Performance Analysis of Low Complexity Soft Detection for BICM MIMO System

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***Abstract*—We consider in this paper high spectral efficiency bit interleaved coded modulation (BICM) MIMO system where, after serial to parallel conversion and per antenna coding, spatial data streams are simultaneously transmitted by using an antenna array. We carry out the performance analysis of an earlier proposed detection algorithm for such system which is based on the combination of linear and non linear detection techniques. This algorithm employs linear MMSE filters to detect the streams which have seen good channel realizations and therefore enjoy higher signal-to-noise-ratios (SNRs). In the performance analysis, we show that the detection of these streams suffer from the loss of diversity order. These streams, after being detected, are subsequently stripped off leading to the max log MAP detection of the streams which have seen comparatively poor realizations of the channel and consequently have lower SNRs. We further show that these streams are benefited from the additional diversity order in the detection. Key idea is therefore to lower the detection complexity at the cost of reduced diversity order for the streams benefiting from good channel realizations and to enhance diversity order at the cost of increased detection complexity for the streams suffering from poor channel realizations.**

***Index Terms*—MIMO, BICM, Max log MAP detector, MMSE detector, Pair wise error probability analysis, Low complexity detection**

I. Introduction

The seminal works in [1] and [2] on multiple antenna elements at the transmitter and receiver show a huge increase in the throughput of this point-to-point channel referred to also as multiple input multiple output (MIMO) system. These promising results motivated the introduction of spatial dimension in future wireless systems as IEEE 802.11n (Wireless LAN) [3], IEEE 802.16m (WiMax) [4] and third generation partnership project long term evolution 3GPP LTE [5]. Enhanced capacity (degrees of freedom) and improved reliability (diversity) are the two significant gains of MIMO communications which can not be realized simultaneously and need to be traded off [6].

Diversity increases the robustness of the system by eliminating the fades; channel-aware diversity raises the average received SNR with the increase in the number of antennas while if the channel is not known, then the diversity hardens the SNR to the mean SNR [7]. Even if the average SNR increases linearly with the number of antennas, the capacity growth is logarithmic, easily verified with Shannons formula $C = B\log_2(1 + \text{SNR})$. Spatial multiplexing, on the other hand, divides the incoming data into multiple parallel substreams and transmits each on a different spatial dimension (e.g. a different antenna). As long as there are at least as many (sufficiently spaced) receive antennas as transmitted streams, spatial multiplexing increases the capacity linearly with the number of streams. The maximum likelihood (ML) detector for the spatially multiplexed MIMO system is characterized by high computational complexity that increases exponentially with the product of the number of transmit antennas and the number of bits per modulation symbol, making it prohibitive, in practice. A significant complexity reduction coupled with a performance degradation can be obtained by employing linear detectors in the place of ML detector especially in uncoded systems. However coding gains can be obtained if the capacity achieving temporal encoders, such as turbo or Low-Density Parity Check (LDPC) code are used in concatenation with spatial multiplexing [8].

Our focus in this paper is on the scenario of coded spatial streams where spatial dimension is used to achieve multiplexing gain while coding is used to exploit the temporal diversity of the channel. For decoding in such a system, it is important to obtain reliability information (soft decisions) for each bit. Direct computation of such soft decisions (log-likelihood ratios - LLRs) involves the summation of a number of terms that grows exponentially with the dimension of and polynomially in the size of the signal constellation. In many cases of practical interest, one resorts to the approximation of replacing the sums with the largest term commonly termed as max log MAP approximation. The problem of computing good soft decisions is fundamentally much more important for slowly fading channels (i.e. when a codeword spans only one realization of the channel) than in fast fading (where a codeword spans many realizations of the channel). In slow fading channels, different spatial streams experience different SNRs as each stream sees a different realization of the channel and the error performance of the system is dominated by the spatial streams with the relatively lower SNRs. Key idea is to lower the detection complexity at the cost of reduced diversity order for the streams benefiting from good channel realizations and to enhance diversity order at the cost of increased detection complexity for the streams suffering from poor channel realizations. The detection algorithm [9] incorporates low complexity linear detectors, characterized by lower diversity order, for the detection of streams which have seen good channel realizations and consequently have higher SNRs. These streams after detection are subsequently stripped off thereby reducing the dimensionality of the system. It is followed by the max log MAP detection, characterized by full diversity order, of the remaining streams which have seen poor channel realizations and therefore have lower SNRs. The algorithm therefore successfully escapes the exponential complexity of MIMO detection and the degraded performance of linear receivers.

