Maximum Weighted Sum Rate Multi-User MIMO Amplify-and-Forward for Two-Phase Two-Way Relaying

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***Abstract*—A base station (BS) transmits (Tx) and receives (Rx) signals to and from multiple mobile users (MU) through a two-way amplify and forward (AF) relay station (RS) using a two-phase protocol. The BS and the RS are both equipped with multiple antennas. In the first phase (time or frequency), the BS and all MU transmit their signals to the RS. In the second phase, the relay transmits towards the BS and MU a transformed signal in a broadcast (BC) fashion. We present a Weighted Sum Rate (WSR) maximizing approach. The optimization problem is similar as e.g., the multi-input multi-output (MIMO) BC where instead of alternating between Tx and Rx filters, one now alternates between Tx filters at BS and RS. Furthermore, we show that in the two-phase relaying considered here, there is not only a rate region for the MU in the downlink, but the coupled optimization of the BS transmitter and the RS receiver/transmitter leads in fact to an uplink/downlink rate region. Different UL and DL rates in this region can be achieved through rate tradeoffs across individual users.**

I. Introduction

Relays have recently attracted great attention due to the potential improvement of system performance: system capacity [1], coverage area. For this reasons the Third Generation Partnership Project (3GPP) consortium, responsible for the standardization of the next generation mobile wireless communication system called Long Term Evolution (LTE) has decided to include relaying as a key technology in the enhanced release of LTE (LTE-Advanced) [2]. Research activity on relays has developed different strategies to enhance system performances. In this work we focus only on the Amplify and Forward (AF) technique, where the Relay Station (RS) transmits directly the received signal without any further signal elaboration. The advantage of this relaying strategy is the reduced complexity at the RS. The main drawback of this system is the noise amplification that makes this approach suboptimal. Multiple Input Multiple Output (MIMO) systems have demonstrated the possibility to enhance the performance of various mobile wireless communication standards, such as High-speed Packet Access (HSPA) and LTE. For this reason MIMO is considered an unavoidable technique also in relaying systems.

When a relay station (RS) operates in half duplex mode, since the RS cannot transmit and receive a signal simultaneously, the resources required to separate the signals are doubled compared to conventional direct communication and spectral efficiency is deteriorated. This is a problem for relay systems. Two-way relays overcome this problem by scheduling all nodes together because they can exchange two messages of two source nodes in less than four time slots whereas it takes four time slots for conventional one-way relay protocols in Time Division Duplex (TDD) mode.

In [3] the authors propose an algorithm to determine the Beamformer at the RS and at each node for a Two-Way relay channel where there are two MIMO transmitting nodes and a MIMO relay station. They use an iterative algorithm that alternately optimize the relay beamformer and the receiver matrix at each nodes via a Weighted Minimum Mean Square Error (WMMSE) assuming a fixed transmitter beamformer for the two nodes. In a second stage they optimize the beamformer for the two nodes according to a generalized waterfilling technique adapted to relay systems assuming fixed the relay beamformer.

The relay system that we study in this paper is an AF two-way relay where there is a set of nodes that want to communicate with each other using a MIMO RS. The communication protocol is divided in two phases. During the first phase all the nodes transmit their messages to the RS at the same time, then the RS will broadcast the received signal to all the nodes in the second phase.

The base station (BS) having multiple antennas and a set of K Mobile Users (MS) that want to exchange message with each other using a MIMO Relay Station (RS). In particular we address the problem of designing the beamformer applied to the RS and the BS in order to maximize the Weighted Sum Rate (WSR). The algorithm that we propose tries to optimize alternately the beamformers at RS and at the BS and the receiver filters applied at MUs and BS. The beamformer at the RS is designed maximizing the total Weighted Sum Rate (TWSR), that includes the downlink (DL) and the uplink (UL) rate while the transmitter filter at the BS is determined maximizing the DL WSR. This iterative algorithm extends the idea described in [4] for a broadcast (BC) channel and in [5] for the MIMO interference channel to a two-way MIMO relay channel. The difference between our algorithm and the one proposed in [3] is that in our case the two beamformers are optimized jointly while in the other one the two transmitter matrices are optimized one after the other neglecting the dependency between the two.

It may be observed that in the two-way two-phase problem discussed here, the UL problem could be seen as a multiple access (MAC) channel irrespective of the two-phase transmission protocol. Indeed if the BS suppresses its own (known) contribution from the Rx signal, the resulting signal is as seen through a MAC. The DL is a different matter (not comparable with a BC) where, due to the AF RS, even after self interference suppression, all MU see other users' signal along with the BS signal. Thus in the two-phase relaying considered here, there is not only a rate region for the MU in the downlink, but the coupled optimization of the BS Tx and the RS Tx leads in fact to a jointly described UL/DL rate region. In the approach considered here, restricted to AF and linear Tx beamforming, the coupled design of the BS and RS Tx filters allows to trade off the UL and DL rates. More precisely, the precise uplink/downlink rate trade off needs to consider the uplink and downlink rates for all users separately. In the K user case, a $2K$ dimensional trade-off is faced. While not further pursued in this paper, such tradeoff amongst UL and DL and amongst users provides an opportunity to cognitively balance users rates in systems where some users may require relaying more than others as is typically the case in relay-assisted cellular networks.

