Diversity Analysis of Equal Gain Transmission for Singleuser and Multiuser MIMO

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***Abstract*—In this paper we look at the diversity of equal gain transmission (EGT) for single user (SU) MIMO and multiuser (MU) MIMO in the context of third generation partnership project long term evolution (3GPP LTE). We look at the low resolution LTE precoders which are based on the principle of EGT. Our analytical results show that EGT has full diversity in SU MIMO while the simulation results show that there is a loss of diversity for EGT in MU MIMO mode.**

I. Introduction

Spatial dimension surfacing from the usage of multiple antennas promises improved reliability, higher spectral efficiency and spatial separation of users [1]. This spatial dimension (MIMO) is particularly beneficial for precoding in single user (SU) systems and in the downlink of multiuser (MU) cellular systems, where spatial resources can be used to transmit higher data rate to one user in the former case and multiple data streams to multiple users simultaneously in the latter case.

Future wireless systems being characterized by the need of higher data rate are considering different precoding strategies both for SU and MU cases. In third generation partnership project long term evolution (3GPP LTE) system [2], several MIMO modes of operation including precoding for SU MIMO and MU MIMO are considered. These low resolution LTE precoders are based on the principle of equal gain transmission (EGT) which has more modest transmit amplifier requirements as compared to the other precoding strategies since it does not require the antenna amplifiers to modify the amplitudes of the transmitted signals. This property allows inexpensive amplifiers to be used at each antenna as long as the gains are carefully matched.

In this paper, we look at this characteristic of the low resolution LTE precoders [2] and investigate their effectiveness both for SU MIMO and MU MIMO modes. For SU case, we derive pairwise error probability (PEP) expressions by using the approach of moment generating function (MGF) and show that EGT has full diversity (a result earlier derived for equal gain combining for BPSK in [3] and for EGT in MIMO systems in [4]). Performance analysis for EGT in MU case appears to have no closed form solution so we resort to simulations for this analysis and show that EGT in MU case looses diversity. For MU case, we consider an earlier proposed algorithm [5] which is based on the geometrical alignment of interference at eNodeB (LTE notation for base station).

Regarding notations, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors and uppercase boldface letters for matrices. The matrix $\mathbf{I}_n$ is the $n \times n$ identity matrix. $|.|$ and $\|.\|$ indicate norm of scalar and vector while $(.)^T$, $(.)^*$ and $(.)^\dagger$ indicate transpose, conjugate and conjugate transpose respectively. $\Re(.)$ and $\Im(.)$ indicate real and imaginary parts. The notation $E(.)$ denotes the mathematical expectation while $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx$ denotes the Gaussian Q-function.

The paper is divided into five sections. In section II we define the system model while in section III we discuss the performance analysis of EGT in SU MIMO systems. Section IV encompasses simulation results where we show the loss of diversity for EGT in MU MIMO case and it is followed by conclusions.

II. LTE - A brief overview

For the downlink of 3GPP LTE, 2×2 MIMO system is assumed as the baseline configuration, however configurations with four transmit or receive antennas are also foreseen and reflected in the specifications [2]. In this paper, we restrict ourselves to the baseline configuration with the eNodeB equipped with two antennas while we consider single antenna UEs. The physical layer technology employed for the downlink is Orthogonal Frequency Division Multiple Access (OFDMA) combined with bit interleaved coded modulation (BICM) [6]. Several different transmission bandwidths are possible, ranging from 1.08 MHz to 19.8 MHz with the constraint of being a multiple of 180 kHz. Resource Blocks (RBs) are defined as groups of 12 consecutive resource elements (REs - LTE acronym for subcarrier) with a bandwidth of 180 kHz thereby leading to the constant RE spacing of 15 kHz. For 5 MHz bandwidth, it is divided into 25 RBs. Approximately 4 RBs form a subband and the feedback is done on per subband basis. The minimum allocation in time-domain is a subframe or transmission time interval (TTI), which has a duration of 1 ms and consists of two slots. One TTI consists of 12 or 14 OFDM symbols depending on normal or extended cyclic prefix. Seven operation modes are specified in LTE downlink, however in this paper, we focus on the following two modes.

