Rate Scaling Laws in Multicell Networks under Distributed Power Control and User Scheduling

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Abstract— We analyze the sum rate performance in multicell single-hop networks where access points are allowed to cooperate in terms of a joint resource allocation. The resource allocation policies considered here combine power control and user scheduling. Although promising from a conceptual point of view, the optimization of the sum of per-link rates hinges on tough issues such as computational complexity and the requirement for heavy receiver-to-transmitter and cell-to-cell channel information feedback. In this paper, however, we show that simple distributed algorithms can scale optimally in terms of rates, when the number of users per cell U is allowed to grow large. We use extreme value theory to provide scaling laws for upper and lower bounds for the network sum-rate (sum of single user rates over all cells), corresponding to zero-interference and worst-case interference scenarios. We show that the scaling is either dominated by path loss statistics or by small-scale fading, depending on the regime and user location scenario. A key result is that the well known $\log \log U$ rate behavior exhibited in i.i.d. fading channels with maximum rate schedulers is transformed into a $\log U$ behavior when path loss is accounted for. Additionally, by showing that upper and lower rate bounds behave in fact identically, asymptotically, our results suggest, remarkably, that the impact of multicell interference on the rate (in terms of scaling) actually vanishes asymptotically, when appropriate resource allocation policies are used.

Index Terms— Cooperation, cellular networks, extreme value theory, sum rate scaling, interference, coordination, distributed, scheduling.

I. INTRODUCTION

The performance of wireless cellular networks with reuse of the spectral resource is limited by the problem of interference. Traditional ways to tackle this problem include careful planning of the spectral resource and the use of interference mitigation or advanced coding/detection techniques combined with fast link adaptation protocols at the physical layer [1], [2]. In a typical approach to resource planning, the system designer aims at the fragmentation of the network geographical area into smaller zones (reuse patterns) using orthogonal spectral resources. Static orthogonal multiple access is acceptable (although suboptimal) at the cell level but is very inefficient across cells because it neglects the natural ability of wireless propagation to alleviate interference through path loss and random fading. More efficient resource allocation protocols include power control [3] and dynamic channel

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assignment methods which exploit the fading information. Due to a heavy legacy from voice-centric network, the majority of existing techniques are designed with the aim of achieving a given signal to interference plus noise ratio (SINR), common to all users, rather than maximizing the spectral efficiency in Bits/Sec/Hz per area [4], [5]. However rate-maximizing resource allocation has been addressed before, e.g. in game theoretic power control algorithms with pricing [6], [7], iterative/greedy techniques combining power control and scheduling [8] to name just a few.

In this paper, we look at interference suppression from the point of view of the diversity benefits provided by resource allocation techniques. We do not assume advanced multiuser or multicell encoding or decoding. In particular, MIMO (multiple input multiple output) based joint encoding at multiple base stations, such as the one considered e.g. in [9], [10], [11], [12] is left out, in order to emphasize less complex and less signaling hungry coordination schemes where various transmitters need not exchange the user data information to achieve cooperation.

The impact of scheduling on so-called multiuser diversity has been researched extensively for the single cell scenario, with or without interference, with single or multiple antennas. Here we revisit the advantages of multiuser diversity for multicell networks, where some level of cooperation between the transmitters is allowed in the form of joint power control and user scheduling across the cells. The positive impact of scheduling in multicell networks is intuitively well understood and has been addressed, sometimes in conjunction with beamforming [11], [13]. In [14], the gain related to intercell scheduling is analyzed with the means of extreme value theory with the emphasis on the extra multi-user diversity extracted from intercell scheduling when interference is assumed to be eliminated, either with the help of joint multicell DPC encoding/decoding, or orthogonal dynamic frequency reuse.

Here we explore how the scaling of rates (when increasing the number of users per cell) is impacted by interference in a typical cellular network, under joint power control and user scheduling. Single user encoding/decoding is used and no frequency reuse is assumed (i.e. all cells are fully interfering). We are targeting the maximization of the network throughput (sum of rates over the cell). Scaling laws for single cell, MISO and MU-MIMO channels have been analyzed in the recent past [15], [16], exploiting interesting extreme value theoretic tools. Extensions to traffic data model where different users share some of the data were addressed in [17]. Interestingly, some of these results can be readily reused in the multicell network context with i.i.d Rayleigh fading channel models. In [11] it is shown for a simplified interference model (Wyner

model) that the same scaling is obtained for multicell networks with joint linear MIMO precoding as for an optimal precoder, both of these coinciding with the scaling reached in a single cell network (i.e. in $\log \log U$). However other channel models accounting for the path loss effects and cell user locations require additional tools and bring fundamental changes in the scaling performance. Interestingly, there is other work on the scaling law of capacity in interference-limited networks, for which path loss is a key factor, including [18]. Up to our knowledge the existing analysis considers the rate with asymptotically growing number of links (cells) rather than users, thus yielding quite different interpretations, mostly targeting ad hoc networks.

Specific contributions of this paper include the finding of scaling laws of rates in different interference scenarios in cellular networks. We show in particular that the impact of multicell interference on the scaling of rates in the network asymptotically vanishes when sum rate-optimal resource allocation strategies are used. Another important point is that while a $\log \log U$ scaling law is obtained for networks with symmetric i.i.d Rayleigh channels (much akin to single cell results [15], [16]), a much higher growth rate in $\log U$ is achieved when path loss is accounted for.

II. NETWORK AND SIGNAL MODELS

We consider a wireless network featuring a number N of transmit-receive active pairs, which are simultaneously selected for transmission by the scheduling protocol at any considered instant of time, others remaining silent. All active links interfere with each other. This setup, an instance of the interference channel [19] can be observed in e.g. a cellular network with reuse factor one, such as the upcoming IEEE 802.16e (WiMax) and 3GPP (LTE) wireless standards. We assume each of the N cells is equipped with an access point (AP) and that APs communicate with the users in a single-hop fashion. We also assume the APs are time-synchronized. In this paper we focus on the performance of downlink communication from the AP to the users. However we believe our analysis carries over to the uplink without great difficulty.

Let U_n be the number of users randomly distributed over cell n, for $n = 1, \ldots, N$. We will assume these users are uniformly randomly distributed over either a circle or a disk around their access point.

Since we focus on the impact of inter-cell rather than intra-cell interference, we consider an orthogonal multiple access scheme *within the cell* so that a *single* user per cell is supported on any given spectral resource slot (time slot, frequency slot, code slot, etc.). For instance, in OFDMA-based WiMax or LTE standards, a resource slot is represented by a unique time/frequency slice. For ease of exposition, single antenna devices are considered. On any given spectral resource slot, shared by all N cells, we denote by $u_n \in \{1, \ldots, U_n\}$ the index of the user that is granted access to the slot (i.e. scheduled) in cell n. An example of such a situation is depicted for a simple two cell network in Fig.1.

