Training Sequence-Based Multiuser Channel Estimation for Block-Synchronous CDMA

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Abstract

Uplink channel estimation for a block-synchronous chip-asynchronous CDMA system as proposed for the time-division duplex (TDD) option of 3rd generation cellular systems is considered. Training midambles are employed for joint channel estimation of all users. An *unstructured* approach based on modeling the effective user channels as unknown FIR filters is compared with two *structured* methods that exploit *a priori* knowledge about the user channels such as the maximum delay spread, the transmit chip-shaping pulse and the path delays. A low-complexity, high-performance delay estimator based on maximum-likelihood is proposed which explicitly estimates the delays of the individual multipath components of each user.

Keywords: DS/CDMA, channel estimation, multiuser detection.

1 Introduction

One of the proposals for UMTS, the European 3rd generation mobile communication system, is based on hybrid TDMA/CDMA (briefly denoted as T-CDMA) [1, 2, 3]. In this scheme, multiple access is regulated by a TDMA slot structure similar to GSM [4], and time-division duplexing (TDD) is adopted. User signals are direct-sequence spread-spectrum, with limited processing gain. In standard TDMA, one single user per cell is allowed to transmit over a slot. However, in T-CDMA multiple users per cell are allowed to transmit over the same time slot.

In the uplink (mobile-to-base), users have a coarse common timing reference and are able to align their signal "blocks" with the slot reference of the base station. Due to imperfect synchronization and to the delay-spread of the multipath channels, the signal blocks experience some misalignment. This misalignment is compensated for by inserting guard intervals of appropriate duration. Therefore, the system is block-synchronous, but cannot be considered chip-synchronous.

Provided that the blocks are sufficiently short with respect to the channel coherence time [5], the user channels can be considered time-invariant over each slot. On the other hand, due to the TDMA dynamic user allocation over the slots and to the bursty nature of transmission, tracking the channels from block to block might be infeasible. Hence, we shall consider blockwise channel estimation, where the receiver estimates the user channels block-by-block without tracking across different blocks.

We consider training-based joint channel estimation of all uplink users as in [3]. Each signal block contains a training sequence of known chips, in a fixed nominal position (typically, in the middle of the block [4]). In classical LS (or ML) training-based channel estimation, the user channels are modeled as FIR filters with unknown coefficients [3], and no *a priori* information about the channels is exploited. We refer to this approach as "unstructured" channel estimation. More recently, *a priori* knowledge of the structure of the overall use channel impulse responses has been exploited in order to improve estimation [6, 7, 8, 9, 10, 11]. In some works, the special feature of rectangular chip-shaping pulses is exploited to estimate directly the channel physical parameters (delays and path gains). In this way, a minimal number of unknowns needs to be estimated. On the other hand, these methods do not generalize easily to arbitrary chip-shaping pulses (typically, root-raised-cosine (RRC) pulses with a given roll-off factor α [5]). Other approaches exploit the fact that the channel vectors must lie in the column-space of an *a priori* known convolution matrix determined only by the chip-shaping pulse and by the maximum delay-spread. This approach can be applied to arbitrary approximately bandlimited chip-shaping pulses, but requires in general more unknowns than the methods based on "physical" channel parameterization.

In this paper, we propose two types of "structured" channel estimators which can be applied to any arbitrary (approximately bandlimited) chip-shaping pulse. Our first method is essentially the multiuser version of [10], and exploits only the coarse information represented by the maximum delay-spread and by the chip-shaping pulse. Our second method is based on the "physical" channel parameterization and exploits the knowledge of the path delays for each user. In practice, this information is not available. Thus, we propose a two-step approach where first the path delays are explicitly estimated, and then used in the structured channel estimator. Starting from a ML approach, we derive a low-complexity delay estimator with very good performance, which can be used in the first step of structured channel estimation.

2 Signal model

We consider the uplink of a CDMA system with K block-synchronous users. The baseband receiver front-end is an ideal Low-Pass Filter with (one-sided) bandwidth $W/2$. The k-th user's total channel impulse response $g_k(t)$ is given by the convolution of the chip-shaping pulse $\psi(t)$, common to all users, with the k-th user multipath channel response $c_k(t)$ and the LPF. The pulse $\psi(t)$ is bandlimited with bandwidth $(1+\alpha)/(2T_c) \leq W/2$, such that $\int \psi(\tau)\psi(\tau+t)^* d\tau$ satisfies the Nyquist criterion [5]. The noise $\nu(t)$ after LPF is complex circularly-symmetric Gaussian with autocorrelation function $E[\nu(t)\nu(t-\tau)^*] = N_0 \text{sinc}(W\tau)$.

We do not make an explicit distinction between the delay introduced by non-ideal user synchronization and the delay introduced by multipath propagation. These effects are incorporated in the channel impulse response

$$c_k(t) = \sum_{p=0}^{P-1} c_{k,p} \delta(t - \tau_{k,p}) \tag{1}$$

where we let Δ denote the maximum delay-spread (over all users) accounting for synchronization errors and multipath delay-spread. All users have the same *transmit* average energy per chip: different average *received* signal-to-noise ratios (SNR) are taken into account as an effect of the channels, by multiplying the chanel gains of each user k by the corresponding amplitude factor.

