# Weighted Sum Rate Maximization in the MIMO Interference Channel

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***Abstract*—Centralized algorithms for weighted sum rate (WSR) maximization for the $K$-user frequency-flat MIMO Interference Channel (MIMO IFC) with full channel state information (CSI) are considered. Maximization of WSR is desirable since it allows the system to cover all the rate tuples on the rate region boundary for a given MIMO IFC. First, we propose an iterative algorithm to design optimal linear transmitters and receivers. The transmitters and receivers are optimized to maximize the WSR of the MIMO IFC. Subsequently, we propose a *greedy user selection* algorithm based on the maximum WSR algorithm that can be applied to select a subset of transmit-receive pairs that cooperate in the interest of maximizing the sum-rate of the resulting cooperative network. To the best of our knowledge this is the first time user selection has been proposed in the context of the MIMO IFC.**

***Index Terms*—MIMO, MMSE, weighted sum rate, Interference Channel, linear transmitter, linear receiver, interference alignment**

## I. Introduction

In cellular systems where spectrum scarcity/cost is a major concern, a frequency reuse factor of 1 is desirable. Such systems however have to deal with the additional problem of inter-cell interference which does not exist in isolated point-to-point systems. Interference is being increasingly identified as the major bottleneck limiting the throughput in wireless communication networks. Traditionally, the problem of interference has been dealt with through careful planning and (mostly static) radio resource management. With the widespread popularity of wireless devices following different wireless communication standards, the efficacy of such interference avoidance solutions is fairly limited. A systematic study of the performance of cellular communication systems where each cell communicates multiple streams to its users while enduring/causing interference from/to neighboring cells due to transmission over a common shared resource comes under the purview of MIMO interference channels (MIMO IFC). A $K$-user MIMO-IFC models a network of $K$ transmit-receive pairs where each transmitter communicates multiple data streams to its respective receiver. In doing so, it generates interference at all other receivers. While the interference channel has been the focus of intense research over the past few decades, its capacity in general remains an open problem and is not well understood even for simple cases. Recently, it was shown that the concept of interference alignment (IA) [1], allows each receiver to suppress more interfering streams than it could otherwise cancel in interference channels. In a frequency-flat MIMO IFC,at least in the high-SNR regime, the network (comprising of $K$ user pairs) performance can be maximized (i.e, the sum-rate can be maximized) using IA since aligning the streams at the transmitter will now allow the maximization of the capacity pre-log factor in a $K$-user IFC. A distributed algorithm that exploits the reciprocity of the MIMO IFC to obtain the transmit and receiver filters in a $K$-user MIMO IFC was proposed in [2]. It is was shown there that IA is a suboptimal strategy at finite SNRs. In the same paper, the authors propose a signal-to-interference-plus-noise-ratio (SINR) maximizing algorithm which outperforms the IA in finite SNRs and converges to the IA solution in the high SNR regime. However, this approach can be shown to be suboptimal for multiple stream transmission since it allocates equal power to all streams. Moreover, the convergence of this iterative algorithm has not been proved. Thus an optimal solution for MIMO IFC at finite SNR remains an open problem.

Some early work on the MIMO IFC was reported in [3] by Ye and Blum for the asymptotic cases when the interference to noise ratio (INR) is extremely small or extremely large. It was shown there that a ”greedy approach” where each transmitter attempts to maximize its individual rate regardless of its effect on other un-intended receivers is provably suboptimal.There have been some attempts to port the solution concepts of the MIMO BC and MIMO MAC to the MIMO IFC. For instance, the problem of joint transmitter and receiver design to minimize the sum-MSE of a multiuser MIMO uplink was considered in [4] where iterative algorithms that jointly optimize precoders and receivers were proposed. Subsequently [5] applied this algorithm to the MIMO IFC where each user transmits a single stream and a similar iterative algorithm to maximize the sum rate was proposed in [6].