We denote the complex downlink flat-fading channel gain between the i-th AP and user u_n of cell n by $\alpha_{u_n,i}$. In practice
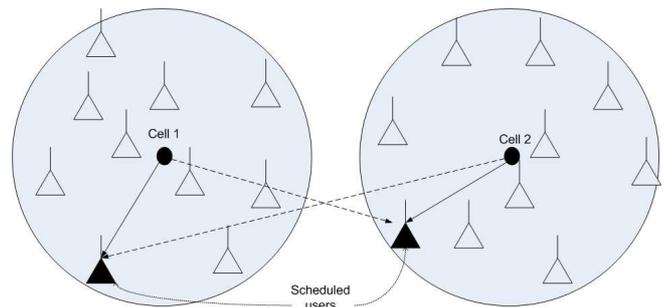
Fig. 1. A two-cell network diagram example. Direct and interfering links toward the scheduled user (black) are indicated in solid and dashed arrows respectively. Users are located randomly over a cell of radius R around their access point.

the flat fading channel model may be obtained at the subcarrier level in an OFDM setting. The local channel state information (CSI) is assumed perfect at the receiver side. This information is also fed back perfectly to the control unit responsible for resource allocation, either in a centralized or distributed manner (this point crucial when it comes to applicability, as discussed later). The study of how much degradation is incurred by the capacity in the case of imperfect feedback is interesting, yet beyond the scope of this paper. The received signal Y_{u_n} at user u_n is given by

$$Y_{u_n} = \alpha_{u_n,n} X_{u_n} + \sum_{i \neq n}^{N} \alpha_{u_n,i} X_{u_i} + Z_{u_n},$$

where X_{u_n} is the message-carrying signal from the serving AP, subject to a peak (per block) power constraint P_{max}. $\sum_{i \neq n}^{N} \alpha_{u_n,i} X_{u_i}$ is the sum of interfering signals from other cells and Z_{u_n} is the additive noise or extra interference. Z_{u_n} is modeled for convenience as white Gaussian with power $\mathbb{E}|Z_{u_n}|^2 = \sigma^2$. Note that a single power level is applied at each AP in this notation. This will allow us to ease the exposition of our analysis. In the OFDMA case however, a possibly unequal power level may be applied on each subcarrier, leading to the optimization of a *power vector*, under sum power constraint, rather than a *scalar* power level at each AP. The analysis in that case however leads to similar conclusions on the rate scaling and is skipped in this paper.

III. THE MULTICELL RESOURCE ALLOCATION PROBLEM

As stated above, intra-cell multiple access is orthogonal, while intercell multiple access is simply superposed, due to full reuse of spectrum. The resource allocation problem considered here consists in *power allocation* and *user scheduling* subproblems. Importantly we focus on *rate maximizing* resource allocation policies, rather than *fairness-oriented* ones [20]. As is the case with known single cell protocols, multicell scheduling protocols can be enhanced to offer some desired performance-fairness trade-off, however this is outside the focus of this paper. Fairness issues are touched upon in [8]. In our setting the optimization of resource in the various resource slots decouples and we can consider the power allocation and user scheduling maximizing the rate in any one slot, independently of other slots. A few useful definitions follow.

Definition 1: A **scheduling vector U** contains the set of users simultaneously scheduled across all N cells in the same slot:

$$U = [u_1 \, u_2 \, \cdots \, u_n \, \cdots \, u_N]$$

where $[U]_n = u_n$. Noting that $1 \leq u_n \leq U_n$, the constraint set of scheduling vectors is given by $\Upsilon = \{U \mid 1 \leq u_n \leq U_n \,\forall\, n = 1, \ldots, N\}$.

Definition 2: A **transmit power vector P** contains the transmit power values used by each AP to communicate with its respective user:

$$P = [P_{u_1} \, P_{u_2} \, \cdots \, P_{u_n} \, \cdots \, P_{u_N}]$$

where $[P]_n = P_{u_n} = \mathbb{E}|X_{u_n}|^2$. Due to the peak power constraint $0 \leq P_{u_n} \leq P_{\max}$, the constraint set of transmit power vectors is given by $\Omega = \{P \mid 0 \leq P_{u_n} \leq P_{\max} \,\forall\, n = 1, \ldots, N\}$.

A. Rate optimal resource allocation

The merit (or utility) associated with a particular choice of a scheduling vector and power allocation vector is measured via the set of SINRs observed by all scheduled users simultaneously. $\Gamma([U]_n, P)$ refers to the SINR experienced by the receiver u_n in cell n as a result of power allocation in all cells, and is given by:

$$\Gamma([U]_n, P) = \frac{G_{u_n, n} P_{u_n}}{\sigma^2 + \sum_{i \neq n}^{N} G_{u_n, i} P_{u_i}}, \qquad (1)$$

where $G_{u_n, i} = |\alpha_{u_n, i}|^2$ is the channel power gain from cell i to receiver u_n. This expression corresponds to the use of orthogonal multiple-access schemes (TDMA, FDMA, etc.) within the cell but non orthogonal access from cell to cell. This might be considered as a first step toward a more general analysis taking into account both intra-cell and inter-cell interference simultaneously.

Assuming that (i) the transmitters cannot afford to perform cooperative encoding, (ii) single user decoding, and Gaussian interference, we consider the average of rates achieved over all cells as our utility [19]:

$$\mathcal{C}(U, P) \triangleq \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \Gamma([U]_n, P)\right). \qquad (2)$$

The sum-rate optimal resource allocation problem can now be formalized simply as:

$$(U^*, P^*) = \arg\max_{\substack{U \in \Upsilon \\ P \in \Omega}} \mathcal{C}(U, P), \qquad (3)$$

The optimization above can be seen as generalizing known approaches in two ways. First the capacity maximizing scheduling problem has been considered (e.g. [21]), but in general not jointly over multiple cells. Second, the problem above extends the classical multicell power control problem (which usually rather aims at achieving SINR balancing) to include joint optimization with the scheduler.

The problem in (3) presents us with many degrees of freedom for optimizing system capacity but also with several serious challenges. First the problem above is non convex (as a mixed integer-non linear problem) and standard optimization techniques do not apply directly. On the other hand an exhaustive search of the (U, P) pairs over the constraint set is prohibitive. Finally, even if computational issues were to be resolved, the optimal solution still requires a central controller updated with instantaneous inter-cell channel gains which would create acute signaling overhead issues in practice. The central question addressed by this paper can be formulated as follows: Can we approach the gains related to multicell resource allocation within reasonable complexity and signaling constraints? Our study provides a positive answer to this problem, at least from the point of view of rate scaling.

IV. NETWORK SUM-RATE: MODELS AND BOUNDS

Let us consider a system with a large number of users in each cell. For the sake of exposition we shall assume $U_n = U$ for all n, where U is asymptotically large, while N remains fixed. We expect a growth of the sum-rate $\mathcal{C}(U^*, P^*)$ with U thanks to the *multicell* multiuser diversity gain[1]. Thus we are interested in how the *expected* sum-rate scales with U. To this end we shall use several interpretable bounding arguments. We consider two channel gain models. The first considers a symmetric distribution of gains to all users from their serving AP. Although not very practical, this assumption has the merit of creating a strong parallel with the single cell MU-MIMO rate analysis carried out in [15], [16], allowing us to readily exploit these results. Later on, we are considering a more general model where an additional random distance-dependent path loss is accounted for. In this case however, existing analysis does not apply and special extreme value theoretic tools are developed.