The scope of the problem can be broadened to investigate more practical settings: MU served from and directly talking to BS and relay serving users with poor coverage from BS. Independent designs of BS Tx and RS Tx are of relevance in such an arena where the two optimize different optimization problems. The RS can therefore *cognitively* leverage rate tradeoff previously mentioned to serve the needier users. These cognitive aspects though of great interest will be touched upon in this paper only briefly due to space limitations. We shall nevertheless mention that various rate points in this UL/DL rate region are attainable for example extreme ones where the RS Tx only caters for downlink BF design, leaving the uplink to live with whatever results from such design. The paper therefore mainly focus on the on the BS/RS design for WSR optimization.

II. Signal Model

Fig. 1 depicts the communication scenario that we consider in this paper. In the multi-user MIMO two-way relaying system there is a Base Station (BS) that wants to communicate to the K single antenna Mobile Stations (MS) through the Relay (RS) and vice versa. We assume that all the channels are flat fading and all the nodes have full channel state information (CSIT). Transmission is divided in two phases with equal time duration. Because MSs and BS are transmitting and receiving at the same time there is no direct link between them.

In the first transmission phase BS and all MSs transmit at the same time their messages to the RS.

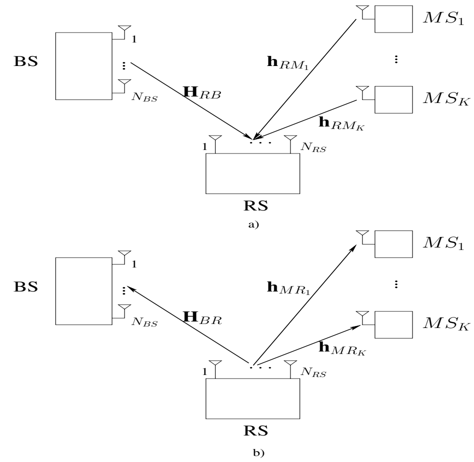

Fig. 1: Two-Way Relay System: a) Phase 1, b) Phase 2

The received signal at RS can be written as:

$$\begin{aligned}\mathbf{y}_{RS} &= \mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS} + \sum_{k=1}^{K}\mathbf{h}_{RM_k}s_{MS_k} + \mathbf{n}_{RS} \\ &= \mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS} + \mathbf{H}_{RM}\mathbf{s}_{MS} + \mathbf{n}_{RS} \qquad (1)\end{aligned}$$

where $\mathbf{H}_{RB}$ is the $N_{RS}\times N_{BS}$ matrix that contains the channel coefficients between the BS and the RS, $\mathbf{G}_{RB}$ is the beamformer that is applied to the BS and has dimension $N_{BS}\times K$. The $N_{RS}\times 1$ channel vectors between the k-th MS and the RS are denoted by $\mathbf{h}_{RM_k}$ and they are stacked in the $N_{RS}\times K$ matrix $\mathbf{H}_{RM}$. Vectors $\mathbf{s}_{BS}$ and $\mathbf{s}_{MS}$ represent the transmitted symbols from the BS and the MSs respectively with zero mean and correlation matrix given by $\mathbf{Q}_{BS} = \mathbb{E}\{\mathbf{s}_{BS}\mathbf{s}_{BS}^H\} = \mathbf{I}$ and $\mathbf{Q}_{MS} = \mathbb{E}\{\mathbf{s}_{MS}\mathbf{s}_{MS}^H\} = diag\{P_{MS_1},\ldots,P_{MS_K}\}$, where P_{MS_i} is the TX power at MS number i . We denoted by $\mathbf{n}_{RS}$ the vector that contains the $N_{RS}\times 1$ white noise samples with covariance matrix $\mathbf{R}_{n_{RS}} = \sigma_{RS}^2\mathbf{I}$.

In the second phase the relay will broadcast the received signal in the previous time slot to all terminals. The transmitted signal vector passes through a beamformer $\mathbf{G}_{RS}$ that has dimension $N_{RS}\times N_{RS}$, $\mathbf{x}_{RS} = \mathbf{G}_{RS}(\mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS}+\mathbf{H}_{RM}\mathbf{s}_{MS}+\mathbf{n}_{RS})$.

The received signal at the k-th MS is written as:

$$\begin{aligned}y_{MS_k} = \;& \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS} + \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{H}_{RM}\mathbf{s}_{MS} \\ & + \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{n}_{RS} + n_{MS_k} \qquad (2)\end{aligned}$$

where $\mathbf{h}_{MR_k}$ represents the channel coefficients between the RS and the k-th MS, n_{MS_k} is the zero mean noise sample with variance $\sigma_{MS_k}^2$. Analogously the received signal vector at the BS is:

$$\begin{aligned}\mathbf{y}_{BS} = \;& \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS} + \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{H}_{RM}\mathbf{s}_{MS} \\ & + \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{n}_{RS} + \mathbf{n}_{BS} \qquad (3)\end{aligned}$$

$\mathbf{H}_{BR}$ is the channel matrix from the RS to the BS and the zero mean white noise, with covariance matrix $\mathbf{R}_{BS} = \sigma_{BS}^2\mathbf{I}$, is given by $\mathbf{n}_{n_{BS}}$.