## II. Signal Model

Fig. 1 depicts a $K$-user MIMO interference channel with $K$ transmitter-receiver pairs. The $k$-th transmitter and its corresponding receiver are equipped with $M_k$ and $N_k$ antennas respectively. The $k$-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the $\mathbb{C}^{N_k \times 1}$ received signal $\mathbf{y}_k$ at the $k$-th

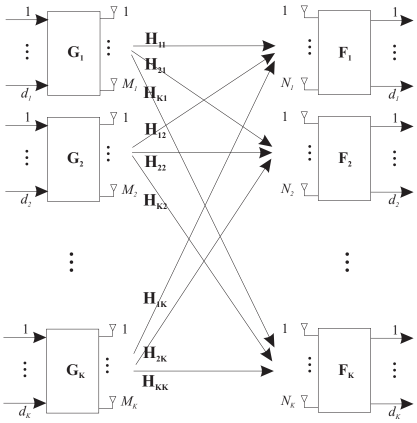


Fig. 1: MIMO Interference Channel

receiver, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \tag{1}$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the $l$-th transmitter and $k$-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the $k$-th transmitter and the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel matrices corresponding to its direct link and all the other cross-links in addition to the transmitter power constraints and the receiver noise covariances.

We denote by $\mathbf{G}_k$, the $\mathbb{C}^{M_k \times d_k}$ precoding matrix of the $k$-th transmitter. Thus $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where $\mathbf{s}_k$ is a $d_k \times 1$ vector representing the $d_k$ independent symbol streams for the $k$-th user pair. We assume $\mathbf{s}_k$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_k \sim \mathcal{N}(0, \mathbf{I}_{d_k})$. The $k$-th receiver applies $\mathbf{F}_k \in \mathbb{C}^{d_k \times N_k}$ to suppress interference and retrieve its $d_k$ desired streams. The output of such a receive filter is then given by

$$\mathbf{r}_k = \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k\mathbf{n}_k$$

Note that $\mathbf{F}_k$ does not represent the whole receiver but only the reduction from a $N_k$-dimensional received signal $\mathbf{y}_k$ to a $d_k$-dimensional signal $\mathbf{r}_k$, to which further (possibly optimal) receive processing is applied.

## III. Weighted sum rate maximization for the MIMO IFC

The stated objective of our investigation is the maximization of the WSR of MIMO IFC. For a given MIMO IFC, the maximization of the weighted sum rate (WSR) allows to cover all the rate tuples on the rate region boundary. It is for this reason that, in this paper we consider the weighted sum rate maximization problem for a $K$-user frequency-flat MIMO IFC and propose an iterative algorithm for linear precoder/receiver design. With full CSIT, but only knowledge of $\mathbf{s}_k$ at transmitter $k$, it is expected that linear processing at the transmitter should be sufficient. On the receive side however, optimal WSR approaches may involve joint detection of the signals from multiple transmitters. In this paper we propose to limit receiver complexity by restricting the modeling of the signals arriving from interfering transmitters as colored noise (which is Gaussian if we consider Gaussian codebooks at the transmitters). As a result, linear receivers are sufficient. For the MIMO IFC, one approach to linear transmit precoder design is the joint design of precoding matrices to be applied at each transmitter based on channel state information (CSI) of all users. Such a *centralized* approach [3] requires (channel) information exchange among transmitters. Nevertheless, studying such systems can provide valuable insights into the limits of perhaps more practical *distributed* algorithms [7] [8] that do not require any information transfer among transmitters.

The WSR maximization problem can be mathematically expressed as follows.

$$\{\mathbf{G}_k^\star, \mathbf{F}_k^\star\} = \arg \min_{\{\mathbf{G}_k, \mathbf{F}_k\}} \mathcal{R} \quad \mathrm{s.t}\ \mathrm{Tr}(\mathbf{G}_k^H\mathbf{G}_k) = P_k \ \ \forall k \tag{2}$$

where

$$\mathcal{R} = \sum_k -u_k R_k.$$

with $u_k \geq 0$ denoting the weight assigned to the $k$-th user's rate and $P_k$ it's transmit power constraint. We use the notation $\{\mathbf{G}_k, \ \mathbf{F}_k\}$ to compactly represent the candidate set of transmitters $\mathbf{G}_k$ and receivers $\mathbf{F}_k$ $\forall k \in \{1, \ldots, K\}$ and the corresponding set of optimum transmitters and receivers is represented by $\{\mathbf{G}_k^\star, \mathbf{F}_k^\star\}$. Assuming Gaussian signaling, the $k$-th user's achievable rate is given by

$$\begin{aligned} R_k &= \log|\mathbf{E}_k|, \\ \mathbf{E}_k &= \mathbf{I}_k + \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k(\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k)^H(\mathbf{F}_k\mathbf{R}_{\bar{k}}\mathbf{F}_k^H)^{-1} \end{aligned} \tag{3}$$

where the interference plus noise covariance matrix is define as:

$$\mathbf{R}_{\bar{k}} = \mathbf{R}_{n_k n_k} + \sum_{l \neq k} \mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H.$$

