Error Exponents for Multi-Source Multi-Relay Parallel Relay Networks with Limited Backhaul Capacity

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***Abstract*—In this paper, we assess the random coding error exponents (EEs) corresponding to decode-and-forward (DF), compress-and-forward (CF) and quantize-and-forward (QF) relaying strategies for a parallel relay network (PRN), consisting of two sources, two relay stations (RSs) and single destination where the RSs access to the destination via orthogonal, error-free, limited-capacity backhaul links. Among these relaying strategies, the DF and QF studied in this paper differ from their well-known conventional versions in certain aspects. In the DF relaying, each RS applies maximum-likelihood (ML) detection and sends the message corresponding to the detected signal along with a *reliability information* to the destination which finalize the decision on the transmitted message. In QF relaying, as opposed to the Gaussian codebook and vector quantization (VQ) theoretical model used for deriving bounds, we consider a simple and practical relaying strategy consisting of finite-alphabet constellations (i.e., M-QAM) at the sources and symbol-by-symbol uniform scalar quantizers (uSQs) at the RSs.**

We also show, through numerical analysis, that the proposed QF relaying can provide better EEs than the others when the modulation constellation sizes selected by the sources match to the network conditions, i.e., operating signal-to-noise ratio (SNR), and the backhaul capacity is sufficient. This behavior is due to the structure inherent in the considered modulation alphabets, which Gaussian signaling lacks. [1]

I. Introduction

In this paper, we focus on a parallel relay network (PRN) consisting of two sources and two relay stations (RSs) wherein an error-free finite capacity backhaul connection between each RS and the destination is assumed (see Fig. 1). This network model with single source was first studied by Schein [1] where he derived several outer bounds and achievable rates. The PRN we consider in this paper can find *applications* in cellular networks for UL communications, in long-range sensor networks, and in rapidly deployable infrastructure networks for military or civil applications.

In wireless networks consisting of RSs the system's reliability and achievable rate performance is highly dependent on the processing capabilities of RSs. In the literate, regarding relay based networks, most of the research has been conducted on the achievable rate performance (see [2] and references therein). However, in order to have thorough characterization of a system's performance, knowing the capacity (or achievable rate) of the system is not sufficient alone. Hence, in this paper we want to shed light on the reliability issues in PRN setup and consider the random coding error exponent (EE) [3], which is also defined as channel reliability function and represents a decaying rate in the decoding error probability as a function of codeword length, as our system performance metric. Moreover, we investigate whether it is possible to have good reliability performance by using simple and cheap RSs with limited backhaul connections to the destination.

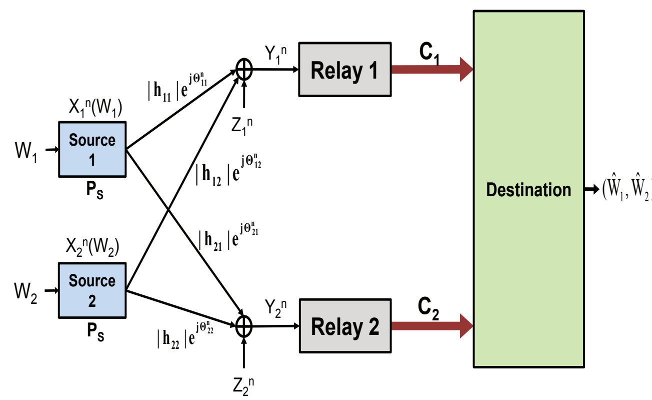

Fig. 1. A 2 sources, 2 relays PRN setup with orthogonal error-free finite-capacity backhaul links between the RSs and the destination, where C_k in [bits/transmission] is the link capacity between the k-th RS and the destination, $k = 1, 2$.