A. Bounds on multicell sum-rate

The simple bounds below hold in the asymptotic and non asymptotic regimes as well.

Upper bound: An upper bound (ub) on the rate for a given resource allocation vector (not necessarily an optimal one) is obtained by simply ignoring intercell interference effects:

$$\mathcal{C}(U, P) \leq \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \frac{G_{u_n, n} P_{u_n}}{\sigma^2}\right). \qquad (4)$$

In the absence of interference, the optimal rate is clearly reached by transmitting at a level equal to the power constraint, i.e. $\mathbf{P}_{max} = [P_{max}, \ldots, P_{max}]$ and selecting the user with the largest channel gain in each cell (maximum rate scheduler), thus giving the following upper bound on rate:

$$\mathcal{C}(U^*, P^*) \leq \mathcal{C}^{ub} \qquad (5)$$

where

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[1]The multicell multiuser diversity gain can be seen as a generalization of the conventional multiuser diversity [21] to multicell scenarios with joint scheduling

$$C^{ub} = \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \Gamma_n^{ub}\right). \tag{6}$$

and where the upper bound on SINR Γ_n^{ub} is given by the maximum rate scheduler:

$$\Gamma_n^{ub} = \max_{u_n=1\ldots U} \{G_{u_n,n}\} P_{max}/\sigma^2 \tag{7}$$

Lower bound: A lower bound (lb) on the optimal rate (in the presence of interference) $\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)$ can be derived by restricting the domain of optimization. Namely, by restricting the power allocation vector to full power P_{max} in all transmitters, we have

$$\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*) \geq \mathcal{C}^{lb} \tag{8}$$

where

$$\mathcal{C}^{lb} = \mathcal{C}(\boldsymbol{U}_{FP}^*, \mathbf{P}_{max}) \tag{9}$$

and where $\boldsymbol{U}_{FP}^*$ denotes the maximum rate scheduling vector when assuming full power everywhere. This scheduling vector is defined by

$$\boldsymbol{U}_{FP}^* = \arg\max_{\boldsymbol{U}\in\Upsilon} \mathcal{C}(\boldsymbol{U}, \boldsymbol{P}_{max}), \tag{10}$$

Note that the user selected in the n-th cell, designated by $[\boldsymbol{U}_{FP}^*]_n$, is found via:

$$[\boldsymbol{U}_{FP}^*]_n = \arg\max_{\boldsymbol{U}\in\Upsilon} \frac{\{G_{u_n,n}\}P_{max}}{\sigma^2 + \sum_{i\neq n}^{N} G_{u_n,i} P_{max}} \tag{11}$$

The SINR corresponding to the selected user, denoted by Γ_n^{lb}, is therefore given by:

$$\Gamma_n^{lb} = \max_{u_n=1\ldots U} \frac{\{G_{u_n,n}\}P_{max}}{\sigma^2 + \sum_{i\neq n}^{N} G_{u_n,i} P_{max}} \tag{12}$$

Finally the lower bound on rate $\mathcal{C}^{lb}$ may be rewritten as:

$$\mathcal{C}^{lb} = \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \Gamma_n^{lb}\right). \tag{13}$$

B. Distributed vs. centralized scheduling

For large networks, it is important that scheduling algorithms can operate on a distributed mode, that is, the choice of the optimal user set should be done by each cell on the basis of locally available information only. This is in principle difficult task because the achievable rates observed in different cells are coupled together through the interference terms. Therefore a crucial question is how much performance can one reach by sticking to power control and scheduling algorithms that only require local CSI? This problem is a difficult one in the general case, but some light is shed in some asymptotic cases. A first step in this direction consists in noting that if the scheduler is based on maximizing the upper bound of network sum-rate given by (6), then each cell only needs to know the realization of the direct gain $G_{u_n,n}$, and the scheduler is trivially distributed. Alternatively, to obtain a scheduler maximizing the lower bound of rate given by (13),

each cell must collect the worst case SINR for each of its users. The worst case SINRs are computed during e.g. a common preamble phase where all APs are asked to transmit pilot or data symbols at full power. This makes the scheduler of (11) also distributed. Note that "worst case" is here understood in terms relative to the power control policy, not the scheduler.

C. Channel models

We now detail our assumptions regarding the fading and path loss models. Some of these assumptions are mainly technical, serving to simplify the analysis but could be relaxed without altering the fundamental results, as discussed later. As mentioned above we assume a cellular network where APs are regularly located with cell radius R. In this sense, the cells are assumed to be circular with each base being at the center of it, although this assumption is not critical to this study (i.e. similar conclusions can be obtained for hexagonal cell etc.) as explained below.

The basic channel model consists in the product between a variable representing the path loss and a variable representing the fast fading coefficient: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1\ldots U, i = 1\ldots N$ be the set of power gains where $\gamma_{u_n,i}$ is the path loss between user u_n (selected in cell n) and the access point in cell i. $h_{u_n,i}$ is the corresponding normalized complex fading coefficient. A generic path loss model is given by [22]:

$$\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon} \tag{14}$$

where β is scaling factor, ϵ is the path loss exponent (usually with $\epsilon > 2$), and $d_{u_n,i}$ is the distance between user u_n and AP i.

Note that we assume as preamble a user-to-AP assignment strategy resulting in all users being served by the AP with the smallest path loss. This means, as is usually the case in current network design, that the AP assignment operates on a time scale which is not fast enough to provide diversity against fast fading.

We consider in turn two basic user location scenarios, and a hybrid one. As it will be made clear later, the user location scenario has significant impact on the analysis of the network sum-rate. In the first scenario, denoted as *symmetric network*, all users served by a given AP are assumed to be located at the same distance from that AP. This idealized situation results in all users experiencing the same average signal-to-noise ratio (SNR), an assumption often made by previous authors in this area, and for which several interesting results of the existing literature can be reused. This scenario is illustrated in Fig.2.

In the second, more realistic, scenario, denoted simply as *non symmetric network*, the users are located randomly over a cell given by a disk of radius R around each of the serving APs. Finally, a hybrid scenario mixing the two scenarios above is discussed later in the paper.

Note that the actual cell shape will not be a disk in reality. However we argue that, when it comes to studying the scaling laws of network sum-rate with maximum-rate user scheduling, the actual shape taken by the cell borders has in fact little impact on the result. The main reason is that since the user's
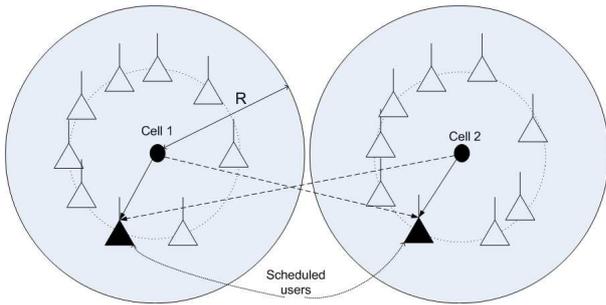
Fig. 2. A two-cell idealized symmetric network diagram example. Direct and interfering links toward the scheduled user (black) are indicated in solid and dashed arrows respectively. In this idealized case, users a located over a circle, a fixed distance away from their access point.

direct links is subject to a location dependent path loss, the distance to the serving AP will affect its chances of being selected by the scheduler. As a consequence the users located in the inner region of the cell (i.e. close to the access point) bear the vast majority of the traffic and are the drivers for the rate scaling laws. Therefore an accurate modeling for the location of cell-edge users is unimportant here.