We use here the standard notation $|\ .\ |$ to denote the determinant of a matrix. The MIMO IFC rate region is known to be non-convex. The presence of multiple local optima complicates the computation of optimum precoding matrices to be applied at the transmitter in order to maximize the weighted sum rate. What is known however, is that, for a given set of precoders, linear minimum mean squared error (LMMSE) receivers (4) are optimal in terms of interference suppression.

$$\mathbf{F}_k^{LMMSE} = \mathbf{G}_k^H\mathbf{H}_{kk}^H(\mathbf{R}_{\bar{k}} + \mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H)^{-1} \tag{4}$$

### A. Gradient of weighted sum rate for the MIMO IFC

Consider the WSR maximization problem in (2). Let

$$\mathbf{E}_k = (\mathbf{I}_k + \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k(\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k)^H(\mathbf{F}_k\mathbf{R}_{\bar{k}}\mathbf{F}_k^H)^{-1})^{-1}. \tag{5}$$

Expressing the WSR in terms of $\mathbf{E}_k$, we have

$$\mathcal{R} = \sum_{k=1}^{K} -u_k \log|\mathbf{E}_k^{-1}|$$

The Karush-Kuhn-Tucker (KKT) conditions for this optimization problem obtained by setting the gradient of the WSR w.r.t $\mathbf{F}_k$ proves difficult to solve. Therefore, we consider a (more tractable) optimization problem where MMSE processing at the receiver is implicitly assumed. The rationale for this assumption will become clear as we proceed in this section. For now, we simply state that this assumption allows us to leverage a connection between the weighted sum MSE minimization problem and the WSR maximization problem that is the focus of our investigation. The alternative optimization problem that we consider is expressed as

$$\{\mathbf{G}_k^{\star}\}=\arg\min_{\{\mathbf{G}_k\}} \sum_{k=1}^{K} -u_k \log|\mathbf{E}_k^{-1}| \ \text{ s.t } \ \mathrm{Tr}(\mathbf{G}_k^H\mathbf{G}_k) = P_k \ \forall k \tag{6}$$

where $\mathbf{E}_k$ is given by

$$\mathbf{E}_k = (\mathbf{I} + \mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k)^{-1}. \tag{7}$$

In order to obtain the stationary points for the optimization problem (6), we solve the Lagrangian:

$$J\left(\{\mathbf{G}_k, \lambda_k\}\right) = \sum_{k=1}^{K} -u_k \log|\mathbf{E}_k^{-1}| + \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} - P_k)$$

Now setting the gradient of the Lagrangian w.r.t. the transmit filter $\mathbf{G}_k$ to zero, we have:

$$\begin{gathered}\frac{\partial J(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = 0 \\ \sum_{l\neq k} u_l \mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\mathbf{G}_l\mathbf{E}_l\mathbf{G}_l^H\mathbf{H}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\mathbf{G}_k \\ -u_k\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k\mathbf{E}_k + \lambda_k\mathbf{G}_k = 0\end{gathered} \tag{8}$$

Notice that it is now possible to derive the gradient of the WSR expression w.r.t $\mathbf{G}_k$ for fixed $\mathbf{F}_i$ and $\mathbf{E}_i$ $\forall i \neq k$. However, direct computation of $\lambda_k$ that satisfies the KKT conditions now becomes complex. For single antenna receivers in a broadcast channel, a solution for transmit filter design that minimizes the MSE at the receiver was proposed in [9]. The key idea was to allow for scalars to compensate for transmit power constraints. Our approach to the design of the WSR maximizing transmit filters for the MIMO IFC is inspired by this idea. Before we explain the computation of $\lambda_k$ any further, we digress in order to highlight an important connection between the WSR maximization and the weighted sum mean squared error (WSMSE) minimization problem that we exploit in our iterative algorithm.