- Transmission mode 5. MU MIMO
- Transmission mode 6. Closed-loop precoding for rank=1 or SU MIMO

In these transmission modes, one data stream is transmitted to each UE using LTE precoders. Time-frequency resources are orthogonal to different UEs in transmission mode 6 while in transmission mode 5, two parallel data streams are transmitted simultaneously by eNodeB (eNodeB has two antennas) to two UEs sharing the same time-frequency resources. Note that LTE restricts transmission of one stream to each UE in MU-MIMO mode. For this mode, eNodeB selects the precoding matrix such that each data stream is transmitted to the corresponding UE with maximum throughput and the interference between data streams is minimized.

Low resolution LTE precoders for transmission modes 5 and 6 are based on the principle of EGT. For the case of eNodeB with two antennas, LTE proposes the use of following four precoders for mode 5 basing on two bits feedback from the UEs.

$$\mathbf{p}=\frac{1}{\sqrt{4}}\begin{bmatrix}1\\1\end{bmatrix},\frac{1}{\sqrt{4}}\begin{bmatrix}1\\-1\end{bmatrix},\frac{1}{\sqrt{4}}\begin{bmatrix}1\\j\end{bmatrix},\frac{1}{\sqrt{4}}\begin{bmatrix}1\\-j\end{bmatrix} \tag{1}$$

The number of precoders increases to sixteen in the case of four transmit antennas however in this paper we restrict to the case of two transmit antennas. The precoders remain same for mode 6 except with a change of scaling factor for normalizing the power i.e.

$$\mathbf{p}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix},\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix} \tag{2}$$

III. System Model

We first consider the system model for transmission mode 5 i.e. MU MIMO. During the transmission for UE-1, code sequence $\mathbf{c}_1$ is interleaved by π_1 and then is mapped onto the signal sequence $\underline{\mathbf{x}}_1 \in \chi_1$ where $\chi_1 \subseteq \mathcal{C}$. Bit interleaver for UE-1 can be modeled as $\pi_1 : k^{'} \rightarrow (k,i)$ where $k^{'}$ denotes the original ordering of the coded bits $c_{k'}$, k denotes the RE of the symbol $x_{1,k}$ and i indicates the position of bit $c_{k'}$ in the symbol $x_{1,k}$. Note that each RE corresponds to a symbol from a constellation map χ_1 for UE-1 and χ_2 for UE-2. Selection of normal or extended cyclic prefix for each OFDM symbol converts downlink frequency selective channels into parallel flat fading channels. Cascading IFFT at the eNodeB and FFT at the UE with cyclic prefix extension, transmission at k-th RE for UE-1 in mode 5 can be expressed as

$$y_{1,k}=\mathbf{h}_{1,k}^{\dagger}\mathbf{p}_{1,k}x_{1,k}+\mathbf{h}_{1,k}^{\dagger}\mathbf{p}_{2,k}x_{2,k}+z_{1,k}$$

where $y_{1,k}$ is the received symbol at UE-1 and $z_{1,k}$ is zero mean circularly symmetric complex white Gaussian noise of variance N_0. Complex symbols $x_{1,k}$ and $x_{2,k}$ are assumed to be independent and of variances σ_1^2 and σ_2^2 respectively. $\mathbf{h}_{n,k}^{\dagger} \in \mathbb{C}^{1\times 2}$ symbolizes the spatially uncorrelated flat Rayleigh fading MISO channel from eNodeB to n-th UE $(n = 1, 2)$ at k-th RE. $\mathbb{C}^{1\times 2}$ denotes the two dimensional complex space. $\mathbf{p}_{n,k}$ denotes the precoding vector for n-th UE at k-th RE and is given by (1).

In transmission mode 6 i.e. SU MIMO, as only one user will be served in one time-frequency resources so the system equation will be modified as

$$y_{1,k}=\mathbf{h}_{1,k}^{\dagger}\mathbf{p}_{1,k}x_{1,k}+z_{1,k}$$

where $\mathbf{p}_{n,k}$ is given by (2)

IV. Performance Analysis

As already highlighted that LTE transmission is based on the principle of EGT so we carry out the performance analysis of EGT in the case of SU MIMO. As EGT in MU MIMO mode does not seemingly have closed form expression, so we analyze this transmission mode by simulation results in the subsequent section.

