS algorithm. This algorithm is therefore serve as a comparison base for our proposed algorithm.

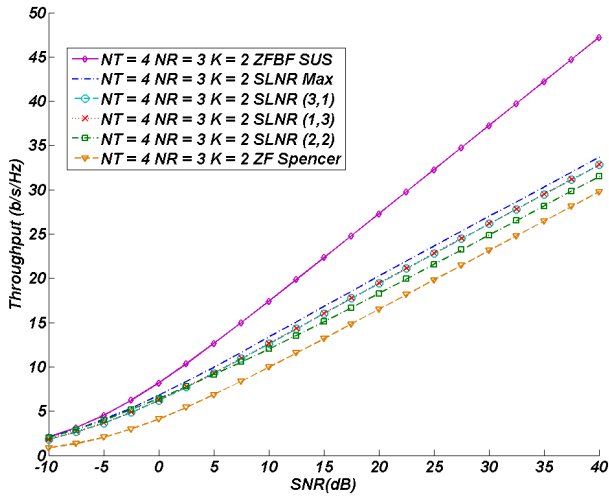


Fig. 1. Comparing existing algorithms

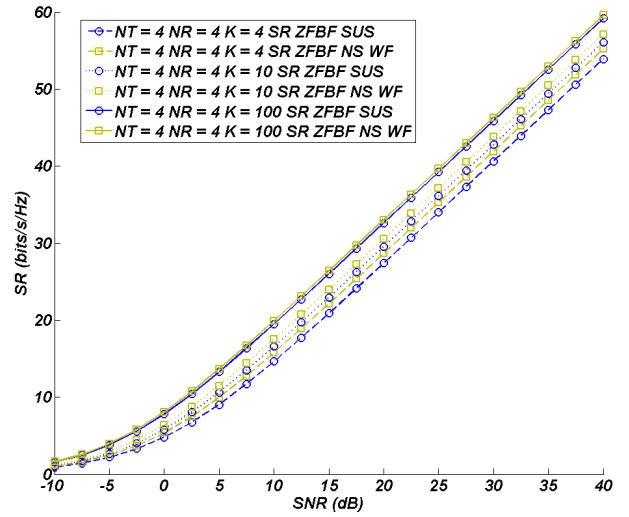


Fig. 3. Throughput for $K = 4, K = 10, K = 100$

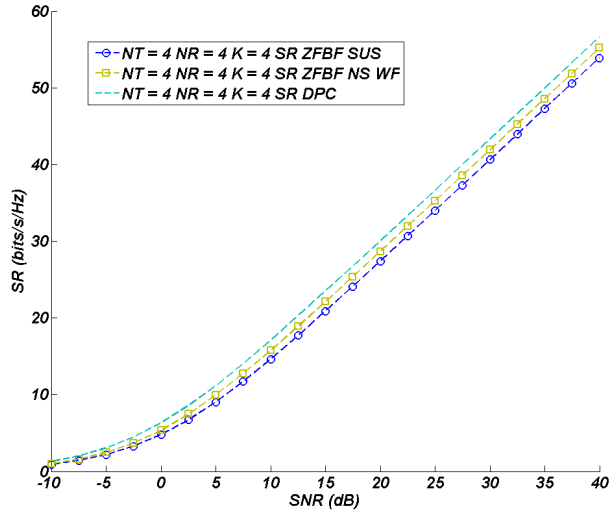


Fig. 2. Throughput as a function of SNR

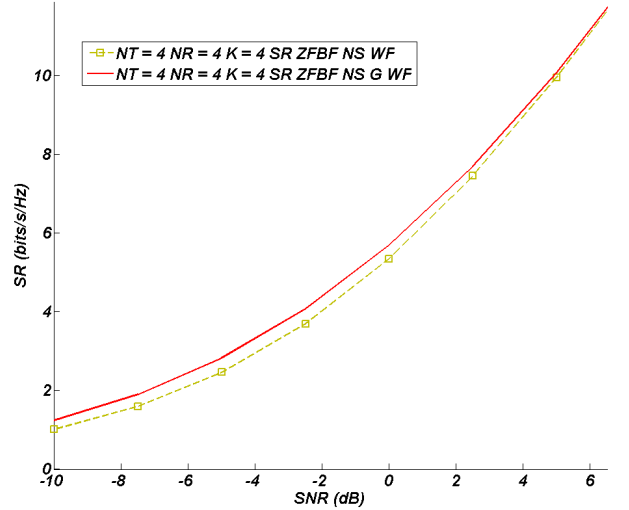


Fig. 4. Gains with Greedy Selection for $K = 4$

Curves in figures 2 represent the performances obtained with $K = 4$ users. Simulations results show that the proposed algorithm **Algorithm 1** is getting closer to the channel capacity (represented by the DPC curve) and remains better than the ZFBF-SUS curves for all SNRs.

The next figure 3 confirms the superiority of ZFBF-NS compared to the ZFBF-SUS even for a growing number of considered users. We can also note here that the difference is getting smaller with a growing number of users K .

The next curves represented in figure 4 show the gain that can be obtained for a $K = 4$ system by applying the greedy selection **Algorithm 2**, and thus confirm the expected gains and resources savings. What is important here is the relative gains.