Consider the problem where it is desired to optimize the transmit filters so as to minimize the WSMSE across all users (assuming MMSE receivers). Denote by $\mathbf{W}_k$ the weight matrix of the $k$-th user. Then this problem can be expressed as

$$\arg\min_{\{\mathbf{G}_k\}} \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbf{E}_k\} \ \text{ s.t. } \mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} = P_k \ \ \forall k$$

and the corresponding Lagrangian reads

$$L(\{\mathbf{G}_k, \lambda_k\}) = \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbf{E}_k\} + \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} - P_k)$$

Deriving $L(\{\mathbf{G}_k, \lambda_k\})$ respect to $\mathbf{G}_k$ we have

$$\begin{gathered}\frac{\partial \mathrm{L}(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = 0 \\ \sum_{l\neq k} \mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\mathbf{G}_l\mathbf{E}_l\mathbf{W}_l\mathbf{E}_l\mathbf{G}_l^H\mathbf{H}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\mathbf{G}_k \\ -\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k\mathbf{E}_k\mathbf{W}_k\mathbf{E}_k + \lambda_k\mathbf{G}_k = 0\end{gathered} \tag{9}$$

Comparing the gradient expressions for the two Lagrangians (8) and (9) we see that they can be made equal if

$$\mathbf{W}_k = u_k\mathbf{E}_k^{-1} \tag{10}$$

In other words, with a proper choice of the weighting matrices, a stationary point for the weighted sum minimum mean square error objective function is also a stationary point for the maximum WSR problem. This is the extension of [10] to the MIMO IFC. We exploit this relationship to henceforth compute the $\mathbf{G}_k$ that minimizes the WSMSE when $\mathbf{W}_k = u_k\mathbf{E}_k^{-1}$ instead of directly maximizing the WSR. We are now ready to extend the solution in [9] to MIMO IFC problem at hand. Since we are interested in minimizing the WSMSE, we have

$$\min \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbb{E}[(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)^H]\}$$
$$\text{s.t. } \ \mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} \ = \ P_k$$

where the $\alpha_k$ allows to compensate for the (scalar) transmit-filter power constraint. Assuming $\mathbb{E}\{\mathbf{d}\mathbf{d}^H\} = \mathbf{I}_k$, the MSE covariance matrix becomes:

$$\begin{aligned}\boldsymbol{\mathcal{E}}_k = \ & \mathbb{E}[(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)^H]\} \\ = \ & \mathbf{I} - \alpha_k^{-1}\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H - \alpha_k^{-1}\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k \\ & +\alpha_k^{-2}\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H \\ & +\alpha_k^{-2}\sum_{l\neq k}\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k^H + \alpha_k^{-2}\mathbf{F}_k\mathbf{R}_{n_kn_k}\mathbf{F}_k^H\end{aligned} \tag{11}$$

The corresponding Lagrangian can be written as:

$$J(\{\mathbf{G}_k, \alpha_k, \lambda_k\}) = \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\boldsymbol{\mathcal{E}}_k\} - \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} - P_k) \tag{12}$$

Optimizing for $\alpha_k$ we get:

$$\alpha_k = 2\frac{\mathrm{Tr}\{\sum_{l=1}^{K}\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k^H + \mathbf{W}_k\mathbf{F}_k\mathbf{R}_{n_kn_k}\mathbf{F}_k^H\}}{\mathrm{Tr}\{\mathbf{W}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\} + \mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\}}. \tag{13}$$

Interestingly, fixing the receivers to be MMSE leads to the simplified expression $\alpha_k = 1 \ \ \forall k$. Assuming LMMSE Rx filter for the Lagrange multiplier $\lambda_k$, we have:

$$\begin{aligned}\lambda_k = \ & \frac{1}{P_k}\left(\sum_{l\neq k}\mathrm{Tr}\{\mathbf{W}_l\mathbf{F}_l\mathbf{H}_{lk}\mathbf{G}_k(\mathbf{F}_l\mathbf{H}_{lk}\mathbf{G}_k)^H\}\right) \\ & -\frac{1}{P_k}\left(\sum_{l\neq k}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l(\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l)^H\}\right) \\ & -\frac{1}{P_k}\left(\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{R}_{n_kn_k}\mathbf{F}_k^H\}\right).\end{aligned} \tag{14}$$