The LPF output $r(t)$ is sampled at rate $W = N_c/T_c$ (N_c is an integer), without any explicit timing reference. We define the polyphase representation of the discrete-time low-pass filtered channel impulse response as $g_{k,\ell}[m] \triangleq g_k((mN_c+\ell)/W)$ for $\ell = 0, \ldots, N_c-1$. and we assume that, for all possible channel realizations and user synchronization errors, there exists an integer Q such that $g_{k,\ell}[m]$ is negligible for $m \notin \{0, \ldots, Q-1\}$. Under this finite-memory assumption, we define the channel vectors $\mathbf{g}_{k,\ell} = (g_{k,\ell}[0], \ldots, g_{k,\ell}[Q-1])^T$.

Users transmit sequences of known chip-symbols of length $M + Q - 1$ starting at a given nominal point of the block. Without loss of generality, we let $n = -Q + 1$ be the training sequence starting point. In order to estimate the user channels, the receiver forms the vectors of received signal samples

$$\mathbf{r}_\ell = \mathbf{A}\mathbf{g}_\ell + \boldsymbol{\nu}_\ell \tag{2}$$

where $\mathbf{g}_\ell = (\mathbf{g}_{1,\ell}^T, \ldots, \mathbf{g}_{K,\ell}^T)^T$ is the total user channel vector corresponding to the sampling phase ℓ; $\boldsymbol{\nu}_\ell = (\nu[\ell], \nu[N_c + \ell], \ldots, \nu[(M-1)N_c + \ell])^T$ is a vector of i.i.d. noise samples; and $\mathbf{A} = [\mathbf{A}_1, \ldots, \mathbf{A}_K]$ is a $M \times KQ$ block matrix containing only training chip-symbols, whose k-th $M \times Q$ block is given by

$$\mathbf{A}_k = \left[\begin{array}{cccc} a_k[0] & a_k[-1] & \cdots & a_k[-Q+1] \\ a_k[1] & a_k[0] & \cdots & a_k[-Q+2] \\ \vdots & & \ddots & \vdots \\ a_k[M-1] & a_k[M-2] & \cdots & a_k[M-Q] \end{array} \right] \tag{3}$$

3 Unstructured ML channel estimation

Since the noise is i.i.d. and Gaussian, ML estimation is equivalent to the simple LS estimation [12]:

$$\widehat{\mathbf{g}}_\ell = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{r}_\ell \tag{4}$$

for $\ell = 0, \ldots, N_c - 1$, where we assume that $\mathbf{A}$ rank$(\mathbf{A}) = KQ$. We refer to the above estimation method as "unstructured", since we treat $\mathbf{g}_\ell$ as a vector of KQ unknowns.

The resulting normalized estimation MSE of the unstructured estimator is given by

$$\epsilon^2_{\text{unstr}} \triangleq \frac{1}{KQN_c}\sum_{\ell=0}^{N_c-1} E[|\widehat{\mathbf{g}}_\ell - \mathbf{g}_\ell|^2] = \frac{N_0}{KQ}\text{Tr}\left((\mathbf{A}^H\mathbf{A})^{-1}\right) \tag{5}$$

It is well-known [13, 14, 3], that optimal training sequences minimizing $\epsilon^2_{\text{unstr}}$ must satisfy $\mathbf{A}^H\mathbf{A} \propto \mathbf{I}$. The construction of optimal training sequences satisfying $\mathbf{A}^H\mathbf{A} \propto \mathbf{I}$ has been investigated in several papers (see [3, 13, 14, 15, 16, 17] and references therein). Most related research considers sequences constructed from simple symbol alphabets, like BPSK and QPSK. Unfortunately, this requirement is too restrictive and optimal sequences cannot be found for most training lengths M. In this paper, we require the chip-symbols to belong to a N-th root-of-unity alphabet $\mathcal{A}_N = \{e^{j2\pi i/N} : i = 0, \ldots, N-1\}$, for some integer N. This choice is enough to obtain optimal training sequences for any desired training length M, while preserving the constant envelope. In particular, we propose to derive the training sequences for all K users from a single Perfect Root-of-Unity Sequence (PRUS) [16]:

Definition: PRUS. *The sequence* $\mathbf{x} = (x_0, \ldots, x_{M-1}) \in \mathbb{C}^M$ *is a PRUS if* $x_m \in \mathcal{A}_N$*, for all* $m = 0, \ldots, M-1$ *and some integer* N*, and if its periodic autocorrelation satisfies*

$$\phi_x(n) = \sum_{m=0}^{M-1} x_m x^*_{[m-n \mod M]} = M\delta_{n,0}$$

$\square$

Let $\mathbf{x}$ be a PRUS of length $M \geq KQ$. Then, the k-th user training sequence $(a_k[-Q+1], \ldots, a_k[M-1])$ is obtained from $\mathbf{x}$ as

$$a_k[m] = \sqrt{\mathcal{E}_c} x_{[m-(k-1)Q \mod M]} \tag{6}$$

for all $k = 1, \ldots, K$ and $m = -Q+1, \ldots, M-1$, where $\mathcal{E}_c$ is the transmit chip-energy. By using (6) into (3), it is easy to check that the columns of $\mathbf{A}$ are distinct cyclic shifts of the same PRUS $\mathbf{x}$. By construction, we obtain $\mathbf{A}^H\mathbf{A} = M\mathcal{E}_c\mathbf{I}$, as desired, and the resulting minimum estimation error is given by

$$\epsilon^2_{\text{unstr}} = \frac{L}{M}\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1} \tag{7}$$

where $\mathcal{E}_s = L\mathcal{E}_c$ is the nominal average transmit energy per symbol and L is the spreading factor (number of chip/symbol).

4 Structured channel estimation

We define the k-th sampled LPF channel response $\widetilde{\mathbf{c}}_k \triangleq (\widetilde{c}_k[0], \ldots, \widetilde{c}_k[D-1])^T$ with j-th element $\widetilde{c}_k[j] \triangleq \frac{1}{\sqrt{W}} \int c_k(t)\text{sinc}(j - t/W)dt$, the k-th channel gain vector $\mathbf{c}_k \triangleq (c_{k,0}, \ldots, c_{k,P-1})^T$, the $Q \times D$ convolution matrix $\mathbf{\Psi}_\ell$ with (i,j)-th element $\psi\left(\frac{iN_c+\ell-j}{W}\right)$ and the $D \times P$ interpolation matrix $\mathbf{\Phi}_k$ with (j,p)-th element $\text{sinc}\left(j - \tau_{k,p}W\right)/\sqrt{W}$. Then, we let $\widetilde{\mathbf{c}} \triangleq (\widetilde{\mathbf{c}}_1^T, \ldots, \widetilde{\mathbf{c}}_K^T)^T$, $\mathbf{c} \triangleq (\mathbf{c}_1^T, \ldots, \mathbf{c}_K^T)^T$, $\mathbf{\Phi} \triangleq \text{diag}(\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_K)$ and we define the Kronecker product matrix $\underline{\mathbf{\Psi}}_\ell \triangleq \mathbf{I}_K \otimes \mathbf{\Psi}_\ell$, the block vector $\mathbf{g} \triangleq (\mathbf{g}_0^T, \ldots, \mathbf{g}_{N_c-1}^T)^T$ and the block matrix $\underline{\mathbf{\Psi}} \triangleq [\underline{\mathbf{\Psi}}_0^T, \ldots, \underline{\mathbf{\Psi}}_{N_c-1}^T]^T$. Eventually, we can write

$$\mathbf{g} = \underline{\mathbf{\Psi}}\widetilde{\mathbf{c}} = \underline{\mathbf{\Theta}}\mathbf{c} \tag{8}$$

Equations (8) define the *a priori* structure of the channel impulse response. The matrix $\underline{\mathbf{\Psi}}$ is determined by the chip-shaping pulse $\psi(t)$ and by the maximum delay-spread Δ, therefore it is always known by the receiver. The matrix $\underline{\mathbf{\Theta}}$ is determined also by the path delays $\{\tau_{k,p} : p = 0, \ldots, P-1\}$ for all $k = 1, \ldots, K$, and by the number of paths P, which are generally not known by the receiver.

4.1 Type I structured estimation

Let $\mathbf{r} \triangleq (\mathbf{r}_0^T, \ldots, \mathbf{r}_{N_c-1}^T)^T$ and $\boldsymbol{\nu} \triangleq (\boldsymbol{\nu}_0^T, \ldots, \boldsymbol{\nu}_{N_c-1}^T)^T$. Then,

$$\mathbf{r} = [\mathbf{I}_{N_c} \otimes \mathbf{A}]\,\mathbf{g} + \boldsymbol{\nu} \tag{9}$$

where $\mathbf{A}$ is the same as defined by (3). From the first equality of (8), we have that the desired channel vector $\mathbf{g}$ lies in the column-space of the *a priori* known matrix $\underline{\boldsymbol{\Psi}}$. Since $KQN_c > KD$, this has a non-trivial null-space and this additional information can be exploited to improve channel estimation. By using the singular-value decomposition (SVD) [18] $\underline{\boldsymbol{\Psi}} = \mathbf{U}'\mathbf{S}'(\mathbf{V}')^H$, where $\mathbf{S}'$ is $\rho' \times \rho'$ diagonal, ρ' is the rank of $\underline{\boldsymbol{\Psi}}$, and $\mathbf{U}'$, $\mathbf{V}'$ are rectangular matrices with orthonormal columns and dimension $KQN_c \times \rho'$ and $KD \times \rho'$, respectively, we get the following channel estimator:

1. Obtain the ML estimate of $\mathbf{d}'$ from the observation $\mathbf{r}$ as

$$\widehat{\mathbf{d}} = \arg \min_{\mathbf{d}} |\mathbf{r} - [\mathbf{I}_{N_c} \otimes \mathbf{A}]\, \mathbf{U}'\mathbf{d}|^2 \tag{10}$$

2. Obtain the Type I structured estimate of $\mathbf{g}$ as $\widehat{\mathbf{g}} = \mathbf{U}'\widehat{\mathbf{d}}$.

Since $\underline{\boldsymbol{\Psi}}$ is known *a priori*, no real-time SVD computation is required.