Regarding notations, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors and uppercase boldface letters for matrices. The matrix $\mathbf{I}_n$ is the $n \times n$ identity matrix while $\text{vec}(\mathbf{A})$ denotes the vectorization operator which stacks the columns of $\mathbf{A}$. $|.|$ and $\|.\|$ indicate norm of scalar and vector while $(.)^T$ and $(.)^\dagger$ indicate transpose and conjugate transpose respectively. The notation $E(.)$ denotes the mathematical expectation while $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx$ denotes the Gaussian Q-function. The paper is organized as follows. In section II we provide the system model while section III elaborates the low complexity detection algorithm accompanied by its diversity analysis. Section IV describes simulation results which is followed by the conclusions.

II. System Model

We consider a MIMO system which is a $n_t \times n_r$ ($n_r \geq n_t - 1$) BICM MIMO system with n_t spatial streams. x_j is the symbol of j-th stream over a signal set χ_j. During the transmission of j-th stream, code sequence $\mathbf{c}_j$ is interleaved by π_j and then is mapped onto the signal sequence. Bit interleaver for j-th stream can be modeled as $\pi_j : k' \rightarrow (k, i)$ where k' denotes the original ordering of the coded bits $c_{k'}$ of j-th stream, k denotes the time ordering of the signal x_j and i indicates the position of the bit $c_{k'}$ in the symbol x_j. We consider frequency flat slow fading channel and assume that the channel remains constant for the transmission of a frame. Well known baseband model of the system at the k-th channel use is given as:-

$$\begin{aligned} \mathbf{y}_k &= \mathbf{h}_1 x_{1,k} + \mathbf{h}_2 x_{2,k} + \cdots + \mathbf{h}_{n_t} x_{n_t,k} + \mathbf{z}_k \qquad (1) \\ \mathbf{y}_k &= \mathbf{H}\mathbf{x}_k + \mathbf{z}_k \end{aligned}$$

where n_t is the total number of spatial streams/transmit antennas and $\mathbf{y}_k, \mathbf{z}_k \in \mathbb{C}^{n_r}$ are the vectors of the received symbols and circularly symmetric complex white Gaussian noise of variance N_0 at the n_r receive antennas. $\mathbf{h}_j \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from the j-th transmitting antenna to n_r receive antennas and x_j is the complex symbol of j-th stream transmitted by j-th transmit antenna with $E\left[|x_j|^2\right] = \sigma^2$. It is assumed that each channel path between the transmitter and the receiver is independent. The complex symbols $x_1, \cdots, x_{n_t}$ of n_t streams are also assumed to be independent. $\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \cdots \mathbf{h}_{n_t}]$ is the channel matrix with complex channel gains $E\left[|h_{ij}|^2\right] = 1$. The average SNR at each receiver branch is $\frac{n_t \sigma^2}{N_0}$.

III. Detection Algorithm

The algorithm earlier presented in [9] is based on the idea of imparting more diversity to the spatial streams which necessitate higher diversity in the detection but at the cost of enhanced complexity and providing less diversity to the spatial streams which do not need higher diversity with the benefit of low detection complexity. The algorithm exploits low complexity of the linear detectors and improved diversity of max log MAP detectors. The fact that the linear detectors as MMSE have significantly lower complexity as compared to the brute-force search (max log MAP) but exhibit degraded performance particularly at lower SNRs is exploited by detecting the spatial streams which have seen good realizations of the channel and therefore enjoy relatively higher SNR using these linear detectors. These detected streams are subsequently stripped off leading to a group of yet undetected streams which have seen comparatively poor realizations of the channel and therefore have relatively lower SNR. This group of streams is then detected using max log MAP detectors. So the detection algorithm can be divided into 3 stages.