Thus, assuming MMSE receivers, from (11) (12) and (14)

$$\mathbf{G}_k = \left(\sum_{l=1}^{K} \mathbf{H}_{lk}^H \mathbf{F}_l^H \mathbf{W}_l \mathbf{F}_l \mathbf{H}_{kl} - \frac{1}{P_k}\left(\left(\left(\sum_{l\neq k} \mathrm{Tr}\{\mathbf{W}_l \mathbf{J}_l^{(k)}\} - \mathrm{Tr}\{\mathbf{W}_k \mathbf{J}_k^{(l)}\}\right) - \mathrm{Tr}\{\mathbf{W}_k \mathbf{N}_k\}\right)\mathbf{I}\right)\right)^{-1} \mathbf{H}_{kk}^H \mathbf{F}_k^H \mathbf{W}_k \tag{15}$$

$$\mathbf{J}_l^{(k)} = \mathbf{F}_l \mathbf{H}_{lk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{lk}^H \mathbf{F}_l^H; \quad \mathbf{J}_k^{(l)} = \mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H \mathbf{F}_k^H; \quad \mathbf{N}_k = \mathbf{F}_k \mathbf{R}_{n_k n_k} \mathbf{F}_k^H$$

we have the expression (15) for the transmit filter $\mathbf{G}_k$. We therefore have the following two-step iterative algorithm to compute the precoders that maximize the weighted sum rate for a given MIMO IFC (c.f Table **Algorithm 1**).

**Algorithm 1** MWSR Algorithm for MIMO IFC

Fix an arbitrary initial set of precoding matrices $\mathbf{G}_k, \quad \forall \in k = \{1, 2 \ldots K\}$
set $n = 0$
**repeat**
  $n = n + 1$
  Given $\mathbf{G}_k^{(n-1)}$, compute $\mathbf{F}_k^n$ and $\mathbf{W}_k^n$ from (4) and (10) respectively $\forall k$
  Given $\mathbf{F}_k^n$ and $\mathbf{W}_k^n$, compute $\mathbf{G}_k^n$ $\forall k$ using (15)
**until** convergence

For further details on the proposed algorithm refer to [11].

*B. Hassibi-style Solution*

An alternative approach is the extension of [12] to the MIMO IFC and involves normalizing the transmit filter so as to satisfy the power constraint. i.e.,

$$\bar{\mathbf{G}}_k = \sqrt{P_k}\,\frac{1}{\sqrt{\mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\}}}\,\mathbf{G}_k = \sqrt{P}_k\,\beta_k\,\mathbf{G}_k \tag{16}$$

This converts the constrained optimization problem considered so far to an unconstrained optimization problem, thereby avoiding the introduction of Lagrange multipliers. The solution proposed in [12] was for a MISO BC problem. To extend it properly to a MIMO case (here IFC), it suffices to follow thread one of the philosophy of [10], as mentioned in Section III. In the case of the MISO case, the $\mathbf{F}_k$, $\mathbf{E}_k$, which are frozen during the optimization over the $\mathbf{G}_k$, are scalars. In [12], two different but equivalent sets of scalars are considered.The sum rate expression with the normalized beamformers can be written as

$$\mathcal{R} = \sum_{k=1}^{K} u_k \log|\mathbf{I}_k + P_k \beta_k^2 \mathbf{H}_{kk}\mathbf{G}_k(\mathbf{H}_{kk}\mathbf{G}_k)^H \mathbf{R}_{\bar{k}}^{-1}|$$

where $\mathbf{R}_{\bar{k}}$ is now given by

$$\mathbf{R}_{\bar{k}} = \mathbf{R}_{n_k n_k} + \sum_{l\neq k} P_l \beta_l^2 \mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H.$$

To find the optimal transmit filter we derive the WSR expression first w.r.t. $\mathbf{G}_k$, and absorb the scalar contribution $P_k\beta_k$ of the resulting equation in $\bar{\mathbf{G}}_k$.

$$\frac{\partial \mathcal{R}(\mathbf{G}_k)}{\partial \mathbf{G}_k^*} = 0$$

$$\begin{aligned} -u_k \tfrac{1}{P_k}\bar{\mathbf{G}}_k \mathrm{Tr}\{\mathbf{E}_k \bar{\mathbf{G}}_k^H \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\bar{\mathbf{G}}_k\} + u_k \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\bar{\mathbf{G}}_k\mathbf{E}_k \\ + \textstyle\sum_{l\neq k} u_l \tfrac{1}{P_k}\bar{\mathbf{G}}_k \mathrm{Tr}\{\mathbf{E}_l \bar{\mathbf{G}}_l^H \mathbf{H}_{ll}^H \mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\bar{\mathbf{G}}_k\bar{\mathbf{G}}_k^{\,H}\mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}^H\bar{\mathbf{G}}_l\} \\ - \textstyle\sum_{l\neq k} u_l \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\bar{\mathbf{G}}_l\mathbf{E}_l\bar{\mathbf{G}}_l\tilde{\mathbf{H}}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\bar{\mathbf{G}}_k = 0 \end{aligned} \tag{17}$$