The MSs and the BS know their own transmitted signal so they can subtract it out from the received signal. The signal at the k-th MS, after the self-interference cancellation, can be written as:

$$r_{MS_k} = y_{MS_k} - \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{h}_{RM_k}s_{MS_k}$$
$$= \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{H}_{RB}\mathbf{g}_{BS_k}s_{BS_k} + \sum_{l\neq k}\mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{H}_{RB}\mathbf{g}_{BS_l}s_{BS_l}$$
$$+\sum_{l\neq k}\mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{h}_{RM_l}s_{MS_l} + \mathbf{h}_{MR_k}\mathbf{G}_{RS}\mathbf{n}_{RS} + n_{MS_k} \quad (4)$$

where $\mathbf{g}_{BS_k}$ is the k-th column of the beamformer at the BS. At the BS we have:

$$\begin{aligned}\mathbf{r}_{BS} = & \ \mathbf{y}_{BS} - \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{s}_{BS} \\ = & \ \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{H}_{RM}\mathbf{s}_{MS} + \mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{n}_{RS} + \mathbf{n}_{BS_k}\end{aligned} \quad (5)$$

III. Beamformers Optimization

In this section we derive an iterative algorithm for the optimization of the beamformers applied at the RS and at the BS. The objective function that we use for the optimization of the precoder of the RS is the Total Weighted Sum Rate (TWSR) (equation (8)) that incorporates the downlink (DL) WSR (from BS to MUs) and the uplink (UL) weighted rate (from MUs to BS). For the optimization of the beamformer at the BS we maximize only the DL WSR.

We consider this two cost functions because the introduction of some weights $\{u_i\}_{i=1}^K$, in the DL rate, and u_{BS}, in the UL rate, allows us to cover all the rate tuples on the rate region boundary. This optimization should take into account the fact that there is always a power constraint at the transmitter. In order to commute this constrained optimization problem into an unconstrained one it turns out to be useful (refers to [4],[5] for more details) to normalize the beamformers in order to satisfy always the power constraints with equality:

$$\mathbf{G}_{BS} = \frac{\sqrt{P_{BS}}}{\sqrt{Tr\{\tilde{\mathbf{G}}_{BS}\tilde{\mathbf{G}}_{BS}^H\}}}\tilde{\mathbf{G}}_{BS} = \beta_{BS}\tilde{\mathbf{G}}_{BS} \quad (6)$$

where P_{BS} is the maximum transmit power at the BS. For the RS we can write:

$$\mathbf{G}_{RS} = \frac{\sqrt{P_{RS}}}{\sqrt{Tr\{\tilde{\mathbf{G}}_{RS}\mathbf{Q}_{RS}\tilde{\mathbf{G}}_{RS}^H\}}}\tilde{\mathbf{G}}_{RS} = \beta_{RS}\tilde{\mathbf{G}}_{RS} \quad (7)$$

P_{RS} is the maximum transmit power at the RS and $\mathbf{Q}_{RS} = \mathbf{H}_{RB}\mathbf{G}_{BS}\mathbf{G}_{BS}^H\mathbf{H}_{RB}^H + \mathbf{H}_{RM}\mathbf{Q}_{MS}\mathbf{H}_{RM}^H + \mathbf{R}_{n_{RS}}$ is the covariance matrix of the received signal at the RS (1).

A. RS Precoder Design

First we show how to find the expression for the precoder used at the RS, $\mathbf{G}_{RS}$, considering the beamformer at the BS $\mathbf{G}_{BS}$ fixed. To determine the optimal BF matrix $\mathbf{G}_{RS}$ we derive the TWSR, given in (8), w.r.t. the beamformer at the RS. To make the analysis more manageable we will decompose the total derivative as the sum of two terms using the linearity property of the derivative.

$$\frac{\partial\mathcal{R}_{tot}}{\partial\tilde{\mathbf{G}}_{RS}^*} = \frac{\partial\mathcal{R}_{DL}}{\partial\tilde{\mathbf{G}}_{RS}^*} + \frac{\partial\mathcal{R}_{UL}}{\partial\tilde{\mathbf{G}}_{RS}^*} = 0 \quad (9)$$

We first derive the DL WSR, the first term in the sum in (8), w.r.t. $\tilde{\mathbf{G}}_{RS}$, this derivative can be written as:

$$\frac{\partial\mathcal{R}_{DL}}{\partial\tilde{\mathbf{G}}_{RS}^*} = \sum_{i=1}^K u_i[\frac{1}{den_i}\mathbf{h}_{MR_i}^H\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RB}\mathbf{g}_{BS_i}\mathbf{g}_{BS_i}^H\mathbf{H}_{RB}^H +$$
$$\frac{num_i}{den_i(num_i+den_i)}\mathbf{h}_{MR_i}^H\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{h}_{RM_i}p_{MS_i}\mathbf{h}_{RM_i}^H -$$
$$\frac{num_i}{den_i(num_i+den_i)}[\mathbf{h}_{MR_i}^H\mathbf{h}_{MR_i} + \frac{\sigma_{MS}^2}{P_{RS}}]\tilde{\mathbf{G}}_{RS}\mathbf{Q}_{RS}] \quad (10)$$

where num_i and den_i represent respectively the numerator and the denumerator of the argument of the logarithm in the rate expression. Deriving the UL rate, the second term in (8), we obtain:

$$\frac{\partial\mathcal{R}_{UL}}{\partial\tilde{\mathbf{G}}_{RS}^*} = u_{BS}[\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\mathbf{W}_{BS}^{-1}\mathbf{H}_{RM}^H -$$
$$\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\mathbf{W}_{BS}^{-1}\mathbf{H}_{RM}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{R}_{n_{rs}} -$$
$$Tr\{\mathbf{W}_{BS}^{-1}\mathbf{H}_{RM}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\mathbf{R}_{n_{BS}}\mathbf{R}_{\bar{BS}}^{-1}\mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\}\frac{\tilde{\mathbf{G}}_{RS}}{P_{RS}}\mathbf{Q}_{RS}] \quad (11)$$

where:

$$\mathbf{W}_{BS} = (\mathbf{Q}_{MS}^{1/2}\mathbf{E}\mathbf{Q}_{MS}^{H/2})^{-1}$$
$$\mathbf{E} = (\mathbf{I} + \mathbf{Q}_{MS}^{H/2}\mathbf{H}_{RM}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\mathbf{Q}_{MS}^{1/2})^{-1} \quad (12)$$

As we can see all terms in (9) depend on the optimization variable, this means that in order to solve the expression in (9) w.r.t. the precoder at the RS we need to identify some quantities in (10) and (11) that should be taken fixed to obtain a linear equation in the unknown. In (10) it is simple because we can assume fixed all the scalar quantities but in (11) is more complex due to the fact that matrices there are involved. To simplify this analysis we can introduce an MMSE receiver filter at each MU and at the BS. The scalar MMSE receiver filter applied at the i-th MU is given by

$$f_{MS_i} = \beta_{RS}^{-1}\mathbf{g}_{BS_i}^H\mathbf{H}_{RB}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H(\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{Q}_{RS}\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H -$$
$$\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{h}_{RM_i}p_{MS_i}\mathbf{h}_{RM_i}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H + \beta_{RS}^{-2}\sigma_{n_{MS}}^2)^{-1} \quad (13)$$

The MMSE receiver filter matrix at the BS can be written as:

$$\mathbf{F}_{BS} = \beta_{RS}^{-1}\mathbf{Q}_{MS}^{1/2}\mathbf{E}\mathbf{Q}_{MS}^{H/2}\mathbf{H}_{RM}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1} \quad (14)$$

Introducing the receivers in expression (10) and (11) we obtain:

$$\frac{\partial\mathcal{R}_{tot}}{\partial\tilde{\mathbf{G}}_{RS}^*} = \sum_{i=1}^K u_i e_i f_{MS_i}^H\mathbf{h}_{MR_i}^H\mathbf{g}_{BS_i}^H\mathbf{H}_{RB}^H +$$
$$\sum_{i=1}^K u_i e_i f_{MS_i}^H f_{MS_i} w_i p_{MS_i}\mathbf{h}_{MR_i}^H\mathbf{h}_{RM_i}^H -$$
$$\sum_{i=1}^K u_i e_i f_{MS_i}^H f_{MS_i}[\mathbf{h}_{MR_i}^H\mathbf{h}_{MR_i} + \frac{\sigma_{MS}^2}{P_{RS}}]\mathbf{G}_{RS}\mathbf{Q}_{RS}$$
$$u_{BS}\{\mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{H}_{RM}^H - \mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{W}_{BS}\mathbf{F}_{BS}\mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{R}_{n_{RS}}$$
$$-\frac{1}{P_{RS}}Tr\{\mathbf{F}_{BS}\mathbf{R}_{n_{BS}}\mathbf{F}_{BS}^H\mathbf{W}_{BS}\}\mathbf{G}_{RS}\mathbf{Q}_{RS}\} = 0 \quad (15)$$

where $e_i = \frac{num_i+den_i}{den_i}$ and $w_i = \mathbf{h}_{MR_i}\mathbf{G}_{RS}\mathbf{h}_{RM_i}$. In the equation reported above we have absorbed the normalization

$$\mathcal{R}_{tot} = \sum_{i=1}^{K} u_i \log(1 + \frac{\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RB}\mathbf{g}_{BS_i}\mathbf{g}_{BS_i}^H\mathbf{H}_{RB}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H}{\mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}[\mathbf{Q}_{RS} - \mathbf{h}_{RM_i}p_{MS_i}\mathbf{h}_{RM_i}^H]\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H - \mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RB}\mathbf{g}_{BS_i}\mathbf{g}_{BS_i}^H\mathbf{H}_{RB}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{h}_{MR_i}^H + \beta_{RS}^{-2}\sigma_{n_{MS}}^2}) + u_{BS}\log(\det(\mathbf{I} + \mathbf{Q}_{MS}^{H/2}\mathbf{H}_{RM}^H\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H\mathbf{R}_{\bar{BS}}^{-1}\mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\mathbf{Q}_{MS}^{1/2})) \quad (8)$$

$$\mathbf{R}_{\bar{BS}} = \mathbf{H}_{BR}\tilde{\mathbf{G}}_{RS}\mathbf{R}_{n_{RS}}\tilde{\mathbf{G}}_{RS}^H\mathbf{H}_{BR}^H + \beta_{RS}^{-2}\mathbf{R}_{n_{BS}}$$

factor β_{RS} in the BF $\mathbf{G}_{RS}$. To simplify the expression of the gradient it is useful to group some quantities, in particular we define:

$$\begin{aligned}
\mathbf{\Delta}_{MS_1} &= \sum_{i=1}^{K} u_i e_i f_{MS_i}^H \mathbf{h}_{MR_i}^H \mathbf{g}_{BS_i}^H \mathbf{H}_{RB}^H \\
\mathbf{\Delta}_{MS_2} &= \sum_{i=1}^{K} u_i e_i f_{MS_i}^H f_{MS_i} w_i p_{MS_i} \mathbf{h}_{MR_i}^H \mathbf{h}_{RM_i}^H \\
\mathbf{\Delta}_{MS_3} &= \sum_{i=1}^{K} u_i e_i f_{MS_i}^H f_{MS_i} [\mathbf{h}_{MR_i}^H \mathbf{h}_{MR_i} + \frac{\sigma_{MS}^2}{P_{RS}}]
\end{aligned} \quad (16)$$