4.2 Type II structured estimation

If the delays $\tau_{k,p}$ and the number of paths P are known, the receiver can compute $\underline{\boldsymbol{\Theta}}$ and impose that the channel vector $\mathbf{g}$ must lie in its column space (see the second equality of (8)). Again, it is convenient to re-parameterize the problem by using the SVD $\underline{\boldsymbol{\Theta}} = \mathbf{U}''\mathbf{S}''(\mathbf{V}'')^H$, where $\mathbf{S}''$ is $\rho'' \times \rho''$ diagonal, ρ'' is the rank of $\underline{\boldsymbol{\Theta}}$, and $\mathbf{U}''$, $\mathbf{V}''$ are rectangular matrices with orthonormal columns and dimension $KQN_c \times \rho''$ and $KP \times \rho''$, respectively. Since in general neither the $\tau_{k,p}$'s nor the number of paths P are *a priori* known, these parameters must also be estimated from the received signal. We propose a two-step approach where first an estimate $\widehat{\tau}_{k,p}$ of delays $\tau_{k,p}$ is obtained and then Type II structured estimator is computed assuming $\tau_{k,p} = \widehat{\tau}_{k,p}$. We have:

1. Obtain an estimate the maximum number of paths per user $\widehat{P}$ and of the delays $\{\widehat{\tau}_{k,p} : p = 0, \dots, \widehat{P} - 1\}$, for all $k = 1, \dots, K$.

2. Based on the estimated delays, compute an estimate $\widehat{\underline{\boldsymbol{\Theta}}}$ of $\underline{\boldsymbol{\Theta}}$ and the $KQN_c \times \widehat{\rho}''$ factor $\widehat{\mathbf{U}}''$ in its SVD.

3. Under the assumption $\mathbf{U}'' = \widehat{\mathbf{U}}''$, obtain the ML estimate of $\mathbf{d}''$ from the observation $\mathbf{r}$ as

$$\widehat{\mathbf{d}} = \arg \min_{\mathbf{d}} \left| \mathbf{r} - [\mathbf{I}_{N_c} \otimes \mathbf{A}]\, \widehat{\mathbf{U}}''\mathbf{d} \right|^2 \tag{11}$$

4. Obtain the Type II structured estimate of $\mathbf{g}$ as $\widehat{\mathbf{g}} = \widehat{\mathbf{U}}''\widehat{\mathbf{d}}$.

Since $\widehat{\underline{\boldsymbol{\Theta}}}$ is not known *a priori*, a real-time SVD per block is needed. In general, since $\mathbf{U}'' \neq \widehat{\mathbf{U}}''$ the Type II estimator is biased.

Next, we find optimal training sequence sets for Type I and Type II structured estimators (assuming perfect knowledge of the delays for the latter). The solution of the LS estimation (10) and (11) is given by

$$\widehat{\mathbf{d}} = \left(\mathbf{U}^H \left[\mathbf{I}_{N_c} \otimes (\mathbf{A}^H\mathbf{A})\right] \mathbf{U}\right)^{-1} \mathbf{U}^H[\mathbf{I}_{N_c} \otimes \mathbf{A}^H]\mathbf{r}$$

where $\mathbf{U} = \mathbf{U}'$ (resp., $\mathbf{U} = \mathbf{U}''$) for Type I (resp., Type II) estimation. The error vector in the estimation of $\mathbf{d}$ is given by $\mathbf{e} = \widehat{\mathbf{d}} - \mathbf{d}$, $\sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\Sigma} \stackrel{\Delta}{=} E[\mathbf{e}\mathbf{e}^H] = N_0 \left(\mathbf{U}^H \left[\mathbf{I}_{N_c} \otimes (\mathbf{A}^H\mathbf{A})\right] \mathbf{U}\right)^{-1}$$

The error vector in the estimation of $\mathbf{g}$ is given by $\mathbf{U}\mathbf{e}$, $\sim \mathcal{NC}(\mathbf{0}, \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H)$. The resulting normalized estimation MSE is given by

$$\epsilon^2_{\rm struc} \stackrel{\Delta}{=} \frac{1}{KQN_c}\mathrm{Tr}\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H\right) = \frac{1}{KQN_c}\mathrm{Tr}(\boldsymbol{\Sigma}) \tag{12}$$

where we used the fact that $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Optimal training sequences should satisfy $\boldsymbol{\Sigma} \propto \mathbf{I}$. Fortunately, since $\mathbf{U}$ has orthonormal columns, we have immediately that if $\mathbf{A}^H\mathbf{A} \propto \mathbf{I}$ then also $\boldsymbol{\Sigma} \propto \mathbf{I}$. We conclude that optimal sequences for unstructured estimation are also optimal for Type I and Type II structured estimation (assuming ideal delay estimation for the latter). In particular, the construction of training sequences from a single PRUS of length $M \geq KQ$, as indicated in (6), can be successfully applied here. The resulting minimum estimation error is given by $\epsilon^2_{\rm struc} = \frac{\rho}{KQN_c}\epsilon^2_{\rm unstr}$ where $\rho = \rho'$ (resp., $\rho = \rho''$) for Type I (resp., Type II) estimation. Then, the ratio between the minimum structured and unstructured estimation errors is exactly equal to the ratio between the number of structured and unstructured unknown parameters to be estimated (this does not hold in general, for suboptimal training sequences).