A. Optimal Detection Ordering

System performance is affected by the order in which these spatial streams are detected. We consider V-BLAST criteria by simply ordering the streams as per the decreasing post detection SNR. This approach is termed as the "best first" cancellation approach within the multi-user community. The spatial streams to be detected by linear MMSE filter [10] are decided by choosing the rows of MMSE filter with the smallest norms. Depending on the system parameters, a threshold for the row norm of the MMSE filter can be decided as a criteria for the detection using MMSE filters.

B. Detection based on linear MMSE filters

After ordering, first group of spatial stream is detected by MMSE filters. The MMSE filter for detecting j-th stream is given as

$$\mathbf{m}_j = \left(\mathbf{h}_j^\dagger \mathbf{R}_j^{-1} \mathbf{h}_j + 1/\sigma^2\right)^{-1} \mathbf{h}_j^\dagger \mathbf{R}_j^{-1} \qquad (2)$$

where

$$\mathbf{R}_j = \sigma^2 \mathbf{h}_1 \mathbf{h}_1^\dagger + \cdots + \sigma^2 \mathbf{h}_{j-1} \mathbf{h}_{j-1}^\dagger + \sigma^2 \mathbf{h}_{j+1} \mathbf{h}_{j+1}^\dagger + \cdots + \sigma^2 \mathbf{h}_{n_t} \mathbf{h}_{n_t}^\dagger + N_0 \mathbf{I} \qquad (3)$$

Application of MMSE filter on the received vector $\mathbf{y}_k$ yields

$$\mathbf{m}_j \mathbf{y}_k = y_k = \alpha_k x_{j,k} + z_k \qquad (4)$$

So the bit metric for bit b at i-th location of j-th spatial stream is given as

$$\lambda_j^i(\mathbf{y}_k, b) \approx \min_{x_j \in \chi_{j,b}^i} \left[\frac{1}{N_j} |y_k - \alpha_k x_j|^2\right] \qquad (5)$$

where $N_j = \mathbf{m}_j \mathbf{R}_j \mathbf{m}_j^\dagger$ and $\chi_{j,b}^i$ denotes the subset of the signal set $x_j \in \chi_j$ whose labels have the value $b \in \{0, 1\}$ in the position i. Bit LLRs are calculated using these bit metrics which are then deinterleaved and are subsequently fed to the decoders for the detection of j-th stream. The detected stream is then again encoded, interleaved, modulated and convolved with the channel and is consequently subtracted from the received signal.

C. PEP Analysis

Consider the detection of j-th stream codeword $\mathbf{c}_j$ by MMSE filter. The conditional PEP i.e. $P\left(\mathbf{c}_j \rightarrow \hat{\mathbf{c}}_j|\mathbf{H}\right) = \mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j}$ is given as

$$\mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j}=P\left(\sum_{k'} \min_{x_j\in\chi^i_{j,c_{k'}}} \frac{\left|y_k-\alpha_k x_j\right|^2}{N_j} \geq \sum_{k'} \min_{x_j\in\chi^i_{j,\hat{c}_{k'}}} \frac{\left|y_k-\alpha_k x_j\right|^2}{N_j}\right) \tag{6}$$

For the worst case scenario once $d\left(\mathbf{c}_j-\hat{\mathbf{c}}_j\right) = d_{free}$, the inequality on the right hand side of (6) shares the same terms on all but d_{free} summation points for which $\hat{c}_{k'} = \bar{c}_{k'}$ where $\bar{(.)}$ denotes the binary complement. Let

$$\tilde{x}_{j,k}=\arg\min_{x_j\in\chi^i_{j,c_{k'}}} \frac{\left|y_k-\alpha_k x_j\right|^2}{N_j},\ \hat{x}_{j,k}=\arg\min_{x_j\in\chi^i_{j,\bar{c}_{k'}}} \frac{\left|y_k-\alpha_k x_j\right|^2}{N_j}$$