V. NETWORK SUM-RATE: SCALING LAWS

A. Capacity scaling with large U in symmetric network

We analyze the scaling of rate $\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)$ via the scaling of the bounds $\mathcal{C}^{lb}$ and $\mathcal{C}^{ub}$, with increasing U. We just focus on the performance in cell n, as other cells are expected to behave similarly under equal number of users U and isotropic conditions throughout the network. For the symmetric network, users experience an equal average SNR, thus $\gamma_{u_n,n} = \gamma_n$ is a constant independent of the user index.

Interestingly, for this particular case, we show we can reuse extreme value theory results [23] developed specifically in the context of single cell opportunistic beamforming [15], [16] and transposed here to the case of networks with multicell interference. For the case asymmetric network, specific results are developed in later sections.

First, the following results provide insight into the interference-free scaling of SINR and rates respectively.

1) Scaling laws for interference-free case: The interference-free multicell rate scaling boils down to studying the scaling in each cell independently. Further assuming an isotropic network (i.e. all cells experience the same channel statistics) we can simplify the analysis by exploiting known results on single cell rate scaling, as done below.

Lemma 1: Let $G_{u_n,n} = \gamma_{u_n,n}|h_{u_n,n}|^2$, $u_n = 1 \ldots U, n = 1 \ldots N$, where $\gamma_{u_n,n} = \gamma_n$. Assume $|h_{u_n,n}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$) (i.e. $h_{u_n,n}$ is a unit-variance complex normal random variable). Assume the $|h_{u_n,n}|^2$ are independent and identically distributed (i.i.d.) across users. Then for fixed N and U asymptotically large, the upper bound on the SINR in cell n scales like

$$\Gamma_n^{ub} \approx \frac{P_{max}\gamma_n}{\sigma^2} \log U \qquad (15)$$

where the symbol $\approx$ means that the ratio of the left hand side and right hand side terms converges to one almost surely, as U goes to infinity.

Proof: This result is a reuse of a now well known result for single cell opportunistic scheduling. This states that the maximum of U i.i.d. $\chi^2(2)$ random variables behaves like $\log U$ for large U. See for instance [15], itself building on classical extreme value theory results [23]. We omit the proof here and refer the readers to these references.

From the SNR scaling, we obtain the scaling of the interference-free rate shown in (6). This is stated in the following theorem, again building on known single cell results but stated here for convenience, with our own notations:

Theorem 1: Let $G_{u_n,n} = \gamma_{u_n,n}|h_{u_n,n}|^2$, $u_n = 1 \ldots U, n = 1 \ldots N$, where $\gamma_{u_n,n} = \gamma$. This means that all cells are assumed to enjoy an identical link budget. Assume $|h_{u_n,n}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,n}|^2$ are i.i.d. across users. Then for fixed N and U asymptotically large, the average of the upper bound on the network sum-rate scales like

$$\mathbb{E}(\mathcal{C}^{ub}) \approx \log \log U \qquad (16)$$

where the expectation is taken over the complex fading gains.

Proof: Under isotropic network conditions, we have from (6):

$$\mathbb{E}(\mathcal{C}^{ub}) = \mathbb{E}\left(\log\left(1 + \Gamma_n^{ub}\right)\right) \qquad (17)$$

Once the scaling of Γ_n^{ub} is obtained, the scaling of the expected value of $\log(1 + \Gamma_n^{ub})$ is readily obtained from published results in the context of single cell maximum rate user scheduling, found in [15], [16] among others. For a detailed proof, see e.g. [16], Theorem 1.

2) Scaling laws for full-powered interference case: We now turn to the behavior of interference limited networks by exploring the lower bounds given for SINR and rates. The initial intuition would be that the analysis of the lower bound given in (12) provides us with a very pessimistic view of the network performance as it assumes interference coming at full power from every AP in the network. The interesting aspect behind our findings below is that it is not. In fact the negative impact of interference at the user on network sum-rate can be made arbitrarily small while not sacrificing transmission rates to the assigned APs, as shown per the following theorems. In the results below, remember we assume each user is assigned to a serving AP which is the one with minimum path loss. As a consequence, since the region of coverage under study is limited to a disk of radius R around the serving AP, the distance between a user and any interfering AP is greater than R. As a result we have from (14):

$$G_{u_n,i} \leq \beta R^{-\epsilon}|h_{u_n,i}|^2 \quad \text{for any } i \neq n \qquad (18)$$

The lemma below gives the scaling law for the worst case SINR (12).

Lemma 2: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1 \ldots U, n = 1 \ldots N$, where $\gamma_{u_n,n} = \gamma_n$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume

$|h_{u_n,i}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d. across users, cells. Then for fixed N and U asymptotically large, the lower bound on the SINR in cell n scales like

$$\Gamma_n^{lb} \approx \frac{P_{max}\gamma_n}{\sigma^2}\log U \qquad (19)$$

Proof: To obtain this result, one uses the fact that users in cell n are served by their closest AP. Following (18), an upper bound on the interference power then given by $\sum_{i\neq n}^{N}\beta R^{-\epsilon}|h_{u_n,i}|^2 P_{max}$. This gives a further lower bound on Γ_n^{lb} given by

$$\Gamma_n^{lb} \geq \Gamma_n^{lb2} \qquad (20)$$

where Γ_n^{lb2} is corresponds to the SINR assuming pessimistically that all sources of interferences are located on the edge of the cell of interest, calculated by:

$$\Gamma_n^{lb2} = \gamma_n P_{max} \max_{u_n=1...U} \omega_{u_n} \qquad (21)$$

where ω_{u_n} denotes the normalized SINR at user u_n:

$$\omega_{u_n} = \frac{|h_{u_n,n}|^2}{\sigma^2/P_{max} + \beta R^{-\epsilon}\sum_{i\neq n}^{N}|h_{u_n,i}|^2} \qquad (22)$$

The scaling law of Γ_n^{lb2} is also that of ω_{u_n}, which is the ratio of a Chi-square (2 degrees of freedom) distributed variable and the sum of a fixed noise term and a Chi-square (2N-2 degrees of freedom) variable. Thus the scaling of ω_{u_n} is similar to the scaling of the SINR in the single cell opportunistic beamforming problem with N antennas at the transmitter, studied in [16]. In there, the SINR is the ratio of a direct beam power and a noise plus $N-1$ interfering beam power term. In particular we can find its distribution as:

$$F_W(\omega) = 1 - \frac{e^{-\frac{\omega\sigma^2}{P_{max}}}}{(1+\omega\beta(N-1)R^{-\epsilon})^{N-1}} \qquad (23)$$

([16], Lemma 4) shows that the SINR then scales like $\log U$. This gives in our context:

$$\Gamma_n^{lb2} \approx P_{max}\gamma_n \log U/\sigma^2 \qquad (24)$$

Note that the scaling above is identical to the one reported for the interference-free case (15).