In particular, we assess the random coding EEs corresponding to DF, CF and QF relaying strategies for the PRN setup. Specifically, for the DF we assume Gaussian codebook at the source and maximum-likelihood (ML) decoding at the RSs where each passes its own decision and a corresponding *reliability function* to the destination. For the CF, we assume Gaussian codebook at the source and vector quantization (VQ) at the RSs and ML decoding at the destination. For the QF, it is assumed that each source codeword is selected from a finite alphabet constellation, i.e., M-ary quadratic amplitude modulation (M-QAM), and that each RS performs a *simple*

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and *practical* quantization technique, i.e., symbol-by-symbol uniform scalar quantization (uSQ), as opposed to the CF relaying wherein the VQ is used. We note that in the high resolution regime, with respect to the VQ, the performance loss incurred by symbol-by-symbol uSQ becomes negligible [4]. Moreover, through numerical analysis we show that the EEs corresponding to the proposed QF relaying is better than that of DF and CF relaying strategies when the right constellation size is selected by each source and the backhaul capacity is sufficient.

II. Channel Model and Preliminaries

We study the PRN model shown in Fig. 1 where two sources want to communicate with a destination with the assistance of two RSs. We assume neither direct link between the sources and the destination nor among the RSs. All the channels are modeled as time-invariant, memoryless additive white Gaussian noise (AWGN) channels with constant gain (which may correspond to path-loss between each transmitter and receiver) and ergodic phase fading. The RSs operate in *full-duplex* (FD) mode. Each source encodes its message $w_t \in [1, 2^{nR_t}]$, where R_t is the transmission rate of the t-th source, into the codeword $x_t^n(w_t)$, $t = 1, 2$. All source channel inputs are independent of each other.

The received signals at both RSs are given, in vector form, as follows [2] [3]

$$\begin{aligned}\mathbf{y}_R = \begin{bmatrix} y_{R_1} \\ y_{R_2} \end{bmatrix} &= \begin{bmatrix} |h_{11}|e^{j\Phi_{11}} & |h_{12}|e^{j\Phi_{12}} \\ |h_{21}|e^{j\Phi_{21}} & |h_{22}|e^{j\Phi_{22}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ &= \mathbf{H}\mathbf{x} + \mathbf{z} \\ &= \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{z} = \begin{bmatrix} \mathbf{g}_1^T \\ \mathbf{g}_2^T \end{bmatrix} \mathbf{x} + \mathbf{z} \end{aligned} \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1 \;\; \mathbf{h}_2] = [\mathbf{g}_1 \;\; \mathbf{g}_2]^T$, $\mathbf{x} = [x_1 \;\; x_2]^T$ and $\mathbf{z} = [z_1 \; z_2]^T$. Here x_k is the transmitted signal from the k-th source and y_{R_k} is the received signal at the k-th RS, where $|h_{kt}| \in \mathbb{R}^+$, $\forall k, t \in \{1, 2\}$, is the fixed channel gain from the t-th source to the k-th RS, $z_k \sim \mathcal{CN}(0, \sigma^2)$ is circularly symmetric complex AWGN at the k-th RS. The Φ_{kt}, $\forall\{k, t\}$ denote the set of random phases induced by the channels from the t-th source to the k-th RS. Note that we assume ergodic phase fading where each of Φ_{kt} and Φ_{Dk} is a random variable distributed uniformly over $[-\pi; \pi]$. Random phases are perfectly known to the relevant receivers and unknown to the transmitters. Each source has an average power constraint, i.e., $\mathbb{E}[|x_t(w_t)|^2] = P_s$, $\forall w_t \in [1, 2^{nR_t}]$, $t = 1, 2$. The k-th RS transmits x_{R_k} based on the previously received signals (causal encoding) [5].

For the access channel from the RSs to the destination, we consider lossless orthogonal links with finite capacity between each RS and the destination. Let C_k[bits/transmission], $k = 1, 2$, be the link capacity between the k-th RS and the destination. This assumption might correspond to a cellular telephony system where some of the base stations (acting as RS) connect to a central control unit either via fiber-optic links or via microwave links.

A. Random Coding Error Exponents for Multiple Access Channels

The random coding error exponent (EE) [3] gives insights about how to achieve a certain level of reliability in communication at a rate below the channel capacity. The basic and thorough EE analysis for single antenna point-to-point communications is done by Gallager in [3]. Later on in [6], Gallager also analyzed the EEs of multiple access channels (MACs). The random coding EE for single-user multi-antenna AWGN channel is derived in [7]. In this paper since we consider multiple source PRN, we will follow the basic definitions and procedures given in [6].