Let the channel from eNodeB to UE at k-th RE is given by $\mathbf{h}_{1,k}^{\dagger}=\left[h_{11,k}^{*}\ h_{21,k}^{*}\right]$ where $h_{11,k}^{*}$ and $h_{21,k}^{*}$ are the complex channel coefficients. For EGT in SU MIMO mode, precoder vector is given by

$$\mathbf{p}_{1,k}=\frac{1}{\sqrt{2}}\left[1\quad \frac{h_{21,k}h_{11,k}^{*}}{|h_{21,k}|\,|h_{11,k}|}\right]^{T} \tag{3}$$

So the received signal after normalization by $\frac{h_{11,k}}{|h_{11,k}|}$ is given by

$$y_{1,k}=\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_{1,k}+\frac{h_{11,k}}{|h_{11,k}|}z_{1,k}$$

The max Log MAP bit metric [6] can be written as:-

$$\lambda_1^i\left(y_k,c_{k'}\right)\approx \min_{x_1\in\chi_{1,c_{k'}}^i}\left[\frac{1}{N_0}\left|y_{1,k}-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_1\right|^2\right]$$

where $\chi_{1,c_{k'}}^i$ denotes the subset of the signal set $x_1 \in \chi_1$ whose labels have the value $c_{k'} \in \{0, 1\}$ in the position i. Conditional PEP i.e $P\left(\mathbf{c}_1 \rightarrow \hat{\mathbf{c}}_1|\mathbf{H}_1\right)$ is given as

$$P\left(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1|\mathbf{H}_1\right)=P\left(\sum_k\min_{x_1\in\chi_{1,c_k}^i}\frac{1}{N_0}\left|y_{1,k}-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_1\right|^2\geq \sum_k\min_{x_1\in\chi_{1,\hat{c}_k}^i}\frac{1}{N_0}\left|y_{1,k}-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_1\right|^2\middle|\mathbf{H}_1\right) \tag{4}$$

where $\mathbf{H}_1$ indicates the complete channel for the transmission of codeword $\mathbf{c}_1$. For the worst case scenario once $d\left(\mathbf{c}_1-\hat{\mathbf{c}}_1\right)=d_{free}$ (free distance of the code), the inequality on the right hand side of (4) shares the same terms on all but d_{free} summation points and the summation can be simplified to only d_{free} terms for which $\hat{c}_{k'}=\bar{c}_{k'}$. Let's denote

$$\tilde{x}_{1,k}=\arg\min_{x_1\in\chi_{1,c_{k'}}^i}\frac{1}{N_0}\left|y_k-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_1\right|^2$$
$$\hat{x}_{1,k}=\arg\min_{x_1\in\chi_{1,\bar{c}_{k'}}^i}\frac{1}{N_0}\left|y_k-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_1\right|^2 \tag{5}$$

As

$$\frac{1}{N_0}\left|y_k-\frac{1}{\sqrt{2}}\left(|h_{11}|+|h_{21}|\right)x_{1,k}\right|^2\geq\frac{1}{N_0}\left|y_k-\frac{1}{\sqrt{2}}\left(|h_{11}|+|h_{21}|\right)\tilde{x}_{1,k}\right|^2$$

so it leads to PEP being given as

$$
\begin{aligned}
&P\left(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1|\mathbf{H}_1\right)\\
&\leq P\left(\sum_{k,d_{free}}\frac{1}{N_0}\left|y_{1,k}-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)x_{1,k}\right|^2\geq\right.\\
&\qquad\left.\sum_{k,d_{free}}\frac{1}{N_0}\left|y_{1,k}-\frac{1}{\sqrt{2}}\left(|h_{11,k}|+|h_{21,k}|\right)\hat{x}_{1,k}\right|^2\Bigg|\mathbf{H}_1\right)\\
&=P\left(\sum_{k,d_{free}}\frac{\sqrt{2}\left(|h_{11,k}|+|h_{21,k}|\right)}{N_0}\Re\left(z^*_{1,k}\left(\hat{x}_{1,k}-x_{1,k}\right)\right)\geq\right.\\
&\qquad\left.\sum_{k,d_{free}}\frac{1}{2N_0}\left(|h_{11,k}|+|h_{21,k}|\right)^2\left|\hat{x}_{1,k}-x_{1,k}\right|^2\right)\\
&=Q\left(\sqrt{\sum_{k,d_{free}}\frac{1}{4N_0}\left(|h_{11,k}|+|h_{21,k}|\right)^2\left|x_{1,k}-\hat{x}_{1,k}\right|^2}\right)\\
&\leq\frac{1}{2}\exp\left(-\sum_{k,d_{free}}\frac{1}{8N_0}\left(|h_{11,k}|+|h_{21,k}|\right)^2 d^2_{1,\min}\right)\\
&=\frac{1}{2}\prod_{k,d_{free}}\exp\left(-\frac{1}{8N_0}\left(|h_{11,k}|+|h_{21,k}|\right)^2 d^2_{1,\min}\right)
\end{aligned}
$$