5 Delay estimation

We propose a delay estimator derived from ML estimation of $\boldsymbol{\tau} \stackrel{\Delta}{=} \{\tau_{k,p}\}$ from the observation $\widehat{\mathbf{g}}$, given by the estimated discrete-time low-pass filtered channel impulse responses provided either by unstructured or by Type I structured estimation. This method can be used in the preliminary delay estimation of the Type II structured channel estimator described in Section 4.2. We can write

$$\widehat{\mathbf{g}} = \mathbf{g} + \mathbf{e} = \underline{\boldsymbol{\Theta}}\mathbf{c} + \mathbf{e} \tag{13}$$

where $\mathbf{e} \sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma})$, independent of the vector of channel gains $\mathbf{c}$. We assume Rayleigh fading, so that $\mathbf{c}$ is $\sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Lambda})$. For given delay vector $\boldsymbol{\tau}$, $\underline{\boldsymbol{\Theta}}$ is fixed and $\widehat{\mathbf{g}}$ is conditionally $\sim \mathcal{NC}(\mathbf{0}, \mathbf{R}_g(\boldsymbol{\tau}))$, where the conditional covariance matrix is given by

$$\mathbf{R}_g(\boldsymbol{\tau}) = \underline{\boldsymbol{\Theta}}(\boldsymbol{\tau})\boldsymbol{\Lambda}\underline{\boldsymbol{\Theta}}(\boldsymbol{\tau})^H + \boldsymbol{\Sigma}$$

where we indicate explicitly the dependence on the delay vector $\boldsymbol{\tau}$. With the above statistical model, the ML estimate of $\boldsymbol{\tau}$ based on the observation $\widehat{\mathbf{g}}$ is given by

$$\widehat{\boldsymbol{\tau}} = \arg \max_{\boldsymbol{\tau}} \left\{-\widehat{\mathbf{g}}^H\mathbf{R}_g^{-1}(\boldsymbol{\tau})\widehat{\mathbf{g}} - \log \mathrm{Det}(\mathbf{R}_g(\boldsymbol{\tau}))\right\} \tag{14}$$

This requires the maximization of the log-likelihood function over a KP-dimensional real space. Moreover, the log-likelihood function depends on $\boldsymbol{\tau}$ via a matrix inversion.

In order to decrease complexity, we make some simplifications and assumptions. In particular, we assume that: i) white error vector (i.e., $\boldsymbol{\Sigma} = \sigma_e^2\mathbf{I}$); ii) channel gains for different users and for different delays of the same user are mutually independent (i.e., Λ is diagonal); iii) In order to obtain an additional simplification, we assume that the delays $\boldsymbol{\tau}_k$ are sufficiently separated, so that $\psi(t-\tau_{k,p})$ and $\psi(t-\tau_{k,q})$ have essentially disjoint support for $p \neq q$. Then, it is possible to show [19] that the ML delay estimator for each user k can be put in the form:

$$\widehat{\boldsymbol{\tau}}_k = \arg \max_{\boldsymbol{\tau}_k} \sum_{p=0}^{P-1} \frac{\sigma_{k,p}^2}{\sigma_e^2 + \sigma_{k,p}^2} \left| \boldsymbol{\psi}_{k,p}^H \widehat{\mathbf{g}}^{(k)} \right|^2 \tag{15}$$

where $\boldsymbol{\tau}_k$ and $\mathbf{g}^{(k)}$ are the subvectors of $\boldsymbol{\tau}$ and of $\mathbf{g}$ corresponding to user k, where $\sigma_{k,p}^2 \triangleq E[|c_{k,p}|^2]$ and where we define the column vector $\boldsymbol{\psi}_{k,p}$ with i-th element

$$[\boldsymbol{\psi}_{k,p}]_i = \psi(i/W - \tau_{k,p})/\sqrt{W} \tag{16}$$

for $i = 0, \ldots, QN_c - 1$. We have not escaped a P dimensional maximization and typically independent maximization of each term in (15) yields $\widehat{\tau}_{k,p} = \widehat{\tau}$, for all $p = 0, \ldots, P-1$, where $\widehat{\tau}$ is located with high probability in the vicinity of the maximum peak of the sampled observed channel response $\widehat{\mathbf{g}}^{(k)}$. Also, we observe that in practice, both the delay-intensity profile and P are unknown. However, we can still propose a further approximated algorithm which requires only a sequence of P one-dimensional maximizations.