The curves given in figure 5 compares the DPC version of [13] and **Algorithm 1**. The obtained results show that applying DPC as a precoder to cancel out the LI gives the same gain for both selection algorithms. This result has been verified to be true for all system configurations and demonstrates the real gain obtained thanks to the precoders as in both cases the LI interference is completely removed. This shows that selecting recursively streams present in the

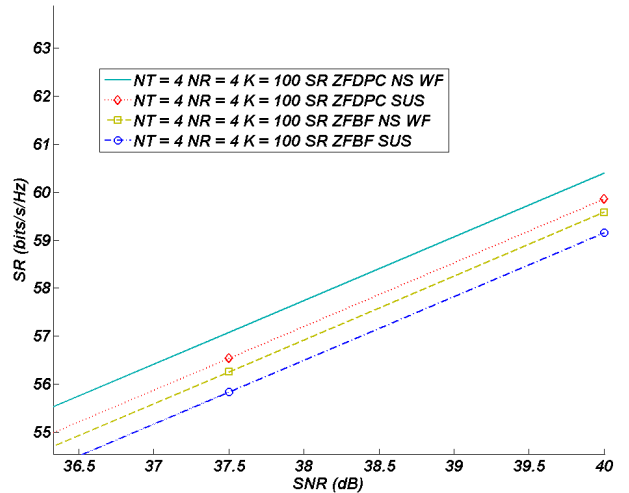


Fig. 5. ZFDPC-NS-WF and ZFDPC-SUS for $K = 100$

null space is much more efficient than selecting the streams based on the SVD decomposition of the original channels.

The next figure, figure 6 represents the performances for

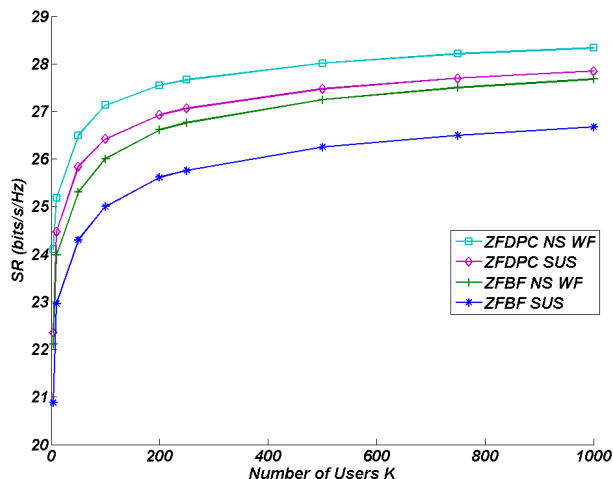


Fig. 6. Performances as a function of K for $SNR = 15dB$

the considered algorithms with DPC and beam forming (BF) precoding represented in function of the number of users K at $SNR = 15dB$. Comparing the algorithms with each other, these curves confirm the previously stated results. Using the same precoding technique, the proposed selection algorithm outperforms the successive selection algorithm presented in [13].

For all the previously presented results, the step 7) of **Algorithm 1** and step 3) of **Algorithm 2** have been executed with option b). Here we present, in figure 7 the impact of δ for option a). This figure is presented for $SNR = 15dB$ and $K = 100$. Analyzing this curve, we observe that at

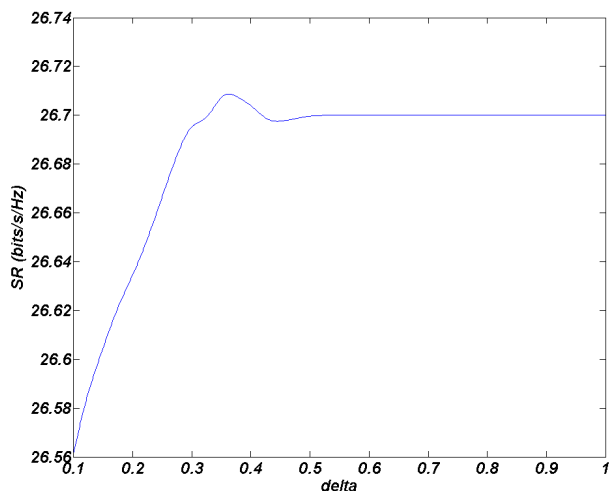


Fig. 7. Performances as a function of δ for $SNR = 15dB$ and $K = 100$

very low values of δ the limited number of streams in the selected set decreases the probability of getting some 'good' virtual projected channels (large eigenvalues for the projected channel and moderate interference). On the other hand, at high values of δ , the selected set contains almost all available streams (in particular those with high interference); this increases the probability of selecting 'bad' streams and decreases the system

performances. Therefore, there is an optimal intermediate value of δ maximizing the SR.

VI. CONCLUSION

In this paper, we present a new CF precoding technique based on a recursive selection of the streams to serve. The selection process is done according to a null space criteria. In fact, the selected streams are those with the best channel located in the null space of the previously selected ones. A further improvement has been introduced by performing a greedy selection. Thus extra unnecessary computations are reduced and further improvements of the total sum-rate are obtained. These selection algorithms (with and without greedy selection) have been compared to the existing successive selection procedure proposed in [13]. The comparison demonstrates that our algorithms outperform the existing one and get closer to DPC.

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