where we have used Chernoff bound $Q\left(x\right)\leq\frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$ and the bound $\left|x_{1,k}-\hat{x}_{1,k}\right|^2\geq d^2_{1,\min}$. Averaging over channel leads to

$$
\begin{aligned}
P(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1)&\leq\frac{1}{2}\prod_{k,d_{free}}E_{\mathbf{h}_{1,k}}\exp\left(-\frac{1}{8N_0}\left(|h_{11,k}|+|h_{21,k}|\right)^2 d^2_{1,\min}\right)\\
&=\frac{1}{2}\prod_{k,d_{free}}E_{\mathbf{h}_{1,k}}\exp\left(\left(-\frac{\breve{d}^2_{1,\min}}{4}\right)\frac{\left(|h_{11,k}|+|h_{21,k}|\right)^2\sigma_1^2}{2N_0}\right)
\end{aligned}
$$

which is led by the channel independence at each RE. Here we have used the notation $d^2_{1,\min}=\sigma_1^2\breve{d}^2_{1,\min}$ with $\breve{d}^2_{1,\min}$ being the normalized minimum distance of the constellation χ_1. Using the MGF of the SNR at the output of two branch equal gain combining as per eq. (23) in [7], PEP at high SNR is upper bounded as

$$
\begin{aligned}
&P\left(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1\right)\leq\frac{1}{2}\prod_{d_{free}}\left(\frac{8\left(\frac{\sigma_1^2}{N_0}\right)^2+\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)^3}{4\left(\frac{\sigma_1^2}{N_0}\right)^2\left(2+\frac{\sigma_1^2\breve{d}^2_{1,\min}}{4N_0}\right)^2}-\right.\\
&\frac{\left(\frac{\breve{d}^2_{1,\min}}{2\sqrt{2}}\right)\left(\frac{\sigma_1^2}{N_0}\right)}{\left(2+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)^{3/2}}\left[\pi-2\sin^{-1}\left(\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}\right)\right]+\\
&\left.\frac{4\left(\frac{\sigma_1^2}{N_0}\right)^2\left(4+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)}{4\left(\frac{\sigma_1^2}{N_0}\right)^2\left(2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2\left(2+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)}\right)
\end{aligned}
\tag{6}
$$

Using the identity $\cos^{-1}\left(x\right)=\frac{\pi}{2}-\sin^{-1}\left(x\right)$, we have

$$
\pi-2\sin^{-1}\left(\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}\right)=2\cos^{-1}\left(\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}\right)
$$

Taylor series expansion [8] of $\cos^{-1}\left(\sqrt{x}\right)$ at $x=1$ is

$$
\cos^{-1}\left(\sqrt{x}\right)=\sqrt{2-2\sqrt{x}}\sum_{k=0}^{\infty}\frac{\left(1-\sqrt{x}\right)^k(1/2)_k}{2^k\left(k!+2kk!\right)}\qquad\text{for }\left|-1+\sqrt{x}\right|<2
\tag{7}
$$

For the case of high SNR, only first term will be dominant i.e.

$$
\cos^{-1}\left(\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}\right)=\sqrt{2-\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}}
$$

Taylor series expansion [8] of $\sqrt{x}$ at $x=1$ is

$$
\sqrt{x}=1+\frac{x-1}{2}-\frac{(x-1)^2}{8}+\frac{(x-1)^3}{16}-\cdots
\tag{8}
$$

Again first two terms being dominant at high SNR leads to

$$
\begin{aligned}
\sqrt{2-\sqrt{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}}}&=\sqrt{2-2\left(1+\frac{\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}-1}{2}\right)}\\
&=\sqrt{-\left(\frac{\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}{2\left(\frac{\sigma_1^2}{N_0}\right)^{-1}+\frac{\breve{d}^2_{1,\min}}{4}}-1\right)}\\
&=\frac{1}{\sqrt{2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)}}
\end{aligned}
$$