Assume P known. First, define a delay discretization step $\Delta\tau$, such that $T_c/\Delta\tau = N_\tau$ is an integer multiple of N_c. Then, for all $j = 0, \ldots, QN_\tau - 1$, define the vectors $\mathbf{v}_j$ of length QN_c with i-th component

$$[\mathbf{v}_j]_i = \psi(i/W - j\Delta\tau)/\sqrt{W}$$

for $i = 0, \ldots, QN_c - 1$. Clearly, $\mathbf{v}_j = \boldsymbol{\psi}_{k,p}$ if $j\Delta\tau = \tau_{k,p}$, for some j, while if $\tau_{k,p}$ is not an integer multiple of $\Delta\tau$, the maximum delay discretization error is $\Delta\tau/2$. Initialize the vector $\mathbf{w}_0 = \widehat{\mathbf{g}}^{(k)}$ and the set of delay indexes $\mathcal{S}_0 = \{0, \ldots, QN_\tau - 1\}$. For $p = 0, \ldots, P-1$, repeat the following steps:

1. Estimate the p-th delay as $\widehat{\tau}_{k,p} = \widehat{j}_p \Delta\tau$, where

$$\widehat{j}_p = \arg \max_{j \in \mathcal{S}_p} \left| \mathbf{v}_j^H \mathbf{w}_p \right|^2 \tag{17}$$

2. Eliminate the effect of the p-th delay from the observed channel impulse response as

$$\mathbf{w}_{p+1} = \mathbf{w}_p - \left(\frac{\mathbf{v}_{\widehat{j}_p}^H \mathbf{w}_p}{|\mathbf{v}_{\widehat{j}_p}|^2} \right) \mathbf{v}_{\widehat{j}_p} \tag{18}$$

3. Update the delay index set as

$$\mathcal{S}_{p+1} = \mathcal{S}_p - \{\widehat{j}_p - N_\tau + 1, \ldots, \widehat{j}_p + N_\tau - 1\} \tag{19}$$

6 Performance with linear detectors

In order to investigate the impact of channel estimation on the receiver performance, we consider some simple FIR (or "one-shot") linear receivers. After low-pass filtering and sampling at rate W, for each n, a window of samples centered around the interval $[nLT_c, (n+1)LT_c]$ is used to detect the n-th symbol. Since transmission is chip-asynchronous and the channel delay-spread can be larger than a symbol interval, the processing window should span more than one symbol interval. Approximated single-user matched filters (SUMF) and linear minimum mean-square error (LMMSE) filters [20] are obtained from the channel estimates produced by the algorithms seen before.

We consider a system with $K = 8$ users, spreading factor $L = 16$, RRC chip-shaping pulse with roll-off $\alpha = 0.22$, truncated over $\kappa = 12$ chips, and maximum channel delay spread of $\Delta = 20T_c$. With this delay-spread, user blocks may be misaligned by more than one symbol and ISI is not at all negligible, in contrast to what is normally assumed in most DS/CDMA literature. The maximum channel length is $Q = 20 + 12 = 32$ chips. We choose training length $M = 256$, i.e., the minimum length necessary for estimating 8 channels of length 32. As a performance measure, we consider the signal-to-interference plus noise ratio (SINR) at the filter output. Since the channel impulse responses change randomly and independently from block to block, the output SINR is a random variable. Therefore, we use Monte Carlo simulation to evaluate the empirical SINR cumulative distribution function (cdf).

Figs. 1 show the SINR cdf for the SUMF receiver in the case of slow and fast power control. In the case of slow power control, the ratio E_k/N_0 is set to 10 dB for all users and no further normalization of the path gains applied. Then, the average received SNR on the shortest path (for which $\sigma^2_{k,0} = 1$) is actually equal to 10 dB for all users, but the instantaneous user received power fluctuates considerably from block to block, because of the uncompensated Rayleigh fading. In the case of fast power control, E_k/N_0 is set as before but the channel gains are normalized such that $\sum_{p=0}^{P-1} |c_{k,p}|^2 = 1$. This could be obtained in practice by a TDD system where each user measures the signal power received on the downlink and exploits reciprocity in order to compensate instantaneously for the Rayleigh fading block by block. The SINR cdf yields the block outage probability, defined as the probability that the SINR is below a fixed threshold γ. The horizontal dashed line corresponds to $F_{\text{sinr}}(\gamma) = 10^{-1}$.

Figs. 2 show analogous results for the LMMSE receiver. Apparently, structured channel estimaiton does not provide considerable advantages with SUMF detection. On the contrary, the MMSE detector is more sensitive to accurate channel estimation and structured methods provide considerable improvements in terms of outage probability.

References

[1] ETSI SMG2#24, "WB-TDMA/CDMA system description performance evaluation," Madrid, Spain, Dec. 15 – 19, 1997.

[2] A. Klein, G. Kawas Kaleh, P. Baier, "Zero forcing and minimum mean-square error equalization for multiuser detection in code-division multiple-access channels," *IEEE Trans. on Vehic. Tech.,* Vol. 45, No. 2, pp. 276 – 287, May 1996.