Thus, Γ_n^{lb} is bounded above and below by two expressions (respectively the interference-free Γ_n^{ub} and Γ_n^{lb2}) which exhibit the same scaling law. Therefore Γ_n^{lb} must satisfy itself the same scaling law.

The following theorem gives the scaling law for the lower bound on rate for an isotropic network.

Theorem 2: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1...U, n = 1...N$, where $\gamma_{u_n,n} = \gamma$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume $|h_{u_n,i}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d. across users, cells. Then for fixed N and U asymptotically large, the average of the lower bound on the network sum-rate scales like

$$\mathbb{E}(\mathcal{C}^{lb}) \approx \log\log U \qquad (25)$$

Proof: From the result in Lemma 2, this result is proved in a way identical with that of ([16], Theorem 1). Therefore the proof is omitted here for space considerations.

From bounding arguments and from theorems 1 and 2 above, the following conclusion is now obtained:

Theorem 3: Let $G_{u_n,i} = \gamma_{u_n,i}|h_{u_n,i}|^2$, $u_n = 1...U, n = 1...N$, where $\gamma_{u_n,n} = \gamma_n$, $\gamma_{u_n,i} = \beta d_{u_n,i}^{-\epsilon}$ for $i \neq n$. Assume $|h_{u_n,i}|^2$ is Chi-square distributed with 2 degrees of freedom ($\chi^2(2)$). Assume the $|h_{u_n,i}|^2$ are i.i.d. across users, cells. Then for fixed N and U asymptotically large, the average of the network sum-rate with optimum power control and scheduling scales like

$$\mathbb{E}(\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)) \approx \log\log U \qquad (26)$$

Proof: The result is readily obtained from writing:

$$\mathbb{E}(\mathcal{C}^{lb}) \leq \mathbb{E}(\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*)) \leq \mathbb{E}(\mathcal{C}^{ub}) \qquad (27)$$

Then, invoking (25) and (16) exhibiting the same scaling law, we obtain a similar law in (26).

Theorems 1 and 2 suggest that, in a multicell network with symmetric users, the rate obtained with optimal multicell scheduling in both an interference-free environment and an environment with full interference power have identical scaling laws in $\log\log U$. This result bears analogy to the results by [16] which indicate that in a single cell broadcast channel with random beamforming and opportunistic scheduling, the degradation caused by inter-beam interference tends becomes negligible when the number of users to choose from becomes large. Here the multicell interference becomes negligible because the optimum scheduler tends to select users on an instantaneous basis who have both a strong direct link to their serving AP and *small* interfering links from surrounding APs. Interestingly, the minimization of the multicell interference term should take away some degrees of freedom in choosing the users with best direct links, however not sufficiently so to affect the overall rate scaling.

Another interpretation of this result is in terms of our ability to find distributed scheduling schemes for maximizing the network sum-rate. The optimal multicell scheduler and power control solution would be hard to implement in practice. However from the observations above, a simple scheme based on each cell measuring the worst case SINR of each of its users (during e.g. a preamble) and selecting the users with the best worst case SINR as per (12), will result in an quasi optimal behavior asymptotically (again, from a scaling perspective). Such a scheme does not require any exchange of information between the cells and the worst case SINR can be measured in one shot by each user and fed back to its serving AP.

These results come as a complement to previously reported findings [24], [18] which propose a near optimal power allocation scheme, for fixed number of users, where a fraction of the transmitters are selected to be turned off while the rest operate at full power. It was observed experimentally [24] there that the fraction of off cells would go to zero when the number of users grows large. Thus in a network with full reuse and greedy user scheduling, the optimal power control policy

should be for all cells to operate at the power constraint. The analysis of scaling of rates provides a theoretical justification to this intuitive result.

We now turn to a non symmetric network where users can experience different average SNR values depending on their position and conduct a similar analysis. However we will see that different capacity scaling rates are obtained compared with the symmetric network case.

B. Capacity scaling with large U in non symmetric network

We assume the path loss is determined by the user's distance to the emitting AP, both serving and interfering. We consider a uniform distribution of the population in each cell. Thus $d_{u_n,n}$ (distance between user u_n and its serving AP) is a random variable with non uniform distribution $f_D(d)$. For a cell radius R, we find easily:

$$f_D(d) = 2d/R^2, \quad d \in [0, R] \tag{28}$$

Further, the random process $d_{u_n,n}$ can be considered i.i.d. across users and cells, if users in each cell are dropped randomly in each disk[2] Assuming $R = 1$ for normalization, the distribution of $\gamma_{u_n,n} = \beta d_{u_n,i}^{-\epsilon}$ is given by (details omitted here):

$$f_\gamma(g) = \begin{cases} \frac{2}{\epsilon}\left(\frac{g}{\beta}\right)^{-\frac{2}{\epsilon}}\frac{1}{g} & \text{with } g \in [\beta, \infty) \\ 0 & \text{with } g \notin [\beta, \infty) \end{cases} \tag{29}$$

In order to get upper and lower bounds on performance, we are interested in the behavior of the following extreme values of product of independent random variables:

$$\max_{u_n=1...U} \gamma_{u_n,n}|h_{u_n,n}|^2 \text{ for the interference-free case and}$$
$$\max_{u_n=1...U} \gamma_{u_n,n}\omega_{u_n} \text{ for the full-powered interference case}$$

where ω_{u_n} is again defined as per (22).

1) Extreme values of heavy-tail random variables: The distribution of $\gamma_{u_n,n}$ shown in (29) is remarkable in that it differs strongly from fast fading distributions, due to its *heavy tail* behavior. Tail behavior clearly plays a fundamental role in shaping the limiting distribution of the maximum value, hence also the scaling of rate. Note that heavy tail is also observed in *large scale* fading models such as log normal shadowing for instance. In order to study the extreme value of a product of random variables involving one heavy tailed variable, we need first to review the properties of so-called *regularly varying* random variables. See e.g. [23] for a definition of such variables, restated below:

Definition 3: A random variable X, with distribution (cdf) given by $F_X(x)$, is said to be regularly varying (at ∞) with exponent $-a$ if and only if:

$$\frac{1 - F_X(x)}{1 - F_X(tx)} \to t^a \quad \text{when } x \to \infty \tag{30}$$

The lemma below shows how the definition above applies to our situation:

---

[2]The considered coverage region can be assimilated with the inside area of each disk, in a disk-packing region of the 2D plane. Users dropped outside the disks can dropped from the analysis, as these will not affect the scaling law.

Lemma 3: Let $X = \gamma_{u_n,n}$. X is regularly varying with exponent $-\frac{2}{\epsilon}$.

Proof: A direct application of the definition above, with a distribution obtained from (29):

$$F_X(x) = 1 - \left(\frac{x}{\beta}\right)^{-\frac{2}{\epsilon}} \quad x \geq \beta. \tag{31}$$

An interesting aspect of regularly varying distributed random variable (R.V.) is that they are stable with respect to multiplication with other independent R.V. with finite moments as pointed out by the following theorem shown by Breiman [25]:

Theorem 4: Let X and Y be two independent R.V. such that X is regularly varying with exponent $-a$. Assuming Y has finite moment $\mathbb{E}(Y^a)$, then the tail behavior of the product $Z = XY$ is governed by:

$$1 - F_Z(z) = \mathbb{E}(Y^a)(1 - F_X(z))(1 + o(1)) \quad \text{when } z \to \infty \tag{32}$$

The idea behind this theorem is that when multiplying a regularly varying R.V. with another one with finite moment, one obtains a heavy tailed R.V. whose tail behavior is similar to the first one, up to a scaling. In other words, heavy tail behavior tends to dominate over other distribution.