Using the fact that $\frac{1}{N_j}\left|y_k-\alpha_k x_{j,k}\right|^2 \geq \frac{1}{N_j}\left|y_k-\alpha_k \tilde{x}_{j,k}\right|^2$, the conditional PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j} \leq Q\left(\sqrt{\sum_{k,d_{free}} \frac{\alpha_k^2}{2N_j}\left|\hat{x}_{j,k}-x_{j,k}\right|^2}\right) \tag{7}$$

We bound $\left|\hat{x}_{j,k}-x_{j,k}\right|^2 \geq \sigma^2 d_{\min}^2$ where d_{min} is the normalized minimum distance of the constellation χ_j. Using the Chernoff bound $Q\left(x\right) \leq \frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$ we get

$$\mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j} \leq \frac{1}{2}\exp\left(-\frac{\sigma^2 d_{\min}^2}{4}\sum_{k,d_{free}} \mathbf{h}_j^{\dagger}\mathbf{R}_j^{-1}\mathbf{h}_j\right) \tag{8}$$

The channel being quasi-static, (8) can be rewritten as

$$\mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j} \leq \frac{1}{2}\exp\left(-\frac{\sigma^2 d_{\min}^2}{4}\left(d_{free}\mathbf{h}_j^{\dagger}\mathbf{R}_j^{-1}\mathbf{h}_j\right)\right) \tag{9}$$

Using unitary transformation $\mathbf{U}_j$, the covariance matrix $\mathbf{R}_j$ (3) can be written as

$$\begin{aligned}\mathbf{R}_j &= \sigma^2\mathbf{H}_j\mathbf{H}_j^{\dagger} + N_0\mathbf{I} \\ &= \mathbf{U}_j^{\dagger}\left(\sigma^2\mathbf{\Lambda}_j + N_0\mathbf{I}\right)\mathbf{U}_j\end{aligned} \tag{10}$$

where $\mathbf{H}_j$ is the submatrix of $\mathbf{H}$ formed by deleting the j-th column i.e. $\mathbf{H}_j = \left[\mathbf{h}_1\cdots\mathbf{h}_{j-1}\mathbf{h}_{j+1}\cdots\mathbf{h}_{n_t}\right]$ and $\mathbf{\Lambda}_j = \text{Diag}\left(\left[\lambda_{j,1},\lambda_{j,2},\cdots,\lambda_{j,n_r}\right]\right)$ is the diagonal matrix containing the eigenvlaues of $\mathbf{H}_j\mathbf{H}_j^{\dagger}$. So PEP can be written as

$$\mathcal{P}_{\mathbf{c}_j|\mathbf{H}}^{\hat{\mathbf{c}}_j} \leq \frac{1}{2}\exp\left(-\frac{\sigma^2 d_{\min}^2}{4}\left(d_{free}\mathbf{h}_j^{\dagger}\mathbf{U}_j\left(\sigma^2\mathbf{\Lambda}_j+N_0\mathbf{I}\right)^{-1}\mathbf{U}_j^{\dagger}\mathbf{h}_j\right)\right) \tag{11}$$

As unitary transformation preserves the distribution so $\mathbf{U}_j^{\dagger}\mathbf{h}_j \sim \mathcal{CN}\left(\mathbf{0},\mathbf{I}\right)$. For a Hermitian quadratic form in complex Gaussian random variable $q = \mathbf{g}^{\dagger}\mathbf{A}\mathbf{g}$ where $\mathbf{A}$ is a Hermitian matrix and column vector $\mathbf{g}$ is a circularly symmetric complex Gaussian vector i.e. $\mathbf{g} \sim \mathcal{NC}\left(\boldsymbol{\mu},\sum\right)$, the MGF is

$$E\left[\exp\left(-t\mathbf{g}^{\dagger}\mathbf{A}\mathbf{g}\right)\right] = \frac{\exp\left[-t\boldsymbol{\mu}^{\dagger}\mathbf{A}\left(\mathbf{I}+t\sum\mathbf{A}\right)^{-1}\boldsymbol{\mu}\right]}{\det\left(\mathbf{I}+t\sum\mathbf{A}\right)} \tag{12}$$