In contrast to a MISO system, solving the above expression for $\bar{\mathbf{G}}_k$ is not straightforward for a general MIMO IFC. In a MISO system, extending [12] is simply, as it corresponds to fixing all scalar quantities involved in the expression thereby allowing us to find the the beamformer by iterating between the beamformer vectors and the fixed scalars. However, in moving from the MISO IFC to the MIMO IFC, the scalars now become matrices ($\mathbf{E}_k$ and $\mathbf{F}_k$) and hence a more structured reasoning is required. In particular what we propose as generalization of the approach proposed in [12] is essentially what we have described in the previous section (refer to [11] for more detail).

Using the riparametrization (16) of the BF in the expression of the WSR and WMMSE it is possible to show also in this unconstrained reformulation of the two optimization problems that choosing properly the weighting matrices (10) a stationary point for the weighted sum MMSE objective function is also a stationary point for the maximization of the WSR problem. This means that to determine the normalized optimal BF matrices we can solve directly the following unconstraint WSMSE minimization problem:

$$\arg\min_{\{\bar{\mathbf{G}}_k\}} \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k \boldsymbol{\mathcal{E}}_k\}.$$

Due to the normalization of the BF in the definition (11) of $\boldsymbol{\mathcal{E}}_k$ is no longer required the introduction of the scalar quantities $\alpha_k$ to compensate for the Tx power constraints. This reduces the number of variable to be optimized.

To determine the expression for the optimal TX filter we derive the optimization function reported above w.r.t. the normalized BF. After some standard steps, that are not reported here due to lack of space, it is possible to show that the expression for the optimal TX filter obtained using the power normalization is exactly the same as the one reported in (15). Thus, the extension of [12] to the MIMO IFC as well as the extension of [10] to the MIMO IFC yield exactly the same solution. Interestingly, it was observed that extending the approach in [12] to the MIMO BC leads to the same solution as that of [10] thus proving the optimality of integrating the [9] solution in the approach proposed in [10] (i.e., iterating between transmit filters and receive filters with corresponding weights). Indeed, it can be shown that the KKT condition $\mathbf{G}_k$ is satisfied when the solution for $\mathbf{G}_k$ and $\lambda_k$ are substituted thereby proving optimality of using the [9] approach both for the MIMO BC and MIMO IFC.

*C. Per-Stream WSR Optimization*

In the previous sections we have introduced an iterative algorithm that tries to determine the BF filters, that maximize the WSR, where all the transmitted streams per each user are treated jointly at the TX and RX side. This leads to a MMSE matrix expression that does not have any particular structure. As shown in [10] in a BC channel it is possible to optimize the BF matrices imposing a diagonal structure on the MMSE matrix, i.e. the streams per each user are assumed to be decoupled. The same reasoning can be extended to the MIMO IFC, the detailed description is not reported in this paper due to page limitation.

In a per stream approach the LMMSE RX filter treats all other streams (including from own user) as interference that needs to be suppressed. In this case the LMMSE RX expression for the $m-$th stream of the $k$-th user, and the interference plus noise covariance matrix are:

$$\mathbf{f}_{k,m} = \bar{\mathbf{g}}_{k,m}^H \mathbf{H}_{kk}^H [\mathbf{R}_{\overline{k,m}} + \mathbf{H}_{kk} \bar{\mathbf{g}}_{k,m} \bar{\mathbf{g}}_{k,m}^H \mathbf{H}_{kk}^H]^{-1} \quad (18)$$

$$\mathbf{R}_{\overline{k,m}} = \sum_{j \neq m}^{d_k} \mathbf{H}_{kk} \bar{\mathbf{g}}_{k,j} \bar{\mathbf{g}}_{k,j}^H \mathbf{H}_{kk}^H + \sum_{l \neq k} \sum_{j=1}^{d_l} \mathbf{H}_{kl} \bar{\mathbf{g}}_{l,j} \bar{\mathbf{g}}_{l,j}^H \mathbf{H}_{lk}^H + \mathbf{R}_{n_k n_k} \quad (19)$$

where $\bar{\mathbf{g}}_{l,j}$ represents the normalized BF vector for the $(l, j)$-th stream.