With this matrices we can rewrite (15) as:

$$\begin{aligned}
\frac{\partial \mathcal{R}_{tot}}{\partial \tilde{\mathbf{G}}_{RS}^*} = & \mathbf{\Delta}_{MS_1} + \mathbf{\Delta}_{MS_2} + u_{BS}\mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{H}_{RM}^H \\
& - \mathbf{\Delta}_{MS_3}\mathbf{G}_{RS}\mathbf{Q}_{RS} - u_{BS}\mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{W}_{BS}\mathbf{F}_{BS}\mathbf{H}_{BR}\mathbf{G}_{RS}\mathbf{R}_{n_{RS}} \\
& - u_{BS}Tr\{\mathbf{F}_{BS}\mathbf{R}_{n_{BS}}\mathbf{F}_{BS}^H\mathbf{W}_{BS}\}\mathbf{G}_{RS}\mathbf{Q}_{RS} = 0
\end{aligned} \quad (17)$$

As we can see in the expression above there are some terms that do not depend on $\mathbf{G}_{RS}$ explicitly and other term that are linear in the unknown matrix but are post multiplied by some other matrices. To solve the equation reported above w.r.t. $\mathbf{G}_{RS}$ we can use the vectorization operation and the property: $\text{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$. In this way we obtain a linear system in the form $\mathbf{Ax} - \mathbf{b} = \mathbf{0}$ with traditional solution

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \quad (18)$$

In our case $\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$, where $\mathbf{A}_1 = \mathbf{Q}_{RS}^T \otimes [\mathbf{\Delta}_{MS_3} + u_{BS}Tr\{\mathbf{F}_{BS}\mathbf{R}_{n_{BS}}\mathbf{F}_{BS}^H\mathbf{W}_{BS}\}\mathbf{I}]$ and $\mathbf{A}_2 = \mathbf{R}_{n_{RS}}^T \otimes u_{BS}\mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{W}_{BS}\mathbf{F}_{BS}\mathbf{H}_{BR}\mathbf{G}_{RS}$. $\mathbf{b} = \text{vec}(\mathbf{\Delta}_{MS_1} + \mathbf{\Delta}_{MS_2} + u_{BS}\mathbf{H}_{BR}^H\mathbf{F}_{BS}^H\mathbf{H}_{RM}^H)$ and $\mathbf{x} = \text{vec}(\mathbf{G}_{RS})$

B. BS Precoder Design

In the following we describe how to determine the expression for the beamformer applied to the BS assuming fixed the precoder at the RS. This precoder is designed in order to maximize the WSR in DL so we derive the DL rate w.r.t. the i-th column of the beamformer $\mathbf{g}_{BS_i}$. For this purpose we rewrite the DL rate in order to explicit the dependency on the normalization factor β_{BS}. In particular the denominator of the DL rate, the first term in (8), becomes:

$$\begin{aligned}
&\textstyle\sum_{l\neq i} \beta_{BS}^2 \tilde{\mathbf{g}}_{BS_l}^H \mathbf{H}_{RB}^H \tilde{\mathbf{G}}_{RS}^H \mathbf{h}_{MR_i}^H \mathbf{h}_{MR_i} \tilde{\mathbf{G}}_{RS} \mathbf{H}_{RB} \tilde{\mathbf{g}}_{BS_l} + \\
&\textstyle\sum_{l\neq i} \mathbf{h}_{MR_i} \tilde{\mathbf{G}}_{RS} \mathbf{h}_{RM_l} p_{MS_l} \mathbf{h}_{RM_l}^H \tilde{\mathbf{G}}_{RS}^H \mathbf{h}_{MR_i}^H \\
&+ \mathbf{h}_{MR_i} \tilde{\mathbf{G}}_{RS} \mathbf{R}_{n_{RS}} \tilde{\mathbf{G}}_{RS}^H \mathbf{h}_{MR_i}^H + \beta_{BS}^{-2}\beta_{RS}^{-2}\sigma_{n_{MS}}^2
\end{aligned} \quad (19)$$

where we have written explicitly the components inside the matrix $\mathbf{Q}_{RS}$. We identify the cascade of $\mathbf{h}_{MB_i} = \mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RB}$ and $\mathbf{h}_{MM_{il}} = \mathbf{h}_{MR_i}\tilde{\mathbf{G}}_{RS}\mathbf{h}_{RM_l}$ as an two equivalent channels between the BS and the i-th MU and between the l-th MU and the i-th MU. As it has been done for the derivation of the beamformer at the RS in section III-A it is possible to introduce at each MS a scalar receiver f_{MS_i} $\forall i$ defined in (12). Using this definition the final expression for the derivative is:

$$\begin{aligned}
&\frac{\partial \mathcal{R}_{DL}}{\partial \tilde{\mathbf{G}}_{BS}^*} = u_i e_i f_{MS_i}^H \mathbf{h}_{MB_i}^H - \sum_{m=1}^{K} d_m \mathbf{h}_{MB_m}^H \mathbf{h}_{MB_m} \mathbf{g}_{BS_i} \\
&- \sum_{m=1}^{K} \frac{d_m}{P_{BS}} \sum_{l\neq m} \mathbf{h}_{MM_{ml}} p_{MS_l} \mathbf{h}_{MM_{ml}}^H \mathbf{g}_{BS_i} \\
&- \sum_{m=1}^{K} \frac{d_m}{P_{BS}} \mathbf{h}_{MR_m} \tilde{\mathbf{G}}_{RS} \mathbf{R}_{n_{RS}} \tilde{\mathbf{G}}_{RS}^H \mathbf{h}_{MR_m}^H \mathbf{g}_{BS_i} - \\
&\frac{\sigma_{MS}^2}{P_{RS}} \sum_{m=1}^{K} d_m \mathbf{H}_{RB}^H \tilde{\mathbf{G}}_{RS}^H \tilde{\mathbf{G}}_{RS} \mathbf{H}_{RB} \mathbf{g}_{BS_i} - \\
&\frac{\sigma_{MS}^2}{P_{BS}P_{RS}} \sum_{m=1}^{K} d_m Tr\{\tilde{\mathbf{G}}_{RS} \mathbf{H}_{RM} \mathbf{Q}_{MS} \mathbf{H}_{RM}^H \tilde{\mathbf{G}}_{RS}^H\} \mathbf{g}_{BS_i} - \\
&\frac{\sigma_{MS}^2}{P_{BS}P_{RS}} \sum_{m=1}^{K} d_m Tr\{\tilde{\mathbf{G}}_{RS} \mathbf{R}_{n_{RS}} \tilde{\mathbf{G}}_{RS}^H\} \mathbf{g}_{BS_i} = 0
\end{aligned} \quad (20)$$

where $d_m = u_m e_m f_{MS_m}^H f_{MS_m}$. From the previous equation we obtain the expression for the $i-$th column of the beamformer at the BS. Introducing the following quantities:

$$\mathbf{H}_{MB} = \begin{bmatrix} \mathbf{h}_{MB_1} \\ \vdots \\ \mathbf{h}_{MB_K} \end{bmatrix} \quad (21)$$

$$\begin{aligned}
\mathbf{\Delta}_{BS_1} &= diag\{u_1 e_1 f_{MS_1}^H, \ldots, u_K e_K f_{MS_K}^H\} \\
\mathbf{D}_{BS} &= diag\{d_1 \sigma_{MS_1}^2, \ldots, d_K \sigma_{MS_K}^2\} \\
\mathbf{\Delta}_{BS_2} &= \sum_{m=1}^{K} d_m \mathbf{h}_{MB_m}^H \mathbf{h}_{MB_m} \tilde{\mathbf{g}}_{BS_i}
\end{aligned} \quad (22)$$

we can determine the expression of the complete BF at the BS, equation (23) in the next page, solving w.r.t. $\mathbf{G}_{BS}$ equation (20). In Table (Algorithm 1) we briefly describe the iterative algorithm that we have derived in this paper to design the beamformers at RS and BS. The algorithm alternates between some quantities that are considered fixed, we assume that they depend on the values of the two beamformers calculated at the previous iteration, and the two beamformers $\mathbf{G}_{BS}$ and $\mathbf{G}_{RS}$. To simplify the algorithm description let us introduce

$$\mathbf{G}_{BS}=[\mathbf{\Delta}_{BS_2}+\sum_{m=1}^{K}\frac{d_m}{P_{BS}}\mathbf{h}_{MR_m}\tilde{\mathbf{G}}_{RS}\mathbf{R}_{n_{RS}}\tilde{\mathbf{G}}_{RS}^{H}\mathbf{h}_{MR_m}^{H}+\sum_{m=1}^{K}\frac{d_m}{P_{BS}}\sum_{l\neq m}\mathbf{h}_{MM_{ml}}p_{MS_l}\mathbf{h}_{MM_{ml}}^{H}+\mathbf{H}_{RB}^{H}\tilde{\mathbf{G}}_{RS}^{H}\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RB}$$
$$+\tfrac{1}{P_{BS}P_{RS}}Tr\{D_{BS}\}Tr\{\tilde{\mathbf{G}}_{RS}\mathbf{H}_{RM}\mathbf{Q}_{MS}\mathbf{H}_{RM}^{H}\tilde{\mathbf{G}}_{RS}^{H}\}+\tfrac{1}{P_{BS}P_{RS}}Tr\{D_{BS}\}Tr\{\tilde{\mathbf{G}}_{RS}\mathbf{R}_{n_{RS}}\tilde{\mathbf{G}}_{RS}^{H}\}]^{-1}\mathbf{H}_{MB}^{H}\mathbf{\Delta}_{BS_1} \quad (23)$$

the following sets:

$$\begin{aligned}\mathcal{F}_{MS}&=\{f_{MS_i}\}_{i=1}^{K}\\ \mathcal{E}_{MS}&=\{e_i\}_{i=1}^{K}\\ \mathcal{W}_{MS}&=\{w_i\}_{i=1}^{K}\\ \mathcal{D}_{BS}&=\{d_i\}_{i=1}^{K}\end{aligned} \quad (24)$$