So using the MGF, PEP is upperbounded as

$$\begin{aligned}\mathcal{P}_{\mathbf{c}_j}^{\hat{\mathbf{c}}_j} &\leq \frac{1}{2\det\left(\mathbf{I}_{n_r}+\frac{\sigma^2 d_{\min}^2 d_{free}}{4}\left(\sigma^2\mathbf{\Lambda}_j+N_0\mathbf{I}\right)^{-1}\right)} \\ &= \frac{1}{2\prod_{l=1}^{n_r}\left(1+\frac{\sigma^2 d_{\min}^2 d_{free}}{4}\left(\sigma^2\lambda_{j,l}+N_0\right)^{-1}\right)}\end{aligned} \tag{13}$$

As $n_r \geq n_t-1$, it can be seen that out of the n_r eigenvalues of $\mathbf{H}_j\mathbf{H}_j^{\dagger}$, (n_r-n_t+1) eigenvalues would be zero as $\mathbf{H}_j$ is a full column rank matrix. Taking expectation w.r.t the eigenvalues, PEP is then further written as

$$\mathcal{P}_{\mathbf{c}_j}^{\hat{\mathbf{c}}_j} \leq \frac{1}{2\left(1+\frac{\sigma^2 d_{\min}^2 d_{free}}{4N_0}\right)^{n_r-n_t+1}\prod_{l=n_r-n_t+2}^{n_r}\left(1+\frac{\sigma^2 d_{\min}^2 d_{free}}{4\left(\bar{\lambda}_{j,l}+N_0\right)}\right)} \tag{14}$$

where $\bar{\lambda}_{j,l} = E\left[\lambda_{j,l}\right]$ and $\bar{\lambda}_{j,l}$ can be found via closed from solutions in some cases which will be discussed later. At high SNR when $\sigma^2\bar{\lambda}_{j,l} >> N_0$, so PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_j}^{\hat{\mathbf{c}}_j} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma^2 d_{\min}^2 d_{free}}\right)^{n_r-n_t+1}\prod_{l=n_r-n_t+2}^{n_r}\frac{4\bar{\lambda}_{j,l}}{d_{\min}^2 d_{free}} \tag{15}$$

which demonstrates the well known result [11] that the diversity order of MMSE detection is n_r-n_t+1. When the number on interferers is restricted to one or two, the random eigenvalues can be found exactly. For single interferer case, the PEP is given as

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma^2}\right)^{(n_r-1)}\left(\frac{4}{\sigma^2}\right)\left(\frac{1}{d_{min}^2 d_{free}}\right)^{n_r}\left(\sigma^2 n_r+N_0\right)$$

For the special case of two interferers, the PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma^2}\right)^{(n_r-2)}\left(\frac{4}{\sigma^2}\right)\left(\frac{1}{d_{min}^2 d_{free}}\right)^{n_r}\left(\sigma^2 n_r+N_0\right)^2$$

D. Max Log MAP

Without loss of generality, let the last K spatial streams $(n_t-K+1,\cdots,n_t)$ are detected using MMSE filters which are then subsequently stripped off. For the remaining n_t-K spatial streams which have experienced poor channel realizations, max log MAP detector is used. The new system equation after nulling the last K detected streams at k-th channel use is

$$\mathbf{y}_k^{'} = \mathbf{H}^{'}\mathbf{x}_k^{'} + \mathbf{z}_k^{'} \tag{16}$$

where $\mathbf{y}_k^{'},\mathbf{z}_k^{'}\in\mathbb{C}^{n_r}$ and $\mathbf{H}^{'}$ is $n_r\times(n_t-K)$ complex matrix and $\mathbf{x}_k^{'}\in\mathbb{C}^{n_t-K}$. The ML bit metric for bit b at i-th location of j-th stream x_j is given as

$$\lambda_j^i(\mathbf{y}_k,b)=\log\sum_{x_1\in\chi_1}\cdots\sum_{x_j\in\chi^i_{j,b}}\cdots\sum_{x_{n_t-K}\in\chi_{n_t-K}}\exp\left(\frac{-1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}^{'}\right\|^2\right)$$