Because all the transmitters have full CSIT, they can pre-multiply their $d_k$ streams with a $d_k \times d_k$ unitary matrix such that using a unitary match filter (MF) RX the noise plus interference pre-whitened channel becomes diagonal. The unitary MF RX is the LMMSE filter described before.

To determine the optimal BF matrices we need to solve the following optimization problem:

$$\max_{\{\bar{\mathbf{g}}_{k,m}\}} \sum_{k=1}^{K} \sum_{m=1}^{d_k} \log(1 + \bar{\mathbf{g}}_{k,m}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{k,m}}^{-1} \mathbf{H}_{kk} \bar{\mathbf{g}}_{k,m}). \quad (20)$$

Working per stream helps us to extend the MISO optimization approach to the MIMO setting even preserving the optimality of the solution. In particular the optimization algorithm iterates between TX and RX filter vectors and scalar quantities. Now deriving the optimization cost function described above w.r.t. the beamformer vector it is possible to show that the expression for the optimal BF matrix $\bar{\mathbf{G}}_k$, once you build the compound BF matrix, in a per-stream approach is similar to (15). Thus the BF optimization in a per stream approach determines to an optimization algorithm in which the optimal solution is obtained iterating between two sets of quantities exactly as described before for the joint optimization, (c.f Table **Algorithm 1**). The difference is that the weighting matrix $\mathbf{W}_k$ is now diagonal and the $m$-th diagonal element is related to the minimum mean squared error for stream number $m$. In addition the $m$-th row of the RX matrix $\mathbf{F}_k$ is the per-stream receiver (18).

## IV. Greedy User Selection Algorithm

The user selection problem arises, for instance, in a multi-cell scenario, when each cell has multiple cell-edge users each experiencing interference from cell-edge users of neighboring cells. One possible solution is to first form *clusters* of cooperating cells. Each cluster now consists of a variable number of cooperating transmitters and a central controller. In this section we address this situation and propose a greedy approach to form such clusters so as to maximize the weighted sum rate in the resulting fully connected MIMO IFC. In the first instance, we assume that each cooperating transmitted services a single user in the downlink. Furthermore, we assume single stream transmission to each user.

Let $K$ be the total number of cells, a subset of which, will form a collaborating cluster. We denote the index set of these cells by $\mathcal{B} = \{1, \ldots, K\}$. Let $L_b$ denote the number of downlink users in each cell $b \in \{\mathcal{B}\}$. Each user $u_{b,l}$ is now identified by a cell index $b$ and user id $l$ (Of these, only one user will be selected for downlink transmission). We propose the following greedy algorithm to decide the cluster size $K$ and select users in each cooperating cell to maximize the weighted sum rate.

1) Initialization: $i = 1$
$$\mathcal{S}_U^{(i)} = \{\emptyset\}$$
$$\mathcal{B} = \{1, \ldots, K\}$$
$$\mathcal{U}_b = \{1, \ldots, L_b\}, \quad \forall b \in \mathcal{B}$$
2) Run the MWSR algorithm described in the previous section to determine the user that maximizes the WSR among all the cells $b \in \{\mathcal{B}\}$.
$$[b^\star, l^\star] = \arg\max_{\substack{b \in \mathcal{B} \\ l \in \mathcal{U}_b}} \text{MWSR}\{\mathcal{S}_U^{(i)}, u_{b,l}\}$$
3) The optimum user is selected as the $u^\star = u_{b^\star,l^\star}$. This user is now included in the set $\mathcal{S}_U$ and removed from the candidate set $\{\mathcal{B}\}$.
$$\mathcal{S}_U^{(i+1)} = \mathcal{S}_U^{(i)} \bigcup \{u_{b^*,l^*}\}$$
$$\mathcal{B} = \mathcal{B} - \{b^*\}$$
4) $i = i + 1$. Repeat steps 2 and 3 until $|\mathcal{S}_U^{(i)}| = K$ or
$$\text{WSR}\{\mathcal{S}_U^{(i)}\} < \text{WSR}\{\mathcal{S}_U^{(i-1)}\}$$

## V. Simulation Results

We provide here some simulation results to compare the performance of the proposed max-WSR algorithm. i.i.d Gaussian channels (direct and cross links) are generated for each user. For a fixed channel realization transmit and receiver filters are computed based on IA algorithm and max-WSR algorithm over multiple SNR points. The non convexity of the problem may lead the algorithm to converge to a stationary point that represents a local optimum instead of the global one which we are interested in. To increase the probability of reaching the optimum a common strategy in non convex problem is to to choose multiple random initial beamforming matrices and adopting the solution of the algorithm that determines the best WSR. Using these filters individual rates are computed. The resulting rate-sum is averaged over several hundred Monte-Carlo runs. The average rate-sum plots are used to compare the performance of the proposed algorithm.