For a given MAC, let $P_{e,sys}(n, R_1, R_2)$ denote the smallest average probability of system error of any length-n block-code and rates R_1, R_2 for source 1 and source 2, respectively. Then, the random coding EE for a MAC is defined as

$$E_{sys}(R_1, R_2) \triangleq \lim_{n\to\infty} -\frac{\log_2 P_{e,sys}(n, R_1, R_2)}{n}. \tag{2}$$

In [6], Gallager derived an upper bound on the average probability of system error using *joint* ML decoding rule at the receiver. Let (w_1, w_2) be the message pair sent from the sources and $(\hat{w}_1, \hat{w}_2)$ be the decoded message pair. Consider an ensemble of $(n, 2^{nR_1}, 2^{nR_2})$ codes where each codeword is selected independently for a given joint input distribution $f(x_1, x_2) = f(x_1)f(x_2)$. Then, the probability of system error can be written as

$$P_{e,sys}(n, R_1, R_2) = P_1 + P_2 + P_3 \tag{3}$$

where

$$\begin{aligned} P_1 &\triangleq P(\hat{w}_1 \neq w_1 \cap \hat{w}_2 = w_2) \\ P_2 &\triangleq P(\hat{w}_1 = w_1 \cap \hat{w}_2 \neq w_2) \\ P_3 &\triangleq P(\hat{w}_1 \neq w_1 \cap \hat{w}_2 \neq w_2). \end{aligned} \tag{4}$$

We have the following bounds on P_i, for $i = 1, 2, 3$ [6]

$$P_i \leq 2^{-n[-\rho R_i + E_{0i}(\rho, f(x_1, x_2))]} \tag{5}$$

for all $\rho, 0 \leq \rho \leq 1$ where $E_{0i}(\rho, f(x_1, x_2))$, for $i = 1, 2, 3$, are given in (6) and (7), respectively, with $f(\mathbf{x}_1, \mathbf{x}_2) = \prod_{i=1}^n f(x_{1i}) f(x_{2i})$ being the joint input distribution and $f(\mathbf{y}|\mathbf{x}_1, \mathbf{x}_2)$ being the channel output distribution conditioned on the inputs, and $R_3 = R_1 + R_2$.

Then for an input distribution $f(x_1, x_2) = f(x_1)f(x_2)$ we can bound the probability of system error as follows

$$\begin{aligned} P_{e,sys}(n, R_1, R_2) &= P_1 + P_2 + P_3 \\ &\leq 2^{-n\left(E_r(R_1, R_2, f(x_1, x_2)) - \frac{\log_2(3)}{n}\right)} \end{aligned} \tag{8}$$

where

$$\begin{aligned} &E_r(R_1, R_2, f(x_1, x_2)) \\ &\quad = \min_{1\leq i\leq 3} \max_{0\leq\rho\leq 1} \left[E_{0i}(\rho, f(x_1, x_2)) - \rho R_i\right]. \end{aligned} \tag{9}$$

[2] Throughout the paper we drop the time index for convenience.

[3] In the paper, $\mathbb{E}[(.)]$ denotes the expectation operator, $\mathbf{I}_m$ is the $m \times m$ identity matrix. $X \sim \mathcal{CN}(\mu, \sigma^2)$ means RV X follows circularly symmetric complex Gaussian distribution with mean μ and variance σ^2.

$$E_{0i}(\rho, f(x_1,x_2)) = -\log_2\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{x}_j)\left(\int_{-\infty}^{\infty} f(\mathbf{x}_i) f(\mathbf{y}|\mathbf{x}_1,\mathbf{x}_2)^{\frac{1}{1+\rho}} d\mathbf{x}_i\right)^{1+\rho} d\mathbf{x}_j d\mathbf{y}\right], \quad (i,j) = \{(i,j) \mid i \neq j, \forall i,j \in \{1,2\}\} \tag{6}$$

$$E_{03}(\rho, f(x_1,x_2)) = -\log_2\left[\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{x}_1) f(\mathbf{x}_2) f(\mathbf{y}|\mathbf{x}_1,\mathbf{x}_2)^{\frac{1}{1+\rho}} d\mathbf{x}_1 d\mathbf{x}_2\right)^{1+\rho} d\mathbf{y}\right] \tag{7}$$

III. Error Exponents for DF relaying

For the DF, we assume a Gaussian codebook at the source and ML decoding at the RSs where each passes its own decision and a corresponding *reliability function* to the destination. We note that for the DF the destination is not required to have channel side information (CSI).