[3] B. Steiner, P. Jung, "Optimum and suboptimum channel estimation for the uplink of CDMA mobile radio systems with joint detection," *European Trans. on Commun.,* Vol. 5, No. 1, pp. 39 – 49, Jan.–Feb., 1994.

[4] T. Rappaport, *Wireless Communications.* Englewood Cliffs, NJ: Prentice-Hall, 1996.

[5] J. G. Proakis, *Digital Communications.* New York, NY: McGraw Hill, 2nd ed., 1989.

[6] S. Bensley, B. Aazhang, "Subspace-based channel estimation for Code Division Multiple Access communication systems," *IEEE Trans. on Commun.,* Vol. 44, No. 8, pp. 1009 – 1020, Aug. 1998.

[7] S. Bensley, B. Aazhang, "Maximum-Likelihood synchronization of a single user for Code Division Multiple Access communication systems'," *IEEE Trans. on Commun.,* Vol. 46, No. 3, pp. 392 – 399, March 1996.

[8] E. Strom, S. Parkvall, S. Miller, B. Ottersten, "Propagation delay estimation in asynchronous Direct-Sequence Code-Division Multiple-Access systems," *IEEE Trans. on Commun.,* Vol. 44, No. 1, pp. 84 – 93, Jan. 1996.

[9] E. Ertin, U. Mitra, S. Siwamogsatham, "Maximum-Likelihood based multipath channel estimation for code-division multiple-access systems," submitted to the *IEEE Trans. on Commun.*, June 1998.

[10] B. C. Ng, M. Cedervall, A. Paulraj, "A structured channel estimation for maximum likelihood sequence detection," *IEEE Commun. Letters,* pp. 52 – 55, March 1997.

[11] B. C. Ng, D. Gesbert, A. Paulraj, "A semi-blind approach to structured channel equalization," Proc. of *IEEE Intern. Conf. on Commun. (ICC) 1997,* pp. 3385 – 3388, Atlanta, 7 – 11 June, 1998.

[12] S. Kay, *Fundamentals of statistical signal processing: Estimation theory.* New York: Prentice-Hall, 1993.

[13] S. Crozier, D. Falconer, S. Mahmoud, "Least sum of squared errors (LSSE) channel estimation," *IEE Proc.- F,* vol. 138, pp. 371 – 378, Aug. 1991.

[14] A. Clark, Z. Zhu, J. Joshi, "Fast start-up channel estimation," *IEE Proc.- F,* vol. 131, pp. 375 – 382, July 1984.

[15] S. Grant, J. Cavers, "Multiuser channel estimation for detection of cochannel signals," Proceedings of *1999 IEEE Intern. Conf. on Commun. ICC '99,* Vancouver, BC, Canada, 6 – 10 June, 1999.

[16] W. Mow, *Sequence design for spread spectrum*, The Chinese University Press, Chinese University of Hong Kong, Shatin, Hong Kong, 1995.

[17] J. Ng, K. Ben Letaief, R. Murch, "Complex optimal sequences with constant magnitude for fast channel estimation initialization," *IEEE Trans. on Commun.,* Vol. 46, No. 3, pp. 305 – 308, March 1998.

[18] G. Golub, C. Van Loan, *Matrix computations.* The John Hopkins University Press, 2nd ed., 1989.

[19] G. Caire, U. Mitra, "Structured multiuser channel estimation for block-synchronous DS/CDMA," submitted to *IEEE Trans. on Commun.,* July 1999.

[20] S. Verdú, *Multiuser Detection.* New York, NY: Cambridge University Press, 1998.