So rewriting (6), we get

$$
\begin{aligned}
&P\left(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1\right)\leq\frac{1}{2}\prod_{d_{free}}\left(\frac{2}{\left(2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2}+\frac{\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)}{4\left(2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2}-\right.\\
&\frac{\left(\frac{\breve{d}^2_{1,\min}}{2\sqrt{2}}\right)\left(\frac{\sigma_1^2}{N_0}\right)}{\left(2+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)^{3/2}}\times\frac{2}{\left(2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)\right)^{1/2}}+\\
&\left.\frac{\left(4+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)}{\left(2+\frac{\breve{d}^2_{1,\min}}{4}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2\left(2+\frac{\breve{d}^2_{1,\min}}{2}\left(\frac{\sigma_1^2}{N_0}\right)\right)}\right)
\end{aligned}
$$

At high SNR, second term converges to $\frac{4}{\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)}$ while the third term converges to $\frac{-4}{\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)}$ thereby canceling each other. So PEP at high SNR is upper bounded as

$$
P\left(\mathbf{c}_1\rightarrow\hat{\mathbf{c}}_1\right)\leq\frac{1}{2}\prod_{d_{free}}\left(\frac{32}{\left(\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2}+\frac{16}{\left(\breve{d}^2_{1,\min}\left(\frac{\sigma_1^2}{N_0}\right)\right)^2}\right)
\tag{9}
$$

(9) clearly shows full diversity of EGT, a result which was earlier derived for equal gain combining for BPSK in [3] and

for EGT in MIMO systems in [4] using approach of metrics of diversity order.

V. Simulations

A. Simulation Settings

We consider earlier proposed algorithm [5] for effective utilization of LTE precoders for both SU MIMO and MU MIMO modes. For SU MIMO mode, the strategy is based on low complexity matched filter (MF) precoders. The UEs compute quantized versions of their respective MF precoders i.e. the UE first measures its channel $\mathbf{h}_1^\dagger = [h_{11}^* \quad h_{21}^*]$ from eNodeB and consequently computes the MF precoder i.e. $[h_{11} \quad h_{21}]^T$ (the normalized version involves a division by $\|\mathbf{h}_1\|$). As LTE precoders are characterized by unit coefficients as their first entry (2) so UE normalizes first coefficient of the MF precoder i.e.

$$\mathbf{p}_{MF} = \frac{h_{11}^*}{|h_{11}|^2} \left[\begin{array}{c} h_{11} \\ h_{21} \end{array} \right] = \left[\begin{array}{c} 1 \\ h_{11}^* h_{21} / |h_{11}|^2 \end{array} \right] \tag{10}$$

where second coefficient indicates the phase between two channel coefficients. Now basing on the minimum distance between $\mathbf{p}_{MF}$ and $\mathbf{p}$ given in (2), one of the four precoders is selected by the UE and the index of that precoder is fed back to eNodeB. In the MU MIMO mode, eNodeB has the knowledge of requested precoders of different UEs and uses a scheduling algorithm. In this algorithm, eNodeB selects second UE on each RE whose requested precoder $\mathbf{p}_2$ is 180° out of phase from the requested precoder $\mathbf{p}_1$ of first UE to be served on the same RE. The selection of the precoder for each UE ensures maximization of its desired signal strength while selection of the UE pairs with out of phase precoders ensures minimization of interference strength seen by each UE. UEs further employ low complexity detectors [9] for the suppression of residual interference.

To analyze the effects of EGT and quantization, we consider some other transmission strategies. Unquantized MF precoder is given as

$$\mathbf{p} = \frac{1}{\sqrt{|h_{11}|^2 + |h_{21}|^2}} \left[\begin{array}{c} h_{11} \\ h_{21} \end{array} \right] \tag{11}$$

For EGT, the unquantized MF precoder is given as

$$\mathbf{p} = \frac{1}{\sqrt{2}} \left[\begin{array}{c} 1 \\ h_{11}^* h_{21} / |h_{11}| \, |h_{21}| \end{array} \right] \tag{12}$$

For the case of unquantized precoders, we divide the spatial space into 4 quadrants and ensure that $\mathbf{h}_1^\dagger$ and $\mathbf{h}_2^\dagger$ for the selected UEs in MU MIMO mode lie in the opposite quadrants. This scheduling for unquantized precoders ensures fairness with the scheduling algorithm for LTE precoders in MU MIMO mode [5]. This division is based on the spatial angle between $\mathbf{h}_1^\dagger$ and $\mathbf{h}_2^\dagger$ which is given as

$$\phi = \cos^{-1}\left(\frac{\left| \mathbf{h}_1^\dagger \mathbf{h}_2 \right|}{\|\mathbf{h}_1\| \, \|\mathbf{h}_2\|} \right) \qquad 0^\circ \leq \phi \leq 90^\circ \tag{13}$$