In the following, we first briefly introduce the EE calculation corresponding to the DF relaying for single-source PRN proposed in [8], and then proceed with the derivation of the EEs for two-source PRN, where we make use of the results of the single source case.

A. Single-source Case

In this section, we briefly explain the DF type relaying for single-source PRN studied in [8] and give the corresponding EE. Assume each RS applies ML detection and sends the message corresponding to the detected signal along with a *reliability information* (which is a scalar variable equal to the logarithm of the Euclidean distance between the received signal and the detected signal) to the destination on orthogonal error- and cost-free limited capacity backhaul links. Moreover, we assume that the backhaul link capacities are at least equal to the source transmission rate, R. Hence, the backhaul links do not create a bottleneck for system performance.

Upon receiving the detected signals and the reliability information, the destination makes its decision by comparing the reliability information: it decides on the codeword which has the minimum reliability information (Euclidean distance). Hence, if the codeword detected at one of the RS is wrong and its corresponding reliability information is smaller, then the ultimate detection will be wrong even if the other RS has made a correct detection (but with greater reliability information).

With the above detection rule we showed in [8] that for single source (transmitting with rate R [bit/transmission]), two RSs PRN with *symmetric* channel gains from the source to the RSs, the average probability of error is upper-bounded as follows [8]

$$\begin{aligned} P_e &\leq P_{ML}^2 + 2P_{ML}\, 2^{-nT(\Gamma)} \\ &\leq 2^{-n\min\left\{2E_r(R), E_r(R)+T(\Gamma)-\dfrac{2}{n}\right\}} \end{aligned} \tag{10}$$

where

$$T(\Gamma) = \frac{\log_2(e)}{2} - \frac{\log_2(e)(1+2\Gamma)^2}{1+(1+2\Gamma)^4} \tag{11}$$

with $\Gamma = \frac{|h|^2 P_s}{\sigma^2}$ with $|h| \in \mathbb{R}^+$ being the channel coefficient from the source to each RS. $P_{ML} = \exp\{-nE_r(R)\}$ is the standard ML error probability [3] at each RS (due to the channel symmetry this expression is the same at each RS [8]). From the definition of random coding EE [3], as $n \to \infty$, the corresponding EE is given by

$$E_{DF}(R) = \min\{2E_r(R), E_r(R)+T(\Gamma)\} \tag{12}$$

which indicates that by the proposed DF relaying allowing multiple RSs (here two) to participate in communications between the source and the destination always provides *diversity gains* (against noise) at all SNR ranges.

B. Multi-source Case

For two-source PRN case, in order to simplify the relay processing, we assume that wireless medium is shared by the sources in an *orthogonal* fashion, i.e., time-division (TD) MAC, with $\alpha_1 n$ duration for source 1 and $\alpha_2 n$ duration for source 2, where $\alpha_1+\alpha_2=1$. During the access of each source, both RSs perform the same steps as in the single-source PRN case wherein Gaussian codebooks are used by each source. For TD-MAC, we have the following probability of system error

$$\begin{aligned} P_{e,DF}(n,R_1,R_2,\alpha_1,\alpha_2) &= P(\hat{w}_1 \neq w_1) + P(\hat{w}_2 \neq w_2) \\ &= P_{e,DF,1}(n,R_1,\alpha_1) + P_{e,DF,2}(n,R_2,\alpha_2) \\ &\leq 2^{-\alpha_1 n E_{DF,1}(R_1,\alpha_1)} + 2^{-\alpha_2 n E_{DF,2}(R_2,\alpha_2)} \end{aligned} \tag{13}$$

and the corresponding EE

$$\begin{aligned} &E_{DF}(R_1,R_2) \\ &\triangleq \lim_{n\to\infty} \max_{\alpha_1+\alpha_2=1} \frac{-\log_2 P_{e,DF}(n,R_1,R_2,\alpha_1,\alpha_2)}{n} \\ &= \max_{\alpha_1+\alpha_2=1} \min\{\alpha_1 E_{DF,1}(R_1,\alpha_1), \alpha_2 E_{DF,2}(R_2,\alpha_2)\} \end{aligned} \tag{14}$$

where $\alpha_1+\alpha_2=1$, $E_{DF,i}(R_i,\alpha_i), i=1,2$, will be specified. We assume that the w_i-th message, $w_i \in \{1,\ldots,2^{\alpha_i n R_i}\}$, is encoded into the codeword $\mathbf{x}_i(w_i)$ of length $\alpha_i n$, $i=1,2$.