Applying log sum approximation [12] we have

$$\lambda_j^i\left(\mathbf{y}_k, b\right) \approx \min_{x_1\in\chi_1\cdots x_j\in\chi_{j,b}^i\cdots x_{n_t-K}\in\chi_{n_t-K}} \left[\frac{1}{N_0}\left\|\mathbf{y}_k^{'} - \mathbf{H}^{'}\mathbf{x}^{'}\right\|^2\right]$$

The computational complexity of this detection process is $\mathcal{O}(|\chi|^{n_t-K})$. The closer K is to n_t, lower is the complexity of detection. Bit LLRs are calculated using these bit metrics and consequently the streams are detected using the corresponding channel decoders.

E. PEP Analysis

Without loss of generality, we consider the detection of first stream by max log MAP detection. The conditional PEP is given as

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} = P\left(\sum_{k'} \min_{x_1\in\chi_{1,c_{k'}}^i, x_2\in\chi_2,\cdots,x_{n_t-K}\in\chi_{n_t-K}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}^{'}\right\|^2 \geq \sum_{k'} \min_{x_1\in\chi_{1,\hat{c}_{k'}}^i, x_2\in\chi_2,\cdots,x_{n_t-K}\in\chi_{n_t-K}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}^{'}\right\|^2\right) \quad (17)$$

We consider the worst case scenario once $d\left(\mathbf{c}_1 - \hat{\mathbf{c}}_1\right) = d_{free}$. Let

$$\tilde{\mathbf{x}}_k^{'} = \arg \min_{x_1\in\chi_{1,c_{k'}}^i, x_2\in\chi_2,\cdots,x_{n_t-K}\in\chi_{n_t-K}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}^{'}\right\|^2$$
$$\hat{\mathbf{x}}_k^{'} = \arg \min_{x_1\in\chi_{1,\bar{c}_{k'}}^i, x_2\in\chi_2,\cdots,x_{n_t-K}\in\chi_{n_t-K}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}^{'}\right\|^2 \quad (18)$$

where $\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}_k^{'}\right\|^2 \geq \left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\tilde{\mathbf{x}}^{'}\right\|^2$. The conditional PEP is given as

$$\begin{aligned}
\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} &\leq P\left(\sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\mathbf{x}_k^{'}\right\|^2 \geq \sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{y}_k^{'}-\mathbf{H}^{'}\hat{\mathbf{x}}_k^{'}\right\|^2\right) \\
&= P\left(\sum_{k,d_{free}} \frac{1}{N_0} 2\Re\left(\mathbf{z}_k^{'\dagger}\mathbf{H}^{'}\left(\hat{\mathbf{x}}_k^{'} - \mathbf{x}_k^{'}\right)\right) \geq \sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{H}^{'}\mathbf{x}_k^{'} - \mathbf{H}^{'}\hat{\mathbf{x}}_k^{'}\right\|^2\right) \\
&= Q\left(\sqrt{\sum_{k,d_{free}} \frac{1}{2N_0}\left\|\mathbf{H}^{'}\left(\hat{\mathbf{x}}_k^{'} - \mathbf{x}_k^{'}\right)\right\|^2}\right) \\
&= Q\left(\sqrt{\sum_{k,d_{free}} \frac{1}{2N_0} \mathrm{vec}\left(\mathbf{H}^{'\dagger}\right)^{\dagger}\left(\mathbf{I}_{n_R} \otimes \left(\hat{\mathbf{x}}_k^{'}-\mathbf{x}_k^{'}\right)\left(\hat{\mathbf{x}}_k^{'}-\mathbf{x}_k^{'}\right)^{\dagger}\right) \mathrm{vec}\left(\mathbf{H}^{'\dagger}\right)}\right) \\
&= Q\left(\sqrt{\frac{1}{2N_0} \mathrm{vec}\left(\mathbf{H}^{'\dagger}\right)^{\dagger}\left(\mathbf{I}_{n_R} \otimes \mathbf{\Delta}\mathbf{\Delta}^{\dagger}\right) \mathrm{vec}\left(\mathbf{H}^{'\dagger}\right)}\right)
\end{aligned} \quad (19)$$