In Fig. 2, we plot the results for a 3-user MIMO IFC. The antenna distribution at the receive and transmit side is $M_k = N_k = 2 \ \forall k$. The max-WSR algorithm results in a DoF allocation of $d_1 = 1 \ d_2 = 1 \ d_3 = 1$ with $u_k = 1 \ \forall k$ In Fig. 3, we plot the results for a 3-user MIMO IFC with $M_k = N_k = 3 \ \forall k$. The resulting DoF allocation is $d_1 = 2 \ d_2 = 1 \ d_3 = 1$ with $u_k = 1 \ \forall k$ Finally, Fig. 4 shows the convergence behavior of our algorithm for the same 3-user MIMO IFC with $M_k = N_k = 4 \ \forall k$ in a given SNR point, SNR=5dB

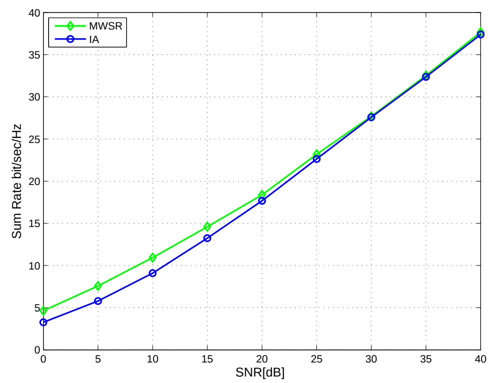


Fig. 2: 3-user MIMO IFC with $M_k = N_k = 2\ \forall k$.

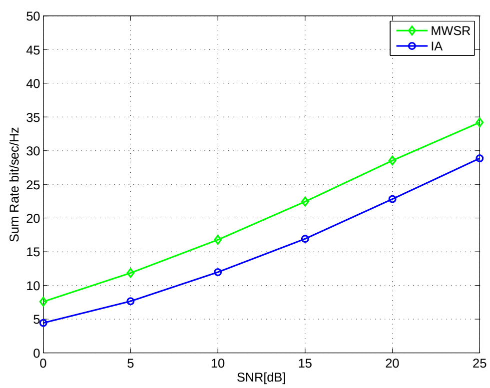


Fig. 3: 3-user MIMO IFC with $M_k = N_k = 3\ \forall k$

## VI. Conclusions

We addressed maximization of the weighted sum rate for the MIMO IFC. We introduced an iterative algorithm to solve this optimization problem. In the high-SNR regime, this algorithm leads to an optimized Interference Alignment (IA) solution In the finite SNR regime the performance of this algorithm is superior to that of IA and all known algorithms since it maximizes the WSR as opposed to previous attempts that maximize the sum rate. Convergence to a local optimum was also shown experimentally. Convergence to local optima is known and is related to the non-convexity of the MIMO IFC rate region. A greedy user selection algorithm based on the MWSR algorithm was proposed that selects a subset of transmitters that cooperate to maximize their weighted sum rate.

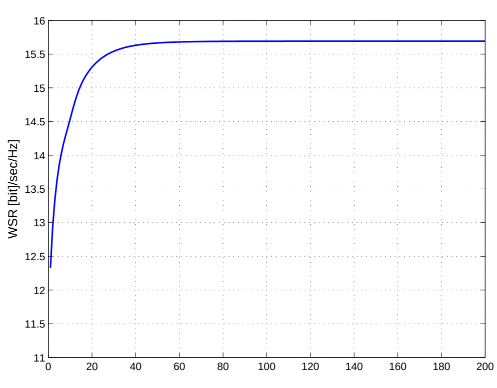


Fig. 4: Convergence behavior for a 3-user MIMO IFC with $M_k = N_k = 4\ \forall k$ at SNR=5dB

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