The average power constraint, due to power control at the transmitting nodes, at the i-th source is P_s/α_i. With symmetric channel assumption from each source to the RSs and using (10), the probability of error for the i-th source can be similarly expressed as follows

$$P_{e,DF,i}(n,R_i,\alpha_i) \leq P_{ML,i}^2 + 2\, P_{ML,i}\, 2^{-\alpha_i n T(\Gamma_i(\alpha_i))} \tag{15}$$

where $T(\cdot)$ is defined in (11), $\Gamma_i(\alpha_i) = \frac{|h_i|^2 P_s}{\alpha_i \sigma^2}$, and $P_{ML,i} = 2^{-\alpha_i n E_{r,i}(R_i,\alpha_i)}$ being the standard ML error probability at each RS. Hence, the corresponding EE, as $n_i' = \alpha_i n \to \infty$ for fixed $\alpha_i > 0$, can be easily expressed as

$$E_{DF,i}(R_i,\alpha_i) = \min\{2\, E_{r,i}(R_i,\alpha_i),\ E_{r,i}(R_i,\alpha_i) + T(\Gamma_i(\alpha_i))\} \tag{16}$$

and the overall EE $E_{DF}(R_1, R_2)$ given in (14) can be calculated accordingly.

For *symmetric* channel gains from each source to the RSs, i.e., $\alpha_1 = \alpha_2 = 1/2$ and $\Gamma_1(1/2) = \Gamma_2(1/2) = \Gamma$, and assuming both users communicate with the same rate $R_1 = R_2 = R$, then the EE becomes

$$E_{DF}(R,R) = \min\left\{E_r(R,1/2), \frac{E_r(R,1/2)+T(\Gamma)}{2}\right\}. \quad (17)$$

IV. Error Exponents for CF relaying

For CF relaying, we assume all the sources access the wireless medium *simultaneously*, hence the system probability of error can be upper bounded as in MAC defined in (3) with modified channel matrices and noise assumptions.

The general input-output relation for the CF is given by

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{y}_R + \mathbf{z}_q = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{z} + \mathbf{z}_q$$

where $z_{q,k} \sim \mathcal{CN}(0, D_k)$ for $k = 1, 2$. Define $\mathbf{W} = \mathrm{diag}\{\sigma^2 + D_1, \sigma^2 + D_2\}$.

Then, for an i.i.d. Gaussian input distribution with $x_i \sim \mathcal{CN}(0, P_i)$, we can bound the probability of system error $P_{e,sys}(n, R_1, R_2)$ given in (8) with the corresponding random coding EEs given by

$$E_{r,CF}(R_1,R_2) = \min_{1\le i\le 3} \max_{0\le\rho\le 1} \left[E_{0i}(\rho) - \rho R_i\right], \quad (18)$$

with $R_3 = R_1 + R_2$ and

$$E_{0i}(\rho) = \rho \log_2 \mathbb{E}_{\mathbf{h}_i}\left|\mathbf{I} + \frac{P_s}{(1+\rho)}\mathbf{W}^{-1}\mathbf{h}_i\mathbf{h}_i^H\right|, \quad i = 1,2,$$
$$E_{03}(\rho) = \rho \log_2 \mathbb{E}_{\mathbf{H}}\left|\mathbf{I} + \frac{P_s}{(1+\rho)}\mathbf{W}^{-1}\mathbf{H}\mathbf{H}^H\right|. \quad (19)$$

As in the process of achievable rate calculation [2], we have the compression rate constraints as follows:

$$\log_2\left(\frac{\sigma^2_{v_k}}{D_k}(1-\zeta^2)\right) \le C_k, \quad k = 1,2,$$
$$\log_2\left(\frac{\sigma^2_{v_1}}{D_1}\frac{\sigma^2_{v_2}}{D_2}(1-\zeta^2)\right) \le C_1 + C_2 \quad (20)$$

where $\sigma^2_{v_k} = (|h_{k1}|^2 + |h_{k2}|^2)\ P_s + \sigma^2 + D_k$, $k = 1, 2$, and $\zeta \in [-1, 1]$ is the correlation factor between v_1 and v_2.