where $\mathbf{\Delta} = \left[\hat{\mathbf{x}}_1^{'} - \mathbf{x}_1^{'} \quad \hat{\mathbf{x}}_2^{'} - \mathbf{x}_2^{'} \quad \cdots \quad \hat{\mathbf{x}}_{k,dfree}^{'} - \mathbf{x}_{k,d_{free}}^{'}\right]$. Note that $\mathbf{\Delta}\mathbf{\Delta}^{\dagger}$ is a $n_t \times n_t$ Hermitian matrix while vec $\left(\mathbf{H}^{'\dagger}\right)$ is a zero mean Gaussian vector with covariance matrix $\mathbf{I}_{n_t n_r}$. Q is the Gaussian Q-function and vec indicates vectorization of a matrix. Using MGF (12) and Chernoff bound, PEP is upper bounded as

$$\begin{aligned}
\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} &\leq \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\mathbf{I}\left(\mathbf{I}_{n_R} \otimes \mathbf{\Delta}\mathbf{\Delta}^{\dagger}\right)\right)} \\
&= \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\left(\mathbf{I}_{n_R} \otimes \mathbf{\Delta}\mathbf{\Delta}^{\dagger}\right)\right)} \\
&= \frac{1}{2\prod_{k=1}^{r}\prod_{l=1}^{n_r}\left(1 + \frac{\sigma^2}{4N_0}\lambda_k\left(\mathbf{\Delta}\mathbf{\Delta}^{\dagger}/\sigma^2\right)\right)}
\end{aligned} \quad (20)$$

where r is the rank of $\mathbf{\Delta}\mathbf{\Delta}^{\dagger}$. Here we have used the identity that for square matrices $\mathbf{A}$ and $\mathbf{B}$ of size n and q respectively with the eigenvalues $\lambda_1\left(\mathbf{A}\right), \cdots, \lambda_n\left(\mathbf{A}\right)$ and $\lambda_1\left(\mathbf{B}\right), \cdots, \lambda_q\left(\mathbf{B}\right)$ of $\mathbf{A}$ and $\mathbf{B}$ respectively then the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ are

$$\lambda_i\left(\mathbf{A}\right)\lambda_j\left(\mathbf{B}\right) \qquad i = 1, \cdots, n,\ j = 1, \cdots, q. \quad (21)$$

For the high SNR approximation, we get

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma^2}\right)^{rn_r}\prod_{k=1}^{r}\frac{1}{\left(\lambda_k\left(\mathbf{\Delta}\mathbf{\Delta}^{\dagger}/\sigma^2\right)\right)^{n_r}} \quad (22)$$

Note that the minimizations in (18) ensure that in $\mathbf{\Delta}$, $\hat{x}_{k,1}^{'} - x_{k,1}^{'}$ for $k = 1, \cdots, d_{free}$ is always non-zero where $\hat{x}_{k,l}^{'} - x_{k,l}^{'}$ for $l = 2, \cdots, (n_t - K)$ can be zero. So in the worst case scenario, $\mathbf{\Delta}$ would have only first row with non-zero elements. Subsequently $\mathbf{\Delta}\mathbf{\Delta}^{\dagger}$ will have only one non-zero entry so its minimum rank will be 1 thereby resulting in the diversity order to be n_r.