We now apply this result to $X = \gamma_{u_n,n}$ and Y given by $Y = |h_{u_n,n}|^2$ for the interference free case and $Y = \omega_{u_n}$ for the full-powered interference case, respectively. Note that in both cases, Y has finite moments. The tail behavior of $Z = XY$ can then be characterized by the following lemma:

Lemma 4: Let $X = \gamma_{u_n,n}$ be a R.V. with distribution given by (29). Let Y be an independent R.V. such that $\mathbb{E}(Y^{\frac{2}{\epsilon}}) < \infty$. Then the tail of $Z = XY$ is governed by:

$$1 - F_Z(z) = \mathbb{E}(Y^{\frac{2}{\epsilon}})\left(\frac{\beta}{z}\right)^{\frac{2}{\epsilon}}(1 + o(1)) \quad \text{when } z \to \infty \tag{33}$$

Proof: A direct application of Theorem 4 using the distribution of X shown in (31).

The lemma above indicates that the tail behavior of the distribution of $X = \gamma_{u_n,n}$, characterized by Lemma 3, carries over to that of the product $Z = XY$. As a consequence, Z is also regularly varying with the same exponent $-\frac{2}{\epsilon}$.

We now complete our study by reviewing existing results on the extreme value of regularly varying R.V. Following [23], a regularly varying variable can be classified to be of *Frechet* type. Extreme values of Frechet (or regularly varying) variables are characterized by use of the Gnedenko theorem, given in appendix I. For comparison, note that the random variables involved in the analysis of previous sections (Sec.V-A and therein), belong to the so-called Gumbel category. In our context, we have the following result:

Lemma 5: Let $Z_{u_n} = \gamma_{u_n,n}Y$ where Y is a R.V. with finite moments, independent of $\gamma_{u_n,n}$. Then we have:

$$\lim \Pr\{\max_{u_n=1...U} Z_{u_n} \leq \beta\mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}}U^{\frac{\epsilon}{2}}t\} = e^{-t^{-\frac{2}{\epsilon}}} \quad \forall t > 0, \tag{34}$$

when $U \to \infty$.

Proof: We invoke Gnedenko's theorem [26] given in appendix I. It is easy to find that $a_U = \beta \mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}} U^{\frac{\epsilon}{2}}$ where a_U is defined in the appendix.

2) Scaling law for interference-free case: The inequality in (5) allows us to characterize the scaling law of rate. Although a characterization in terms similar to those of previous section (i.e. finding a scaling law $l(U)$ for the SINR, such that the ratio of the SINR and $l(U)$ converges towards 1 when $U \to \infty$) may possible when analyzing the rate, such a task is not easy and mathematically involved. Using existing extreme value theoretic tools, we proceed in two steps. First we analyze the wide-sense scaling of SINR in a way that allows us to directly exploit Lemma 5, where the notion of wide-sense scaling is defined precisely. In the second step we proceed to characterize the scaling of rate, this time in the conventional sense of scaling used earlier in this paper, so we can still make key interpretations.

The theorem below gives the wide sense scaling law of SINR for the interference-free case in a non symmetric network. First we give the following definition of wide sense scaling:

Definition 4: Let $U \geq 0$. Let $g(U)$ be a random variable whose distribution depends on parameter U. Let $l(U)$ be a deterministic function of U. $g(U)$ is said to *scale as $l(U)$ in the wide sense*, which is denoted by $g(U) \sim l(U), \quad U \to \infty$ when

$$\Pr(g(U) > v(U)) \to 0, \quad \text{when } U \to \infty$$
$$\Pr(g(U) < w(U)) \to 0, \quad \text{when } U \to \infty$$
$$(35)$$

for any two functions $v(U)$ and $w(U)$ such that $\frac{l(U)}{v(U)} \to 0$ and $\frac{w(U)}{l(U)} \to 0$, respectively.

Note that this notion of scaling can be interpreted as $g(U)$ grows neither significantly faster than $l(U)$, not does it grow significantly slower than $l(U)$. A typical application of wide sense scaling is that $g(U)$ and any other function of the type $g(U)O(U)$ have the same wide sense scaling law.

Theorem 5: Let $h_{u_n,n}$, $u_n = 1 \ldots U$ be i.i.d. Gaussian distributed unit-variance random variables. Assuming that $\gamma_{u_n,n}$ is i.i.d., distributed as per (29), for $n = 1 \ldots N$. Then for fixed N and U asymptotically large, the interference-free SNR scales in the wide sense like:

$$\Gamma_n^{ub} \sim U^{\frac{\epsilon}{2}} \qquad (36)$$

Proof: Let $v(U)$ be any function growing faster than $U^{\frac{\epsilon}{2}}$, i.e. such that $\lim_{U \to \infty} U^{\frac{\epsilon}{2}}/v(U) = 0$. Then let $t = v(U)/(\beta \mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}} U^{\frac{\epsilon}{2}})$. From Lemma 5 we have that

$$\Pr\{\max_{u_n=1\ldots U} Z_{u_n} \leq v(U)\} \to \lim_{U \to \infty} e^{-t^{-\frac{2}{\epsilon}}} = 1 \qquad (37)$$

Equivalently, we have that $\Pr(\max_{u_n=1\ldots U} Z_{u_n} > v(U)) \to 0$. Similarly, we can prove that any function $w(U)$ growing slower than $U^{\frac{\epsilon}{2}}$ will be such that $\Pr\{\max_{u_n=1\ldots U} Z_{u_n} < w(U)\} \to 0$. Thus $\max_{u_n=1\ldots U} Z_{u_n}$ scales as $U^{\frac{\epsilon}{2}}$ in the wide sense.

From the wide sense scaling of SNR above, we can infer the *conventional* scaling law for the upper bound on rate $\mathbb{E}(\mathcal{C}^{ub})$, as shown per the theorem below:

Theorem 6: Let $h_{u_n,n}$, $u_n = 1 \ldots U$ be i.i.d. Gaussian distributed unit-variance random variables. Assuming that $\gamma_{u_n,n}$ is i.i.d., distributed as per (29), for $n = 1 \ldots N$. Then for fixed N and U asymptotically large, the interference-free rate scales like (i.e. the ratio of the two quantities converges to 1 almost surely):

$$\mathbb{E}(\mathcal{C}^{ub}) \approx \frac{\epsilon}{2} \log U \quad \text{for large } U \qquad (38)$$

Proof: See appendix II.

We now proceed to determine the scaling laws in the case of full-powered interference.