In these simulations, we simulate the downlink of LTE system under two different scenarios of fast fading (Fig. 1 - upper row) and slow fading (Fig. 1 - lower row) channels. As a reference we consider fallback transmit diversity scheme (LTE mode 2 - Alamouti space frequency code) and compare it with the SU MIMO and MU MIMO modes employing LTE low resolution precoders along with unquantized MF and MF equal gain precoders. In accordance with 3GPP LTE, we consider BICM OFDM based transmission from eNodeB equipped with two antennas to two single antenna UEs using rate-$1/3$ LTE turbo code [10] with rate matching to rate $1/2$ and $1/4$ [1].

We consider ideal OFDM system (no ISI) and analyze the system in the frequency domain where the channel has iid Gaussian matrix entries with unit variance. For fast fading case, the channel is independently generated for each channel use while for the case of slow fading, channel remains constant during one frame but changes independently between different frames. We assume no power control so two users have equal power distribution. Perfect CSIT is assumed for the case of MF and MF equal gain precoding while error free feedback of 2 bits is assumed for LTE precoders. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We focus on frame error rates (FER) while the frame length is fixed to 1056 information bits. The users employ low complexity detectors [9] which have the inherent ability of exploiting interference structure in the detection of desired stream.

B. Simulation Results

Fig. 1 (upper row) shows that the MU MIMO mode in LTE (transmission mode 5) has more than 3 to 4 dB gain over Alamouti transmit diversity scheme (LTE mode 2) for low spectral efficiencies under fast fading. However the gap reduces for higher spectral efficiencies. SU MIMO schemes (LTE mode 6) perform better than MU MIMO schemes at higher spectral efficiencies. An interesting result is the almost equivalent performance of MF equal gain and LTE precoders which shows that loss with respect to MF precoder is attributed to equal gain transmission rather than the low resolution of LTE precoders.

Fig. 1 (lower row) shows the comparison in slow fading environment. The results show that EGT in SU MIMO mode has full diversity (in conformity with the earlier derived result) but there is a loss in diversity once EGT is applied to MU MIMO. This loss of diversity leads to substantial degradation in the performance of LTE MU MIMO mode (mode 5). Earlier conclusion that the performance loss is attributed to EGT rather than the low resolution quantization of LTE precoders is further confirmed. These results amply manifest the possible employment of SU MIMO and MU MIMO in LTE under different environments. Once not enough diversity is available in the channel, SU MIMO is the preferred option while MU MIMO is the possible choice once channel is rich in diversity.