V. Error Exponents for QF relaying

As in CF relaying case assuming all the sources access the wireless medium *simultaneously*, we will bound the system probability of error defined in (3). The i-th source transmits $(n, R_i), i = 1, 2$, block code where each letter of each codeword is independently selected with probability assignment $p(x_i)$ and M-QAM constellation is used where 2^{nR_i} messages (alphabet size) are encoded over blocks of length n. The received signals at the RSs are simply quantized by using uSQ, where correlation information is discarded (no compression is done). We assume that each symbol $x_i = x_i^R + jx_i^I$ on the M-QAM constellation has equal probability $p(x_i) = 1/M$ ($p(x_i^R) = 1/\sqrt{M}, p(x_i^I) = 1/\sqrt{M}$) with $\mathbb{E}[(x_i^R)^2] = \mathbb{E}[(x_i^I)^2] = \frac{P_s}{2}$ and $\mathbb{E}[x_i^R x_k^I] = 0, \forall i, k \in \{1, 2\}$.

The channel output at each RS can be decomposed into real and imaginary parts as follows

$$\begin{bmatrix} y^R_{R_k} \\ y^I_{R_k}\end{bmatrix} = \begin{bmatrix}\Re\{y_{R_k}\} \\ \Im\{y_{R_k}\}\end{bmatrix} = \begin{bmatrix}\Re\{\mathbf{g}_k^T\} & -\Im\{\mathbf{g}_k^T\} \\ \Im\{\mathbf{g}_k^T\} & \Re\{\mathbf{g}_k^T\}\end{bmatrix}\begin{bmatrix}\Re\{\mathbf{x}\} \\ \Im\{\mathbf{x}\}\end{bmatrix} + \begin{bmatrix}\Re\{z_k\} \\ \Im\{z_k\}\end{bmatrix}$$
$$= \begin{bmatrix}\tilde{\mathbf{g}}^T_{k,1} \\ \tilde{\mathbf{g}}^T_{k,2}\end{bmatrix}\begin{bmatrix}\mathbf{x}^R \\ \mathbf{x}^I\end{bmatrix} + \begin{bmatrix} z_k^R \\ z_k^I\end{bmatrix} \quad (21)$$

where $\tilde{\mathbf{g}}^T_{k,i} \in \mathbb{C}^{1\times 4}$ for $i = 1, 2$, and $\mathbf{x}^R = \Re\{\mathbf{x}\} = [x_1^R\ x_2^R]^T$ and $\mathbf{x}^I = \Im\{\mathbf{x}\} = [x_1^I\ x_2^I]^T$ are the real and imaginary parts of the source signal vector, respectively. We define $\underline{\mathbf{x}} = [\mathbf{x}^{R^T}\ \mathbf{x}^{I^T}]^T$. The noise components have zero mean and covariance matrix $\mathbb{E}[\underline{z}_k \underline{z}_k^T] = \frac{\sigma^2}{2}\mathbf{I}_2$ where $\underline{z}_k = [z_k^R\ z_k^I]^T$.