3) Scaling law for full-powered interference case: We can derive the scaling laws for the lower bound of SINR and rate by following a strategy similar to Sec.V-B.2, simply by replacing the R.V. $|h_{u_n,n}|^2$ by the R.V. ω_{u_n} which also has bounded moments. We obtain the following result:

Theorem 7: Let $h_{u_n,i}$, $u_n = 1 \ldots U, i = 1 \ldots N$ be i.i.d. Gaussian distributed unit-variance random variables. Assuming that $\gamma_{u_n,n}$ is i.i.d., distributed as per (29), for $n = 1 \ldots N$. Then for fixed N and U asymptotically large, the lower bound on SINR scales in the wide sense like:

$$\Gamma_n^{lb} \sim U^{\frac{\epsilon}{2}} \qquad (39)$$

Proof: We use the same proof as for Theorem 5, with $X = \gamma_{u_n,n}$ but this time $Y = \omega_{u_n}$.

Finally, from Theorem 7, we infer that the upper bound on rate for a non symmetric network exhibits an *conventional* scaling law defined as:

$$\mathbb{E}(\mathcal{C}^{lb}) \approx \frac{\epsilon}{2} \log U \qquad (40)$$

The proof for (40) is identical to that of Theorem 6 in Appendix II, but simply replacing Y with ω_{u_n}, which clearly does not change the scaling.

Remarkably, as in the case of the symmetric network, the results above (38) and (40) suggest that multicell interference, no matter how strong, does not affect the scaling of the network sum-rate, if enough users exist *and* rate-optimal scheduling is applied. Furthermore, by virtue of the upper bound and lower bound exhibiting the same scaling law in (38) and (40) respectively, the rate under optimal scheduling and power allocation must behave like

$$\mathcal{C}(\boldsymbol{U}^*, \boldsymbol{P}^*) \approx \frac{\epsilon}{2} \log U \qquad (41)$$

Two remarks are in order. First, in the symmetric network case, a suboptimal but fully distributed resource allocation based on constant (full) power transmission at all transmitters and scheduling policy based on (11) will actually result in the best possible scaling law of sum-rate for the network. Second, we observe that we obtain a much greater rate growth than in the case of the symmetric network. This is due to the amplified multiuser diversity gain due to the presence of unequal path

loss across the user locations in the cell. This results from a scheduler which, in a quite unfair fashion admittedly, tends to select users closer to the access point as more users are added to the network.

C. Discussion on channel models and exclusion area around the AP

Interestingly, the theory on regularly varying variables stipulates that multiplication of the path loss variables by any small scale fading variable with finite moments will preserve its heavy tail behavior. This means that our result shown in (41) is in fact valid for a wider class of fading channel models, such as Nakagami, Rice, etc. On a different note, one may wonder how close users can be assumed to get to the access point in practice. Let us imagine that a small disk of exclusion, with the AP at its center, prevents users to getting too close to the AP. As a by product, the disk also serves the purpose of maintaining the validity of the path loss model, which may not be reasonable in the close vicinity of the AP. In this case, one may expect two successive regimes for the rate scaling as U grows. In the first regime, when the number of users is still moderate, the scaling is dominated by the path loss effect, with a behavior such as shown in (41). In the second regime, when enough users are already accumulated near the exclusion circle, it is the turn of the tail behavior of small scale fading to dominate and the scaling will be characterized by (26). This situation is investigated briefly in one simulation example.

As the growth would be ultimately limited by that the tail of the small-scale random fading in practical situations, one may also wonder how accurately Chi-square distributions model reality in real-world wireless channels. Clearly, this discussion is inherent in all previous studies dealing with scaling laws and asymptotic performance analysis. Nevertheless it is important to keep in mind the basic law of power preservation which indicates that no matter how many users are considered, the most favorable users cannot receive more power than what was actually transmitted. This simple fact will impose a hard limit on the SNR which in turn limits the domain of validity of our scaling in terms of the number of users U. Although we believe a specific analysis of the validity domain will rely on yet unexplored channel model properties (tail properties of the pdf are less explored than the behavior near zero which characterize outage) and is outside the scope of this paper, it remains clear that this domain is wide enough for the analysis to be meaningful since the power preservation limit is reached only when the small scale fading is in the order of the inverse of path loss, which would require very large fading coefficients in practice (several tens of dB).

VI. NUMERICAL EVALUATION

We validate the asymptotic behavior of the multicell sum rate when U grows large with Monte Carlo simulations. We use a network with $N = 4$ cells, unit cell radius and the following parameters $\beta = 1/16$, $\epsilon = 4$, $P_{max} = 1$, $\sigma^2 = 0.02$. I.i.d. flat Rayleigh fading is considered in addition to the path loss based power decay. We consider three scenarios for user location, as mentioned previously in this paper. First, we

consider cells with users located on a circle with distance 0.5 away from the AP (symmetric network). Then we consider a non symmetric distribution of average SNR by drawing users randomly in the cell. Finally we consider an hybrid scenario where users are drawn uniformly randomly over the cell but kept outside an exclusion disk of radius 0.1 around the AP. In all cases, we evaluate the upper and lower bound on per-cell data rates (see Fig.3, Fig.4, Fig.5 and observe the identical rate growth of the lower and upper rate bounds. This also shows that the rate obtained with exhaustive user and power level selection also has the same growth rate. The observed rate growth in $\log \log U$ for the symmetric network and in $\log U$ for the non symmetric network confirms our earlier theoretical claims. In Fig.5, we observe a scaling behavior with two distinct regimes with a $\log U$ in the moderate number of users U and $\log \log U$ for high number of users, thus confirming our intuition for what could happen in a realistic network.

VII. CONCLUSIONS

We present an extreme value theoretic analysis of network sum-rate for maximum sum rate multicell power allocation and user scheduling. We derive scaling laws of rates when the number of users per cell grows large, both in cases where the users have same average SNR and path loss dependent SNR. We show that in both cases, 1-the effect of intercell interference on rate scaling tends to be negligible asymptotically, and 2-should intercell interference be considered, an asymptotically optimal allocation procedure is given based on full power allocation at all transmitters, which is furthermore completely distributed. We show that the growth of rates is exponentially faster in the case of a system with unequal distance-based average SNR.
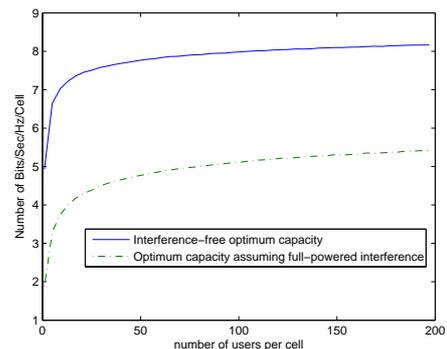
Fig. 3. Scaling of upper and lower bounds of rate versus U for a symmetric network ($N = 4$). The observed scaling for both curves is in $\log \log U$.