Algorithm 1 Algorithm for RS and BS beamformer design

Fix an arbitrary initial set of precoding matrices $\mathbf{G}_{BS}^{(0)}$ and $\mathbf{G}_{RS}^{(0)}$
set $n=0$
repeat
  $n=n+1$
  Given $\mathbf{G}_{BS}^{(n-1)}$ and $\mathbf{G}_{RS}^{(n-1)}$, compute $\mathbf{F}_{BS}^{(n)}$ and $\mathbf{W}_{BS}^{(n)}$ from (14) and (12) respectively and $\mathcal{F}_{MS}^{(n)}$, $\mathcal{W}_{MS}^{(n)}$ and $\mathcal{E}_{MS}^{(n)}$ from (24)
  Given $\mathbf{F}_{BS}^{(n)}$, $\mathbf{W}_{BS}^{(n)}$, $\mathcal{F}_{MS}^{(n)}$, $\mathcal{W}_{MS}^{(n)}$ and $\mathcal{E}_{MS}^{(n)}$ compute $\mathbf{G}_{RS}^{(n)}$ from (18)
  Given $\mathbf{G}_{BS}^{(n-1)}$ and $\mathbf{G}_{RS}^{(n)}$ determine $\mathcal{F}_{MS}^{(n)}$, $\mathcal{E}_{MS}^{(n)}$ and $\mathcal{D}_{BS}^{(n)}$ from (24)
  Given $\mathcal{F}_{MS}^{(n)}$, $\mathcal{E}_{MS}^{(n)}$ and $\mathcal{D}_{BS}^{(n)}$ compute $\mathbf{G}_{BS}^{(n)}$ using (23)
until convergence

IV. Cognitive Relays

In the previous section we have derived an algorithm that allows us to calculate the transmit beamformers applied at the BS and RS in order to maximize the TWSR. It is possible to extend this strategy also to cognitive radio Two-way relay system. In this section we will describe some possible extension of our beamformer design approach to CR scenarios.

A. Coexistence with Primary System

The scenario that we study here is constituted by a Two-way CR relay that has the objective to allow the communication between a set of K Secondary Mobile Users (SMU) and a Secondary Base Station (SBS). In addition to the communication setting described previously we also consider that is present a single antenna Primary User (PU) nearby the secondary system that communicates with a MIMO primary BS. We assume that the primary system uses an FDD transmission strategy with two different subbads for UL and DL while the CR system opportunistically uses only the DL subband of the primary communication using a TDD duplexing scheme. The RS and SBS are equipped with a number of antennas that are greater then the number of SMU that they want to serve, $N_{SBS}, N_{RS} > K$. The level of cognition that we assume is such that the RS and the SBS are able to obtain the channel state information (CSI) on the channels from SBS and PU and from RS and PU. This two nodes will use this pieces of information to design their beamformers in order to cause zero interference to the PU and at the same time to maximize to WSR in the secondary network. Further more we assume that the interference caused by SMUs transmission to the PU is negligible. The fact that the cognitive RS and SBS are fitted with a number of antennas greater than the number of SMU allows us to use the additional degrees of freedom to cancel out the interference towards the PU. It is possible to parametrize each beamformer as the product of two beamformers $\mathbf{G}=\bar{\mathbf{G}}\hat{\mathbf{G}}$.The first one is designed in order to zero force the interference to the PU. At the SBS we have:

$$\mathbf{h}_{PB}\bar{\mathbf{G}}_{BS}=0\Rightarrow\bar{\mathbf{G}}_{BS}=(\mathbf{h}_{PB})^{\perp} \quad (25)$$

where $\mathbf{h}_{PB}$ represents the channel from SBS to PU. In order to keep the power requirements unchanged at the transmitter side we can choose that the ZF beamformer is unitary $\bar{\mathbf{G}}^{H}\bar{\mathbf{G}}=\mathbf{I}$. Analogously we can determine to ZF beamformer at the RS to orthogonalize the signal towards the PU:

$$\mathbf{h}_{PR}\bar{\mathbf{G}}_{RS}=0\Rightarrow\bar{\mathbf{G}}_{RS}=(\mathbf{h}_{PR})^{\perp} \quad (26)$$

Once we have the ZF beamformers it is possible to apply the algorithm determined in the previous section to find out the second set beamformers to optimize the secondary communication. The only necessary modification is to introduce different channels in the system model drew in section II. In particular the channel between SBS and RS in the UL becomes $\bar{\mathbf{H}}_{RB}=\mathbf{H}_{RB}\bar{\mathbf{G}}_{BS}$, also the channel from RS to MUs in the DL should be modified to take into account the ZF beamformer at RS: $\bar{\mathbf{H}}_{MR}=\mathbf{H}_{MR}\bar{\mathbf{G}}_{RS}$.