[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

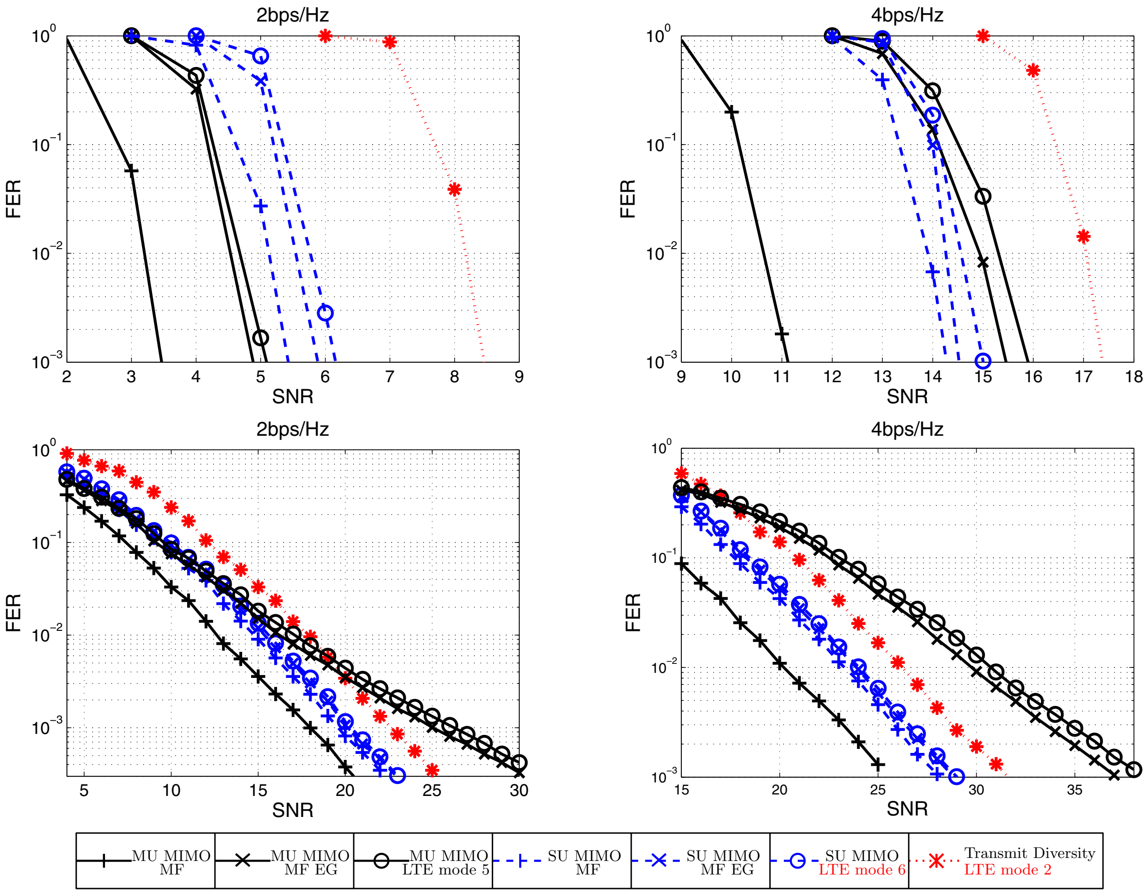

Fig. 1. Downlink with $n_t = 2$ and 2 single antenna users. 3GPP LTE rate $1/3$ turbo code is used with different puncturing patterns. Upper row corresponds to the case of fast fading channel while lower row corresponds to the case of slow fading channel (one channel realization per codeword).

VI. Conclusion

In this paper we have looked at the possible employment of low resolution LTE precoders for SU MIMO and MU MIMO transmission modes. We have shown by PEP analysis that EGT has full diversity in SU MIMO mode while simulation results show that EGT looses diversity in MU MIMO mode. We have also shown that the performance loss of LTE precoders is attributed to their characteristic of EGT rather than their low resolution. This leads to a possible guideline for the employment of SU MIMO and MU MIMO modes in LTE. MU MIMO transmission mode is the preferred option once channel is rich in diversity while SU MIMO mode appears as the preferred transmission mode for slow fading environment.

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References

[1] I. E. Telatar, "Capacity of multiantenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, Nov./Dec 1999.

[2] LTE, *Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Layer Procedures, Release 8, V.8.6.0.* 3GPP TS 36.213, 2009.

[3] Q. Zhang, "Probability of error for equal-gain combiners over rayleigh channels: some closed-form solutions," *Communications, IEEE Transactions on*, vol. 45, no. 3, pp. 270 –273, mar 1997.

[4] D. Love and J. Heath, R.W., "Equal gain transmission in multiple-input multiple-output wireless systems," *Communications, IEEE Transactions on*, vol. 51, no. 7, pp. 1102 – 1110, july 2003.

[5] R. Ghaffar and R. Knopp, "Making Multiuser MIMO work for LTE," in *IEEE 21-st International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2010)*, Istanbul, September 2010.

[6] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Transactions on Information Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[7] X. Qi, M.-S. Alouini, and Y.-C. Ko, "Closed-form analysis of dual-diversity equal-gain combining over rayleigh fading channels," *Wireless Communications, IEEE Transactions on*, vol. 2, no. 6, pp. 1120 – 1125, nov. 2003.

[8] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, USA: Academic Press, 2000.

[9] R. Ghaffar and R. Knopp, "Linear precoders for multiuser MIMO for finite constellations and a simplified receiver structure under controlled interference," in *Asilomar Conference on Signals, Systems and Computers*, Nov. 2009.

[10] LTE, *Evolved Universal Terrestrial Radio Access (E-UTRA); Channel Coding and Multiplexing, Release 8, V.8.6.0.* 3GPP TS 36.212, 2009.