APPENDIX I

The following theorem is due to Gnedenko [26] and states the following property for regularly varying distributions:

Theorem 8: Let Z_i an i.i.d. random process. Then Z_i has a regularly varying distribution with exponent a if and only if

$$\lim \Pr\{\max_{i=1...U} Z_i \leq a_U t\} = e^{-t^{-a}} \quad \forall t > 0 \quad \text{when } U \to \infty$$

(42)

where a_U is a sequence such that $1 - F_Z(a_U) = \frac{1}{U}$.
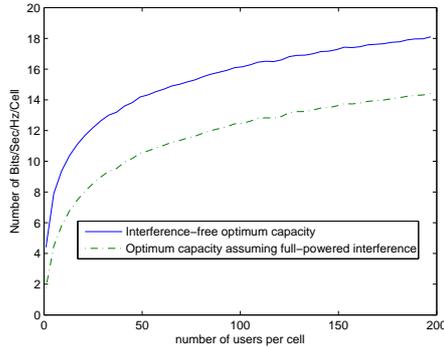
Fig. 4. Scaling of upper and lower bounds of rate versus U for a non symmetric network (unequal average SNR) ($N = 4$). The observed scaling for both curves is in $\log U$.
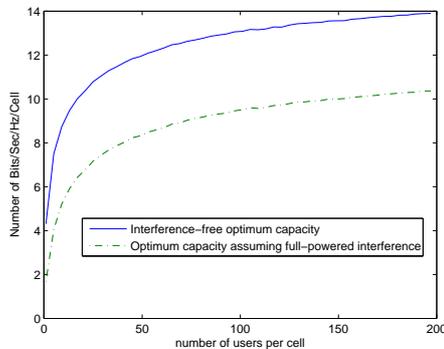
Fig. 5. Scaling of upper and lower bounds of rate versus U for a non symmetric network with an exclusion disk around each AP of radius 0.1. ($N = 4$).The observed scaling in the transitory regime is in $\log U$, then in $\log \log U$ in the asymptotic regime.

APPENDIX II

From Lemma 5 we have that

$$\lim \Pr\{\Gamma_n^{ub} \le \beta \mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}} U^{\frac{\epsilon}{2}} t\} = e^{-t^{-\frac{2}{\epsilon}}} \quad \forall t > 0, \ \text{when } U \to \infty \tag{43}$$

Since the SNR Γ_n^{ub} is growing large in each cell by virtue of Theorem 5, the rate can be approximated by:

$$\mathcal{C}^{ub} \approx \frac{1}{N} \sum_{n=1}^{N} \log \Gamma_n^{ub}. \tag{44}$$

when U grows large. From (43), we write

$$\lim \Pr\{\log \Gamma_n^{ub} \le \log(\beta \mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}}) + \log t + \frac{\epsilon}{2} \log U\} = e^{-t^{-\frac{2}{\epsilon}}}, \tag{45}$$

$\forall t > 0$, when $U \to \infty$. Now, taking $t = \log U$ we infer that

$$\log \Gamma_n^{ub} \le \log(\beta \mathbb{E}(Y^{\frac{2}{\epsilon}})^{\frac{\epsilon}{2}}) + \log \log U + \frac{\epsilon}{2} \log U \tag{46}$$

almost surely when $U \to \infty$.

On the other hand, taking $t = 1/\log U$, we obtain that

$$\log \Gamma_n^{ub} \ge \frac{\epsilon}{2} \log U - \log \log U\} \ \text{ almost surely when } \ U \to \infty \tag{47}$$

From (46) and (47), we conclude that

$$\frac{\log \Gamma_n^{ub}}{\frac{\epsilon}{2} \log U} \to 1 \ \text{ almost surely when } \ U \to \infty \tag{48}$$

From the isotropy of the network, this shows that $\mathcal{C}^{ub}$ (and *a fortiori* its average) scales as $\frac{\epsilon}{2} \log U$.

REFERENCES

[1] A. Goldsmith, *Wireless Communications*. Cambridge Univ. Press, 2005.

[2] S. Verdu, *Multiuser Detection*. Cambridge Univ. Press, 1998.

[3] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. on Veh. Technol.*, vol. 42, pp. 641–646, Nov. 1993.

[4] J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 305–311, Aug. 1992.

[5] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal Sel. Areas on Commun.*, vol. 13, no. 7, pp. 1341–1347, Sept. 1995.

[6] C. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[7] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games in telecommunication," *Comput. Oper. Res.*, vol. 33, pp. 286–311, Feb. 2006.

[8] D. Gesbert, S. Kiani, A. Gjendemsjo, and G. Øien, "Adaptation, co-ordination, and distributed resource allocation in interference-limited wireless networks," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2393–2409, Dec. 2007.

[9] G. J. Foschini, H. Huang, K. Karakayali, R. Valenzuela, and S. Venkatesan, "The value of coherent base station coordination," in *Proc. Conf. on Inform. Sciences and Systems (CISS)*, Baltimore, Maryland, March 2005.

[10] S. Shamai and B. M. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Rhodes, Greece, May 2001, pp. 1745–1749.

[11] O. Somekh, O. Simeone, Y. Bar-Ness, A. Haimovich, and S. Shamai, "Cooperative multi-cell zero-forcing beamforming in cellular downlink," *IEEE Trans. Inform. Theory*, vol. 55, no. 7, pp. 3206–3219, July 2009.

[12] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE Journal Sel. Areas on Commun.*, Dec. 2010, to appear.

[13] X. Tang, S. A. Ramprashad, and H. Papadopoulos, "Multi-cell user scheduling and random beamforming strategies for downlink wireless communications," in *Proc. IEEE Veh. Technol. Conf.*, Anchorage, AK, 2009, pp. 1–5.

[14] W. Choi and J. Andrews, "The capacity gain from intercell scheduling in multi-antenna systems," *IEEE Trans. Wireless Comm.*, vol. 7, no. 2, pp. 714–725, Feb. 2008.

[15] P. Viswanath, D. N. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[16] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channel with partial side information," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[17] T. Y. Al-Naffouri, A. Dana, and B. Hassibi, "Scaling laws of multiple antenna group-broadcast channels," *IEEE Trans. Wireless Comm.*, vol. 7, no. 12, pp. 5030–5038, Dec. 2008.

[18] M. Ebrahimi, M. Maddah-Ali, and A. Khandani, "Throughput scaling in wireless networks with fading channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 11, pp. 4250–4254, Nov. 2007.

[19] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 1991.

[20] D. Park and G. Caire, "Hard fairness versus proportional fairness in wireless communications: The multiple-cell case," *IEEE Trans. Commun.*, Feb. 2008, submitted, [http://arxiv.org/abs/0802.2975].

[21] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE Int. Conf. on Comm.*, Seattle, USA, June 1995, pp. 331–335.

[22] W. C. Y. Lee, *Mobile Communications Engineering*. McGraw Hill, 1982.

[23] H. A. David and H. N. Nagaraja, *Order Statistics, 3rd ed.* John Wiley and Sons, 2003.

[24] S. G. Kiani, G. E. Øien, and D. Gesbert, "Maximizing multi-cell capacity using distributed power allocation and scheduling," in *Proc. IEEE Wireless Comm. and Net. Conf.*, Hong Kong, China, March 2007.

[25] L. Breiman, "On some limit theorems similar to the arc-sin law," *Theory Probab. Appl.*, vol. 10, no. 2, pp. 323–331, Jan. 1965.

[26] B. Gnedenko, "Sur la distribution limite du terme maximum d'une serie aleatoire," *Annals of Mathematics*, vol. 44, no. 3, pp. 423–453, July 1943.

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