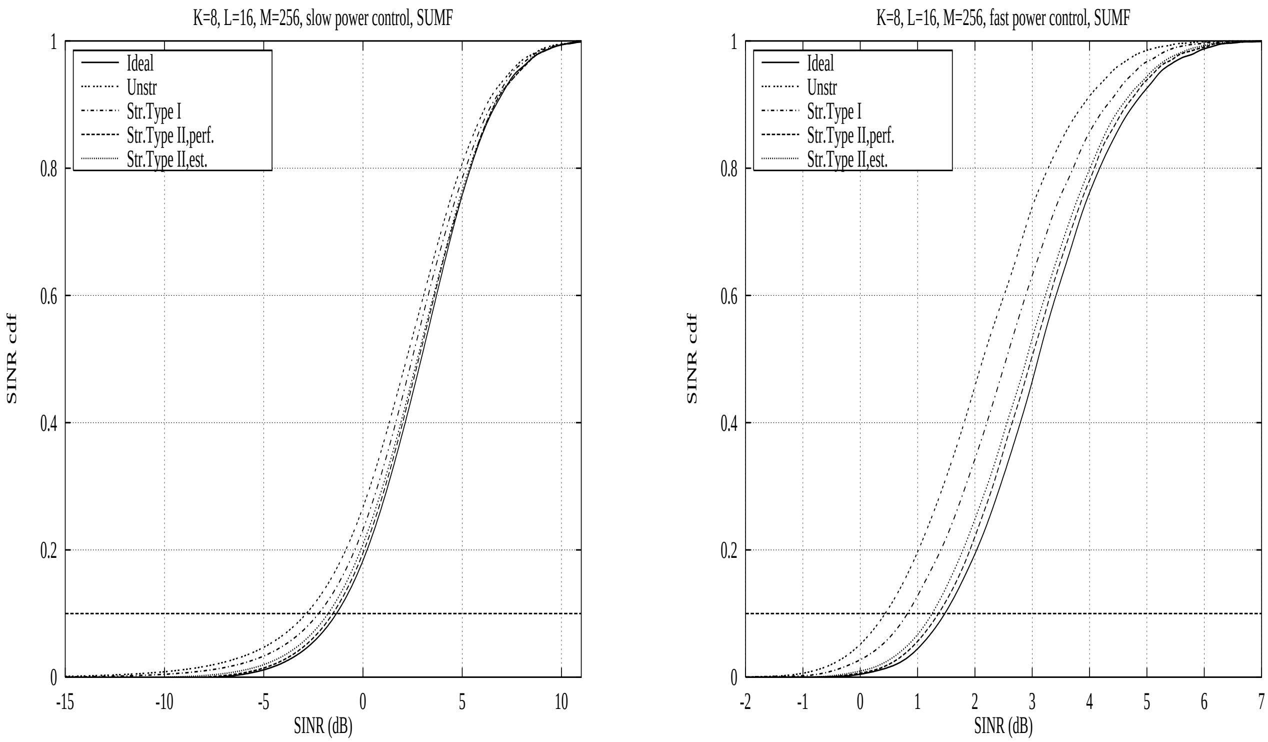

Figure 1: SINR cdf with slow (left) and fast (right) power control and SUMF receiver.

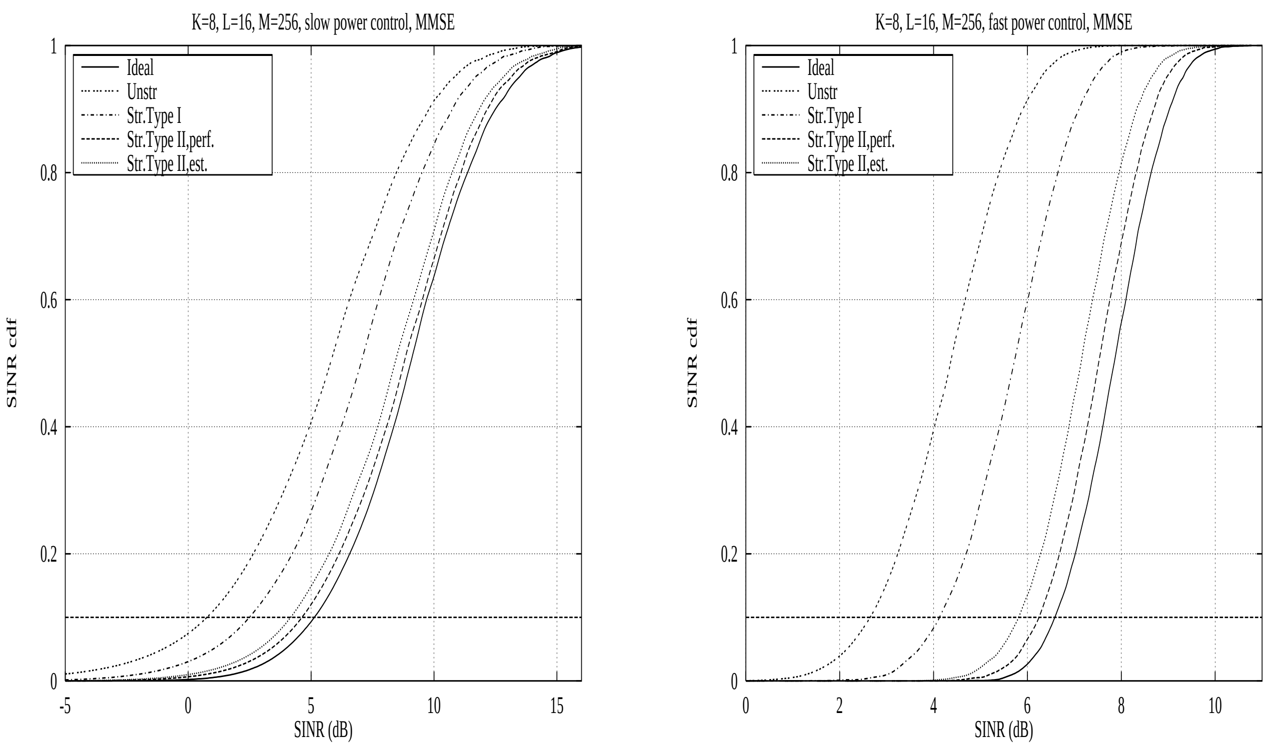

Figure 2: SINR cdf with slow (left) and fast (right) power control and LMMSE receiver.