B. Cognitive Design of BS and RS Beamformers

Here we consider a system in which a MIMO BS wants to transmit to a set of MUs but only some of them can be served directly by the BS. To communicate to the MUs that can not be reached the BS uses a amplify and forward MIMO RS. In this case we can assume to design the beamformer independently at the BS and at the RS using two different cost functions. In particular for the BS we can use the DL WSR in which some users are served passing through the RS. For the precoder optimization at the RS we only maximize the SR of the users that are served by the RS. We have to introduce in this setting two different sets of weights, a set $\{u_i\}$ for the DL WSR, at BS, and one $\{\nu_i\}$ for the TWSR at RS. We have an additional degree of freedom because with different weight settings for the rate profile, it is possible to modify rate distribution among the users. In particular we can choose to serve only a subset of the total number of users or we can give higher rate only to some users that are characterized by some particular proprieties. The level of cognition in this case is given, apart the CSI acquisition, in the possibility of changing the points attained in the rate region varying the weights. A possible extension of this scenario is the coexistence of two neighboring cells, composed by a set of MU and a BS, that want to share the same RS to reach users that can not be served directly by the BS. In this situation using two different cost functions for the BS and RS precoder optimization is expected because the objective of the two communication strategy can be very different.

V. Numerical Results

In this section we present some numerical results to evaluate the performances of the proposed iterative algorithm in term of rate. In our simulations we consider that the noise level at all terminals, BS, MUs and Rs are the same for all of them. In Fig. 2 we analyze the performances of the proposed algorithm when the optimization of the two BF is done jointly (dotted lines) or in a separate manner (solid lines). We report the DL WSR (rate at the MUs), the UL rate (rate at BS) and the TWSR (the sum of the two). With joint optimization

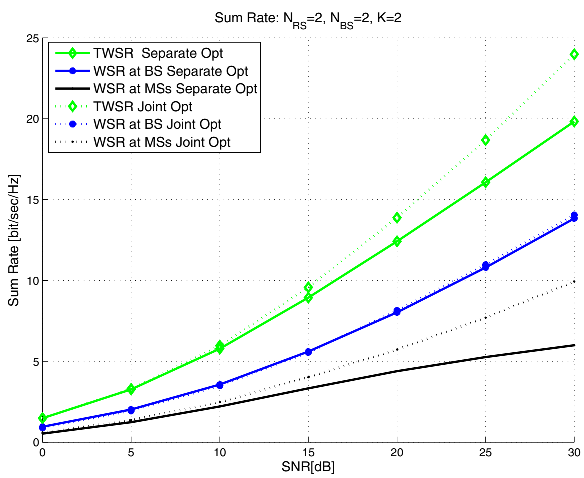

Fig. 2: Comparison of joint optimization against separate optimization: $K = 2$ and $N_{RS} = N_{BS} = 2$

we mean that the two BF are determined one after the other in the same algorithm iteration loop, as described in Table (1). In the other case we first determine the optimal BF at RS considering fixed the BF at BS and once the algorithm has converged we ascertain the optimal BF at BS using the optimal RS's BF. In the system that we consider there are $K = 2$ single antenna MUs, both BS and RS are equipped with the same number of antennas $N_{RS} = N_{BS} = 2$. As we can see the joint optimization determine a significant increase of performances, in particular for the DL rate. This is due to the fact that the DL rate is maximized by both BFs and hence influenced more by the joint optimization. Fig. 3 depicts the convergence behaviour of the algorithm. We report the value of the TWSR as function of the number of algorithm iterations. As we can see the proposed algorithm converges in less than 20 iterations.

VI. Conclusions

This paper presented a multi-user MIMO two-way relay protocol which can exchange the messages between BS and

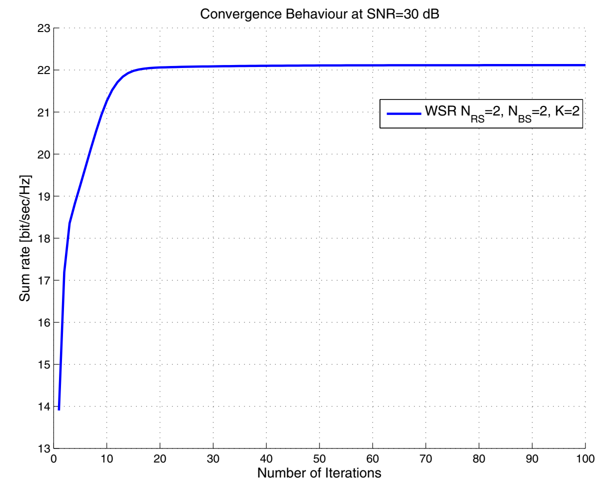

Fig. 3: Convergence behaviour: $K = 2$ and $N_{RS} = N_{BS} = 2$

multiple MU within only two time slots.

In particular we derive an iterative algorithm to designing the Beamformer applied to the RS and the BS that has as objective function to maximize the Weighted Sum Rate (WSR). At each iteration this algorithm alternates between some quantities that are assumed fixed, they depend on the value of the beamformer found in the previous iteration, and the two beamformers. Furthermore, we show that in the two-phase relaying considered here, there is not only a rate region for the MU in the downlink, but the coupled optimization of the BS transmitter and the RS receiver/transmitter leads in fact to an uplink/downlink rate region. Different UL and DL points in this region can be achieved through possibly cognitive rate tradeoffs across individual users.

VII. Acknowledgment

EURECOM's research is partially supported by its industrial members: BMW Group Research & Technology, Swisscom, Cisco, ORANGE, SFR, Sharp, ST Ericsson, Thales, Symantec, Monaco Telecom and by the EU FP7 NoE Newcom++.

The research of EURECOM and Infineon Technologies France is also supported in part by the EU FP7 Future and Emerging Technologies (FET) project CROWN.

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