The analysis shows that the group of streams being detected by max log MAP detectors have higher order of diversity i.e. n_r while the group of streams being detected by MMSE detectors have lesser diversity order i.e. $(n_r - n_t + 1)$. This analysis justifies the basis of the detection algorithm of furnishing higher diversity to the streams necessitating it at the cost of high complexity due to poor channel realizations while trimming the diversity order of the streams with the advantage of low complexity detection benefiting from good channel realizations.

IV. Simulation Results

We now assess the performance of the proposed approach of soft detection of spatially multiplexed MIMO system by means of simulation of the frame-error rates (FER). We focus on target error-rates of the order of 10^{-2}. We consider 3×3 and 4×4 BICM MIMO systems using the *de facto* standard, 64 state $(133, 171)$ rate-$1/2$ convolutional encoder of 802.11n standard and the punctured rate $1/2$ turbo code[1] of 3GPP

[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

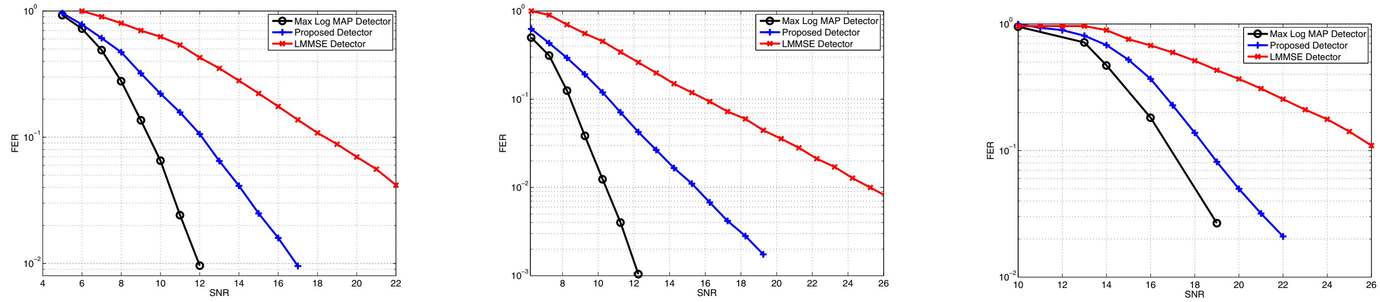

Fig. 1. MIMO system with four spatial streams. Left and center figures are of QPSK with rate $1/2$ convolutional code and rate $1/2$ turbo code respectively while right figure is of QAM16 with rate $1/2$ convolutional code.

LTE [5]. The upcoming WLAN standard 802.11n [3] supports the codeword sizes of 648, 1296, and 1944 bits. For our purposes, we selected the codeword size of 1296 bits. We consider slow fading i.e. MIMO channel remains constant for the duration of one frame. The channel has iid Gaussian matrix entries with unit variance. The channel is independently generated for each frame and perfect CSI at the receiver is assumed. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider the max log MAP detector, MMSE based detector and the proposed detector. Spatial streams of equal rates and equal power are transmitted in a 4×4 system. For the proposed detector, two streams are detected by MMSE detector while the remaining two are detected using max log MAP detector. Fig. 1 indicates improved performance both in terms of frame error rates (FER) and diversity of the proposed approach with respect to that of MMSE based approach. The proposed approach being the fusion of MMSE and the max log MAP approach, has improved diversity than that of MMSE detector while diversity order is less than that of max log MAP detector.

V. Conclusions

We have analyzed the performance of a novel detection scheme for BICM MIMO OFDM system. Being a combination of linear and non linear detectors, the scheme incorporates lower detection complexity at the cost of reduced diversity order for the streams benefiting from good channel realizations and enhanced diversity order at the cost of increased detection complexity for the streams suffering from poor channel realizations.

Acknowledgments

Eurecom's research is partially supported by its industrial partners: BMW, Bouygues Telecom, Cisco Systems, France Télécom, Hitachi Europe, SFR, Sharp, ST Microelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission under SAMURAI and IST FP7 research network of excellence NEWCOM++.

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