The uSQ process at each RS follows the same steps as in [4]. In the following, we let $\mathcal{S}^a_{k,l^a} = (u^a_{k,l^a}, u^a_{k,l^a+1}]$ with $a = \{R, I\}$ where $u^a_{k,l^a}, l^a = 2, 3, \ldots, L^a_k$ represents the transition levels with $u^a_{k,1}$ and u^a_{k,L^a_k+1} being the greatest lower bound and the least upper bound of the received signal $y^a_{R_k}$. L^R_k and L^I_k denote the number of quantization outputs for real and imaginary parts of the received signal at the k-th RS, $k = 1, 2$. Then, for a given source input signal vector $\mathbf{x} = [x_1, x_2]^T$, the probability that the quantizer output is in the $\underline{l} = (l^R, l^I)$-th quantizing interval, i.e., $\underline{V}_k = (V^R_k, V^I_k) = \underline{v}_{k,\underline{l}} = (v^R_{k,l^R}, v^I_{k,l^I})$, $k = 1, 2$, is given by[4]

$$\Pr\left[\underline{V}_k = \underline{v}_{k,\underline{l}} \mid \underline{\mathbf{x}}\right] = \Pr\left[(V^R_k, V^I_k) = (v^R_{k,l^R}, v^I_{k,l^I}) \mid \underline{\mathbf{x}}\right]$$
$$= \Pr\left[y^R_{R_k} \in \mathcal{S}^R_{k,l^R} \mid \underline{\mathbf{x}}\right]\Pr\left[y^I_{R_k} \in \mathcal{S}^I_{k,l^I} \mid \underline{\mathbf{x}}\right]$$
$$= \left[Q\left(\frac{u^R_{k,l^R} - \tilde{\mathbf{g}}^T_{k,1}\underline{\mathbf{x}}}{\sigma/\sqrt{2}}\right) - Q\left(\frac{u^R_{k,l^R+1} - \tilde{\mathbf{g}}^T_{k,1}\underline{\mathbf{x}}}{\sigma/\sqrt{2}}\right)\right] \times$$
$$\left[Q\left(\frac{u^I_{k,l^I} - \tilde{\mathbf{g}}^T_{k,2}\underline{\mathbf{x}}}{\sigma/\sqrt{2}}\right) - Q\left(\frac{u^I_{k,l^I+1} - \tilde{\mathbf{g}}^T_{k,2}\underline{\mathbf{x}}}{\sigma/\sqrt{2}}\right)\right] \quad (22)$$

for $\underline{l} = [1, 2, \ldots, L^R_k] \times [1, 2, \ldots, L^I_k]$.

The destination performs ML decoding on the observations v_1, v_2, which are the representation points corresponding to the received signals at each RS. Then, we have the following EE for the QF relaying with uniform M-QAM at the sources and uSQ at the RSs

$$E_{r,QF}(R_1,R_2) = \min_{1\le i\le 3}\max_{0\le\rho\le 1}\left[E_{0i}(\rho) - \rho R_i\right] \quad (23)$$

with $R_3 = R_1 + R_2$ where $E_{0i}(\rho)$, for all $i = 1, 2, 3$ is defined as in the equations given in (24) and (25) where $p(v_k^R|x_1, x_2)$ and $p(v_k^I|x_1, x_2)$, for $k = 1, 2$, are given by (22). With these settings (23) can be calculated.

VI. Numerical Results

For numerical results we consider two-source, two-relay phase fading AWGN PRN with limited backhaul capacity. We

[4] $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ is the standard tail function for Gaussian RVs

$$E_{0i}(\rho) = -\log_2 \left[\sum_{v_1^R, v_2^R} \sum_{v_1^I, v_2^I} \sum_{\substack{x_j \\ j=\{1,2\}\setminus i}} \frac{1}{M} \left[\sum_{x_i} \frac{1}{M} \left[p(v_1^R|x_1,x_2) p(v_2^R|x_1,x_2) p(v_1^I|x_1,x_2) p(v_2^I|x_1,x_2) \right]^{\frac{1}{1+\rho}} \right]^{1+\rho} \right], \quad i = 1, 2, \tag{24}$$

$$E_{03}(\rho) = -\log_2 \left[\sum_{v_1^R, v_2^R} \sum_{v_1^I, v_2^I} \left[\sum_{x_1} \sum_{x_2} \frac{1}{M^2} \left[p(v_1^R|x_1,x_2) p(v_2^R|x_1,x_2) p(v_1^I|x_1,x_2) p(v_2^I|x_1,x_2) \right]^{\frac{1}{1+\rho}} \right]^{1+\rho} \right] \tag{25}$$

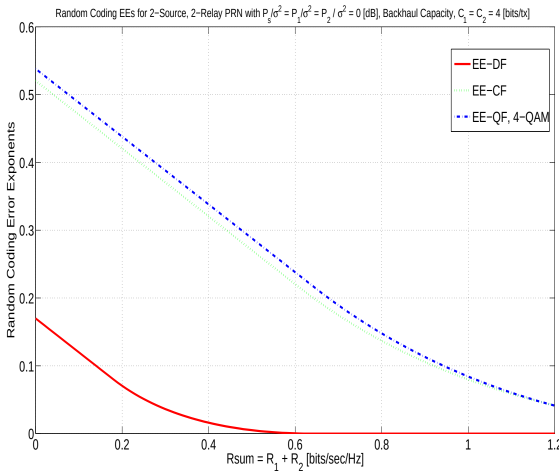

Fig. 2. Random coding EEs for 2-Source, 2-Relay PRN with $\frac{P_s}{\sigma^2} = \frac{P_1}{\sigma^2} = \frac{P_2}{\sigma^2} = 0$ [dB] and $C = C_1 = C_2 = 4$[bits/transmission].

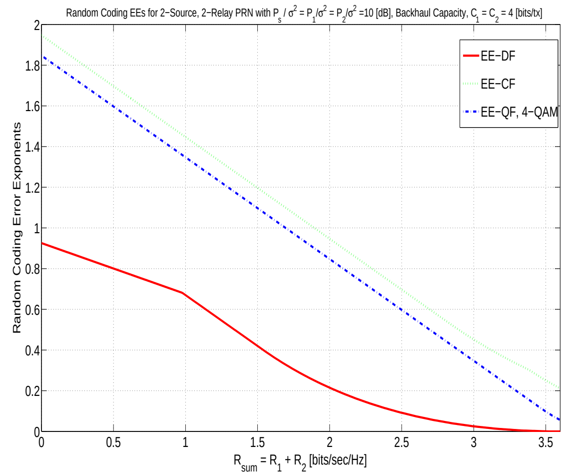

Fig. 3. Random coding EEs for 2-Source, 2-Relay PRN with $\frac{P_s}{\sigma^2} = \frac{P_1}{\sigma^2} = \frac{P_2}{\sigma^2} = 10$ [dB] and $C = C_1 = C_2 = 4$[bits/transmission].

assume the same capacity for backhaul links, i.e., $C = C_1 = C_2$. We take a sample channel matrix from sources to RSs as

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & exp\{-j\pi/3\} \\ exp\{-j2\pi/3\} & 1 \end{bmatrix}. \tag{26}$$

In Fig. 2 and Fig. 3, we plot the EEs given by (17), (18) and (23) corresponding to DF, CF and QF (with 4-QAM at the sources and uSQ at the RSs) relaying strategies with respect to sum-rate $R_3 = R_1 + R_2$ [bits/transmission] for fixed $\frac{P_s}{\sigma^2} = \{0, 10\}$ [dB] where $R_1 = R_2 = R_3/2$. In Fig. 2, which corresponds to a low SNR regime, we see that the proposed simple and practical QF relaying has better EE than both DF and CF over all operating sum-rates. However, from Fig. 3, which corresponds to a high SNR regime, we see that at all rates the proposed QF relaying performs the worse than CF relaying strategy, which could be explained as follows: since the backhaul rate is fixed whilst the SNR is increased the proposed QF strategy cannot fully exploit the structure of the modulation scheme used at the source. From this plot we can also see that the achieved EE with the proposed DF relaying is the worst. In the low-SNR regime, using the proposed QF relaying, which is practical and less complex then the others, provides better EEs by selecting a proper modulation size.

VII. Conclusions and Future Directions

In this paper, we evaluated the random coding EEs corresponding to DF, CF and QF relaying strategies for two-source and two-relay PRN with limited backhaul capacity. We showed through simulations that it is possible to achieve better EEs by using a simple and practical relaying strategy which exploit the inherent structure in the transmitted codewords of the sources.

An interesting future work might be the case where the RSs generate log-likelihood ratios for each source symbol, instead of decoding, and send their quantized versions to the destination which then combines all *soft* information and performs final decoding. The question would be if using quantization on received signal (CF relaying) or on soft information (partial DF relaying) will give the better performance.

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